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Kantowski-Sachs and Bianchi Type Models with a General Non-Canonical Scalar Field

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Abstract—The paper deals with spatially homogeneous and anisotropic Kantowski-Sachs and Bianchi universes with a general non-canonical scalar field with the Lagrangian $L = F(X) - \Omega(\phi)$, where $X = \frac{1}{2}\phi_i\phi^i$. We discuss a general non-canonical scalar field in three different cosmologies: (i) cosmology with a constant potential, $\Omega(\phi) = \Omega_0 =$ const, (ii) cosmology with a constant equation-of-state parameter, i.e., $\gamma_\phi =$ const, and (iii) cosmology with a constant speed of sound, i.e., ${c_s}^2 =$ const. For a constant potential, we have shown that the k-essence Lagrangian and the Lagrangian of the present model are equivalent. Dissipation of anisotropy, when the universe is filled with a general non-canonical scalar field, is investigated. The existence of an average bounce in Kantowski-Sachs and locally rotationally symmetric Bianchi-I and Bianchi-III models is discussed in detail.

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1. INTRODUCTION

Recent astrophysical data including supernovae Ia [1], cosmic microwave background radiation (CMBR) [2], and large-scale structure [3] suggest that the universe is dominated by two dark components, viz., dark matter and dark energy [1, 4]. Dark matter, matter without pressure, is mainly used to explain the galactic curves and large-scale structure formation, while dark energy, an exotic energy with negative pressure, is used to explain the present accelerating cosmic expansion. For dark energy, many candidates have been proposed, such as the cosmological constant [5], quintessence [6], k-essence [7], phantom [8], and so on.

Over the last few years, scalar field models with a non-canonical kinetic term have been attracting much attention. These models are generally motivated by phenomenological considerations, and their theoretical structures are also common in effective field theories. A non-canonical kinetic term appears in supergravity theories [9, 10] to relate the present cosmic acceleration to the onset of matter domination. In models of higher dimension, identification of $\ln\psi$ with the volume of internal space or some appropriate dilaton type field also leads to a non-canonical kinetic term [11–15].

The Lagrangian of a non-canonical scalar field can be parametrized as [16]

$$L = f(\phi)F(X) - \Omega(\phi), \tag{1}$$

where $X = \frac{1}{2}\phi_i\phi^i$. For $f(\phi) =$ const, equation (1) represents quintessence when $F(X) = X$ and a phantom when $F(X) = -X$. Equation (1) reduces to k-essence if $\Omega(\phi) = 0$. The first integral of the k-essence field equation for arbitrary $F(X)$ was obtained in [16, 17] where the potential was taken as an inverse square form or a constant [18]. Then, in [19, 20] it was found that all quintessence models can be viewed as k-essence models generated by appropriate linear kinetic functions $F(X)$. Anisotropic space-times with a k-essence field [17] have been further discussed for Kantowski-Sachs and Bianchi space-times [21] for a constant potential and a linearly varying scalar field. General non-canonical scalar field models, in contrast to k-essence models, have been investigated in [22]. Bounce conditions for Kantowski-Sachs and Bianchi universes in modified gravity theories are studied in [23].

In this paper, we focus on a class of models with the Lagrangian $L = F(X) - \Omega(\phi)$ in some anisotropic space-times and discuss the role of the background geometry in the evolution of cosmologies.

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The paper is organized as follows: the basic theoretical framework is given in Section 2. In Section 3, we study some explicit general non-canonical scalar field solutions. We obtain the general solution of Einstein's equation in three cases: (a) where the potential is constant, (b) where the barotropic index is constant, and (c) where velocity of sound c_s is constant. In Section 4, we study the anisotropy dissipation and the existence of an average bounce. Finally, in Section 5, we present our conclusion.

2. BASIC EQUATIONS

The metric of the models is in the form [24]

$$ds^2 = dt^2 - {a_1}^2 dr^2 - {a_2}^2(d\theta^2 + f^2(\theta)d\phi^2), \qquad (2)$$

where $a_1(t)$ and $a_2(t)$ are the scale factors. The metric (2) reduces to a Kantowski-Sachs (KS) model, a Bianchi-III (B-III) model and a locally rotationally symmetric Bianchi-I (LRS B-I) model at $f(\theta) = \sin\theta$, $f(\theta) = \sinh\theta$, and $f(\theta) = \theta$, respectively. For the metric (2) and a choice of u^i, the field equations can be written in terms of the propagation equations as

