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Particle Acceleration in Rotating Modified Hayward and Bardeen Black Holes

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Abstract—We consider a rotating modified Hayward black hole and construct a rotating modified Bardeen black hole to study particle acceleration of two colliding particles near the horizon. These classes of black holes have new and important parameters with mass dimension, which made crucial differences with the Kerr black hole. We investigate the CM energy of two colliding neutral particles with the same rest masses falling from rest at infinit to near the horizons of the mentioned black holes. We confirm that rotational motion of these black holes is necessary to have infinite CM energy for collision of two particles near the horizon. We also investigate the range of the particles' angular momentum and the orbit of the particle, hence find an infinite region for the case of rotating modified Bardeen black hole and a finite region for the case of a modified Hayward black hole.

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1. INTRODUCTION

Collision of particles around black holes (BHs) is a very interesting topic in the current high energy particle and astroparticle physics. Recently, Banados, Silk and West (BSW) [1] investigated a collision of two particles falling from the rest at infinity in a Kerr BH, which is known as the BSW mechanism. They determined the center of mass (CM) energy in the equatorial plane, which may be high in the limiting case of an extremal BH. Further, it is demonstrated that the CM energy of two colliding particles diverges at the inner horizon of a non-extremal Kerr BH [2, 3]. A general review of BH as particle accelerators is presented by Harada et al. [4]. Wei et al. [5] also studied a collision of two uncharged particles around a Kerr-Newmann BH, depending on the BH spin and charge. Liu et al. studied a collision of two particles around a Kerr-Taub-NUT BH [6]. Subsequently, Zakria and Jamil investigated the CM energy of a collision of two neutral particles with different rest masses falling freely from rest at infinity in the background of a Kerr-Newman-Taub-NUT BH [7]. Till now, several authors [8–15, 17–21] studied collisions of particles near BHs and CM energy of the colliding particles.

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Space-time singularities are a consequence of classical general relativity, while it is a common belief that singularities do not exist in nature. Indeed, we should seek an alternative to general relativity. In that case, models of regular BHs have been studied. For example a nonsingular rotating BH has been studied in [22]. It was argued that the CM energy of colliding particles near a naked singularity diverges [23–26]. Due to gravitational collapse, any astrophysical object can produces a space-time singularity beyond a horizon or a naked one. We know that any classical BH has a singularity. To avoid a singularity, Bardeen [27] proposed the concept of a regular BH, dubbed a Bardeen BH, and subsequently, another type of regular BHs (Hayward BHs) was found [28]. Another kind of regular BH is the Ayon-Beato-Garcia (ABG) BH [29] which consist of a nonlinear electric field as a source. Such electric solutions require different Lagrangians in different parts of space, however, magnetic solutions with the same metric are entirely regular and acceptable [30]. A geodesic study of a regular Hayward BH was discussed by Abbas et al. [31]. The implication of a rotating Hayward BH is discussed in [32]. A modified Hayward BH metric was proposed by Lorenzo et al. [33]. Recently, Amir and Ghosh studied a collision of two particles with equal masses moving in the equatorial plane near the horizon of a rotating Hayward regular BH as a particle accelerator [34]. Using numerical analysis of the case of an extremal BH, they found that the

CM energy diverges near the horizon. On the other hand, for the the case of a non-extremal BH there always exists an upper bound with finite value of CM energy. Also, Pradhan studied regular Hayward and Bardeen BHs as particle accelerators [35]. The collision CM energy of charged particles in a Bardeen BH was studied in [36]. By the above motivations, we now extend their work to rotating modified Hayward and Bardeen black holes. The CM energy and the particles orbits are investigated for two colliding neutral particles of the same rest mass falling from infinity into these BHs. Finally, we conclude the results for a particle accelerator.

The paper is organized as follows. In Section 2, a general rotating BH background is reviewed, and rotating modified Hayward and Bardeen BHs are introduced. In Section 3, the CM energy of colliding particles is calculated. Particle orbits are investigated in Section 4. Extremal limits of the solutions are discussed numerically in Section 5. In Section 6 we give a conclusion and summary of the results.

2. ROTATING BH BACKGROUND

The general form of a rotating BH metric can be written as

$$ds^2 = -g_{tt}dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 + 2g_{t\phi}dt\, d\phi, \tag{1}$$

where

$$\begin{aligned} g_{tt} &= F(r), \\ g_{rr} &= \frac{\Sigma}{G(r)\Sigma + a^2\sin^2\theta}, \\ g_{\theta\theta} &= \Sigma, \\ g_{\phi\phi} &= \sin^2\theta\left[\Sigma + a^2(2 - G(r))\sin^2\theta\right], \\ g_{t\phi} &= a(1 - F(r))\sin^2\theta, \end{aligned} \tag{2}$$

with

$$\Sigma = r^2 + a^2\cos^2\theta, \tag{3}$$

also, $F(r)$ and $G(r)$ are some function of r. In this paper, we are interested in studying two different but approximately similar BHs: rotating modified Hayward and Bardeen BHs, which are introduced in the following subsections.

2.1. Rotating Modified Hayward BH

The metrics under study are non-vacuum solutions of Einstein equations which have some form of exotic fields as a gravitational source. To write the metric of a rotating modified Hayward BH, we begin with the general static spherically symmetric metric given by $g_{t\phi} = 0$ of the line element (1). Then, the Hayward BH is given by [28]

$$\begin{aligned} g_{tt} &= \frac{1}{g_{rr}} = f(r) = 1 - \frac{2Mr^2}{r^3 + 2Ml^2} = 1 - \frac{2m_1}{r}, \\ g_{\theta\theta} &= r^2, \quad g_{\phi\phi} = r^2\sin^2\theta, \\ g_{t\phi} &= 0, \end{aligned} \tag{4}$$

