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Electromagnetic Origin of Particle Masses and Gravitation

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Abstract—We propose an approximate theory describing electromagnetism and gravity as a single direct particle interaction. A new element in this theory is the assumption on real simultaneous (combined) existence of retarded and advanced interactions. Another essential principle of this theory is Mach's principle, according to which the interaction of particles in a certain local region is inextricably connected with the dynamics of all particles in the Universe. A consequence of Mach's principle in this theory is that in terrestrial experiments the advanced electromagnetic interaction is many orders of magnitude smaller than the retarded one (but is not precisely zero). We consider a possible mechanism of emergence of particle masses due to electromagnetic interaction. The proposed theory is relativistic but non-quantum. In this regard, we consider only three types of particles (electrons, protons, and neutrons), and the difference of particle masses is explained only at a qualitative level. The resulting equation of motion for particles is studied in the nonrelativistic approximation. Along with the Lorentz force and the radiative friction force, it contains terms describing the gravitational interaction. Their form is similar to those known in gravielectromagnetism, which is an approximation to general relativity.

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1. INTRODUCTION AND OVERVIEW OF PREVIOUS STUDIES

This work continues the trend of research begun by the author in [1] and earlier [2, 3]. In these papers, the interaction of particles was considered to be direct, i.e., interaction without an intermediary, without an interaction carrier which is called a field in modern physics. Since this concept is little known nowadays (although it was developed by well-known authors in the past), we start with a brief overview of the main works devoted to a direct electromagnetic interaction of particles. The theory of direct electromagnetic interaction, in its most consistent formulation known today, has been discussed and developed since the early 20s of the 20th century by a number of authors, including K. Schwarzschild, H. Tetrode, A.D. Fokker, Ya.I. Frenkel, and later J. Wheeler and R. Feynman joined to this line of research. The mathematical formulation of this theory on the basis of a variational principle was first published apparently by Tetrode [4]. The same paper also marked the difficulties of this theory, which were consistently resolved in the papers by Wheeler and Feynman [5, 6]. We will present the basics of the formalism of this theory.

The theory is built in the background of Minkowski space-time on the basis of a variational principle. The action of a system of electromagnetically interacting particles has the form:

$$S = -c\sum_a m_a \int ds_a - \sum_a \sum_{b<a} \frac{e_a e_b}{c} \int\int u_a^\mu u_{b\mu} \delta(s^2(a,b)) ds_a ds_b, \qquad (1)$$

where $u_a^\mu = dx_a^\mu/ds_a$ is the 4-velocity of a particle number a, e_a and m_a are its charge and mass, c is the speed of light, and the delta function of the squared interval between events on the world lines of particles a and b may be presented in the form

$$\delta(s^2(a,b)) = \frac{1}{2r_{ab}}[\delta(ct_{ab} - r_{ab}) + \delta(ct_{ab} + r_{ab})]. \qquad (2)$$

In expression (1), the first sum is a sum of free actions of the particles, while the second (double) sum describes their electromagnetic interaction. Unlike Maxwell's electrodynamics, in this theory the electromagnetic field or its potential are not regarded as a part of objective reality and are absent in the original expression for the action (1). However, for convenience of calculations and for comparison of this theory with Maxwell's electrodynamics it is possible to introduce an auxiliary mathematical construct corresponding to the electromagnetic field potential. Consider a certain point on the world line of particle

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i. The expression for the potential generated at this point by another particle k can be written as [4, 6]:

$$A_\mu(i,k) = e_k \int u_{k\mu}\delta(s^2(i,k))ds_k. \quad (3)$$

Then, for a single selected particle i, the action can be written in the form

$$S_i = -m_i c \int ds_i - \frac{1}{c}\sum_{k\neq i}\int e_i u_i^\mu A_\mu(i,k)ds_i. \quad (4)$$