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 = -\frac{1}{2}(\rho + 3p), \qquad (3)$$

$$\dot{\sigma} + \Theta\sigma - \frac{1}{2\sqrt{3}}({}^3R) = 0, \qquad (4)$$

$${}^3\dot{R} + \frac{2}{3}\Theta({}^3R) - \frac{2}{\sqrt{3}}({}^3R)\sigma = 0, \qquad (5)$$

where $\Theta = {u^i}_{;i}$ is the expansion scalar,

$$\sigma_{ij} = B_{(ij)} - \frac{1}{3}\theta(g_{ij} + u^i u^j)$$

is the shear tensor, the overdot denotes derivative with respect to cosmic time t, and ${}^3R$ is the 3-curvature. For the metric (2), we have

$$\Theta = \frac{\dot{a}_1}{a_1} + \frac{2\dot{a}_2}{a_2},$$
$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right),$$
$${}^3R = \frac{2k}{{a_2}^2}.$$

The Gauss-Codazzi constraint and the continuity equations are

$${}^3R = -\frac{2}{3}\Theta^2 + 2\sigma^2 + 2\rho, \qquad (6)$$

$$\dot{\rho} + \Theta(\rho + p) = 0. \qquad (7)$$

From Eqs. (3) and (6), we have

$$\frac{2}{3}\dot{\Theta} + \frac{1}{3}\Theta^2 + \sigma^2 + \frac{{}^3R}{6} = -p. \qquad (8)$$

We have assumed $8\pi G = c = 1$ in proper units. Here ρ and p are the energy density and the pressure, respectively. The equation of state is $p = \gamma\rho$. The shear vector $\vec{\sigma}$ has the components [17]

$$\sigma_i = H_i - H, \qquad (9)$$

where $\sigma_i = \sigma_{i0}/V$ and $\sigma^2 = \sigma_0^2/V^2$. Here $H = \dot{a}/a = 1/3(H_1 + H_2 + H_3)$ is the Hubble parameter, and $H_i = \dot{a}_i/a_i$, $i = 1, 2, 3$ are the expansion rates in the three spatial directions. These notations are used in the same sense as in [17]. The three constants σ_{i0} transform as components of a vector in the internal three-dimensional Cartesian space associated with the three axes σ_i [17]. Here $V = a_1{a_2}^2 = \sqrt{-g}$, and σ_i satisfies $\sigma_{10} + \sigma_{20} + \sigma_{30} = 0$ and ${\sigma_{10}}^2 + {\sigma_{20}}^2 + {\sigma_{30}}^2 = {\sigma_0}^2$, where σ_0 is a constant. Thus we have

$$3\sigma^2 = \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)^2. \qquad (10)$$

The Lagrangian density is

$$L = F(X) - \Omega(\phi), \qquad (11)$$

where $\Omega(\phi)$ is a potential, and F is a function of the kinetic term X. We assume that the anisotropic space-time contains an isotropic perfect fluid associated with a spatially homogeneous scalar field ϕ. We are considering such a scalar field with a non-canonical kinetic energy term. The above Lagrangian (11) is a special case of the Lagrangian $L = f(\phi)F(X) - \Omega(\phi)$ for $f(\phi) = \text{const}$. Also,

$$X = \frac{1}{2}g^{ij}\phi_i\phi_j, \quad \phi_i = \frac{\partial\phi}{\partial x^i}.$$

Since ϕ is homogeneous, we have $X = \frac{1}{2}\dot{\phi}^2 > 0$.

In the case of a general non-canonical scalar field, we have

$$\rho_\phi = 2XF_X - F(X) + \Omega(\phi),$$
$$p_\phi = F(X) - \Omega(\phi). \qquad (12)$$

The barotropic index γ_ϕ can be written as

$$\gamma_\phi = \frac{F(X) - \Omega(\phi)}{2XF_X - F(X) + \Omega(\phi)}. \qquad (13)$$

If c_s denotes the speed of sound, then

$${c_s}^2 = \left[1 + \frac{2XF_{XX}}{F_X}\right]^{-1}. \qquad (14)$$

From Eqs. (6) and (8), we have

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + {}^3R = 2(2XF_X - F(X) + \Omega(\phi)), \qquad (15)$$

$$\frac{2}{3}\dot{\Theta} + \frac{1}{3}\Theta^2 + \sigma^2 + \frac{{}^3R}{6} = \Omega(\phi) - F(X). \qquad (16)$$

