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Is the Hubble Constant Scale-Dependent?

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Abstract—An exact determination of the Hubble constant remains one of key problems in cosmology for almost a century. However, its modern values derived by various methods still disagree from each other by almost 10%, larger values being obtained by measurements at relatively small distances (e.g., by Cepheid stars as standard candles), while smaller values are characteristic of the methods associated with huge spatial scales (e.g., from the analysis of cosmic microwave background fluctuations). A reasonable way to resolve this puzzle is to assume that the Hubble constant is inherently scale-dependent. This idea seems to be particularly attractive in the light of the latest observational results on the early-type galaxies, where dark matter halos are almost absent. Therefore, an average contribution of the irregularly distributed dark matter to the rate of the cosmological expansion should be substantially different at various spatial scales. As follows from rough estimates, the corresponding variation of the Hubble constant can be about 10% and even more, which well explains the spread in its values obtained by different methods.

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As is known, measurement of the Hubble constant H_0 represents a long-standing problem of cosmology, whose history lasts for almost a century. The resulting values obtained in this period varied by an order of magnitude, 50 to 500 km/s/Mpc [1]. Although the situation improved in the recent decades, some discrepancies persist till now. The most notable of them is that the value of H_0 derived from the distance scale based on Cepheids is, on the average, 73.24 ± 1.74 km/s/Mpc and for some calibration can even be as large as 76.18 ± 2.37 [2]. On the other hand, the analysis based on measurements of the cosmic microwave background (CMB) by the Planck satellite under the assumption of ΛCDM cosmology leads to the values $H_0 = 66.88 \pm 0.91$ to 67.31 ± 0.96 km/s/Mpc, depending on the data processing method [3]. In other words, these numbers are about 10% smaller than in the first case.[1]

The above-mentioned discrepancy between the direct (by Cepheids) and indirect (by CMB) measurements of H_0 was clearly recognized in the recent years, and it is commonly attributed now either to systematic errors (such as the degeneracy between different quantities in the analysis of CMB) or to an uncertainty in the fitting parameters (e.g., the number and masses of neutrinos, etc.). An especially popular explanation became a modification of the parameter w appearing in the dark energy equation of state, $p = w\varepsilon$ (see, e.g., [4] and references therein), though the resulting values $w < -1$ look quite unrealistic and suspicious from the physical point of view.[2]

However, from our point of view, the spread in values of H_0 can have a much more straightforward astrophysical explanation: this quantity should be inherently scale-dependent. Really, according to the Friedmann equation, the Hubble constant depends on the energy density in the Universe as [6]

$$\begin{aligned} H_0 &= \sqrt{\frac{8\pi G}{3}} \sqrt{\rho_{\mathrm{DE}} + \langle \rho_{\mathrm{DM}} \rangle + \ldots} \\ &= \sqrt{\frac{8\pi G \rho_{\mathrm{DE}}}{3}} \sqrt{1 + \frac{\langle \rho_{\mathrm{DM}} \rangle}{\rho_{\mathrm{DE}}} + \ldots}, \end{aligned} \qquad (1)$$

where G is the gravitational constant, ρ_{DE} is the density of the dark energy, which is assumed to be distributed perfectly uniformly in space, $\langle \rho_{\mathrm{DM}} \rangle$ is the

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[1] This should not be confused with the fact that the Hubble parameter is a continuously decreasing function of cosmological time. So, its value is larger for remote galaxies since we always look into the past.

[2] For example, the values of w somewhat larger than -1, i.e., $|w| < 1$, could be easily interpreted as a result of small-scale spatial irregularities in the equation of state of the scalar field representing dark energy [5], but such an effect cannot lead to $w < -1$.

average density of dark matter (whose value depends on the scale of averaging), and dots denote the contribution from ordinary forms of matter, which is not greater than 5% (and consequently, its contribution to the Hubble constant value is about 2.5%).

Since both the physical origin and spatial distribution of dark matter are actually unknown by now [7, 8], it can be naturally assumed that its contribution to Eq. (1) substantially depends on the spatial scales under consideration. Particularly, according to the recent observational findings [9, 10], dark matter is almost absent in the vicinity of early-type galaxies, located at large redshifts $z \approx 0.6-2.6$. Therefore, averaging over larger spatial scales should result in smaller values of $\langle \rho_{\mathrm{DM}} \rangle$.

As follows from Eq. (1), the corresponding variance of the Hubble parameter, $\delta H_0 = H_0^{(\mathrm{max})} - H_0^{(\mathrm{min})}$, can be as large as

$$\delta H_0 / H_0^{(\mathrm{max})} \approx \frac{1}{2} \big(\rho_{\mathrm{DM}}^{(\mathrm{max})} / \rho_{\mathrm{DE}} \big), \qquad (2)$$

where $H_0^{(\mathrm{min})}$ formally corresponds to $\langle \rho_{\mathrm{DM}} \rangle = 0$, and $H_0^{(\mathrm{max})}$ to $\langle \rho_{\mathrm{DM}} \rangle = \rho_{\mathrm{DM}}^{(\mathrm{max})}$.

Therefore, taking for estimate $\rho_{\mathrm{DM}}^{(\mathrm{max})} / \rho_{\mathrm{DE}} \approx 3/7$ [11], we find that the relative variance $\delta H_0 / H_0^{(\mathrm{max})}$ can reach approximately 20%. In fact, a realistic value should be somewhat smaller because the above estimate was obtained under the simplifying assumption that $\langle \rho_{\mathrm{DM}} \rangle \to 0$ at very large scales. Anyway, the systematic 10% discrepancy in the values of H_0 derived by various methods is not surprising: a "direct" determination of the Hubble constant from the extragalactic distance scale based on the Cepheid variable stars refers to the relatively local part of the Universe, while the "indirect" analysis based on the CMB fluctuations deals with much larger scales. As was already mentioned in the above-cited work by Genzel et al. [10], at such scales dark matter should play a smaller part than in the local Universe.

It is important to emphasize that, since both dark and luminous matter possess the same dust-like equation of state ($w \approx 0$), their temporal evolution (in the cosmological sense) should be the same, i.e., the ratio of their densities should be constant. So, the deficit of dark matter in the vicinity of high-redshift galaxies cannot be merely explained by the fact that they are observed at earlier times.

In summary, we believe that the well recognized discrepancy between different determinations of H_0 could be more naturally explained by the irregularities of matter distribution, not taken into account explicitly in the standard Friedmann equation, rather than by modifications of the equation of state of dark energy or other exotic assumptions in the framework of "uniform" cosmological equations. (In fact, this issue is closely related to the general problem of "excessive extrapolations in cosmology", which was pictorially outlined in the recent paper [12]).

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