where

$$m_1 = \frac{Mr^3}{r^3 + 2Ml^2}, \tag{5}$$

and M is the Hayward BH mass; l is a parameter with the dimension of length and a small scale related to the inverse of the cosmological constant. The Hayward BH behaves as a Schwarzschild BH as $r \to \infty$ ($g_{tt} \approx 1 - 2M/r$) and as de Sitter space-time near the center ($r \to 0$, $g_{tt} \approx 1 - r^2/l^2$). Including one-loop quantum corrections yields a modified Hayward BH [33], which allows a finite time dilation between the center and infinity [37]. In that case, accretion and evaporation of a modified Hayward BH was studied in [37]. The modified Hayward BH is given by the metric (1) with

$$\begin{aligned} g_{tt} &= f(r)h(r) \\ &= \left(1 - \frac{2Mr^2}{r^3 + 2Ml^2}\right)\left(1 - \frac{\mu M}{r^3 + \frac{\mu}{\nu}M}\right), \\ g_{rr} &= \frac{1}{f(r)} = (1 - \frac{2Mr^2}{r^3 + 2Ml^2})^{-1}, \\ g_{\theta\theta} &= r^2, \quad g_{\phi\phi} = r^2\sin^2\theta, \\ g_{t\phi} &= 0, \end{aligned} \tag{6}$$

where μ and ν are positive constants. We can see that the only change is $g_{tt} \to f(r)h(r)$ ($F(r) \to f(r)h(r)$). So, the modified Hayward BH is obtained by adding the function

$$h(r) = 1 - \frac{\mu M}{r^3 + \frac{\mu}{\nu}M}$$

On the other hand, Ref. [32] constructed a general rotating regular BH and considered the Hayward and Bardeen BHs as examples. For a rotating Hayward BH we have the metric (1) with [32],

$$F(r) = G(r) = \tilde{f} = 1 - \frac{2\tilde{m}_1 r}{\Sigma}, \tag{7}$$

where

$$\tilde{m}_1 = M\frac{r^{3+\alpha}\Sigma^{-\alpha/2}}{r^{3+\alpha}\Sigma^{-\alpha/2} + g_1^3 r^\beta\Sigma^{-\beta/2}}, \tag{8}$$

with $g_1^3 = 2Ml^2$, α and β are real numbers. It is easy to check that $\alpha = \beta = a = 0$ yield a nonrotating Hayward BH. It means that a rotating BH is obtained

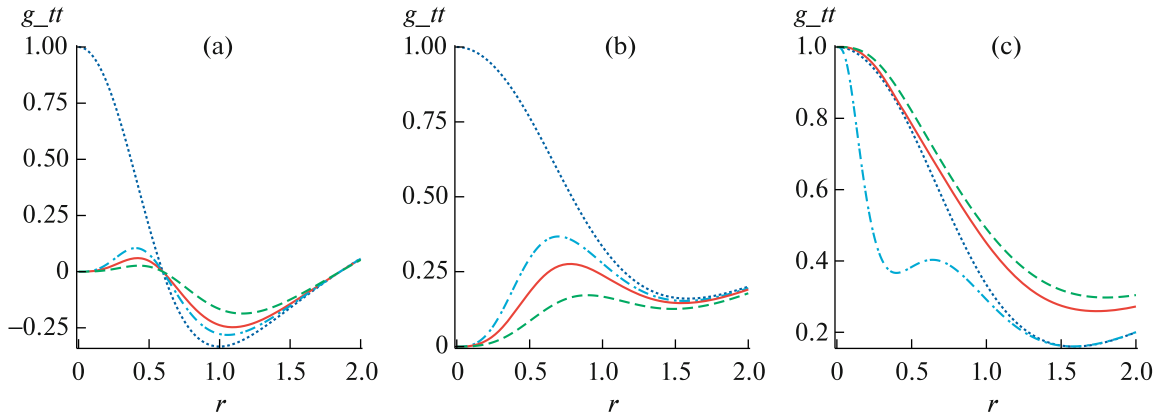

Fig. 1. Rotating modified Hayward BHs, g_{tt} in terms of r for $M = 1$, $\nu = 1$, $\beta = 2$, $\alpha = 1$. **(a)** $\theta = \pi/2$, $a = 1$, $l = 0.5$; and $\mu = 0$ (dot), $\mu = 0.2$ (dot dash), $\mu = 0.4$ (solid), $\mu = 1$ (dash). **(b)** $\theta = \pi/2$, $a = 1$, $l = 1$; and $\mu = 0$ (dot), $\mu = 0.2$ (dot dash), $\mu = 0.4$ (solid), $\mu = 1$ (dash). **(c)** $\theta = \pi/6$, $l = 1$; $a = \mu = 0$ (dot), $a = 2$, $\mu = 0.2$ (dot dash), $a = 0.8$, $\mu = 0.2$ (solid), $a = 1$, $\mu = 0.4$ (dash).

by adding the parameters α, β and a together with the replacement $m_1 \to \tilde{m}_1$ ($f \to \tilde{f}$). In summary we have seen that $F(r) \to f(r)h(r)$ gives a modified Hayward BH, and $F(r) \to \tilde{f}(r)$ gives a rotating Hayward BH. So, it seems that both effects ($F(r) \to \tilde{f}(r)\tilde{h}(r)$, $G(r) \to \tilde{f}(r)$) give us a rotating modified Hayward BH (at least in the first order approximation). So, we suggest the metric (1) as a rotating modified Hayward BH with the metric

$$\begin{aligned} g_{tt} &= \tilde{f}(r)\tilde{h}(r), \\ g_{rr} &= \frac{\Sigma}{\tilde{f}(r)\Sigma + a^2 \sin^2\theta}, \\ g_{\theta\theta} &= \Sigma, \\ g_{\phi\phi} &= \sin^2\theta \left[\Sigma + a^2(2 - \tilde{f}(r))\sin^2\theta\right], \\ g_{t\phi} &= a(1 - \tilde{f}(r)\tilde{h}(r))\sin^2\theta, \end{aligned} \tag{9}$$

where $\tilde{f}(r)$ and $\tilde{m}_1$ are given by Eqs. (7) and (8), and

$$\tilde{h} = 1 - \frac{\mu\tilde{m}_2 r}{\Sigma^2}, \tag{10}$$

$$\tilde{m}_2 = M\frac{r^{3+\alpha}\Sigma^{-\alpha/2}}{r^{3+\alpha}\Sigma^{-\alpha/2} + g_2^3 r^\beta \Sigma^{-\beta/2}}, \tag{11}$$

with $g_2^3 = M\mu/\nu$. This is a regular solution everywhere for $g_2 \neq 0$. The horizon structure of the rotating modified Hayward BH is given by $g_{rr} = \infty$ from (2), which is exactly similar to the rotating Hayward BH discussed in [34].

