What Can the Anthropic Principle Tell Us about the Future of the Dark Energy Universe[#]

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Abstract—An anthropic explanation of the evident smallness of the dark energy (DE) density value implies the existence of a time-dependent component of the scalar field, serving, together with a negative-valued cosmological constant, as one of two components to the overall DE density. The observers (i.e. us) might then only evolve in those regions of the universe where the sum of those two components (the positive and a negative ones) is sufficiently close to zero. However, according to Vilenkin and Garriga, the scalar field component has to slowly but surely diminish in time. In about a trillion years, this process will put a cap to the now-observable accelerated expansion of the universe, leading to a subsequent phase of impending collapse. However, the vanishing scalar field might also produce some rather unexpected singularities with a finite nonzero scale factor. We analyze this possibility using a particular example of a Sudden Future Singularities (SFS) and come to a startling conclusion that the time required for an SFS to arise must be "comparable" to the lifetime of the observable universe.

1. INTRODUCTION

In a seminal work [1] by Garriga and Vilenkin, a very novel "anthropic" approach has been presented, purporting to resolve two enigmas surrounding the "cosmological constant": its smallness and its time coincidence. The key idea there was to view the density of dark energy (DE) ρ_D as a random variable. To be more precise, ρ_D was to be written as a sum of two terms:

$$\rho_D = \rho_\Lambda + \rho_X, \qquad (1)$$

with $\rho_\Lambda \in \mathbb{R}$ being the vacuum energy density, and ρ_X a variable density of a dynamic DE component X. This implies that $|\rho_\Lambda|$ does not have to be small as is usually accepted, for as long as $|\rho_\Lambda| \sim |\rho_X|$ and $\rho_\Lambda \rho_X < 0$, then automatically $|\rho_D| \ll 1$. This, coupled with the anthropic restrictions on the observable value of ρ_D[1] (see [3−5]) provided a radically new

view on the problem of smallness of DE density. In addition to that, the authors of [1] have managed to find a very elegant way of explaining the time coincidence of the cosmological constant with the density of dark matter,[2] as well as to make a number of testable predictions (see also [6]). One of them, being most exciting, but at the same time the hardest to verify, was the prediction that the expansion of the observable universe will, at some time in the future, stop, only to be replaced with a gravitational collapse (however, the earliest time when this can happen is after about a trillion years of accelerated expansion). Here is how this startling conclusion came to pass: first, it was assumed that the term ρ_X in (1) was a contribution of some scalar field $\tilde{\phi}$ with a very low mass [7, 8]. At the next step, one recalls that the conscious observers (i.e. we) can only emerge within

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[1]Interestingly, the very first paper which has suggested the anthropic solution of the cosmological constant problem due to DE dates back to 1986 [2]!

[2]According to the observational data, the present densities of the dark matter (DM) and DE are of approximately the same magnitude. This "coincidence" is actually very puzzling because the evolution of DM is rather distinct from that of DE, and because there should be just a relatively narrow time interval within which their densities would coincide. And yet we somehow happen to be right in the midst of it!

a very narrow spectrum of values of ρ_D:[3]

$$-t_{EI}^{-2} \leq \rho_D \leq t_{EG}^{-2}, \qquad (2)$$

where t_{EI} is the (earliest) time required for intelligence to emerge and develop, and t_{EG} is the time it takes for the earliest galaxies to form (if t_0 is the present time, then $t_{EG} \sim (1 + z_{EG})^{-3/2} t_0$ with redshift $z_{EG} \sim 5$).

With the "band of opportunity" (2) being extremely narrow, the potential $V(\tilde{\phi})$ inside it remains practically constant, and one can estimate that $V(\tilde{\phi}) \sim V(0) + V'(0)\tilde{\phi}$. Therefore, if $\tilde{\phi}$ decreases with time, one is free to use the slow-roll approximation [9−11] to get an estimate on the time t_* which measures how long it will take from the current time t_0 till the beginning of the collapse. In particular, if $\tilde{\phi}_0 = \tilde{\phi}(t = t_0)$ and $V(0) = 0$, then $t_* \sim t_D(\tilde{\phi}_0/m_p)^2$, where $t_D \sim \rho_D^{-1/2}$, and so we end up with $t_* \sim 10^{12}$ yr.

