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Anisotropic Dark Energy Models with Hybrid Expansion Law in Lyra's Manifold

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Abstract—Field equations of the locally rotationally symmetric (LRS) Bianchi type-I metric with anisotropic fluid are constructed in the framework of Lyra's manifold. By assuming a hybrid expansion law (HEL) for the average scale factor that yields power-law and exponential-law cosmologies, we model Bianchi type-I space time for the time-dependent displacement field which is proportional to a power-law form of the Hubble parameter. The model provides an elegant description of the transition from cosmic deceleration to acceleration. We discuss the physical behaviors of the derived models with observational constraints applied to late-time acceleration as well as early stages of the Universe. It is observed that HEL Bianchi type I universe is anisotropic at early stage of evolution and becomes isotropic at late times.

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1. INTRODUCTION

In view of explaining observational cosmology, many researchers are interested in the importance of late-time acceleration of the isotropic and homogeneous Universe which was in an anisotropic phase at early times. But observational evidence such as Lima [1], Perlmutter et al. [2] and Reiss et al. [3] supports that the present Universe is undergoing a phase of accelerated expansion. Cosmologists are considering many alternatives to explain this late-time acceleration. One of the approaches is that some form of dark energy (DE) must exist in the Universe to drive this accelerated expansion. Two main candidates for DE are the cosmological constant and scalar fields. The data indicate that the current standard model of the Universe is dominated by an unclustered fluid with large negative pressure called DE which causes the acceleration. Spergel et al. [4] also assert that DE accounts for 70% of the spatially flat Universe. Scalar field models were invoked to alleviate the problems associated with the cosmological constant. Unfortunately, scalar field models are plagued with similar problems. Caldwell and Kamionkowski [5] studied the mystery of the nature of DE. Several models have been proposed to explain DE [6–12]. An alternative consists in a phenomenological decaying DE density with continuous creation of matter [12] or photons [13, 14]. DE might decay slowly in the course of the cosmic evolution and thus provide a source for matter and radiation.

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Another approach consists in modified theories of gravity to describe the accelerated expansion of the Universe. Among various modification of general relativity, Lyra's geometry is a well-known example of scalar tensor theory. A year after Einstein developed his general relativity (in 1917), in which gravitation is described in terms of geometry, Weyl [15] proposed a more general theory in which electromagnetism is also described geometrically. However, this theory, based on nonintegrability of length transfer, had some unsatisfactory features and did not gain general acceptance. Later Lyra [16] suggested a modification of Riemannian geometry, which may also be considered as a modification of Weyl's geometry, by introducing a gauge function into the structureless manifold which removes the nonintegrability of the length of a vector under parallel transport, and a cosmological constant is naturally introduced from the geometry. In subsequent investigations, Sen [17] and Sen and Dunn [18] formulated a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry. Halford [19] pointed out that a constant displacement vector field ϕ_i in Lyra's geometry plays the role of a cosmological constant in the normal general-relativistic treatment. Halford [20] found that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as general relativity.

Several authors studied cosmological models based on Lyra's manifold with a constant displacement field vector ϕ_i. However, this restriction on the displacement field to a constant is only for convenience, and there is no prior reason for it. Many eminent authors [19, 21–24] investigated cosmological models with constant and time-dependent displacement fields. Recently, an FRW cosmological model in the framework of Lyra's geometry was studied with a variable equation-of-state (EoS) parameter [25]. Singh et al. [26] also investigated bulk-viscous models of the Universe with a variable deceleration parameter in Lyra's Manifold.

Many authors studied power-law and exponential-law cosmologies in general relativity by constraining it with a host of observational data and found that such cosmology is not a complete package for cosmological purposes. In fact, power-law and exponential-law cosmologies can be used only to describe an epoch-based evolution of the Universe because of the constancy of the deceleration parameter. For instance, these cosmologies do not exhibit a transition of the Universe from deceleration to acceleration. Akarsu et al. [27] investigated a simple form of the expansion history of the Universe referred to as the hybrid expansion law (HEL)—a product of power-law and exponential type of functions in the framework of the Brans-Dicke theory.