From Eq. (7) we have

$$(F_X + 2XF_{XX})\ddot{\phi} + F_X\Theta\dot{\phi} + \Omega'(\phi) = 0, \quad (17)$$

$$\frac{d}{dt}\left(\frac{\gamma_\phi + 1}{\dot{\phi}}\right) - \left(\frac{\gamma_\phi + 1}{\dot{\phi}}\right)\gamma_\phi\Theta + \frac{\Omega'(\phi)}{\frac{1}{3}\Theta^2 - \sigma^2 + \frac{1}{2}{}^3R} = 0, \quad (18)$$

where $\Omega'(\phi) = d\Omega(\phi)/d\phi$, $\dot{\phi} \neq 0$, and $F_X = dF/dX$. In terms of geometrical quantities, we have

$$\gamma_\phi = -\frac{4\dot{\Theta} + 2\Theta^2 + 6\sigma^2 + {}^3R}{2\Theta^2 - 6\sigma^2 + 3({}^3R)}. \quad (19)$$

Equations (17) and (19) show that ϕ and γ_ϕ are sensitive to the evolution of the average geometry. Since ${}^3R$ is different for LRS B-I, BIII, and KS models, we obtain different values of γ_ϕ for the corresponding models.

3. SOLVABLE GENERAL NON-CANONICAL SCALAR FIELD COSMOLOGIES

3.1. Cosmology with a Constant Potential

Consider a constant potential, i.e., $\Omega = \Omega_0 =$ const, and investigate the resulting cosmologies in our generalized metric background. For a constant potential, Eq. (18) can be re-written as

$$\frac{d}{dt}\left(\frac{\gamma_\phi + 1}{\dot{\phi}}\right) = \left(\frac{\gamma_\phi + 1}{\dot{\phi}}\right)\gamma_\phi\Theta. \quad (20)$$

Using the geometrical definition of γ_ϕ from Eq. (19) in Eq. (20); after integration, we have

$$\frac{\gamma_\phi + 1}{\dot{\phi}} = \frac{c_1}{V(2\Theta^2 + 3\cdot{}^3R - 6\sigma^2)}, \quad (21)$$

where $c_1 \neq 0, 1$ and is a constant. Equation (21) can be rewritten as $\dot{\phi} = 2(\gamma_\phi + 1)V\rho/(3c_1)$ with $2X = \dot{\phi}^2$, so we have

$$VF_X\dot{\phi} = \frac{3c_1}{2}. \quad (22)$$

From Eqs. (13) and (22), we have

$$\frac{3c_1}{2VF_X} = 2(\gamma_\phi + 1)V\frac{\rho}{3c_1}.$$

Using $\sigma^2 = \sigma_0^2/V^2$ in the previous expression, the barotropic index associated with this general non-canonical scalar field becomes

$$\gamma_\phi + 1 = \left(1 + \frac{4{\sigma_0}^2F_X(\Omega_0 - F)}{9{c_1}^2\sigma^2}\right)^{-1}. \quad (23)$$

The models generated by the set of kinetic functions, with a constant potential satisfying the condition $F_X(\Omega_0 - F)/\sigma^2 \ll 1$ at early times, describe universes which, are, on the average, like dust dominated ones.

With a constant potential, the k-essence Lagrangian can be written as $L_k = -\Omega_0 F_k(X)$, while the Lagrangian in the present model is $L_g = F_g(X) - \Omega_0$. If $F_g(X) = \Omega_0(1 - F_k(X))$, then the two Lagrangians are equivalent. So the present model can reproduce by k-essence models with a constant potential.

Let us consider a Lagrangian of the form $L_g = 1 - \Omega(\phi) - \sqrt{1 - 2X}$, which is considered as the nonlinear Born-Infeld scalar field theory in [25]. If the potential $\Omega(\phi)$ is constant, Ω_0, we have

$$\dot{\phi}^2 = \frac{c_2}{c_2 + V^2}, \quad (24)$$

where $c_2 = 9{c_1}^2/4 =$ const. Also,

$$\rho_\phi = \Omega_0 - 1 + \sqrt{1 + \frac{c_2}{V^2}}, \quad (25)$$

$${c_s}^2 = 1 - \dot{\phi}^2 = \frac{V^2}{c_2 + V^2}. \quad (26)$$

At small V, $\rho_\phi \propto 1/V$, with ${c_s}^2 \simeq 0$, and at large V, $\rho_\phi \simeq \Omega_0$, with ${c_s}^2 \simeq 1$. So the KS model, B-III and LRS-B-I cosmology with a general non-canonical scalar field having a constant potential play the role of models that unify dark matter and dark energy in their respective background geometries.