It has been shown that there are two horizons, the so-called Cauchy and event horizons. Also, the structure and location of the ergosurface may be investigated using $g_{tt} = \tilde{f}(r)\tilde{h}(r) = 0$. A rotating Hayward BH has been studied in [34].

Figure 1 shows the location of the static limit surface for different values of the parameters.

2.2. Rotating Modified Bardeen BH

The Bardeen BH is very similar to the Hayward BH and is obtained by Bardeen [27]. The rotating Bardeen BH has been constructed in [32]. In a similar way with the previous subsection, we can propose a rotating modified Bardeen black hole. Therefore, it is defined by Eqs. (7), (9) and (10), while $\tilde{m}_1$ and $\tilde{m}_2$ are given by

$$\begin{aligned} \tilde{m}_1 &= M\frac{r^{3+\alpha}\Sigma^{-\alpha/2}}{\left(r^{3+\alpha}\Sigma^{-\alpha/2} + Ml^2 r^\beta \Sigma^{-\beta/2}\right)^{3/2}}, \\ \tilde{m}_2 &= M\frac{r^{3+\alpha}\Sigma^{-\alpha/2}}{\left(r^{3+\alpha}\Sigma^{-\alpha/2} + g^2 r^\beta \Sigma^{-\beta/2}\right)^{3/2}} \end{aligned} \tag{12}$$

where, as before, l is a parameter with the dimension of length, α and β are real numbers, and g is a constant parameter with the dimension of mass. So, we can write unified equations for both Bardeen and Hayward BHs in terms of $\tilde{f}$ and $\tilde{h}$, the only difference is in the shape of $\tilde{m}_1$ and $\tilde{m}_2$. Horizons of the BHs are given by the root of the following equation (the denominator of g_{rr} to be zero):

$$\tilde{f}\Sigma + a^2\sin^2\theta = 0. \tag{13}$$

Using Eq. (12), we have

$$\Delta = r^2 + a^2 - \frac{2Mr^{4+\alpha}\Sigma^{-\alpha/2}}{\left(r^{3+\alpha}\Sigma^{-\alpha/2} + Ml^2 r^\beta \Sigma^{-\beta/2}\right)^{3/2}} = 0. \tag{14}$$

The horizon structure of the rotating modified Bardeen BH is shown in Fig. 2. We can see that,

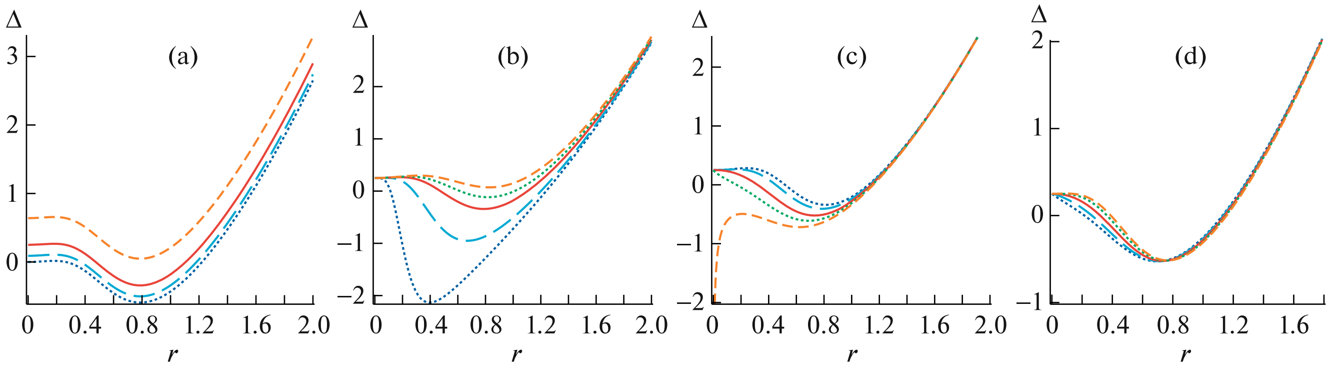

Fig. 2. Rotating modified Bardeen BHs, Δ in terms of r for $M = 1$. **(a)** $\theta = \pi/2$, $\alpha = 1$ and $\beta = 2$, $l = 0.5$, $a = 0$ (space dot), $a = 0.3$ (long dash), $a = 0.5$ (solid), $a = 0.7$ (dot), $a = 0.8$ (dash). **(b)** $\theta = \pi/2$, $\alpha = 1$ and $\beta = 2$, $a = 0.5$, $l = 0.1$ (space dot), $l = 0.3$ (long dash), $l = 0.5$ (solid), $l = 0.6$ (dot), $l = 0.7$ (dash). **(c)** $\theta = \pi/6$, $a = l = 0.5$, $\alpha = 1$, $\beta = 0$ (space dot), $\beta = 0.8$ (long dash), $\beta = 2$ (solid), $\beta = 2.8$ (dot), $\beta = 3.6$ (dash). **(d)** $\theta = \pi/6$, $a = l = 0.5$, $\beta = 2$, $\alpha = 0$ (space dot), $\alpha = 0.4$ (long dash), $\alpha = 1$ (solid), $\alpha = 1.6$ (dot), $\alpha = 2$ (dash).

for a suitable choice of the parameters, there are two horizons $r_+ = r \pm \delta$, where $0 < \delta < 0.5$. The case of $\theta = \pi/2$ is of our interest, although values of α and β are not important. In this case, Δ vs r is plotted in Figs. 2a and 2b.