The equation of motion for particle i was obtained, for example, in [6], by variation of the action (4) with respect to the world line of particle i. The potential $A_\mu(i,k)$ was considered as a *given* function of the coordinates of particle i (see [6], section "The equations of motion"). The equation obtained in this way coincides with the well-known equation of field electrodynamics,

$$m_i c\eta_{\mu\nu}\frac{d^2x_i^\nu}{ds_i^2} = \frac{e_i}{c}u_i^\nu F_{\mu\nu}(i), \quad (5)$$

where the tensor

$$F_{\mu\nu}(i) = \sum_{k\neq i}\left(\frac{\partial A_\nu(i,k)}{\partial x_i^\mu} - \frac{\partial A_\mu(i,k)}{\partial x_i^\nu}\right) \quad (6)$$

is an auxiliary mathematical construction which is an analog of the electromagnetic field tensor in the field formulation of electrodynamics. However, the theory constructed in such a way has two significant differences from Maxwell's generally known electrodynamics, and about 20 years they were a "stumbling block" for this theory.

1. From Eqs. (3) and (2) it can be seen that the potential defined by Eq. (3) is not purely retarded but is half-retarded and half-advanced:

$$A_\mu(i,k) = \frac{1}{2}A_\mu^{\rm ret}(i,k) + \frac{1}{2}A_\mu^{\rm adv}(i,k), \quad (7)$$

where $A_\mu^{\rm ret}(i,k)$ is the Lienard-Wiechert well-known retarded potential, and $A_\mu^{\rm adv}(i,k)$ is a similar advanced potential which differs from the retarded one in that it is determined by the existence and motion of particle k *in the future* relative to the instant when it affected particle i rather than in the past.

2. Equation (5) does not contain the radiative friction force.

However, if one correctly considers both differences together, then everything falls into its proper place. For this purpose, when describing the interaction of two charged bodies, one should consider not only these two bodies but also the rest of the universe (that is, take into account Mach's principle). For example, the first body acts on all other bodies of the Universe by its retarded potential (the same arguments are also correct for the advanced potential). All those other bodies, in turn, are acting with their advanced potentials on the second body. The same is true for the second halves of the potentials and for the second body. If one adds up all the potentials, the resulting potential will be purely retarded, equal to the Lienard-Wiechert potential, and the equation of motion will contain the correct expression for the radiative friction force (also from a sum of the potentials, but this force is only determined by the first particle's motion):

$$m_i c\eta_{\mu\nu}\frac{d^2x_i^\nu}{ds_i^2} = \frac{e_i}{c}u_i^\nu F_{\mu\nu}^{\rm ret}(i)$$
$$+\frac{2e_i^2}{3c}\left(\frac{d^2u_{i\mu}}{ds_i^2} + \frac{du_i^\lambda}{ds_i}\frac{du_{i\lambda}}{ds_i}u_{i\mu}\right). \quad (8)$$

It is this result that was obtained by Wheeler and Feynman [5]. Thus it was shown that the theory of direct electromagnetic interaction of particles is completely equivalent to the field formulation of electrodynamics in the description of the experimentally observed results.

However, later the paper [5] was subject to some revision, first by Hogarth in [7] and then by Hoyle and Narlikar in [8]. In particular, revised were the main points of [5]: when considering the role of the whole Universe in the local electrodynamic phenomena, the advanced interaction is completely eliminated, the retarded one is doubled and is obtained in the same form as in field electrodynamics, while in the equation of motion of a charged particle there appears a correct expression for the force of radiative friction. It was noted that in the derivation of these results, certain assumptions were used which may not be valid for our Universe. In [5], four ways of derivation of these results were proposed, in the ascending order of generality. The first three ways considered too simplified models of the Universe, so these methods do not pretend to prove these statements rigorously. The most general method 4 was based on the concept of an "absolute absorber." To define this notion, we introduce the original notations of [5]. Let $F_{\rm ret}^{(k)}$ and $F_{\rm adv}^{(k)}$ be the electromagnetic "field" tensor created by particle number k at some arbitrary point in Minkowski space. The word "field" is used in quotation marks because it denotes now an auxiliary mathematical construction describing the direct particle interaction; and the tensor indices are omitted here for brevity. The concept of an "absolute absorber" implies that in the three-dimensional coordinate space there is a region of *finite-size*, including *all* charged particles, while outside this region the sum of "fields" created by all particles is exactly zero (see Eq. (33) in [5]):

$$\sum_k\left(\frac{1}{2}F_{\rm ret}^{(k)} + \frac{1}{2}F_{\rm adv}^{(k)}\right) = 0. \quad (9)$$

