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Scalar Field Cosmology in $f(R,T)$ Gravity with Λ

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Abstract—We study the behavior of cosmological parameters, massive and massless scalar fields (normal or phantom) with a scalar potential in $f(R,T)$ theory of gravity for a flat Friedmann-Robertson-Walker (FRW) universe. To get exact solutions to the modified field equations, we use the $f(R,T) = R + 2f(T)$ model by Harko et al. (T. Harko et al., Phys. Rev. D **84**, 024020 (2011)), where R is the Ricci scalar and T is the trace of the energy momentum tensor. Our cosmological parameter solutions agree with the recent observational data. Finally, we discuss our results with various graphics.

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1. INTRODUCTION

According to recent observations from type Ia Supernova (SNe Ia), baryon acoustic oscillations (BAO), cosmic microwave background (WMAP7) [1–3], it was noticed that our universe is accelerating [4]. Recently, many researchers have been working on alternative gravitation theories and getting their solutions showing the cosmic acceleration. Some well-known alternative theories to the Einstein theory are: $f(R)$ cosmology [5], the Brans-Dicke theory [6], Lyra cosmology [7] etc. In 2011, Harko et al. [8] put forward another alternative gravitation theory known as $f(R,T)$ gravitation theory. After the work of Harko et al., various researchers investigated $f(R,T)$ theory for different matter distributions and universe models [9–11]. In this context, Mirza and Oboudiat have analyzed a dynamical system of $f(R,T)$ gravity [12]. Sahoo et al. [13] have studied a Bianchi type universe model with string theory in $f(R,T)$ theory. Zubair et al. [14] studied $f(R,T)$ gravity admitting conformal Killing vectors. Also Zubair et al. studied $f(R,T)$ gravity with various space-time models [15–17]. Singh et al. studied a Bianchi type III universe model with a cosmological constant in $f(R,T)$ theory [18]. Also Singh and Bishi [19] obtained a quadratic equation of state solutions with Λ in $f(R,T)$ theory. Ramesh and Umadevi [20] studied an FRW cosmological model in the presence of a perfect fluid source, with a linearly varying deceleration parameter in $f(R,T)$ gravity. Rudra [21] studied the relation between $f(R,T)$ gravity, dark matter and dark energy. Moraes et al. studied a transition from decelerated to accelerated phase of the universe [22], compact stars [23], brane-world cosmology [24] in $f(R,T)$ gravity. Reddy et al. investigated a Kantowski-Sachs bulk-viscous string cosmological model [25] and a Bianchi type-III dark energy model [26] in $f(R,T)$ gravity. Also, Aygün et al. studied strange quark matter (SQM) solutions for Marder's universe [27] and magnetized SQM solutions for an FRW universe [28] in $f(R,T)$ gravity with Λ. Rao and Neelima [29] obtained perfect fluid solutions for an Einstein-Rosen universe model in $f(R,T)$ gravity. However, scalar fields (SF) are important in cosmology because they have a fundamental role to explanation of dark matter, inflation, also late time acceleration [8].

Burko and Gaurav [30] studied a massive scalar field (MSF) in black-hole universe models. They obtained that MSFs have the same late-time behavior in all black-hole models. Shadar and Piran [31]

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studied gravitational collapse of a massive scalar field and decay in Reissner-Nordström space-time. Singh and Rani [32] studied a MSF in Lyra geometry for a Bianchi type III universe model. Aygün et al. [33] investigated MSF solutions and found that the MSF decays to SFs in Riemann and Lyra geometries. Besides, Singh et al. [34] investigated a SF and time-varying Λ in $f(R,T)$ gravity. Sharif and Zubair [35] studied anisotropic space-times with SF and perfect fluid in $f(R,T)$ gravity. Santos and Ferst [36] investigated a Gödel type universe model with a perfect fluid and also a perfect fluid plus a SF in $f(R,T)$ gravity. Singh and Singh [37] also studied SF solutions with $f(R,T)$ gravity. We normally build our universe models with the a perfect fluid matter distribution. However, it is clear from recent studies and observations that the universe needs other matter fields to create negative pressure, and the cosmic dynamics cannot be clarified by only standard matter [34, 38].