On the other hand, recent experimental data and critical arguments support the existence of an anisotropic phase of the cosmic expansion that approaches an isotropic one with the age of the Universe. Balloon-borne experiments such as BOOMERGang [28] and MAXIMA [29] have detected the anisotropy spectrum of the cosmic microwave background (CMB) radiation in a flat Universe. Spatially homogeneous and isotropic Universes represented by Friedmann-Robertson-Walker (FRW) models are considered to be the most suitable for studying the large-scale structure of the Universe. But, they have higher symmetries than the real Universe, and therefore they are probably poor approximations for a very early Universe. The isotropic and homogeneous Universe on larger scales is well described observationally by the ΛCDM model, but the latter shows a poor fit to the CMB temperature power spectrum at low multipoles. This shows that the essential features of the early Universe are not characterized by isotropy and homogeneity. Also, the WMAP observations hint towards the presence of some anisotropic energy source in the Universe with anisotropic pressures. This prediction motivates us to describe the early stages of the Universe with anisotropic models. Some Bianchi cosmologies, for example, are natural hosts of large-scale magnetic fields, and therefore, their study can shed light on the implications of cosmic magnetism for galaxy formation.

The simplest anisotropic models of the Universe are Bianchi type-I homogeneous models whose spatial sections are flat but the expansion or contraction rates are direction-dependent. Thus it would be worthwhile to explore anisotropic DE models in the framework of modified gravity theory to generate cosmic acceleration with a regular fluid. Koivisto and Mota [30] also purposed cosmological model with anisotropic and viscous DE in order to explain an anomalous cosmological observation in the cosmic microwave background (CMB) at the largest angles. Various attempts are made to construct acceptable DE models in different directions such as the traditional cosmological constant, quintessence or phantom models, a dark fluid with complicated equation of state, string or M theory, higher dimensions, braneworld models, etc. The anisotropy of the cosmic expansion which is supposed to be damped out in the course of cosmic evolution is an important quantity in the history of evolution of the Universe. Saha [31] discussed a binary mixture of a perfect fluid and DE for Bianchi type-I and for Bianchi type-V. The accelerated expansion provides information about the major part of the Universe which has a large negative pressure but without telling anything about the number of cosmic fluids in the Universe. This may be explained by considering the accelerating expansion with a single fluid and an EoS acting like DE. The main benefit of this approach is that a suitable EoS can be obtained, and the observational data can be fitted.In General relativity (GR) that people have investigated by assuming a fluid with an anisotropic EoS.

Akarsu and Kilinic [32] studied Bianchi type-III models in the presence of a single imperfect fluid with anisotropic DE. They found that the anisotropy of DE does not always promote the anisotropy of expansion. Sharif and Zubair [33] studied Bianchi type VI_0 cosmological models in the presence of an electromagnetic field and anisotropic DE. Bianchi type models have been studied by several authors in an attempt to understand better the observed small amount of anisotropy in the Universe. The same models have also been used to examine the role of certain anisotropy sources during the formation of large-scale structures we see in the Universe today. Singh et al. [26] investigated bulk-viscous cosmological models with a variable deceleration parameter in Lyra's manifold. Singh et al. [34] also discussed Bianchi type-I space-time in the presence of bulk viscosity and a Chaplygin gas in the context of Lyra's geometry. It is observed that the considered form of bulk viscosity has a similar qualitative behavior to that of constant and variable bulk viscosities. Surendra et al. [35] investigated power-law inflation with an anisotropic fluid in Lyra's manifold.

In this work, our intention is to investigate a Bianchi type-I cosmological model dominated by an anisotropic fluid in Lyra's geometry for a time-dependent displacement field by assuming a hybrid expansion law (HEL) of the Universe. We will investigate the consistency of the derived HEL cosmology with observations according to the latest data, and then studied the kinematics and dynamics of the HEL Universe in detail. In Section 2, we present the field equations and their solutions. In Section 3, we discuss a model with a hybrid expansion law. Some observational parameters of the derived models are investigated in Section 4, and a concluding remark is written in Section 5.