Equation (9) is valid everywhere outside the region under consideration (outside the absorber). Then at a sufficiently large distance from the absorber, the first part in the sum (9) describes electromagnetic "waves" propagating from the absorber, while the second part describes "waves" converging from infinity towards the inner region of the absorber. Since a destructive interference between such waves simultaneously at all points of space is impossible, it follows that outside the absorber the two sums are equal to zero each separately:

$$\sum_k F^{(k)}_{\text{ret}} = 0, \tag{10}$$

$$\sum_k F^{(k)}_{\text{adv}} = 0. \tag{11}$$

It then follows that everywhere outside the absorber the following equality holds:

$$\sum_k \left(\frac{1}{2} F^{(k)}_{\text{ret}} - \frac{1}{2} F^{(k)}_{\text{adv}} \right) = 0. \tag{12}$$

But the left-hand side of this equation is a solution to Maxwell's equations without sources. Therefore, if it is equal to zero outside the absorber, it is equal to zero in the whole space including the points of the world lines of all particles. But then the tensor in the right-hand side of Eq. (5) can be transformed using the following equality:

$$\sum_{k\neq i} \left(\frac{1}{2} F^{(k)}_{\text{ret}} + \frac{1}{2} F^{(k)}_{\text{adv}} \right) = \sum_{k\neq i} F^{(k)}_{\text{ret}} + \left(\frac{1}{2} F^{(i)}_{\text{ret}} - \frac{1}{2} F^{(i)}_{\text{adv}} \right) - \sum_k \left(\frac{1}{2} F^{(k)}_{\text{ret}} - \frac{1}{2} F^{(k)}_{\text{adv}} \right). \tag{13}$$

In the right-hand side of this equation, the first term gives a purely retarded interaction, the second term gives the radiative friction force in accordance with Dirac's work [9], and the third term is exactly equal to zero. Thus Eq. (5) passes on to Eq. (8).

It is clear that the above model of an "absolute absorber" does not correspond to the cosmological models discussed when the later papers [7] and [8] were written (the early 1960s). The authors of [7] and [8] developed more consistent theories of an absorber (i.e., theories taking into account the ambient world in the electromagnetic interaction of two particles). Hoyle and Narlikar [8] first built a generalization of the theory of direct electromagnetic interaction of particles for curved Riemannian space-time. After that, a theory of the absorber was built, generalizing the ideas of [5] to the case of Riemannian space-time. Hoyle and Narlikar noted that the quantity in the left-hand side of Eq. (9) cannot be put exactly equal to zero outside a certain finite region of space. Instead, it is necessary to study the asymptotic behavior of the "fields" $F^{(k)}_{\text{ret}}$ and $F^{(k)}_{\text{adv}}$ at an infinite distance from the event creating them. During this study, the authors of [8] formulated criteria (necessary and sufficient conditions) under which the interaction in local electrodynamic phenomena is either purely retarded or purely advanced (Eqs. (63) in [8] and the remark following them). If none of these conditions holds, the interaction is mixed (includes both retarded and and advanced parts, possibly not equal in magnitude). Next, the authors considered two cosmological models: the Einstein-de Sitter model and the stationary universe model whose foundations had been provided by G. Bondi and T. Gold [10] and F. Hoyle [11]. The authors came to the conclusion that in the first model the interaction is purely advanced, and in the second model it is purely retarded. Apparently, at that time they decided that the problem of advanced and retarded interactions was solved, because they themselves were inclined to use the second model and continued its development. All calculations were carried out for the flat version of the Friedmann metric, although it was pointed out that a simple generalization of these results could extend them to an arbitrary Friedmann metric.