Scalar fields are one of the important entities is superstring and Kaluza-Klein cosmology [39]. In various modified gravitation theories, such as Lyra, self-creation, Brans-Dicke and inflationary models, SFs are basic components. Also, SFs are a good nominee for dark matter in spiral galaxies [40]. They are in harmony with measurements in weak gravitational fields [33]. So it is very important to investigate solutions that contain massive and massless SFs in physics. There are a few massless scalar field models in the literature, but there is no MSF solutions in $f(R,T)$ theory.

The purpose of this article is to study massive and massless scalar field cosmological models in $f(R,T)$ gravitation theory in the framework of a flat FRW universe model, because FRW space-time well describes today's universe. For this purpose we will use $f(R,T) = R + 2h(T)$. In Section 2, we derive the basic formalism of $f(R,T)$ gravitation theory and describe FRW universe models. In Section 3, we obtain massless scalar field solutions for the $R + 2h(T)$ model in $f(R,T)$ gravity with Λ. In Section 4, we obtain MSF solutions for $f(R,T) = R + 2h(T)$ with Λ. Finally, we discuss the results in Section 5.

2. $f(R,T)$ MODIFIED FIELD THEORY

According to Harko et al. [8], the action of the new modified $f(R,T)$ gravity is given by

$$S = \int \left(\frac{f(R,T)}{16\pi G} + L_m \right) \sqrt{-g} d^4x, \quad (1)$$

where R is the Ricci scalar, T is the trace of $T_{\alpha\beta}$, g is the determinant of $g_{\alpha\beta}$, and $f(R,T)$ is an arbitrary function of R and T. Also, L_m is the matter Lagrangian. $T_{\alpha\beta}$ is defined as [8]

$$T_{\alpha\beta} = g_{\alpha\beta} L_m - \frac{2\partial L_m}{\partial g^{\alpha\beta}}. \quad (2)$$

By varying Eq. (1), we get

$$\begin{aligned} &f_R(R,T) R_{\alpha\beta} - \frac{1}{2} f(R,T) g_{\alpha\beta} \\ &+ (g_{\alpha\beta}\Box - \nabla_\alpha \nabla_\beta) f_R(R,T) = 8\pi T_{\alpha\beta} \\ &- f_T(R,T) T_{\alpha\beta} - f_T(R,T) \Xi_{\alpha\beta} + \Lambda g_{\alpha\beta}, \end{aligned} \quad (3)$$

where $f_R(R,T)$ and $f_T(R,T)$ are derivatives of $f(R,T)$ with respect to R and T, respectively, and ∇_α is the covariant derivative; $\Box = \nabla_\alpha \nabla^\alpha$; $\Xi_{\alpha\beta}$ is

$$\Xi_{\alpha\beta} = -2T_{\alpha\beta} + g_{\alpha\beta} L_m - 2g^{ik} \frac{\partial^2 L_m}{\partial g^{\alpha\beta} g^{ik}}. \quad (4)$$

If we contract Eq. (3), we obtain

$$\begin{aligned} &f_R(R,T) R + 3\Box f_R(R,T) - 2f(R,T) \\ &= 8\pi T - f_T(R,T) T - f_T(R,T) \Xi + \Lambda g_{\alpha\beta}, \end{aligned} \quad (5)$$

where $\Xi = g^{\alpha\beta} \Xi_{\alpha\beta}$. From Eqs. (3) and (5), we get the gravitational field equations as follows [8]:

$$\begin{aligned} &f_R(R,T) \Big(R_{\alpha\beta} - \frac{1}{3} R g_{\alpha\beta} \Big) + \frac{1}{6} f(R,T) g_{\alpha\beta} \\ &= 8\pi \Big(T_{\alpha\beta} - \frac{1}{3} T g_{\alpha\beta} \Big) - f_T(R,T) \Big(T_{\alpha\beta} - \frac{1}{3} T g_{\alpha\beta} \Big) \\ &- f_T(R,T) \left(\Xi_{\alpha\beta} - \frac{1}{3} \Xi g_{\alpha\beta} \right) \\ &+ \nabla_\alpha \nabla_\beta f_R(R,T) + \Lambda g_{\alpha\beta}. \end{aligned} \quad (6)$$