In this paper we are going to describe another unusual prediction that can be made by studying the behavior of the dynamical component of DE. To that end, let us introduce a new field variable ϕ, such that in terms of this variable (1) can be formally rewritten as

$$\rho_D = V(\phi). \qquad (3)$$

Then, let us assume that, in accordance with [1], that the field ϕ varies but very slowly, and that the way the effective potential in (3) depends on ϕ can be approximately described by a power law:

$$V(\phi) \sim A^2 \phi^n, \qquad (4)$$

where all the variables are real-valued, including n, which in general is *not necessarily* an integer. Such potentials has been extensively studied in [12] by Barrow and Graham, who drew one particularly interesting conclusion: the potentials (4) might lead to singularities spontaneously emerging in finite time as $\phi \to 0$.

In particular, let M be an integer, such that

$$M < n < M + 1, \qquad (5)$$

then as $\phi \to 0$ the scalar field develops a divergence in the $M + 2$-th derivative with respect to time: $d^{M+2}\phi/dt^{M+2} \to (-1)^{M+1} \times \infty$, while all lower-order derivatives remain finite. Such a behavior is characteristic of class IV singularities by the Nojiri-Odintsov-Tsujikawa classification [13] and can be characterized as a "weak" singularity according to the terminology of [14, 15]. For example, if $M = 0$,

then only the second-order derivative $\ddot{\phi}$ will diverge, whereas both the density ρ and the pressure p remain finite.

The presence of such peculiar singularities in an otherwise very realistic field model with the with a power-law potential leads to yet another interesting observation: the prediction by Garriga and Vilenkin might actually be a lot more verifiable then was originally thought, since the emergence of those singularities happens much sooner than the aforementioned time scale $t_* \sim 10^{12}$ yr (see Sec. 3).

To get a grasp on the characteristic time that has to pass before the emergence of such a singularity, we will consider a simple integrable cosmological model. In Sec. 2 we will describe the method of superpotential that will be useful in determining the dynamics of singular solutions and will subsequently discover an even stronger singularity—a Sudden Future Singularity (SFS)—arising in finite time, provided the potential has the form

$$V(\phi) = C_1 \phi^n + C_2 \phi^m, \qquad (6)$$

where $nm < 0$. This observation, coupled with the fact that the potential (6) is quite permissible in cosmology, makes us conclude that the Garriga-Vilenkin hypothesis can actually predicate an emergence of SFS. Following this line of reasoning, in Sec. 3 we actually build such a model under the approximation of a slow-varying ϕ. A detailed analysis of that model leads us to something quite unexpected: the conclusion that the time scale in which an SFS emerges must be comparable to the lifetime of the observable universe! In other words, if an SFS should happen, it will be very soon (that is, "soon" in cosmological terms, of course!).

2. SOME SINGULARITIES ARE SUDDEN

We begin by pondering the potential (4). In the flat Friedmann-Lemaître-Robertson-Walker (FLRW) model the dynamics of the universe is given by the system:

$$H^2 = \frac{1}{2}\dot{\phi}^2 + V(\phi),$$
$$\ddot{\phi} = -3H\dot{\phi} - V'(\phi), \qquad (7)$$

which in the case of the power-law potential (4) turns into:

$$H^2 = \frac{1}{2}\dot{\phi}^2 + A^2\phi^n, \qquad (8)$$

$$\ddot{\phi} = -3H\dot{\phi} - nA^2\phi^{n-1}. \qquad (9)$$

Let $\phi_0 > 0$, $H_0 > 0$, and $\dot{\phi}_0 > 0$. Then the r.h.s. of (9) is strictly negative, i.e, $\ddot{\phi} < 0$, and $\dot{\phi}$ shall decrease, eventually turning to zero. If that happens

in finite time (which is easy to verify), then the scalar field ϕ itself will begin to decrease. And if $0 < n < 1$, then at some time $\phi \to 0$ and $\ddot{\phi} \to -\infty$. Despite that, $\dot{\phi}$ remains finite, and so do $\rho = H^2$ and $p = \dot{\phi}^2/2 - V(\phi)$. In other words, we end up with a scale factor whose third (and higher) order derivative with respect to t diverges at some moment of time (for more details cf. [12]).