It is easy, however, to present examples of cosmological models for which none of the criteria (Eqs. (63) in [8]) is satisfied. The author of the present paper asked a question of whether or not some of these criteria holds in the modern ΛCDM model. Due to a limited size of this paper, we will present the results of the analysis without intermediate calculations because it is not the main subject of this work. In the framework of Friedmann's flat metric, the existence criterion of a purely retarded interaction can be reduced to a study of the time dependence of the scale factor $a(t)$ as $t \to +\infty$. The necessary and sufficient condition of a purely retarded interaction is an increasing of a as $t^{1/3}$ or slower as $t \to +\infty$. Otherwise (if a grows faster than $t^{1/3}$ at large t) the condition does not hold. According to the modern ideas, in the modern epoch the Universe expands with acceleration (i.e., a grows faster than t^1). Thus the existence of a purely retarded interaction requires that in the future the expansion of the Universe should slow down. By modern concepts on matter filling the Universe, such a scenario is extremely unlikely. Accordingly, *in this model, the interaction cannot be purely retarded.* We conclude that the interaction is mixed. This result was one of the points motivating the present work and one of the main ideas used in it. We will assume that the *in our Universe, in local* *electrodynamic phenomena, there are manifestations of both retarded and advanced interaction, but the advanced interaction is by many orders of magnitude smaller than the retarded one, and therefore we do not notice it in the known phenomena.*

Hugo Martin Tetrode was one of the first who admitted the real existence of an advanced electromagnetic interaction along with the retarded one and studied a theory with such a mixed interaction in his paper [4]. In such a theory, all conclusions of special relativity are valid, with one important reservation: in such a theory, the causality concept is changed. Events A and B involving charged particles can be causally connected if they lie on the light cones of each other (i.e., the interval between them is equal to zero). But on a fundamental level, *there is no separation of events into causes and effects, the causal relationship is mutual and symmetric*. In the theory, this fact is expressed in the symmetry between the advanced and retarded interactions in the original equations, and in the invariance of these equations under time reversal. It was shown that such a theory is intrinsically free of contradictions. Such a theory may contain seeming paradoxes such as the paradox of predicting the future. It may seem that the presence of an advanced interaction can in principle allow one to predict some event in the future, which further on he is able to willfully change. A solution to this paradox lies in the fact that the theory in Tetrode's version [4], with appropriately specified initial conditions, is deterministic. Therefore, even if we can predict something, we will not be able to change it. The consistency of a theory of this kind was also discussed by Wheeler and Feynman in [6] (see the section entitled "The paradox of advanced interactions').

Let us now consider the well-known theories of direct gravitational interaction of particles. The first and one of the most famous ones among them was the Hoyle-Narlikar theory formulated in [12, 13]. After almost 40 years, this theory was reviewed by Narlikar in [14]. A concise but sufficiently informative review of this theory can be found in [1]. This theory is based on Mach's principle which was interpreted as the assertion that the mass of each particle is determined by the whole set of all other particles in the Universe. The action functional of the Hoyle-Narlikar theory has a simple form coinciding with the form of a free action in the classical theory of gravity:

$$S = -c\sum_a \int m_a ds_a = c\lambda \sum_a \sum_{b\neq a} \int\int G(A,B) ds_a ds_b. \qquad (14)$$

However, this action, as is seen from the second equality, contains a fundamental difference from the free action: the mass m_a of each particle is not fixed but is connected with the distribution and properties of motion of all other particles in the Universe via Green's function G, satisfying a certain equation in Riemannian curved space. But one of the points of criticism of this theory was its eclecticism: the mass was presented in it by a direct scalar interaction of particles, whereas gravity was still described using a geometric approach. As in general relativity, gravity was described by space-time curvature rather than a direct interaction of particles, although related to it. Other versions of the theory of direct gravitational interaction of particles were developed by Soviet authors: Granovsky and Pantyushin [15, 16], Pyragas and Zhdanov [17, 18], Turygin and Vladimirov [19, 20]. The theories presented in [15–18] were approximate: the gravitational interaction was described there by linear differential equations whereas the Einstein equations for the metric in general relativity are nonlinear. A more complete and consistent theory of direct interactions of particles was initiated in [19] and then developed and described in detail in the book by Vladimirov and Turygin [20]. This theory was constructed by iteration (a method of successive approximations, the first of which was a linear theory, as in [15–18]). After introduction of an effective metric ("geometrization" of the theory) it was shown that this theory is completely equivalent to Einstein's classical theory of gravity. However, gravity was completely described as a direct interaction of particles in the background of Minkowski space. The possibility of such a description is of fundamental importance for the present work since here we will use a similar approach.