On the other hand, in the case $\theta = \pi/6$ we can see the effect of α and β on BH horizons in Figs. 2c and 2d. For example, from solid lines in Fig. 2a and 2b we can see $r_+ \approx 1.1$ and $r_- \approx 0.5$ for $\alpha = 1$, $\beta = 2$ and $a = 0.5$ (a Bardeen BH). Also, one can obtain $r_+ \approx 1.65$ and $r_- \approx 0.9$ with $\alpha = 1$, $\beta = 2$ and $a = 0.5$ for the case of a Hayward BH.

It is easy to check that the behavior of $g_{tt} = \tilde{f}\tilde{h}$ is approximately similar to the previous case. So, we can see, for example, Fig. 1 as a typical behavior of the ergo-surface.

3. THE CENTER OF MASS ENERGY

In this section we consider motion of particles with rest mass m_0 falling from infinity in the background of a rotating modified Hayward or Bardeen BHs. The Hamilton-Jacobi equation governs the geodesic motion in these space-times and can written as

$$\frac{\partial S}{\partial \tau} = -\frac{1}{2} g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu}, \qquad (15)$$

where τ is an affine parameter along the geodesics, and S is the Jacobi action, so one can consider the following ansatz [19, 34]:

$$S = \frac{1}{2} m_0^2 \tau - Et + L\phi + S_r(r) + S_\theta(\theta), \qquad (16)$$

where $S_r(r)$ and $S_\theta(\theta)$ are functions of r and θ, respectively. Since equatorial motion ($\theta = \pi/2$) is assumed, $S_\theta(\theta) = C$ is a possible choice, where C is an arbitrary constant. Moreover, $E = -P_t$ and $L = P_\phi$ are the conserved energy and angular momentum, respectively. One can obtain the null geodesic in the form

$$\dot{t} = \frac{1}{\xi} \left[aL(1 - \tilde{h}\tilde{f}) - a^2 E(\tilde{f} - 2) + E\Sigma \right], \qquad (17)$$

$$\dot{\phi} = \frac{1}{\xi} \left[L\tilde{h}\tilde{f} - a(1 - \tilde{h}\tilde{f})E \right], \qquad (18)$$

where

$$\xi \equiv a^2 \left(\tilde{h}\tilde{f}^2(\tilde{h} - 1) + 1 \right) + \tilde{h}\tilde{f}\Sigma, \qquad (19)$$

and the dot denotes a derivative with respect to the parameter τ. Using the Hamilton-Jacobi equation (15) and the relation (16), one can obtain

$$-m_0^2 = g^{tt} E^2 - 2g^{t\phi} EL + g^{\phi\phi} L^2 + g^{rr} R^2(r), \qquad (20)$$

where

$$R(r) = \frac{dS_r(r)}{dr}, \qquad (21)$$

$$g^{tt} = \frac{a^2(\tilde{f} - 2) - \Sigma}{\xi},$$

$$g^{rr} = \frac{a^2 + \tilde{f}\Sigma}{\Sigma},$$

$$g^{\phi\phi} = \frac{\tilde{h}\tilde{f}}{\xi},$$

$$g^{t\phi} = \frac{a(1 - \tilde{h}\tilde{f})}{\xi}. \qquad (22)$$

It is easy to find

$$R(r) = \left(\frac{\Sigma}{a^2 + \tilde{f}\Sigma} \right)^{1/2}$$

$$\times\Big[\frac{2ELa(1-\tilde{h}\tilde{f})-E^2(a^2(\tilde{f}-2)-\Sigma)-L^2\tilde{h}\tilde{f}}{\xi} - m_0^2\Big]^{1/2}. \quad (23)$$

So, we have,

$$\dot{r}=\frac{a^2+\tilde{f}\Sigma}{\Sigma}R(r). \quad (24)$$

Therefore, we have all nonzero 4-velocity components given by Eqs. (17), (18), and (24). Hence, we are able to obtain the CM energy of two neutral particles' collision near the rotating modified Hayward or Bardeen BHs. We suppose that the two particles have the same rest mass (m_0) with the angular momenta per unit mass L_1 and L_2 and energies per unit mass E_1 and E_2, respectively. Thus the CM energy is given by

$$\epsilon\equiv\tilde{E}_{\rm CM}=\sqrt{1-g_{\mu\nu}u_1^\mu u_2^\nu}, \quad (25)$$

where $u_i^\mu=(\dot{t}_i,\dot{r}_i,0,\dot{\phi}_i)$, $i=1,2$, and $E_{\rm CM}=\tilde{E}_{\rm CM}\sqrt{2}m_0$. After some calculations we can find

$$\tilde{E}^2_{\rm CM}=\frac{1}{\xi^2}\Big(\xi^2+\mathcal{A}L_1L_2 + \mathcal{B}E_1E_2+\mathcal{C}(E_1L_2+E_2L_1)-H_1H_2\Big), \quad (26)$$

where

$$\mathcal{A}=a^2\tilde{f}\tilde{h}(\tilde{f}\tilde{h}+\tilde{f}^2\tilde{h}-\tilde{f}^2\tilde{h}^2-1)-\tilde{f}^2\tilde{h}^2\Sigma,$$
$$\mathcal{B}=\tilde{f}\tilde{h}[a^2(\tilde{f}-2)-\Sigma]^2 + a^2(1-\tilde{f}\tilde{h})[a^2(\tilde{f}-2)-\Sigma],$$
$$\mathcal{C}=a(1-\tilde{f}\tilde{h})\Big(a^2(1-\tilde{f}\tilde{h}) - \tilde{f}\tilde{h}[a^2(\tilde{f}-2)-\Sigma]\Big), \quad (27)$$

and

$$H_i=\sqrt{\xi}\Big\{\xi m_0^2-2a(1-\tilde{f}\tilde{h})E_iL_i + [a^2(\tilde{f}-2)-\Sigma]E_i^2+\tilde{f}\tilde{h}L_i^2\Big\}^{1/2}, \quad i=1,2. \quad (28)$$

We will give a numerical analysis of $\tilde{E}_{\rm CM}$ (26) for both cases of rotating modified Hayward and rotating modified Bardeen BHs.