2. FIELD EQUATIONS AND THEIR SOLUTIONS

The metric of a homogeneous and anisotropic Bianchi type-I space-time is

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \tag{1}$$

where the scale factors A and B are functions of cosmic time t only. The field equations in Lyra's manifold as obtained by Sen [17] are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \frac{3}{2}\phi_\mu\phi_\nu - \frac{3}{4}g_{\mu\nu}\phi_m\phi^m = -T_{\mu\nu}, \tag{2}$$

where ($8\pi G = 1$ and $c = 1$) $\phi_\mu = (0, 0, 0, \beta(t))$ is the timelike displacement field vector, $R_{\mu\nu}$ is the Ricci tensor, and R is the Ricci scalar, with comoving coordinates, so that $g_{\mu\nu}u^\mu u^\nu = 1$ and $u^\mu = (0, 0, 0, 1)$. We choose ϕ_μ in this form to be time dependent. Here we apply the fact that the ansatz choosing the coordinate system with matter requires the vector field happens to be in this particular form exactly in the matter-comoving coordinates. The essential difference between the cosmological theories based on Lyra's geometry and the Riemannian geometry lies in the fact that the constant vector displacement field β arises naturally from the concept of gauge in Lyra's geometry whereas the cosmological constant Λ was introduced in an ad-hoc fashion in the usual treatment. Here $T_{\mu\nu} = T^{(m)}_{\mu\nu} + T^{(\mathrm{de})}_{\mu\nu}$ is the total energy-momentum tensor with $T^{(m)}_{\mu\nu}$ and $T^{(\mathrm{de})}_{\mu\nu}$ the energy momentum tensors of matter and anisotropic DE fluid, respectively. We can parametrize the components of $T^{(\mathrm{de})}_{\mu\nu}$ in the following ways:

$$\begin{aligned} T^{(\mathrm{de})}_{\mu\nu} &= \mathrm{diag}[\rho^{(\mathrm{de})}, -p_x^{(\mathrm{de})}, -p_y^{(\mathrm{de})}, -p_z^{(\mathrm{de})}] \\ &= \mathrm{diag}[1, -\omega_x^{(\mathrm{de})}, -\omega_y^{(\mathrm{de})}, -\omega_z^{(\mathrm{de})}]\rho^{(\mathrm{de})} \\ &= \mathrm{diag}[1, -\omega^{(\mathrm{de})}, -(\omega^{(\mathrm{de})}+\delta), -(\omega^{(\mathrm{de})}+\delta)]\rho^{(\mathrm{de})}, \end{aligned} \tag{3}$$

where ρ is the energy density of the fluid, $p_x^{(\mathrm{de})}$, $p_y^{(\mathrm{de})}$ and $p_z^{(\mathrm{de})}$ are the pressure s on the x, y and z axes, respectively. Here $\omega^{(\mathrm{de})} = p^{(\mathrm{de})}/\rho^{(\mathrm{de})}$ is the deviation-free EoS parameter of the fluid while $\omega_x^{(\mathrm{de})}$, $\omega_y^{(\mathrm{de})}$ and $\omega_z^{(\mathrm{de})}$ are the directional EoS parameters on the three axes. Now, we parametrize the deviation from isotropy by setting $\omega_x^{(\mathrm{de})} = \omega_y^{(\mathrm{de})} = \omega_z^{(\mathrm{de})} = \omega^{(\mathrm{de})}$ (the deviation-free EOS parameter of DE), then introducing the skewness parameter δ which is the deviation from $\omega^{(\mathrm{de})}$ on y and z axis. Here $\omega^{(\mathrm{de})}$ and δ are not necessarily constants and can be functions of cosmic time t. Similarly, the energy momentum tensor of matter is given by

$$T^{(m)}_{\mu\nu} = \mathrm{diag}[1, -\omega^{(m)}, -\omega^{(m)}, -\omega^{(m)}]\rho^{(m)} \tag{4}$$

where $\rho^{(m)}$ and $p^{(m)}$ are the energy density and pressure of matter while the EoS parameter is given by $\omega^{(m)} = p^{(m)}/\rho^{(m)}$. In a comoving coordinate system, the above field equations (2) in anisotropic space-time (1), with Eqs. (3) and (4), yield

$$2\frac{\dot{A}\dot{B}}{AB} + \left(\frac{\dot{B}}{B}\right)^2 - \frac{3}{4}\beta^2 = \rho^{(m)} + \rho^{(\mathrm{de})}, \tag{5}$$

$$2\frac{\ddot{B}}{B} + \left(\frac{\dot{B}}{B}\right)^2 + \frac{3}{4}\beta^2 = -\omega^{(m)}\rho^{(m)} - \omega^{(\mathrm{de})}\rho^{(\mathrm{de})}, \tag{6}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{3}{4}\beta^2 = -\omega^{(m)}\rho^{(m)} - (\omega^{(\mathrm{de})} + \delta)\rho^{(\mathrm{de})}, \tag{7}$$

where the overhead dot denotes a derivative with respect to t. The directional Hubble parameters along the x, y and z axes for the LRS Bianchi type-I metric are

$$H_x = \frac{\dot{A}}{A}, \qquad H_y = H_z = \frac{\dot{B}}{B} \tag{8}$$

The mean Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{\dot{V}}{3V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) = \frac{1}{3}(H_x + H_y + H_z). \tag{9}$$