3.2. Cosmology with $\gamma_\phi = const$

In this section, we assume that the equation-of-state parameter $\gamma_\phi = \gamma_0 =$ const. From Eq. (7), we have

$$\rho = \frac{V_0}{V^{1+\gamma_0}}, \quad (27)$$

where V_0 is an integration constant. From Eq. (13), we have

$$\left(\frac{2\gamma_0}{1 + \gamma_0}\right)XF_X - F(X) + \Omega(\phi) = 0. \quad (28)$$

It is obvious from the above equation that the form of $F(X)$ in our model will depend on the potential. For a constant potential Ω_0, we have

$$F(X) = c_3X^{\left(\frac{1+\gamma_0}{2\gamma_0}\right)} + \Omega_0, \quad (29)$$

where c_3 is an integration constant. To check the stability of the solution with a constant γ_0, we let γ_ϕ vary with time. Differentiating $\gamma_\phi + 1 = (\rho_\phi + p_\phi)/\rho_\phi$ with respect to t, we get

$$\dot{\gamma}_\phi = \gamma_\phi\left(\Theta(\gamma_\phi + 1) + \frac{\dot{p}}{p}\right). \quad (30)$$

In equation (30), we have two critical points: $\gamma_0 = 0$ or γ_0 satisfying

$$\Theta(\gamma_0 + 1) + \frac{\dot{p}}{p} = 0. \tag{31}$$

If the condition in the above equation holds, the potential $\Omega(\phi)$ and the function $F(X)$ satisfy

$$p = F(X) - \Omega(\phi) = \frac{c_4}{V^{1+\gamma_0}}, \tag{32}$$

where c_4 is an integration constant. From Eqs. (30) and (31), we have

$$\dot{\gamma}_\phi = \gamma_\phi(\gamma_\phi - \gamma_0)\Theta. \tag{33}$$

From Eq. (33), we get

$$\gamma_\phi = \frac{c_5\gamma_0}{c_5 - V^{\gamma_0}}, \tag{34}$$

where c_5 is an integration constant. For the expanding universe and $\gamma_0 < 0$, the barotropic index γ_ϕ has the asymptotic limit γ_0. For $\gamma_0 > 0$, the barotropic index γ_ϕ approaches the asymptotic limit 0. At $\gamma_0 = 0$, we have

$$\gamma_\phi = \frac{-1}{\log c_6 V}, \tag{35}$$

where c_6 is an integration constant. The above equation shows that $\gamma_0 = 0$ is also a stable point in an expanding universe. The solutions with a constant barotropic index are attractors in the case $\gamma_0 \leq 0$, and the solution $\gamma_0 = 0$ separates the stable region from the unstable one in the phase plane.

3.3. Cosmology with ${c_s}^2 = const$

The speed of sound c_s is the propagation speed of a perturbation of the background scalar field, which can affect the CMB power spectrum. In this section, we are taking the speed of sound as a constant quantity. From Eq. (14), we have

$$2{c_s}^2 X F_{XX} = (1 - {c_s}^2)F_X. \tag{36}$$

From Eq. (36), we get

$$F(X) = \frac{2{c_s}^2}{1 + {c_s}^2} c_7 X^{\frac{1+{c_s}^2}{2{c_s}^2}} + c_8, \tag{37}$$

where c_7 and c_8 are integration constants. In this case, ${c_s}^2$ is independent of the potential $\Omega(\phi)$ and is the same as that in a k-essence model. Also, if $F_{XX} = 0$, we have ${c_s}^2 = 1$.

4. GENERAL ISSUES

4.1. Anisotropy Dissipation

In this section, we study dissipation of the anisotropy when the universe contains a general noncanonical scalar field. Defining $D = \sigma^2/\rho_\phi$, the evolution equation for the ratio D can be written as

$$\dot{D} + \left[\Theta(1 - \gamma_\phi) - \frac{{}^3RV}{\sigma_0\sqrt{3}}\right] D = 0. \tag{38}$$