Near the horizon limit $r\to r_+$ we see that $\xi|_{r\to r_+}=(\tilde{h}-1)(\tilde{h}\tilde{f}^2-a^2)$. Therefore, the CM energy will be infinite if we have $\tilde{h}=1$ or $\tilde{h}\tilde{f}^2=a^2$.

In the plots of Fig. 3 we can see the behavior of $\tilde{E}_{\rm CM}$ for both rotating modified Hayward and Bardeen BHs.

Figure 3a shows the CM energy for $E_1=E_2$ and $L_1=L_2$. We can see that for both Hayward ($\mu=0,a=0$) and modified Hayward ($\mu\neq0,a=0$) BHs the CM energy has a finite constant value.

Then, we can see the growth of this energy near the horizon for $a\neq0$. For example, in the case of $a=0.1$ one finds $r_-\approx0.6$ and $r_+\approx1.85$ (blue), or in the case of $a=0.5$ one finds $r_-\approx0.9$ and $r_+\approx1.65$ (green), we have infinite CM energy. In the Fig. 3b we can see the effect of μ (corresponding to a modified Hayward BH). We find that the value of CM energy of a Hayward BH is larger than for a modified Hayward BH.

In Fig. 3c we give a plot of CM energy with $L_1=-L_2$, arbitrary E_1 and E_2 and $\mu=1$ to see that the case of $a=0$ yields a finite CM energy near the event horizon (r_+) while $a\neq0$ gives an infinite CM energy near the event horizon. Cyan curves corresponds to $\mu=0$, and $g_1=0$ (Kerr BH). It is clear that the modified Hayward BH has a larger CM energy than the Kerr BH. Moreover, $g_1=0$ and $a=0$ together with $\mu=0$ give a Schwarzschild BH with finite CM energy, as expected.

An important situation is that one particle due to its momentum strongly turns around the BH while the other has no such turn, therefore the radial velocity of the second one is close to zero while the first particle has a velocity close to the speed of light. The growing Lorentz factor for the relative velocity leads to growth of the CM energy. It is illustrated by Fig. 3d. We can see a divergence of the CM energy which depends on the particles' properties.

In the Figs. 3e–3h we can see the CM energy for a modified Bardeen BH. We see similar results to those for a Hayward BH.

4. PARTICLE ORBITS

To specify the range of the particles' angular momentum, we should calculate the effective potential to describe the motion of test particles. Here, we are interested to the motion of test particles in the equatorial plane ($\theta=\pi/2$), where the radial equation of motion for the timelike particles moving along the geodesics is

$$\frac{1}{2}\dot{r}^2+V_{\rm eff}=0, \quad (29)$$

which gives the effective potential

$$V_{\rm eff}=-\frac{\dot{r}^2}{2}=-\frac{a^2+\tilde{f}\Sigma}{2\Sigma}\Big\{\frac{1}{\xi}\Big[2ELa(1-\tilde{h}\tilde{f}) - E^2(a^2(\tilde{f}-2)-\Sigma)-L^2\tilde{h}\tilde{f}\Big]-m_0^2\Big\}, \quad (30)$$

where we have used (23) and (24). The circular orbit of particles is obtained using the relations