In this paper we will study the $f(R,T) = R + 2h(T)$ model for a SF in a homogeneous and isotropic flat FRW universe. Its metric is given by

$$ds^2 = dt^2 - A^2 [dr^2 + r^2 (d\theta^2 + \sin^2\theta d\vartheta^2)], \quad (7)$$

where A is a function of t. In this study, we consider the source of gravity as a SF coupled to gravity. Then our new SF ϕ with a self-interacting potential $V(\phi)$ is described by

$$T_{\alpha\beta} = \varepsilon \partial_\alpha \phi \partial_\beta \phi - [\frac{\varepsilon}{2} \partial_l \phi \partial^l \phi - V(\phi)] g_{\alpha\beta}, \quad (8)$$

where $\varepsilon = \pm 1$ and corresponds to phantom and normal SFs, respectively [37]. The SF Lagrangian is

$$L_\phi = -\frac{1}{2} \varepsilon \dot{\phi}^2 + V(\phi), \quad (9)$$

where the dot denotes an ordinary derivative in t. Using Eqs. (8) and (9) in (4), we get

$$\Xi_{\alpha\beta} = -2T_{\alpha\beta} - \Big(\frac{1}{2} \varepsilon \dot{\phi}^2 - V(\phi) \Big) g_{\alpha\beta}, \quad (10)$$

and the trace of $T_{\alpha\beta}$ is

$$T = 4V(\phi) - \varepsilon \dot{\phi}^2. \quad (11)$$

3. SOLUTIONS FOR A MASSLESS SCALAR FIELD IN $f(R,T)=R+2h(T)$ MODEL WITH Λ

For the choice $h(T)=\mu T$ ($\mu=\text{const}$), we use Eqs. (6)–(8), (10), and (11) for the $f(R,T)=R+2h(T)$ model with Λ, where R is a function of cosmic time, and $2h(T)$ describes the gravitational interaction between curvature and matter [8]). We get:

$$\frac{2\ddot{A}}{A}+\frac{\dot{A}^2}{A^2}=4\pi\varepsilon\dot{\phi}^2-8\pi V(\phi)+\mu\varepsilon\dot{\phi}^2-4\mu V(\phi)-\Lambda, \tag{12}$$

$$\frac{3\dot{A}^2}{A^2}=-4\pi\varepsilon\dot{\phi}^2-8\pi V(\phi)-\mu\varepsilon\dot{\phi}^2-4\mu V(\phi)-\Lambda. \tag{13}$$

We have four unknowns A, ϕ, $V(\phi)$, Λ and two modified field equations. To solve the system, we will firstly use the deceleration parameter as follows:

$$q=-\frac{A\ddot{A}}{\dot{A}^2}=\text{const} \tag{14}$$

if we integrate Eq. (14), we get

$$A=(at+b)^{1/(1+q)}, \tag{15}$$

where $a\neq 0$ and b are integration constants. Equation (15) mentions that the condition for expansion of the universe is $1+q>0$.

Now, making the calculation accessible and without loss of generality, we take $a=1, b=0$ in our solutions. Also, we get the metric potential as $A=t^{1/(1+q)}$. Secondly, we will use different SF models: ($V(\phi)=V_0$ and $V(\phi)=V_0e^{-\beta\phi(t)}$), to obtain exact solutions of the field equations as discussed below.

In power-law cosmology, the authors have constrained the parameter H_0 and q. With the help of 14 points of $H(z)$ data and 557 data points of SNe Ia data, Kumar [41] has constrained the H_0 and q parameters. Gumjudpai [42] also constrained the H_0 and $1/(1+q)$ parameters with WMAP7 and WMAP7 + BAO + $H(z)$ data sets. In 2015, Rani et al. [43] used 29 points of the latest $H(z)$ data and 580 data points from Union 2.1 SNe Ia data to constrain these parameters. Recently Singh and Singh [37] used the value of q and $1/(1+q)$ to study the SF cosmology in $f(R,T)$ gravity. In the present study we also use the values of q and $1/(1+q)$ for different data sets [37], presented in Table 1.