The energy conservation equation $T^{\mu\nu}_{;\nu} = 0$ gives

$$\dot{\rho}^{(m)} + 3H\rho^{(m)}(1 + \omega^{(m)}) + \dot{\rho}^{(\mathrm{de})} + 3H\rho^{(\mathrm{de})}(1 + \omega^{(\mathrm{de})}) + 2\delta\rho^{(\mathrm{de})}H_y = 0. \tag{10}$$

The energy momentum tensors of noninteracting matter and DE fluids can be conserved separately:

$$\dot{\rho}^{(m)} + 3H\rho^{(m)}(1 + \omega^{(m)}) = 0, \tag{11}$$

$$\dot{\rho}^{(\mathrm{de})} + 3H\rho^{(\mathrm{de})}(1 + \omega^{(\mathrm{de})}) + 2\delta\rho^{(\mathrm{de})}H_y = 0. \tag{12}$$

If matter is a non-relativistic pressureless fluid such as cold dark matter, then the EoS parameter of matter is $\omega^{(m)} = 0$. To study a dynamical nature of DE,

$\omega^{(\mathrm{de})}$ is allowed to vary with the evolution of Universe while the EoS parameter of matter fluid is taken to be $\omega^{(m)}=0$. As $\omega^{(m)}=0$, integration of Eq. (11) gives

$$\rho^{(m)}=C_0a^{-3}. \tag{13}$$

The spatial volume of the Universe for this model is given by

$$V=a^3=AB^2,\quad a=(AB^2)^{1/3}, \tag{14}$$

where a is the average scale factor. Subtracting (6) from (7), we get

$$\frac{d}{dt}\Big(\frac{\dot A}{A}-\frac{\dot B}{B}\Big)+\Big(\frac{\dot A}{A}-\frac{\dot B}{B}\Big)\frac{\dot V}{V}=-\delta\rho^{(\mathrm{de})}.$$

Integration of this equation gives

$$\frac{\dot A}{A}-\frac{\dot B}{B}=\frac{\lambda}{V}\exp\int\frac{\delta\rho^{(\mathrm{de})}}{(\dot B/B-\dot A/A)}dt \tag{15}$$

where λ is a constant of integration. But since we are looking for physically viable models of the Universe consistent with observations, to find the exact solution of the above equation (15), we assume

$$\frac{\delta\rho^{(\mathrm{de})}}{\dot B/B-\dot A/A}=\frac{1}{t}+3\gamma, \tag{16}$$

where γ is a nonnegative constant. Using Eq. (16) in (15), we obtain

$$\frac{\dot A}{A}-\frac{\dot B}{B}=\frac{\lambda\, t\, e^{3\gamma t}}{V}. \tag{17}$$

To examine whether the expansion of the Universe is anisotropic or not, the anisotropic expansion parameter Δ is defined as

$$\Delta=\frac{1}{3}\sum_{i=1}^{i=3}\left(\frac{H_i-H}{H}\right)^2.$$

If $\Delta=0$, then the Universe expands isotropically. Further, any anisotropic model of the Universe with diagonal energy-momentum tensor approaches isotropy if $\Delta=0, V\to\infty$, and $\rho>0$ as $t\to\infty$. The volumetric deceleration parameter is

$$q=-\frac{a\ddot a}{\dot a^2} \tag{18}$$

The deceleration parameter q measures the rate of expansion of the Universe. If $q<0$ or $q>0$, then it represents inflation or deflation of the Universe, respectively, while $q=0$ shows expansion with a constant rate.