The potentials (4) are generally nonintegrable, which significantly complicates our explorations. Fortunately, we have in our possession a versatile tool called the *method of superpotential* [16], which would allow us to overcome this difficulty by producing a suitable completely integrable model. Here is how it works.

First, we introduce a *superpotential* $W(\phi)$, defined as

$$W(\phi) = \frac{1}{2}\dot{\phi}^2 + V(\phi). \qquad (10)$$

Using (10), it is easy to see that the FLRW equations (7) get reduced to a much simpler system:

$$\frac{d\phi}{dt} = -\frac{1}{3}\frac{W'(\phi)}{\sqrt{W(\phi)}}, \qquad (11)$$

$$V(\phi) = W(\phi) - \frac{1}{9}\frac{(W'(\phi))^2}{W(\phi)}. \qquad (12)$$

The key to the next step would be to introduce the desired power-law relationship, except that we do it *not* for the potential $V(\phi)$, but for the superpotential $W(\phi)$ instead. Then, letting $W(\phi) = A^2\phi^n$ and integrating (11), we end up with:

$$\phi(t) = C\left(t_s - t\right)^{2/(4-n)}, \quad 0 < n < 4, \qquad (13)$$

where we have introduced an integration constant t_s, and

$$C = \left(\frac{An(4-n)}{6}\right)^{2/(4-n)}.$$

What kind of potential V are we dealing with? This can be answered by looking at (12):

$$V(\phi) = A^2\phi^n - \frac{A^2n^2}{9}\phi^{n-2}, \qquad (14)$$

i.e., we have a potential that will be exactly like (6), provided that $0 < n < 2$. If we set $2 < n < 4$, then the potential would still be (6) but with a positive power $m > 0$. It is now easy to see that

(i) if $0 < n < 2$, then $\dot{\phi}$ diverges as $t \to t_s$, i.e., $\dot{\phi} \to -\infty$;

(ii) if $2 < n < 3$, then $\ddot{\phi} \to \infty$ as $t \to t_s$, but $\dot{\phi}$ remains finite;

(iii) if $3 < n < 10/3$, then $\dddot{\phi} \to -\infty$ as $t \to t_s$, while both $\dot{\phi}$ and $\ddot{\phi}$ remain finite. Note that since we are only interested in the noninteger powers of ϕ, we thereby restrict ourselves to strict inequalities.

Naturally, when $n > 2$ we end up with the very solutions that were discovered and described in [12]. However, for the purpose of this paper we will instead concentrate our effort on case (i), which is the only case that does not fit into the class of singularities studied in [12]. The reason for not fitting in is obvious: the class (i) singularity is an SFS! We can verify it by directly calculating the density and pressure:

$$\rho = H^2 = A^2C^n\eta^{\frac{2n}{4-n}}, \qquad (15)$$

$$p = \frac{A}{3(4-n)}\eta^{-\frac{2(2-n)}{4-n}}\left(2nC^{n/2}\right.$$

$$\left. - 3AC^n(4-n)\eta^{\frac{4}{4-n}}\right), \qquad (16)$$

where $\eta = t_s - t$. So, as $t \to t_s$ we have $\rho \to 0$ and $p \to +\infty$ (again, we assume that $0 < n < 2$), which proves that the strong ($\rho + 3p \geq 0$) and the weak ($\rho + p \geq 0$) energy conditions remain valid.

The revelation that an SFS can be generated by the potentials like (6) with $m < 0$ (of which (14) is but an example) actually should not come as too big of a surprise. It is easy to see that all polynomial potentials

$$V(\phi) = \sum_k c_k\phi^{n_k}, \qquad (17)$$

with $n_k > 0$ are unable of producing SFS. Indeed, the requirement that $p \to \infty$, while $\rho \to \rho_s < \infty$ as $t \to t_s$, can be satisfied if and only if the kinetic term and the potential were to diverge at the same time albeit with opposite signs, whereas $\phi \to \phi_s < \infty$ (recall that by assumption $\phi_s = 0$). But the potential (17) with $n_k > 0$ is regular at $\phi \to 0$. Accidentally, this explains why there were no SFS in [12].