(i) Constant scalar potential, $V(\phi)=V_0$**.** To solve the field equations, we use a constant potential $V(\phi)=V_0$ similarly to [34, 37]. Then we get the physical parameters as follows.

$$\phi=\frac{\ln(t)}{\sqrt{-\varepsilon(4\pi+\mu)(1+q)}}+c_1.$$

Table 1. Constraints on q and $1/(1+q)$ from different data sets [37]

Data	q	$1/(1+q)$
$H(z)$	$-0.04^{+0.05}_{-0.05}$	–
SNe Ia	$-0.36^{+0.05}_{-0.05}$	–
$H(z)$ + SNe Ia	$-0.21^{+0.04}_{-0.04}$	–
WMAP7	–	$0.99^{+0.04}_{-0.04}$
WMAP7 + BAO + $H(z)$	–	$0.99^{+0.02}_{-0.02}$

This expression, for a suitable choice of c_1 can also be recast as

$$\phi=\ln(c_1t^{\phi_1}), \tag{16}$$

$$\Lambda=\frac{(q-2)}{(1+q)^2t^2}-4V_0(2\pi+\mu), \tag{17}$$

where $c_1>0$ is a constant and

$$\phi_1=1/\sqrt{-\varepsilon(4\pi+\mu)(1+q)}.$$

Figure 1 shows variation of the scale factor A, and Figure 2 variation of the SF ϕ with time. Figure 3 shows variation of Λ with time for $V(\phi)=V_0$ in the $f(R,T)=R+2h(T)$ model.

(ii) Exponential scalar potential, $V(\phi)=V_0e^{-\beta\phi(t)}$**.** Now we choose an exponential scalar potential, $V(\phi)=V_0e^{-\beta\phi(t)}$, where V_0 and β are nonnegative constants [37, 44]. From Eqs. (12), (13), (15) and the form of the potential, we get the unknown physical parameters as follows:

$$\phi=\frac{\ln(t)}{\sqrt{-\varepsilon(4\pi+\mu)(1+q)}}+c_2.$$

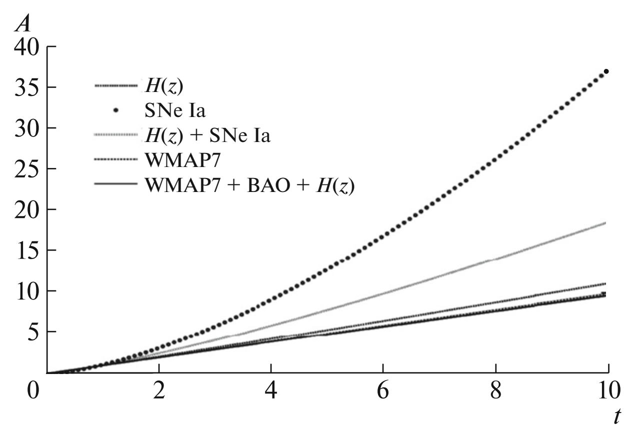

Fig. 1. Variation of the scale factor against time for different observational values of q.

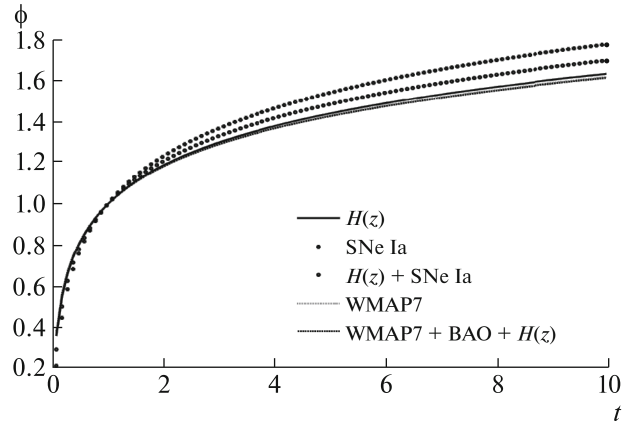

Fig. 2. Variation of ϕ against time for $\varepsilon = -1$, $\mu = 1$, $c_1 = 2.71$ (Eq. 16).