3. MODEL WITH A HYBRID EXPANSION LAW

We have three independent equations in seven unknowns, viz., $A, B, \rho^{(\mathrm{de})}, \omega^{(\mathrm{de})}, \beta$ and δ. Therefore we need more relations among the variables in order to obtain a unique solution. In order to solve the equations completely, we consider the following hybrid expansion law [27]:

$$a=a_0t^ne^{\alpha t}, \tag{19}$$

where $a_0>0$, $n\geq 0$ and $\alpha\geq 0$ are constants. This generalized form of the scale factor is referred to as the hybrid expansion law (HEL) which unifies power-law and exponential expansions of the Universe. Power-law and exponential expansion are obtained with $\alpha=0$ and $n=0$, respectively. Thus $n>0$ and $\alpha>0$ gives a new cosmology arising from HEL. The parameter n determines the initial kinematics of the Universe since $a\sim a_0t^n$ at $t\sim 0$, while the very late time kinematics of the Universe is determined by the parameter α since $a\sim a_0e^{\alpha t}$ at $t\sim\infty$. Using (19), we obtain the volumetric deceleration parameter q as

$$q=\frac{n}{(\alpha t+n)^2}-1 \tag{20}$$

It is observed that the HEL model evolves with a variable deceleration parameter if α and n are positive, and a transition from deceleration to acceleration takes place at $t=(\sqrt{n}-n)/\alpha$ which restricts n to the range $0<n<1$. Equation (14) becomes

$$V=a^3=AB^2=a_0^3t^{3n}e^{3\alpha t} \tag{21}$$

Using the above relation, taking $\gamma=\alpha$ in (17) in order to find a viable solution, Eq. (17) can be integrated to give

$$A=BC_1\exp\left[\frac{\lambda}{a_0^3(2-3n)}t^{2-3n}\right], \tag{22}$$

where C_1 is an integration constant. Using (21) and (22), we get the scale factors as

$$A=a_0C_1^{\frac{2}{3}}t^n\exp\left[\alpha t+\frac{2\lambda}{3a_0^3(2-3n)}t^{2-3n}\right], \tag{23}$$

$$B=\frac{a_0}{C_1^{\frac{1}{3}}}t^n\exp\left[\alpha t-\frac{\lambda}{3a_0^3(2-3n)}t^{2-3n}\right]. \tag{24}$$

It is observed that the scale factors expand along x, y and z axes with different rates with the realistic condition $2/3<n<1$. The directional Hubble parameters in this model are

$$H_x=\alpha+\frac{n}{t}+\frac{2\lambda}{3a_0^3}t^{1-3n}, \tag{25}$$

$$H_y=H_z=\alpha+\frac{n}{t}-\frac{\lambda}{3a_0^3}t^{1-3n},$$

$$H = \alpha + \frac{n}{t}. \tag{26}$$

Here, the directional parameters are extremely large at the beginning of the Universe and decrease monotonically with its age. Such a scenario provides information that our Universe is highly anisotropic in the past and becomes more isotropic later. The Universe starts evolving with different expansion rates H_x, H_y, H_z along x, y and z directions, has high anisotropy and shear at the beginning. In this model, we assume the vector field $\phi_\mu = (0, 0, 0, \beta(t))$, where $\beta = \beta_0 H^\kappa$,

$$\beta = \frac{n^\kappa \beta_0}{t^\kappa}, \tag{27}$$

where $\kappa > 0$. The displacement vector field β is infinite at the beginning and decreases with time. Here $\beta \to 0$ as $t \to \infty$. Thus the concept of Lyra's geometry will not be valid in the future evolution of Universe. The anisotropy parameter of the expansion is found as

$$\Delta = \frac{2\lambda^2 t^{2-6n}}{9a_0^6}\left(\alpha + \frac{n}{t}\right)^{-2} \tag{28}$$

One can check that this behavior of Δ is equivalent to the ones obtained for power law expansion in Bianchi type-I [36] and Bianchi type-V [37, 38] cosmological models with an isotropic fluid. The anisotropy parameter approaches zero for $1 > n > 2/3$ as $t \to \infty$, which shows isotropic expansion, while for $n < 2/3$ its behavior is switched. The expansion scalar Θ is

$$\Theta = 3H = 3\left(\alpha + \frac{n}{t}\right). \tag{29}$$

It also shows that the Universe initially evolves with an infinite expansion rate and shows a constant expansion at later epoch. The shear scalar σ^2 is given by

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{i=3} H_i^2 - 3H^2\right) = \frac{3}{2}\Delta H^2 = \frac{\lambda^2 t^{2-6n}}{3a_0^6}. \tag{30}$$