2. NEW APPROACH: A STUDY OF THE ADVANCED INTERACTION

Hoyle and Narlikar [12] built the theory of gravity discussed above, where the Einstein-Hilbert action, from which, by a variational method, a classical theory of gravity is constructed, was replaced by a simpler action in the form of a sum of double integrals over the world lines of particles (the right-hand side of Eq. (14)). In the same paper, Hoyle and Narlikar advanced a hypothesis on the possibility of constructing a unified theory of gravitation and electricity, in which the action would have the form of a similar sum of double integrals ([12], p. 193, the paragraph before the new section). This problem was partly solved in [1]. However, the paper [1] was subject to criticism which was understood by its author himself. Firstly, the resulting theory was not entirely unified since electromagnetism and gravity were described as two separate interactions rather than as two manifestations of a single interaction. Secondly, the theory was eclectic since electromagnetism and gravitation were finally described in different ways and in different paradigms. Electromagnetism was described as a direct interaction of particles, while to describe gravity it was still necessary to use another, geometric approach using a Riemannian curved space. To build a more consistent theory, we had to make a choice: either to geometrize the electromagnetic interaction, or to present gravity as a direct interaction of particles without introducing a coordinate-dependent metric. Adhering to the concept of direct particle interaction, we choose the second alternative. We will abandon using a Riemannian space and describe the gravitational effects as those of direct interaction of particles in Minkowski space.

The initial equations of the theory, that is, the equations of motion of charged particles, are obtained on the basis of the extremal action principle. The action of this theory has a form completely identical to (1) but without the free action. Thus we follow the idea of Hoyle and Narlikar that the full action of a system of particles is described by a sum of double integrals like (14), and the mass of each particle is not a given constant but is determined by all other particles of the Universe. Meanwhile, we wish to describe the electromagnetism, the origin of particle masses and gravity as a single interaction, and, moreover, we choose Green's function in precisely the same as in (1), following the simplest assumption. However, an important distinction of the new theory from the one developed in [5, 6] is that we admit a joint existence of retarded and advanced interactions. A consideration of the response of the Universe to the interaction of any two particles leads to the fact that the advanced interaction is many orders of magnitude weaker than the retarded one but is not strictly zero.

The variational problem statement, as well as a formulation of the initial conditions for the equations of motion have in this theory features of their own. As noted by Tetrode in [4], to describe the motion of a system of charged particles in such a theory, it is not sufficient to specify the initial coordinates and velocities of particles and the total strengths of the "fields" (or potentials) only at initial time. Instead, one can specify segments of the world lines of all particles in a certain period of time. Moreover, this period of time is finite only in the case where all particles are concentrated in a finite region of space. Let D_m be the size of a three-dimensional region of space in which the particles move within a time interval T (D_m is the greatest of all distances between the location points of the particles within the time T). Then the initial conditions for the equations of motion will be set correctly if $cT \geq D_m$, c being the speed of light. Moreover, Tetrode has noted that the equations of motion will have a solution not for all initial conditions (not for any form of the of set of segments of world lines in the interval T).

In connection with the above-said, one cannot put variations of the world lines equal to zero only at two time instants, the initial and final ones. Moreover, we do not know in advance, whether or not we can assume that variations of world line segments of finite length are equal to zero. This is because we do not know in advance whether or not we can consider the Universe as a set of particles concentrated in a finite volume, or the particles fill an infinite space. Therefore, we will proceed as follows. We will assume that somewhere in the remote past there is a time instant T_{past} before which all world lines of particles are specified (i.e., we put their variations equal to zero). Similarly, somewhere in the remote future there is a time instant T_{future} after which all particle world lines will be fixed. We will suppose that these parts of the world lines are specified in such a way that the equations of motion we want to obtain have solutions on the time interval $(T_{\text{past}}; T_{\text{future}})$. Then we can correctly formulate the variational problem.