Nevertheless, the SFS popping into existence by the vanishing scalar field poses a very interesting question: could this scenario be replicated in the Garriga–Vilenkin model by slow evolution of the dynamical component of a scalar field? In other words, can it be that the very reason for the existence of the conscious observers is an almost complete compensation of the vacuum energy by the scalar field—is something that might in close future render it inhospitable? That would certainly a case of a silver lining conjuring its own cloud! Let us look at this possibility in the next section.

3. IS A COLLAPSE IMMINENT?

Let us assume that we were right in our assumptions: that the Garriga-Vilenkin anthropic model is correct and that there is a slowly vanishing scalar field which (eventually) produces an SFS. Let us move a little further by assuming a more general class of SFS, where, as $t \to t_s$, we have $p \to +\infty$, but $\rho \to \rho_s \neq 0$ [17]. As we have discussed in Sec. 1, the anthropic principle restricts the possible ρ_D to a very narrow band (2), where ρ_D can be defined as (3). This implies the following dynamics: ρ_D essentially behaves as a constant (ρ_s) up to the imminent arrival of an SFS. The same can be said about the pressure $p_D \sim -\rho_D$, although at $t = t_s$ we have $p_D \to +\infty$, unlike ρ_D which remains finite. Such a model can be easily constructed if we assume that

$$\rho = \Lambda + \frac{C_{DM}}{a^3}, \qquad (18)$$

$$p = -\Lambda + \alpha_S^2 \delta\left(\frac{t_s - t}{T}\right), \qquad (19)$$

where δ is the Dirac delta-function. Before we move on, a remark would be in order, regarding this function. At the first glance, it might seem counterintuitive to incorporate such an object into the pressure function, especially in view of an SFS being a "soft" singularity. However, it is instructive to recall that the second derivative of a scale factor at the very moment of sudden singularity ceases to be an analytic function. The same can be said about the pressure, courtesy of the FLRW equation (recall that both a and ρ remain finite during an SFS):

$$\ddot{a} = -\frac{1}{2}(\rho + 3p)a. \qquad (20)$$

Furthermore, it has been recently shown by Fernández-Jambrina and Lazkoz [18−20] that sudden singularities, like those introduced in [21], have one interesting property: any geodesic in a Friedmann universe does not end at a sudden singularity but safely passes through it. This implies that there might exist a unique description of the universe which unites the pre- and post-SFS cosmological dynamics as one unbroken evolution, passing through the sudden singularity. In such a framework, a discontinuity in the pressure dynamics would appear as an infinitesimally thin but infinitely high jump in a pressure plotted as a function of time. This naturally leads us to a function akin to (19). Finally, in order to fit the definition of an SFS, we should satisfy the following three conditions: at $t = t_s$ we should have [22]:

$$0 < a(t_s) < \infty,$$
$$0 < \dot{a}(t_s) < \infty,$$
$$\lim_{t \to t_s} \ddot{a} = -\infty.$$

As we shall see, *all* of these conditions are indeed satisfied by (19), so we do have a bona fide sudden singularity.

Now let us return back to the model (18), (19). When $\alpha_S^2 = 0$, our model morphs into a ΛCDM model, where $\rho_D = \Lambda$, and $p_D = p$, owing to the fact that there is no contribution to the pressure from dark and baryonic matter (note that baryons accounted for in the C_{DM} coefficient in (18), being a small perturbation of the DM term). The system (18), (19) perfectly satisfies our requirements, so the corresponding equations can be used to qualitatively describe a very slow dynamics of the X component in the anthropic model of Garriga and Vilenkin. As such, we expect that the estimates we are about to make using (18), (19) will not change in a significant way even for more difficult and more realistic models, provided those realistic models still satisfy the Garriga-Vilenkin postulates.