$$V_{\rm eff}=0, \quad (31)$$

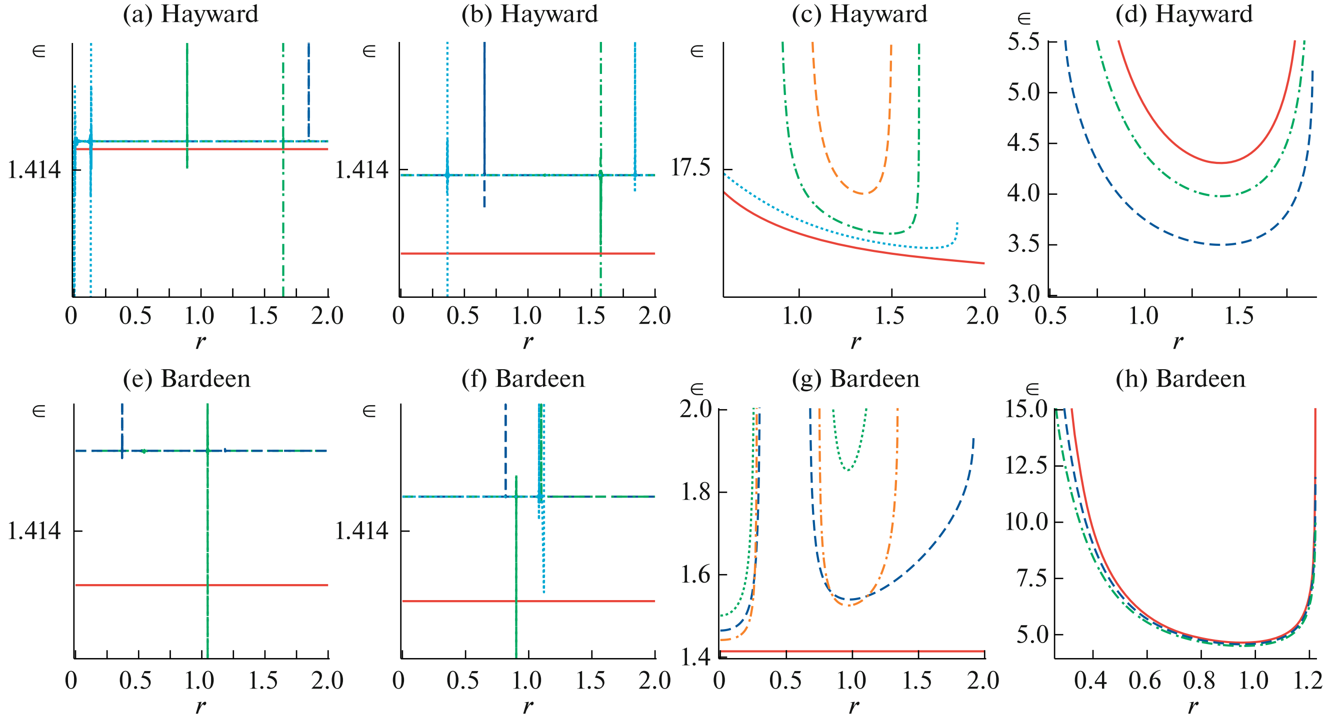

Fig. 3. $\epsilon \equiv \tilde{E}_{\rm CM}$ in terms of r for $\alpha = 1$ and $\beta = 2$ with $M = 1$, $l = 0.5$, $\nu = 1$ and $\theta = \pi/2$, for rotating modified Hayward (a–d) and Bardeen (e–h) BHs. **(a)** $L_1 = L_2 = 0$, $E_1 = E_2 = 1$, $\mu = 0$, $a = 0$ (solid), $a = 0.1$ (dash), $a = 0.5$ (dash dot), $a = 0.5$ with $g_1 = 0$ (dot). **(b)** $L_1 = L2 = 0$, $E_1 = E_2 = 1$, $\mu = 1$, $a = 0$ (solid), $a = 0.1$ (dash), $a = 0.5$ (dash dot), $a = 0.5$ with $g_1 = 0$ (dot). **(c)** $E_1 = E_2 = 1$, $L_1 = -L_2 = -6$, $\mu = 1$. $a = 0$ (solid), $a = 0.5$ (dash dot), $a = 0.6$ (dash), $a = 0.5$ with $g_1 = 0$ (dot). **(d)** $L_1 \gg L_2$, $E_1 1$, $E_2 = 0.2$, $\mu = 0.5$, $g = 0.5$, $a = 0$ (solid), $a = 0.1$ (dash dot), $a = 0.3$ (dash). **(e)** $L_1 = L_2 = 0$, $E_1 = E_2 = 1$, $\mu = 0$, $g = 0.5$, $a = 0$ (solid), $g = 0.5$, $a = 0.3$ (dash), $g = 0.5$, $a = 0.6$ (dash dot). **(f)** $L_1 = L_2 = 0$, $E_1 = E_2 = 1$, $\mu = 1$, $g = 0.5$, $a = 0$ (solid), $g = 0.5$, $a = 0.2$ (dash), $g = 0.5$, $a = 0.4$ (dash dot), $a = 0.3$ with $g_1 = 0$ (dot). **(g)** $g = 0.5$, $a = 0.5$, $L_1 = L_2 = 2$, $E_1 = E_2 = 1$ (solid), $E_1 = 2$, $E_2 = 1$ (dash), $E_1 = 0$, $E_2 = 1$ (dot), $E_1 = 0.4$, $E_2 = 1$ (dash dot). **(h)** $L_1 \gg L_2$, $E_1 = 0.2$, $E_2 = 1$, $\mu = 1$, $g = 1$, $a = 0$ (solid), $a = 0.167$ (dash), $a = 0.333$ (dash dot), $a = 0.3$ (dot).

$$W \equiv dV_{\rm eff}/dr = 0. \tag{32}$$

The first condition (31) satisfied at the BH horizon. Figures 4a–4f show the behavior of W. For the unit values of a and E we can see that the condition (32) is satisfied for the modified Hayward black hole with negative L (see Fig. 4a.

In the other plots (Figs. 4b, 4c we can see the effect of a and E.

On the other hand, Figs. 4d–4f show that rotating modified Bardeen BHs have no restriction on negative L. For any values of L we can have particle circles.

It is important to find the innermost stable circular orbit. Using the relation (24), one can obtain the angular momentum per unit mass on a circular orbit, which satisfy $\dot{r} = 0$:

$$L_+ = \frac{Ea(1-\tilde{f}\tilde{h})}{\tilde{f}\tilde{h}}\left[1 \pm \sqrt{1 + \frac{\tilde{f}\tilde{h}[\xi - E^2(a^2(\tilde{f}-2) - \Sigma)]}{E^2 a^2 (1-\tilde{f}\tilde{h})^2}}\right]. \tag{33}$$

The reality condition for the above solution tells us that

$$E^2 \geq \frac{\tilde{f}\tilde{h}\xi}{a^2[\tilde{f}^2\tilde{h} - \tilde{f}^2\tilde{h}^2 - 1] - \tilde{f}\tilde{h}\Sigma}. \tag{34}$$

The equality hold for $L_+ = L_- \equiv L_0$, where,

$$L_0 = \frac{1-\tilde{f}\tilde{h}}{\tilde{f}\tilde{h}\sqrt{\tilde{f}(1-\tilde{h}) - 1/(\tilde{f}\tilde{h}) - \Sigma/a^2}}. \tag{35}$$

In that case there is no stable circular orbit. To have such an orbit, the angular momentum must be in the range $L_- < L < L_+$. It is possible to suppose that $L_1 = L_+$ and $L_2 = L_- + \varepsilon$, where $0 \leq \varepsilon\, leq L_+ - L_-$. In other words, the first particle is a target and the second particle on a circular orbit collides with the target. ε is a small drift of the second particle from a circular orbit. Therefore, using the relation (33) for