For the LRS-BI model, ${}^3R = 0$, thus Eq. (38) reduces to

$$\dot{D} + \Theta(1 - \gamma_\phi)D = 0. \tag{39}$$

If $\gamma_\phi < 1$, D is a positive-definite quantity, and for an average expanding cosmology ($\Theta > 0$), a solution of Eq. (39), $D = 0$ is asymptotically stable. This model becomes isotropic at late times, and the geometry tends to that of a FRW model, although it depends on the choice of the potential. From Eqs. (6) and (8), we have

$$\dot{\Theta} + \Theta^2 = \frac{3}{2}(\rho - p). \tag{40}$$

For $\gamma_\phi < 1$, we have $\dot{\Theta} + \Theta^2 > 0$, which finally, gives $\rho > p$. So, with a fluid obeying the DEC (Dominant Energy Condition), the initial anisotropy dissipates. By contrast, in the case $\gamma_\phi > 1$, shear dominates over the fluid, the DEC is getting violated, and the quantity D increases asymptotically.

For the KS model, ${}^3R = 2/a_2^2$, and Eq. (38) takes the form

$$\dot{D} + \left[\Theta(1 - \gamma_\phi) - \frac{2a_1}{\sigma_0\sqrt{3}}\right] D = 0. \tag{41}$$

Whenever D is a positive-definite quantity and $\gamma_\phi < 1$, for $\Theta > 0$ with $\Theta(1 - \gamma_\phi) > 2a_1/(\sigma_0\sqrt{3})$, the solution $D = 0$ is asymptotically stable. From Eqs. (6) and (8), for $\gamma_\phi < 1$, we have $\dot{\Theta} + \Theta^2 + 2/a_2^2 > 0$, which finally gives $\rho > p$. So, a fluid obeying the DEC dissipates the initial anisotropy.

For the B-III model, ${}^3R = -2/a_2^2$, thus (38) takes the form

$$\dot{D} + \left[\Theta(1 - \gamma_\phi) + \frac{2a_1}{\sigma_0\sqrt{3}}\right] D = 0. \tag{42}$$

If D is a positive-definite quantity and $\gamma_\phi < 1$, then with $\Theta > 0$ the solution $D = 0$ is asymptotically stable. The model becomes isotropic at late times, and the geometry tends to that of the FRW model, although it depends on the choice of the potential. For $\gamma_\phi < 1$, we have $\dot{\Theta} + \Theta^2 > 2/a_2^2$, thus finally, $\rho > p$.

4.2. Average Bounce

By a bouncing universe, we mean a universe that undergoes a collapse, attains a minimum and then subsequently expands. The FRW universe undergoing a "bounce" attains a minimum, and at this minimum, the Strong Energy Condition (SEC) of classical gravity must be violated. Though, a violation of the SEC is a necessary but not sufficient condition [26]. We define the occurrence of a bounce at a time $t = t_b$ by the conditions

$$\text{(i) } \dot{V} = 0, \qquad \text{(ii) } \ddot{V} > 0,$$

i.e., (i) $\Theta(t_b) = 0$ and (ii) $\dot{\Theta}(t) > 0$ for $t \in (t_b - \epsilon, t_b) \cup (t_b, t_b + \epsilon)$ for small $\epsilon > 0$. The expansion scalar $\Theta(t) > 0$ for all t in this interval. The conditions may not be sufficient for a nonsingular bounce. In the present model, we have more than one scale factor, the above conditions of a bounce should be understood as those characterizing a bounce in the average scale factor $V = (a_1 {a_2}^2)^{1/3}$. However, one can also consider a more generic situation where a bounce can occur in any of the directional scale factors a_i. We can make this precise by defining the directional Hubble parameters $H_i = \dot{a}_i/a_i$. So a bounce in a_i will occur at $t = t_b$ if (i) $H_i(t_b) = 0$ and (ii) $\dot{H}_i(t_b) > 0$. It is clear that although it may be possible to have a bounce in any one of the scale factors but not the other, this does not lead to a new expanding universe region. We therefore require that a bounce occurs in all a_i's at $t = t_b$, even though they may in general occur at different times [23]. These conditions may not be sufficient for a non-singular bounce.

We here take that a scale factor satisfies the necessary condition of a bounce in volumetric expansion, therefore, we call it an average bounce. The conditions derived below just address the necessary conditions of a bounce in anisotropic models given by the metric (2). The idea of a bounce in a spatially flat or open universe may be understood if we recall that the quantity $\dot{\Theta}$ gives a measure of the deviation of matter world lines. In this sense, the bounce conditions simply mean that there exists a phase in which the separation between matter world-lines decreases to a minimum and then increases again. Since this phenomenon is independent of a spatial geometry of the model, the bounce itself is independent of it [27, 28].