We will also take into account that we want to obtain a self-consistent set of equations of motion for all particles rather than an equation of motion of a single particle in a given potential. In a theory of the type under consideration, in the general case, one cannot neglect the influence of a single selected particle on the ambient world. Therefore we vary the action with respect to the world lines of all particles simultaneously. The equations of motion having been obtained, it is possible to use various approximations in order not to solve a set of a large number of equations in practice. A solution of the variational problem posed will differ from (5) only in that the left-hand side of the equation will be equal to zero (due to absence of a free action term, as mentioned above). Consider a fixed particle number i. Its equation of motion will be written as

$$0 = \frac{e_i}{c} u_i^{\mu} \sum_{k \neq i} \left(\frac{1}{2} F_{\nu\mu}^{\text{ret}}(i,k) + \frac{1}{2} F_{\nu\mu}^{\text{adv}}(i,k) \right). \quad (15)$$

According to the idea of Wheeler and Feynman [5, 6], this particle acts on all other particles of the Universe by its retarded potential, and all other particles act on it in response by their advanced potential. A result of summing all the potentials is doubling of the retarded potential and emergence of the radiative friction force f_{ν}^{rad} (with the remark that in [5, 6] each particle had a given mass, while in our work we are only going to obtain it; however, the fourth way of proving the above statement in [5] does not rely on the presence of fixed particle masses):

$$0 = \frac{e_i}{c} u_i^\mu \sum_{k\neq i} F^{\rm ret}_{\nu\mu}(i,k) + f^{\rm rad}_\nu. \qquad (16)$$

But let us suppose that this result holds not exactly but approximately, and, in fact, after summing all potentials there remains a weak but nonzero advanced interaction. We will denote all quantities characterizing this advanced interaction by letters with a tilde. In this case, instead of Eq. (16), we obtain an equation with an additional term:

$$0 = \frac{e_i}{c} u_i^\mu \sum_{k\neq i} F^{\rm ret}_{\nu\mu}(i,k) + f^{\rm rad}_\nu + \frac{e_i}{c} u_i^\mu \sum_{k\neq i} \tilde{F}^{\rm adv}_{\nu\mu}(i,k). \qquad (17)$$

Let us denote the sum of advanced potentials at some point along the world line of particle i created by all other particles of the Universe by the symbol $\tilde{A}^{\rm adv}_\mu$. In agreement with the concept of Wheeler and Feynman on the response of the universe, the quantity $\tilde{A}^{\rm adv}_\mu$ may be represented as a function of many variables, i.e., of the retarded potentials acting from particle i on all other particles (this is possible due to the deterministic nature of this theory, which is a key point in our reasoning). Let us emphasize that $\tilde{A}^{\rm adv}_\mu$ is a function of only retarded potentials induced by source i, as was mentioned by Wheeler and Feynman [5, p. 160]. This function is complex since the response of the Universe cannot be presented as a sum of simple processes in which particle i acts on particle k, while particle k acts back on particle i. Instead, we should also consider more complex chains of interactions, where, for example, i acts on a, a acts on b, b acts on c, c is acts again on i. Wheeler and Feynman [5] took into account these chains by introducing a medium with a certain refractive index and attenuation factor. But we assume that the conclusions of [5] are approximate, and wish to find a correction to them. Suppose that the function to be found is decomposable into a Taylor series with tensor coefficients, and in some approximation we can restrict ourselves to the linear term in the expansion. Then for the quantity $\tilde{A}^{\rm adv}_\mu$ we can write

$$\tilde{A}^{\rm adv}_\mu = \sum_{k\neq i} K_{\mu\sigma} A^{{\rm ret}\ \sigma}(k,i), \qquad (18)$$

where $K_{\mu\sigma}$ are tensor expansion coefficients. To find the form of these coefficients, it is necessary to make some assumptions about the cosmological model. It is the cosmological model that, according to the idea of Hoyle and Narlikar [5], is responsible for the asymmetry of retarded and advanced interactions. Since we are building a new theory of gravitation which will not operate in curved space-time, the cosmological model should also be built anew. We will not do that in this paper and restrict ourselves to some basic assumptions about the Universe. The Universe is homogeneous and isotropic at large scales, and we can introduce some characteristic radius R corresponding to the horizon radius of the Universe in the usual theory. If the right-hand side of (18) contains terms that decay with distance r to particle k more slowly than $1/r^2$, then the main contribution to a sum of such terms will be obtained from particles more distant (of order R) from particle i. We will consider particles separated by a large distance. For them, due to isotropy and homogeneity of the Universe, the tensor expansion coefficients can be written in a simple form:

$$K_{\mu\sigma} = k\eta_{\mu\sigma}, \qquad (19)$$

where k is a certain number, and $\eta_{\mu\sigma}$ is the metric tensor of Minkowski space. Hence the third term in the right-hand side of (17) contains a sum of terms of the form

$$u_i^\mu \left(\frac{\partial A^{\rm ret}_\mu}{\partial x_i^\nu} - \frac{\partial A^{\rm ret}_\nu}{\partial x_i^\mu}\right) = u_i^\mu \frac{\partial A^{\rm ret}_\mu}{\partial x_i^\nu} - \frac{dA^{\rm ret}_\nu}{ds_i}$$
$$= u_i^\mu \frac{\partial A^{\rm ret}_\mu}{\partial x_i^\nu} - \frac{e_i}{R^\lambda u_{i\lambda}}\frac{du_{i\nu}}{ds_i} + \frac{e_i u_{i\sigma} u_{i\nu}}{(R^\lambda u_{i\lambda})^2}\frac{dR^\sigma}{ds_i} + \frac{e_i u_{i\nu}}{(R^\lambda u_{i\lambda})^2} R^\sigma \frac{du_{i\sigma}}{ds_i}, \qquad (20)$$

where $R^\lambda = \{c(t_k - t_i), \vec{r}_k - \vec{r}_i\}$ is the 4-vector joining events on the world lines of the interacting particles, and in the last transformation for the retarded potential induced by particle i on the world line of particle k we have used the representation

$$A^{\rm ret}_\mu(k,i) = e_i \frac{u_{i\mu}}{R^\sigma u_{i\sigma}}. \qquad (21)$$

Consider the expression obtained in the transformation (20). We will assume that the particle i, for which we wish to obtain the equation of motion, is moving at speed much smaller than the speed of light. In this case, the main contribution to the expression $R^\sigma u_{i\sigma}$ is made by the product of temporal components, and if we, as mentioned above, consider a particle very distant from particle i, we can assume this expression to be approximately equal to the horizon radius of the universe,: $R^\sigma u_{i\sigma} \approx R$. Then, while summing the expressions (20) over all remote particles, the first and the last terms in (20) can be neglected due to isotropy of the Universe and smallness of the particle i velocity ratio to the speed of light. Indeed, in the first and last terms there are vectors related to particle i (u_i^μ and $du_{i\nu}/ds_i$), which are contracted with a tensor and a vector involving the position and velocity of some remote particle k. When summing such contractions over all remote particles, the spatial components of the resulting sum will be equal to zero due to isotropy of the Universe, while the temporal component includes the ratio v/c which we neglect. The term before last in (20) decreases as $1/R^2$ at large R, therefore it can be neglected as compared with the terms decreasing as $1/R$. and there remains the second term in (20), of utmost interest for us. Under the above assumptions, it approximately equals to $(-e_i/R)du_{i\nu}/ds_i$. Substituting this expression into (17) carrying out the summation over all remote particles k and transferring the resulting term to the left-hand side of the equation, we get

$$\frac{e_i^2 kN}{cR}\cdot\frac{du_{i\nu}}{ds_i} = \frac{e_i}{c}u_i^\mu\sum_{k\neq i}F_{\nu\mu}^{\rm ret}(i,k) + f_\nu^{\rm rad} + \frac{e_i}{c}u_i^\mu\sum_n \tilde{F}_{\nu\mu}^{\rm adv}(i,n), \qquad (22)$$