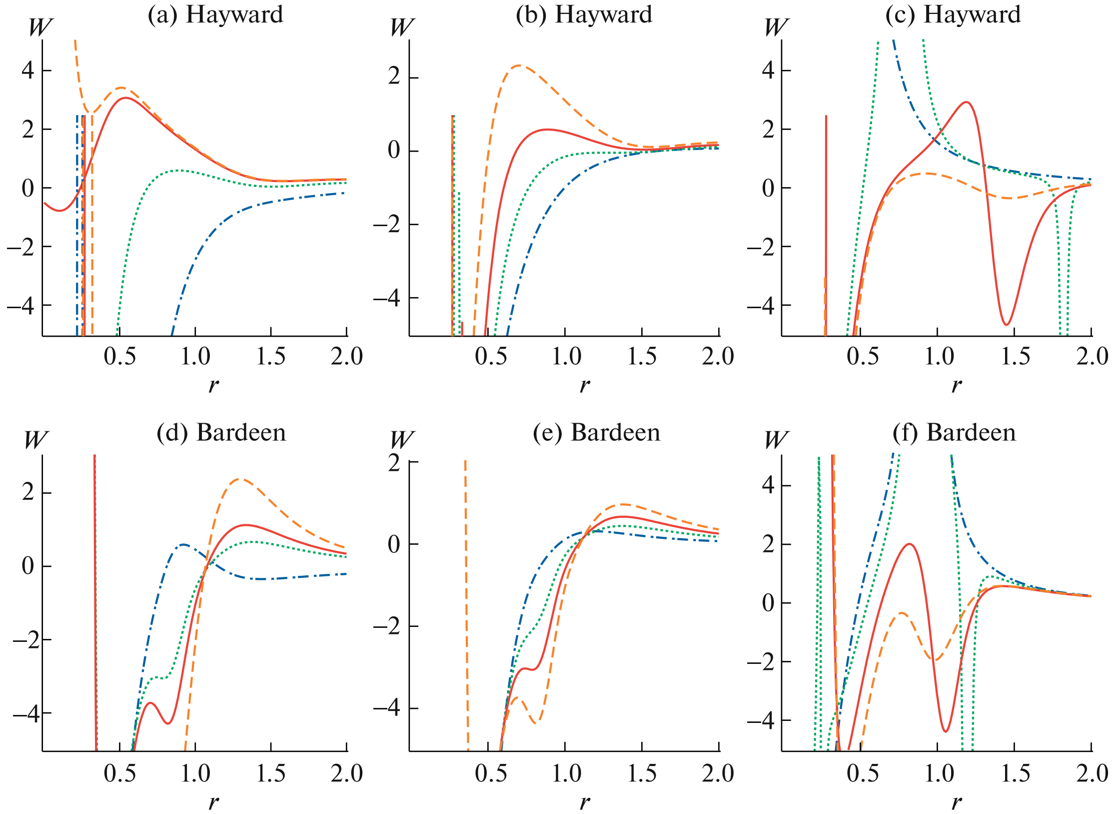

Fig. 4. $W \equiv dV_{\rm eff}/dr$ in terms of r for $\alpha = 1$ and $\beta = 2$ with $M = 1$, $l = 0.5$, $g = 0.25$ and $\theta = \pi/2$, for Hayward (a–c) and Bardeen (d–f) BHs. **(a)** $a = 1$, $E = 1$, $L = -2$ (dash dotted), $L = -0.4$ (dot), $L = 0$ (solid), $L = 0.02$ (dash). **(b)** $a = 1$, $L = -0.4$, $E = 0$ (dash dotted), $E = 0.8$ (dot), $E = 1$ (solid), $E = 1.2$ (dash). **(c)** $E = 1$, $L = -0.4$. $a = 0$ (dash dotted), $a = 0.2$ (dot), $a = 0.6$ (solid), $a = 0.8$ (dash). **(d)** $a = 1$, $E = 1$, $L = -2$ (dash dotted), $L = -0.4$ (dot), $L = 0$ (solid), $L = 0.8$ (dash). **(e)** $a = 1$, $L = -0.4$, $E = 0$ (dash dotted), $E = 0.8$ (dot), $E = 1$ (solid), $E = 1.2$ (dash). **(f)** $E = 1$, $L = -0.4$. $a = 0$ (dash dotted), $a = 0.2$ (dot), $a = 0.6$ (solid), $a = 0.8$ (dash).

the CM energy (26), we can obtain $\tilde{E}_{\rm CM}$ at a circular orbit. In the case $\varepsilon = 0$ we have a maximum of $\tilde{E}_{\rm CM}$, and in the case $\varepsilon = L_+ - L_-$ we have a minimum of $\tilde{E}_{\rm CM}$, i.e,

$$\tilde{E}_{{\rm CM}_{\rm min}} < \tilde{E}_{\rm CM} < \tilde{E}_{{\rm CM}_{\rm max}}. \qquad (36)$$

For the appropriate choice of r (at a special place like that near a horizon), we can see that $\tilde{E}_{{\rm CM}_{\rm max}}$ tends to infinity. The innermost circle with $\tilde{E}_{\rm CM}$ may serve as observable phenomena. We can see that behavior of $\tilde{E}_{\rm CM}$ for both rotating modified Hayward and Bardeen BHs in Fig. 5.

There is also another interesting state with

$$\tilde{f}\tilde{h}[\xi - E^2(a^2(\tilde{f} - 2) - \Sigma)] = 0, \qquad (37)$$

where $L_- = 0$, so the second particle has an infinitesimal angular momentum $L_2 = \varepsilon$. For L_+ there are two possible cases. In the first case,

$$\xi - E^2(a^2(\tilde{f} - 2) - \Sigma) = 0, \qquad (38)$$

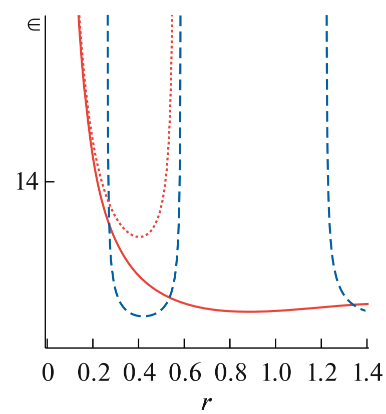

Fig. 5. A typical behavior of $\epsilon \equiv \tilde{E}_{\rm CM}$ in terms of r for $M = 1$, $E_1 = E_2 = 1$, $\varepsilon = 1$, $\alpha = 1$ and $\beta = 2$. Solid and dotted lines represent the case of a rotating modified Hayward BH for $l = 1$ and $l = 0.5$, respectively, while the dashed line represents a rotating modified Bardeen BH with $g = 0.8$.