In a comoving coordinate system with $T^i_j = \mathrm{diag}(\rho, -p, -p, -p)$, the energy conditions can be characterized as follows:

Null Energy Condition (NEC)

$$\Leftrightarrow\ \rho + p \geq 0.$$

Weak Energy Condition (WEC)

$$\Leftrightarrow\ \rho \geq 0, \quad \rho + p \geq 0.$$

Strong Energy Condition (SEC)

$$\Leftrightarrow\ \rho + p \geq 0, \quad \rho + 3p \geq 0.$$

From Eqs. (3) and (6), the equations for a general non-canonical scalar field can be written as

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 = -(F + XF_X - \Omega(\phi)), \qquad (43)$$

$$\frac{2}{3}\Theta^2 - 2\sigma^2 + {}^3R = 2(2XF_X - F + \Omega(\phi)). \qquad (44)$$

At $t = t_b$, Eqs. (43) and (44) can be rewritten as

$$\dot{\Theta} + 2\sigma^2 = \Omega(\phi) - (F + XF_X), \qquad (45)$$

$$\frac{1}{2}({}^3R) - \sigma^2 = 2XF_X - F + \Omega(\phi). \qquad (46)$$

In Eq. (45), the l.h.s. is nonnegative at the bounce point, thus

$$\Omega(\phi) \geq F + XF_X. \qquad (47)$$

From Eq. (47), at $t = t_b$, we have $\rho + 3p < 0$. Thus the SEC is violated at a bounce. For the LRS B-I model, ${}^3R = 0$, thus Eq. (46) takes the form

$$-\sigma^2 = 2XF_X - F + \Omega(\phi). \qquad (48)$$

As $\sigma^2 \geq 0$, at $t = t_b$ we have $\rho_\phi \leq 0$. At the bounce, $D = -1$. Therefore in an average bounce, the relation between energy density and shear is constant. Hence we can reach a bounce avoiding a final singularity, but we have a residual anisotropy in this scenario.

For the KS model, ${}^3R = 2/a_2^2$, thus Eq. (46) takes the form

$$\frac{1}{{a_2}^2} - \sigma^2 = 2XF_X - F + \Omega(\phi). \qquad (49)$$

As $\sigma^2 \geq 0$, if $1/a_2^2 \geq \sigma^2$ at $t = t_b$. We have $\rho_\phi \geq 0$. i.e. ,

$$2XF_X + \Omega(\phi) \geq F. \qquad (50)$$

Therefore at bounce, ${}^3R$ must be dominating over σ^2. Also,

$$D = \frac{1}{{a_2}^2(2XF_X - F + \Omega(\phi))} - 1. \qquad (51)$$

In Eq. (51), $\rho_\phi \geq 0$, and we have $D > 0$ whenever ${}^3R > 2\rho_\phi$. However, D depends on ${}^3R$ at the bounce point.

For the B-III model, ${}^3R = -2/a_2^2$, thus Eq. (46) takes the form

$$\frac{1}{{a_2}^2} + \sigma^2 = -(2XF_X - F + \Omega(\phi)). \qquad (52)$$

At bounce, the l.h.s. is ≥ 0 in the above equation, thus

$$F \geq 2XF_X + \Omega(\phi). \qquad (53)$$

Thus at a bounce point in B-III space-time with a general non-canonical scalar field, there is violation of the WEC and SEC. Now,

$$D = \frac{-1}{{a_2}^2(2XF_X - F + \Omega(\phi)} - 1. \tag{54}$$

From Eq. (54), we get $D > 0$ whenever ${}^3R > 2\rho_\phi$ (as at a bounce point $\rho_\phi \leq 0$ and ${}^3R < 0$).

5. CONCLUSIONS

We have studied spatially homogeneous and anisotropic universes with a general non-canonical scalar field. We have discussed such a field in three different cosmologies: (i) cosmology with a constant potential, (ii) cosmology with a constant equation-of-state parameter and (iii) cosmology with a constant speed of sound. We have shown that the general non-canonical scalar field with a constant potential in the background of an anisotropic universe plays the role of a model that unifies dark matter and dark energy. The model with a constant barotropic index γ_ϕ was investigated. We analyzed the stability of a constant γ_ϕ model and found it to be stable at $\gamma_0 = 0$ and to be an attractor for $\gamma_0 \leq 0$. Dissipation of the anisotropy when the universe contains a general non-canonical scalar field was investigated, as well as the existence of an average bounce in KS, B-I, and B-III models.

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