Let us now move on towards actually building those estimates. The solutions of the FLRW equations will have the form

$$a(t < t_s) = a_-(t) = a_0 \left(\frac{\Omega_{DM}(t_0)}{\Omega_\Lambda(t_0)} \sinh^2 \frac{t}{T}\right)^{1/3}$$

$$= B \sinh^{2/3}\left(\frac{t}{T}\right), \qquad (21)$$

$$a(t > t_s) = a_+(t) = a_0 \left(\frac{\Omega_{DM}(t_0)}{\Omega_\Lambda(t_0)} \sinh^2 \frac{2t_s - t}{T}\right)^{1/3}$$

$$= B \sinh^{2/3}\left(\frac{2t_s - t}{T}\right), \qquad (22)$$

where

$$\Omega_\Lambda(t_0) = \frac{\Lambda}{H^2(t_0)}, \qquad \Omega_{DM}(t_0) = \frac{C_{DM}}{a^3(t_0)H^2(t_0)},$$

$$T = \frac{2}{2\sqrt{\Lambda}}, \qquad B = a_0 \left(\frac{\Omega_{DM}(t_0)}{\Omega_\Lambda(t_0)}\right)^{1/3}.$$

As we have discussed, we are interested in the dynamics of the universe where the scale factor and its first order derivative are both continuous functions everywhere. This implies that at $t = t_s$ we have to impose matching conditions for (21), (22):

$$a_+(t_s) = a_-(t_s) = B \sinh^{2/3} \tau_s, \qquad (23)$$

$$\dot{a}_+(t_s) = -\dot{a}_-(t_s) = -\frac{2B \cosh \tau_s}{3T(\sinh \tau_s)^{1/3}}, \qquad (24)$$

where $\tau_s = t_s/T$. Here we again note that such "stitching" together of two seemingly different solutions (pre- and post-SFS) is not just a mathematical sleigh of hand, but a direct application of the results of Fernández-Jambrina and Lazkoz [18−20] regarding the possible existence of a unique description of a

universe enduring a sudden singularity and surviving it. The matching conditions (23), (24) is then just a mathematical manifestation of this hypothesis.

Now, if we look at our model at $t = t_s$, we shall see that the density remains constant, the pressure blows up (a standard SFS behavior), and the Hubble parameter changes its sign:

$$H_+(t_s) = -H_-(t_s) = -\frac{2}{3T}\coth\frac{t_s}{T}, \qquad (25)$$

so starting out from $t > t_s$, the universe enters a phase of collapse, instigated by the SFS at $t = t_s$.

We still have two undetermined parameters, though: a positive-definite "SFS coupling constant" α_S^2 and t_s, the moment of SFS emergence. We can find them as follows. First, we substitute (18) and (19) into the Friedmann equation (20), then we integrate it in the interval $t \in (t_s - \epsilon, t_s + \epsilon)$ and take a limit $\epsilon \to 0$. Our reward would be the following important relationship:

$$\Lambda\coth\frac{t_s}{T} = \alpha_S^2. \qquad (26)$$

If we choose $\Lambda \sim 0.7 \times 10^{-29}$ g/cm^{-3}, then $T \sim 10$ Gyr. Since we are making a sensible assumption that an SFS will happen in the future, then $t_s > T$, and we have an estimate:

$$1 < \alpha_S^2/\Lambda < \coth 1 \sim 1.31. \qquad (27)$$

As the exact value of this fraction is arbitrary, we can use the Bayes approach (a.k.a. the mediocrity principle) and say that α_S^2/Λ should lie somewhere around 1.155, i.e. in the middle of the interval (27). If this is so, then the most probable estimate we can make is

$$t_s \sim T \times \coth^{-1} 1.155 \sim 13.16 \text{ Gyr.} \qquad (28)$$

Of course, this result should be taken with a grain of salt, being just an approximation. Still, it looks very unlikely that better estimates would differ from T by orders of magnitude. In fact, for more accurate estimates to predict $t_s \gg T$, it would either require a presence of some hitherto unknown "fine tuning" of cosmological parameters, or indicate the work of some undiscovered physical laws behind the choice of α_S^2, the "SFS coupling constant". Both these possibilities seem very unlikely (at least at this juncture) which makes us believe that our result is sufficiently general for those anthropic Garriga-Vilenkin models with a slowly vanishing scalar field that contain SFS.