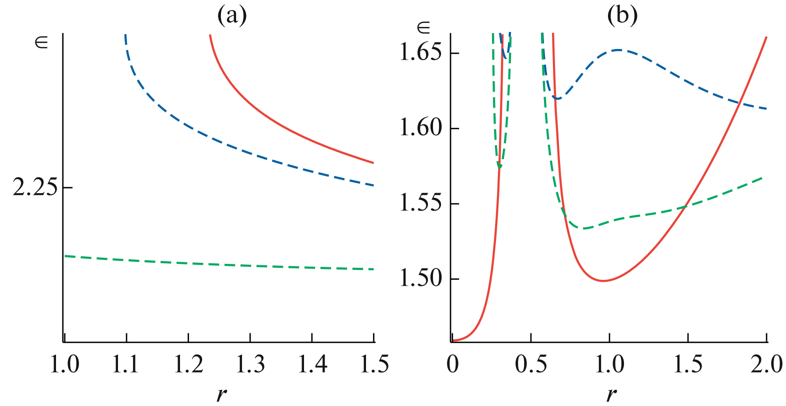

Fig. 6. $\epsilon \equiv \tilde{E}_{\rm CM}$ in terms of r for $\alpha = 1$ and $\beta = 2$ with $M = 1$, $l = 0.5$, $\mu = \nu = 1$ and $\theta = \pi/2$. **(a)** Extremal ($a = 0.65$) rotating modified Hayward BH with $r_+ = r_- \approx 1.25$. $L_1 = 2, L_2 = -2$, $E_1 = E_2 = 1$ (solid), $L_1 = 2$, $L_2 = -2$, $E_1 = 1$, $E_2 = 1.2$ (dash), $L_1 = 0$, $L_2 = -2$, $E_1 = 1$, $E_2 = 2$ (dot). **(b)** Extremal ($a = 1.22$) rotating modified Bardeen BH with $r_+ = r_- \approx 0.6$. $L_1 = L_2 = 2$, $E_1 = 1$, $E_2 = 2$ (solid), $L_1 = 0$, $L_2 = 2$, $E_1 = 1$, $E_2 = 2$ (dash), $L_2 = -2$, $L_1 = 0$, $E_2 = 1$, $E_1 = 2$ (dot).

we have

$$L_+ = \Big[a^2\tilde{f}(r_c)[\tilde{h}(r_c) - 1] + r_c^2 + a^2 - E^2[\tilde{f}(r_c)a^2 - r_c^2 - 2a^2]\Big] Ea \times \Big[E^2[\tilde{f}(r_c)a^2 - r_c^2 - 2a^2] - a^2\Big]^{-1}, \tag{39}$$

where r_c is a root of Eq. (38).

In the second case $\tilde{f}\tilde{h} = 0$, we have $L_+ \to \infty$.

5. THE EXTREMAL LIMIT

It may be interesting to study the extremal solution where $\delta = 0$ and $r_+ = r_-$. It will be obtained using the appropriate choice of α, β and a.

For example, the extremal limit of a rotating modified Hayward BH may be given by $\alpha = 1$, $\beta = 2$ and $a = 0.65$ (with other parameters fixed as previously), leading to $r_+ = r_- \approx 1.25$.

The extremal limit of a rotating modified Bardeen BH may be given by $\alpha = 1$, $\beta = 2$, $g = 0.5$, and $a = 1.22$ (with other parameters fixed as previously), and then $r_+ = r_- \approx 0.6$. It is clear from Fig. 6 that an infinite CM energy near the BH horizon will be obtained for both rotating modified Hayward and Bardeen BHs.

Figure 6a presents $\tilde{E}_{\rm CM}$ in the case of an extremal rotating modified Hayward BH for various values of E_1, E_2, L_1 and L_2. We find that $E_1 = E_2$ is a crucial condition for having an infinite CM energy near the horizon. It is clear that a maximum range of CM energy is seen near $r \approx 1.25$, which is indeed the location of the event horizon.

On the other hand, Fig. 6b presents $\tilde{E}_{\rm CM}$ for an extremal rotating modified Bardeen BH. We can see for various values of E_1, E_2, L_1 and L_2 that we have an infinite CM energy near the BH horizon.

6. CONCLUSIONS

The BSW mechanism [1] has demonstrated that the extremal Kerr BH can be considered as a particle accelerator with infinite CM energy near the BH horizon. Since there is no definite quantum theory of gravity, the BH interior has not yet been understood completely, hence we need regular BHs which are motivated by quantum theories. The Hayward and Bardeen BHs are such regular models considered as classical BHs. The rotating modified Hayward BH has new parameters μ and ν, while the rotating modified Bardeen BH has a new parameter g which make systems different from the Kerr BH. Like [34], we have found the effect of these new parameters on the CM energy, hence we can obtain information about the BH structure. Indeed, in this work we have assumed two types of regular BHs, rotating modified Hayward and Bardeen BHs, as particle accelerators. The horizon structure of a rotating modified Hayward BH specified by $g_{rr} = \infty$, which is exactly similar to the rotating Hayward BH discussed in [34]. The horizon structure of a rotating modified Bardeen black hole is specified by Fig. 2, and Figs. 2a,b show Δ vs. r for $\theta = \pi/2$, while Figs. 2c,d show Δ vs. r for $\theta = \pi/6$. We have investigated the CM energy of two colliding neutral particles with the same rest masses falling from the rest at infinity to near the horizons of the mentioned BHs. Figs. 3 show the CM energy vs. r for rotating modified Hayward and Bardeen BHs on different cases. We have also investigated the range of particles' angular momentum and the particle orbit. Figs. 4a–4c and 4d–4f show W vs. r for rotating modified Hayward and Bardeen BHs, respectively. We have obtained the innermost stable circular orbits of particles. We have also studied the CM energy corresponding to extremal BHs and obtained an infinite

CM energy for appropriate BH parameters, which is in agreement with the result of Zaslavskii [38]. It that case it is interesting to study the BSW mechanism for another kind of BHs like the Gödel BH [39–41], two-dimensional BHs [42–44], Schr'odinger [45] or string BHs [46] with hyperscaling violation [47–49], and the Myers-Perry BHs [50, 51].

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