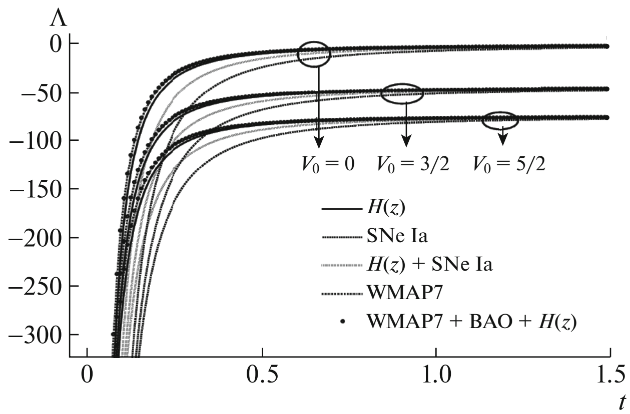

Fig. 3. Variation of Λ against time for $\varepsilon = -1$, $\mu = 1$ and different V_0 (Eq. 17).

This expression, for a suitable choice of c_2 can also be recast as

$$\phi = \ln\left(c_2 t^{\phi_1}\right), \tag{18}$$

$$\Lambda = \frac{(q-2)}{(1+q)^2 t^2} - \frac{4V_0(2\pi + \mu)}{c_2^{\beta} t^{\beta\phi_1}}, \tag{19}$$

where $c_2 > 0$ is a constant. Figure 4 shows variation of Λ with time for $V(\phi) = V_0 e^{-\beta\phi(t)}$ in the $f(R,T) = R + 2h(T)$ model.

4. MASSIVE SCALAR FIELD SOLUTIONS IN $f(R,T)$ GRAVITY

In this section we study flat FRW space-time with MSF matter. The MSF energy-momentum tensor is given by

$$T_{\alpha\beta} = \frac{1}{4\pi}\left[\partial_\alpha\phi\partial_\beta\phi - \frac{1}{2}g_{\alpha\beta}\left(\partial_l\phi\partial^l\phi - M^2\phi^2\right)\right], \tag{20}$$