4. CONCLUSION

Since our final result is admittedly rather unusual, it would be a good idea to once again briefly go over the reasoning that had led us here.

(1) We began with the anthropic solution to the cosmological constant problem, developed in [1]. Its core assumption was that DE consists of two components: the vacuum energy and a slowly varying dynamic component. Then the vacuum energy density might actually satisfy the predictions imposed by the string theory,: being large and negative-valued. As long as the density of a second, dynamic, component is sufficiently large and positive-definite, the anthropic principle will do the rest, i.e. select those universes where the sum of both components is significantly small to ensure the universe in question be habitable, thus producing DE density that is compatible with the observational data.

(2) On the other hand, the model of [1] predicts in uncertain terms that the dynamic component of DE must be slowly decreasing, predicting in a distant future a change in the dynamics of the universe. After about 10^{12} years of accelerated expansion, the universe is supposed to stop growing and begin to contract. If the variable component is produced by a scalar filed (another alternative would be a four-form field strength, which can vary through nucleation of membranes [7, 23]), then the DE density will play the role of an effective self-action potential of this very slowly abating scalar field.

(3) It is sensible to assume that this potential should have a form $V \sim \phi^n$. On the other hand, there are no reasons to a priori set the constant n to be an integer—or even a positive number. But if n is *not* integer, then, according to [12] the vanishing scalar field might give rise to some previously unforseen new singularities. In particular, since we cannot rule out the effective potentials (6), the abating field might lead to an SFS, a Sudden Future Singularity (see Sec. 2). Further on, we have specifically concentrated on this type of singularity, being the most extreme and therefore more interesting (the "weaker" type of singularities will be considered in subsequent papers).

(4) To study the emergence of SFS in the Garriga-Vilenkin approach, we have constructed a very simple intuitive model (Sec. 3) that contained both necessary ingredients for the scenario of [1], and an SFS. Taking into account that the resulting model contains a bare minimum of initial assumptions, it is reasonable to expect that the qualitative estimates made in it would also hold for more realistic models.

(5) Most interestingly, when the model is equipped with the current observational data (see Sec. 3), it predicts an imminent arrival of SFS, for its time of emergence ends up being comparable with the age

of the observable universe (about 10 Gyr). It is important to note that the models of the universe undergoing a relatively abrupt (as compared to the trillions of years) halt in accelerated expansion and collapsing thereafter are not unusual in the framework of supergravity theory. For example, it has been demonstrated in [6, 24] that in such cosmological models the DE density eventually evolves to negative values, after which the universe totally collapses in times comparable with the observable age of the universe. An even more general class of such models was considered in [25], where it was shown that any *eternally* expanding model of a DE universe must have a number of counterparts, where the observable expansion reverts to contraction in characteristic times of $10^{10}-10^{11}$ years. And while those "alternative" models might appear too complex and not very natural, we nevertheless have to take them into account, if for nothing else, but for a simple fact that all of them by design satisfy the current observations (see also [26]).

Returning to models with SFS, we would also like to mention Ref. [27], where the authors, while studying models with a Big Brake singularity (introduced in [28]), endeavoured to compare their pre-singular evolution with the supernovae data and managed to select some sets of initial parameters compatible with these data. The authors then numerically analyzed the duration of time that should pass between the moment of (contemporary) observation and the Big Brake singularity, and showd it to be not very large.

In conclusion, we cannot resist mentioning one last remarkable circumstance. In 2007 the authors of [29] made an attempt to estimate a possible time of arrival of an SFS using the observational data on Ia type Supernovae. They arrived at an astonishing conclusion that "a sudden singularity may happen in the very near future (e.g. within ten million years)"! It is important to point out that the cosmological model used in [29] was very different from the one studied in this paper. And yet the conclusions of [29] and ours are quite alike. Most fascinating, isn't it?

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