Using Eqs. (23), (24) and (27) in the Eqs. (5)–(7), we obtain the deviation parameter, the energy density and the deviation-free EoS parameter for the model as follows:

$$\delta = -\frac{\left(3\alpha + \frac{1}{t}\right)\frac{\lambda}{a_0^3 t^{3n-1}}}{3\left(\alpha + \frac{n}{t}\right)^2 - \frac{\lambda^2}{3a_0^6 t^{6n-2}} - \frac{3n^{2\kappa}\beta_0^2}{4t^{2\kappa}} - \{C_0(a_0 t^n e^{\alpha t})\}^{-3}}, \tag{31}$$

$$\rho^{(\mathrm{de})} = 3\left(\alpha + \frac{n}{t}\right)^2 - \frac{\lambda^2}{3a_0^6 t^{6n-2}} - \frac{3n^{2\kappa}\beta_0^2}{4t^{2\kappa}} - \frac{1}{C_0(a_0 t^n e^{\alpha t})\}^3}, \tag{32}$$

$$\omega^{(\mathrm{de})} = -1 + \frac{\frac{2n}{t^2} + \frac{2\lambda\alpha}{a_0^3 t^{3n-1}} - \frac{2\lambda^2}{3a_0^6 t^{6n-2}} + \frac{2\lambda}{3a_0^3 t^{3n}} - \frac{3n^{2\kappa}\beta_0^2}{2t^{2\kappa}} - \{c_0(a_0 t^n e^{\alpha t})\}^{-3}}{3\left(\alpha + \frac{n}{t}\right)^2 - \frac{\lambda^2}{3a_0^6 t^{6n-2}} - \frac{3n^{2\kappa}\beta_0^2}{4t^{2\kappa}} - \{C_0(a_0 t^n e^{\alpha t})\}^{-3}}. \tag{33}$$

Since in this model, we have many alternatives for choosing values of $\lambda, \kappa, \beta_0, a_0$ and n, it is just sufficient to look for suitable values of these parameters such that the physically viable cosmological models are satisfied. For $1 > n > 2/3$, $\lambda = 0.001$, $\alpha = 0.5$, $a_0 = 0.2$, $\kappa = 8$, $c_0 = 3$ and $\beta_0 = 2$, the positivity condition of energy density is satisfied. i.e., $\rho > 0$, and it is a decreasing function of time; ω and δ are dynamical. Here $\delta \to 0$ as $t \to \infty$ provided $n \geq 2/3$. Thus the Universe approaches isotropy with the age of Universe. One may obtain a model with a constant displacement field if $\beta = \beta_0$.

4. KINEMATICS AND PHYSICAL BEHAVIOR OF THE MODEL

The observational setting $a(t) = 1/(1+z)$, z being the redshift, gives the expression between time and redshift as

$$t = \frac{n}{\alpha} W\left[\frac{\alpha}{n}\left(\frac{1}{a_0(1+z)}\right)^{1/n}\right] \tag{34}$$

where W denotes the Lambert W function, also known as the omega function or product logarithm. Using (29), the parameters of the derived model can be expressed in terms of the redshift. Such a relation

Asymptotic behavior of the model parameters for $2/3 < n < 1$

Parameters	$t \to 0 (z \to \infty)$	$t \to \infty (z \to -1)$
A, B, a	0	∞
$\rho^{(de)}$	∞	3α
$\omega^{(de)}$	Indeterminate	-1
q	$\frac{1}{n} - 1$	-1
H, H_x, H_y, H_z	∞	α
$\Delta, \beta, \sigma, \Theta$	∞	0

is useful for testing the model with observational data. Also, one has the liberty to test the behavior of the parameters with respect to cosmic time or redshift. Observations confirm that DE would have been too small to counteract the gravity of matter in the Universe, and the expansion would have initially slowed. DE would dominate in the future accelerating Universe. The cosmologists observed that the Universe transition from deceleration to acceleration is described by a cosmic jerk. The jerk parameter is important to figure out what DE is. The deceleration to acceleration phase of the Universe occurs for different models with a negative value of the deceleration parameter and a positive value of the jerk parameter [39, 40]. The dimensionless jerk parameter j is the third derivative of the scale factor with respect to cosmic time t and provides a perfect diagnosis of how much the DE model is close to ΛCDM dynamics. Flat ΛCDM models have a constant jerk $j = 1$. The statefinder parameters (j, s) are defined by Sahni et al. [40] as

$$j(t) = \frac{\dddot{a}}{H^3 a} = 1 + \frac{(2 - 3\alpha t - 3n)n}{(\alpha t + n)^3}, \quad (35)$$

$$s = \frac{j - 1}{3(q - \frac{3}{2})} = \frac{2n(3\alpha t + 3n - 2)}{3(\alpha t + n)[5(\alpha t + n)^2 - 2n]}. \quad (36)$$