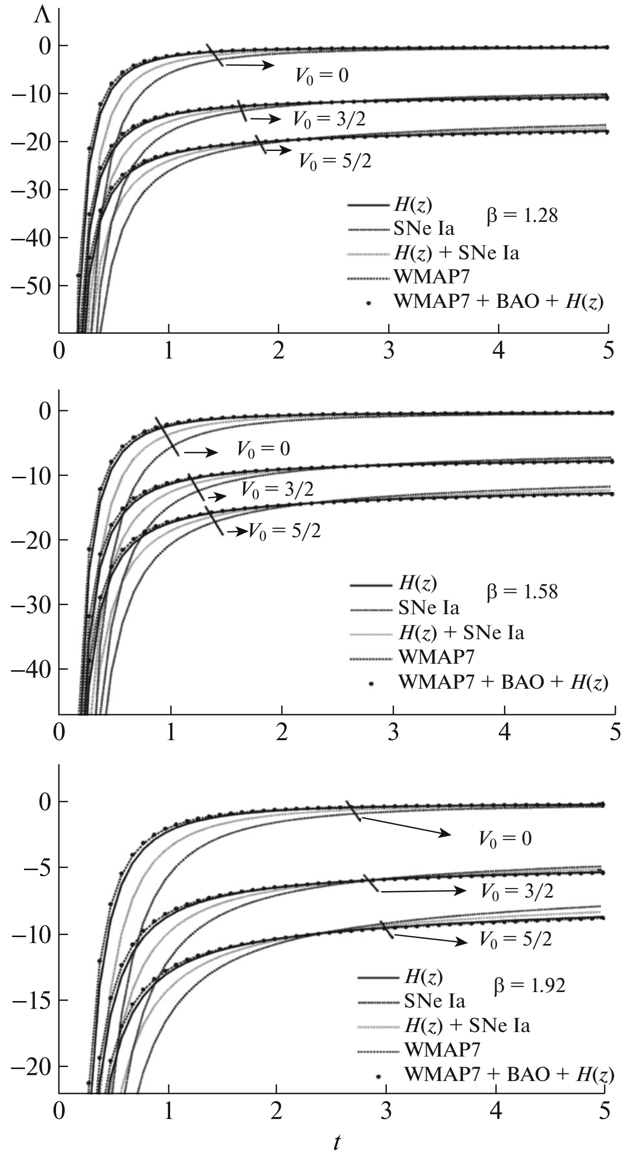

Fig. 4. Variations of Λ with time for $\varepsilon = -1$, $\mu = 1$, $c_2 = 2.71$ and different V_0 and β (Eq. 19).

where M is related to the mass m of a zero-spin particle by $M = 2\pi m/h$, h being Planck's constant [47]. The Lagrangian of the MSF is

$$L_\phi^M = \frac{1}{2}\left(\partial_l\phi\partial^l\phi - M^2\phi^2\right), \tag{21}$$

where the dot denotes an ordinary derivative in t. Using this in Eq. (4), we get

$$\Xi_{\alpha\beta} = -2T_{\alpha\beta} - \frac{1}{2}\left(\partial_l\phi\partial^l\phi - M^2\phi^2\right)g_{\alpha\beta}, \tag{22}$$

and the trace of $T_{\alpha\beta}$ is given by

$$T = \frac{2M^2\phi^2 - \dot{\phi}^2}{4\pi}. \tag{23}$$

4.1. Solutions for a Massive Scalar Field

For the choice $h(T) = \mu T$ ($\mu = \text{const}$), using Eqs. (6), (7), (20), (22), and (23) for the $f(R,T) = R + 2h(T)$ model with Λ, we get the modified Einstein field equations as follows:

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = \dot{\phi}^2(1+\mu) - M^2\phi^2(1+\mu) - \frac{3\mu M^2\phi^2}{4\pi} + \frac{\mu\dot{\phi}^2}{2\pi} - \Lambda, \tag{24}$$

$$\frac{3\dot{A}^2}{A^2} = \dot{\phi}^2(\mu-1) - M^2\phi^2(1+\mu) - \frac{3\mu M^2\phi^2}{4\pi} - \Lambda, \tag{25}$$

where we have two equations and $A, \phi(t)$, and Λ as three unknown parameters. Using Eq. (15), we get the physical parameters as follows:

$$\phi = \frac{2\sqrt{\pi}\ln(t)}{\sqrt{-(1+q)(\mu+4\pi)}} + c_3.$$

This expression, for a suitable choice of c_3 can also be recast as

$$\phi = \ln\left(c_3 t^{\phi_2}\right), \tag{26}$$

$$\Lambda = \frac{\Lambda_N}{16\pi(t)^2(\frac{\mu}{4}+\pi)(1+q)^2}, \tag{27}$$

where

$$\phi_2 = \frac{2\sqrt{\pi}}{\sqrt{-(1+q)(\mu+4\pi)}},$$

$$\Lambda_1 = 16\left[(\mu+1)\pi^{3/2} + \frac{3\mu\sqrt{\pi}}{4}\right]M^2\ln(t) \times t^2\ln c_3(1+q)\sqrt{-(1+q)(\mu+4\pi)},$$

$$\Lambda_2 = 16\Big[\Big(\pi+\frac{3}{4}\Big)\mu+\pi\Big]M^2\pi\ln(t)^2t^2(1+q),$$

$$\Lambda_3 = 16\left[\left(\pi+\frac{3}{4}\right)\mu+\pi\right]M^2t^2(1+q)^2 \times \ln c_3^2\left(\frac{\mu}{4}+\pi\right),$$

$$\Lambda_4 = 16\left[\left(\frac{3}{4}+(1+q)\pi\right)\mu - \pi(q-2)\right]\pi,$$

$$\Lambda_N = \Lambda_1 + \Lambda_2 - \Lambda_3 - \Lambda_4,$$

where $c_3 > 0$ is a constant, and Fig. 5 presents variation of the SF function ϕ with time, while Fig. 6 shows time variations of Λ in the $f(R,T) = R + 2h(T)$ model.