In the above definition of s, there is 3/2 instead of 1/2 in the original definition $s = \frac{j-1}{3(q-1/2)}$ by Sahni et al. [40]. This is to avoid the divergence of the parameter s when the HEL model passes through $q = 1$ or $q = 1/2$ as in Akasu, et al.[27]. This model overlaps with flat ΛCDM models for $n \to \infty$ as (j, s) becomes $(1, 0)$ for $2/3 < n < 1$. This is in agreement with recent observations [41].

Table 1 presents the asymptotic behavior of the parameters of the derived model. We observe that the directional scale factors A, B and the spatial volume a^3 vanish while other parameters $\rho, H, \Delta, \sigma, \beta$ diverge as $t \to 0$. This shows that the early Universe evolves from an initial singularity and anisotropy. The scale factors evolve with different expansion rates in x, y and z directions. As $t \to \infty$, we have $H_x \sim H_y \sim H_z \sim \alpha$, $\Delta \sim \sigma \sim \beta \sim 0$, which shows that the Universe approaches isotropy at a late stage of its evolution. This is consistent with observations which advocate an isotropic Universe on large scales. We also observe that $q \sim -1$, $\rho \sim 3\alpha$, $\omega^{(de)} \sim -1$ as $t \to \infty$,this shows that the Universe achieves an asymptotically de-Sitter phase and hence DE dominates the evolution at late times. We plot the variation of $\omega^{(de)}$ versus time in Fig. 1 and j versus time in Fig. 2. Here, we observe that $\omega^{(de)}$ and j asymptotically approach -1 and 1, respectively, which shows that the Universe will be dominated at late times by DE that drives the cosmic acceleration. Moreover, our results show that the cosmic jerk parameter of the derived model is positive throughout the entire history of the Universe.

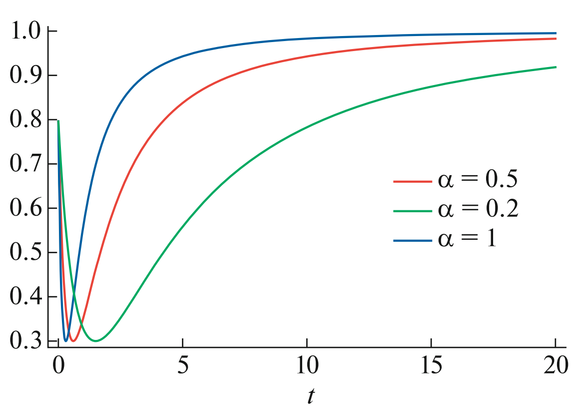

Fig. 1. Time variation of j for $n = 0.7$ and $\alpha = 0.2, 0.5, 1$.

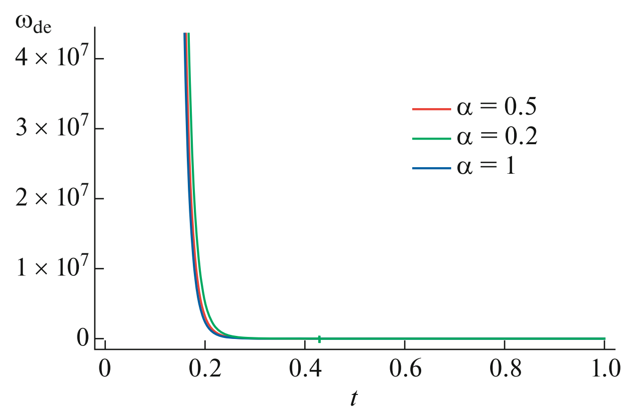

Fig. 2. Time variation of $\omega^{(de)}$ for $n = 0.7$, $\lambda = 0.001$, $\alpha = 0.5$, $a_0 = 0.2$, $\kappa = 20$, $c_0 = 3$ and $\beta_0 = 2$.

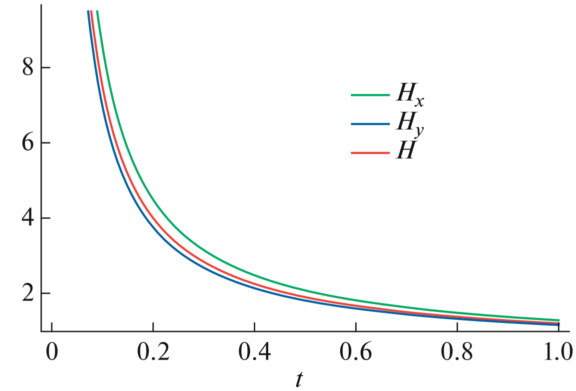

Fig. 3. Time variation of H_x, H_y and H for $n = 0.7$ and $\alpha = 0.5$, $\lambda = 0.001$, $a_0 = 0.2$.

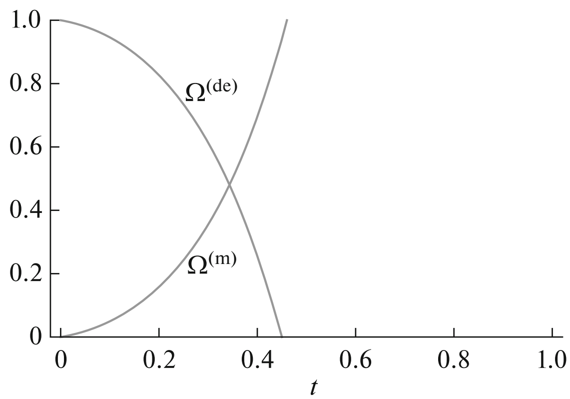

Fig. 4. Time variation of $\Omega^{(\mathrm{de})}$ and $\Omega^{(m)}$ for $n = 0.7$, $\lambda = 0.001$, $\alpha = 0.2, 0.5, 1$, $a_0 = 2$, $\kappa = 0.001$, $c_0 = 3$ and $\beta_0 = 0.001$.

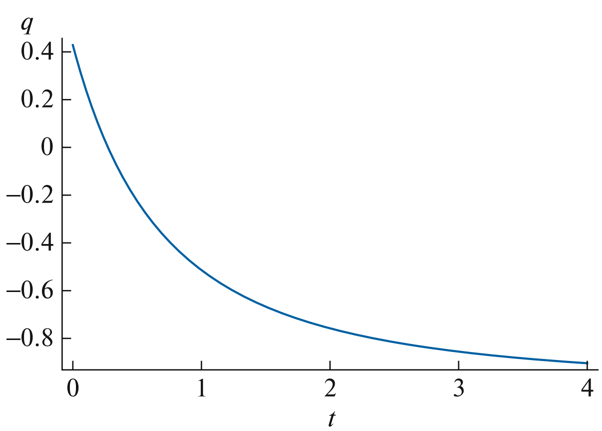

Fig. 5. Time evolution of q for $n = 0.7$, $\alpha = 0.5$.

5. CONCLUSION

In this paper, we have obtained exact solutions for a Bianchi type I model due to anisotropic DE in Lyra's geometry of the Universe. The models are investigated for a hybrid expansion law of the Universe. The model represents accelerated expansion of the Universe with $V \to \infty$ as $t \to \infty$ and is in good agreement with observations as represented by the type Ia Supernova (Perlmutter et al., 1999; Reiss et al., 1998) and WMAP data (Spergel et al., 2007). The physical behavior of the dynamic quantities depends on the value of n. We can only discuss $2/3 < n < 1$ for later times. The expansion scalar shows that the expansion rate is infinite at the beginning but approaches a uniform constant as time passes. The HEL Universe exhibits a transition from deceleration to acceleration, which is an essential feature of the dynamic evolution of the Universe. Interestingly, the model exhibits an initial singularity with high anisotropy. The anisotropy parameter vanishes for $2/3 < n < 1$ at late epoch of the Universe, which shows that the Universe expands isotropically at late times. It may help us to understand the isotropization mechanism which could be responsible for the transition from a possible prior anisotropic phase to the present isotropic epoch we live in. The energy density decreases monotonically with time. In the HEL Bianchi type I model, the displacement field vanished in the future. The HEL model mimics the concordance ΛCDM behavior of the Universe at late epochs. The model developed in this paper may be fruitful while dealing with the issues of CMB anisotropy, structure formation in the early Universe, etc.

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