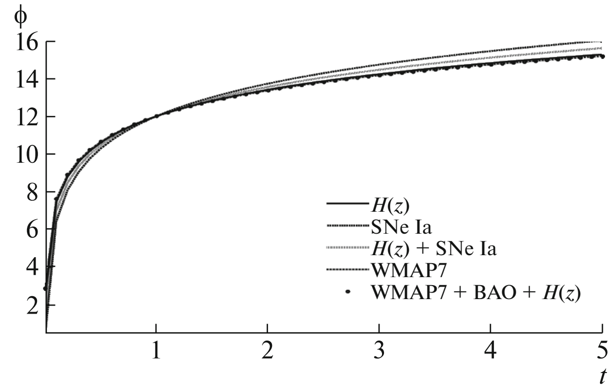

Fig. 5. Time variation of the scalar field for $\mu = -5\pi$, $c_3 = e^{12}$ and different observational value of q (Eq. 26).

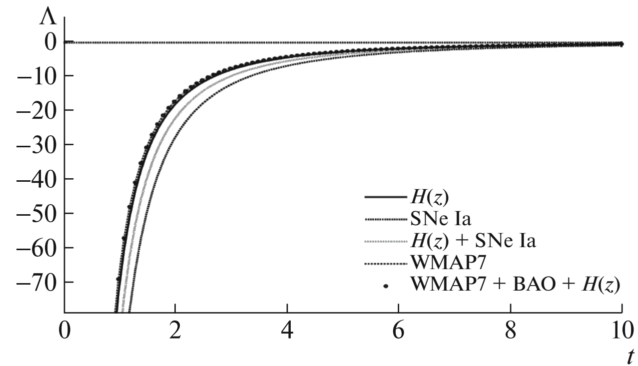

Fig. 6. Time variation of Λ for $\mu = -5\pi$, $c_3 = e^{12}$, $M = 0.01$, and different observational values of q (Eq. 27).

5. DISCUSSION

In this paper, we have studied the modified $f(R,T)$ gravitation theory with massive and massless SF matter distributions for flat FRW universe models. For this purpose we have considered $f(R,T) = R + 2h(T)$. To solve the $f(R,T)$ modified field equations in massless scalar field (normal and phantom) matter distributions, we have used a constant deceleration parameter and constant and exponential SF functions. We also used a constant deceleration parameter for the solution of a MSF. We have compared our solutions with observations of $H(z)$, SNe Ia, $H(z)$+SNe Ia, $WMAP7$ and $WMAP7 + BAO + H(z)$. For a massless scalar field matter distribution, we get the following results: For a constant scalar potential $V(\phi) = V_0$, we find that Λ is negative, ε does not affect Λ, which decreases with an increase of V_0 and cosmic time t in the $R + 2h(T)$ model. We also obtain a real SF ϕ for $\varepsilon = -1$ and $4\pi + \mu < 0$ in our model.

In the situation with an exponential scalar potential $V(\phi) = V_0 e^{-\beta\phi(t)}$: It turns out that ε is effective on the cosmological parameter, when t and V_0 increase, Λ decreases. Also we get a real ϕ value for $\varepsilon = -1$ and $4\pi + \mu < 0$ in our $f(R,T) = R + 2h(T)$ model. However, we get same results for $c_1 = c_2$ in Eqs. (16), (18). For a MSF distribution we get a value of Λ based on the mass value M. As t increases, the value of ϕ increases. We also obtain positive real ϕ values for $4\pi + \mu < 0$ and $c_3 > e^{12}$ in the $f(R,T) = R + 2h(T)$ model. Our cosmological parameter solutions agree with the recent observations.

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