where N is the number of particles within the horizon of the Universe, and the second sum in the right-hand side is taken over all particles n at distances from particle i much smaller than the characteristic scale R of the Universe. The left-hand side of this equation contains a product of a constant and the 4-acceleration of particle i. Suppose that it is, as in the well-known equation of motion of a charged particle, equal to $m_i c\, du_{i\nu}/ds_i$, where m_i is the mass of particle i. Since particles of different sorts have different masses, the values of k must be different for them. Consider an electron as particle i. It can be assumed that for a proton and a neutron the constant k has other values due to a composite structure of these particles (they consist of quarks), and in a description of leptons and quarks of higher generations it is necessary to use a quantum theory. We introduce the classical electron radius r_e defined by the equality $mc^2 = e^2/r_e$, where m and e are the electron mass and charge. Then we can write the following chain of equalities:

$$mc^2 = \frac{e^2 kN}{R} = \frac{e^2}{r_e}\frac{kNr_e}{R} = mc^2\frac{kNr_e}{R}. \qquad (23)$$

This shows that the fraction in the right-hand side must be equal to one. Let us estimate the dimensionless constant k. To do that, we take into account one of the well-known cosmological coincidences, the so-called Eddington formula, which we will accept as an empirical fact:

$$R/r_e \approx \sqrt{N}. \qquad (24)$$

From (23) and (24) it follows that the constant k is approximately expressed through the number of particles inside the horizon of the Universe:

$$k \approx 1/\sqrt{N}. \qquad (25)$$

Let us now consider a small region of the Universe, whose linear size is much smaller than the characteristic scale R, and transform the last term in the right side of (22). Due to smallness of the advanced interaction, we can again, as in Eq. (18), represent the total potential in the form of a Taylor series expansion and restrict ourselves to the linear approximation:

$$\tilde{A}_\mu^{\rm adv} = \sum_n Q_{\mu\sigma}A^{{\rm ret}\ \sigma}(n,i), \qquad (26)$$

where $Q_{\mu\sigma}$ are some new tensor expansion coefficients. Since the region under consideration is very small, we can again write due to the homogeneity and isotropy of the Universe:

$$Q_{\mu\sigma} = q\eta_{\mu\sigma}, \qquad (27)$$

where q is a new dimensionless constant (just as k, this constant can be different for particles of different sorts). For convenience, let us introduce two tensors which do not contain the charge of particle i:

$$G_\mu^{\rm ret}(n,i) = \frac{1}{e_i}A_\mu^{\rm ret}(n,i), \qquad (28)$$

$$G_{\nu\mu}^{\rm ret}(n,i) = \frac{\partial G_\mu^{\rm ret}(n,i)}{\partial x_i^\nu} - \frac{\partial G_\nu^{\rm ret}(n,i)}{\partial x_i^\mu} = -\left(\frac{\partial G_\mu^{\rm ret}(n,i)}{\partial x_n^\nu} - \frac{\partial G_\nu^{\rm ret}(n,i)}{\partial x_n^\mu}\right). \qquad (29)$$

The last equality holds due to representation (21) for the potential $\tilde{A}_\mu^{\rm ret}$. The expression inside the parentheses in (29) can be interpreted as a "field" which acts on particle n due to particle i. Then the equation of motion of particle i takes the form

$$m_i c\frac{du_{i\nu}}{ds_i} = \frac{e_i}{c}u_i^\mu\sum_{k\neq i}F_{\nu\mu}^{\rm ret}(i,k) + f_\nu^{\rm rad} - \frac{qe_i^2}{c}u_i^\mu\sum_n G_{\nu\mu}^{\rm ret}(n,i). \qquad (30)$$

We have obtained an equation having rather an unusual form: in the last term of its right-hand side there is the tensor $G_{\nu\mu}^{\rm ret}(n,i)$ instead of $G_{\nu\mu}^{\rm ret}(i,n)$. Note that this term does not depend on the sign of the charge of particle i (it is proportional to the squared charge). Unlike the first term on the right-hand side of (30), which depends on the product of particle charges, the last term describes an interaction which is always either attractive or repulsive, depending on the sign of the constant q (we mean an analogy with the electrostatic interaction). It is natural to suppose that this additional interaction is nothing else than gravity.

In this paper we restrict ourselves to the the nonrelativistic limit of Eq. (30), assuming that the velocity of particle i under consideration is much smaller than the speed of light. Then, in the SI system of units, the spacelike part of (30) is rewritten in the form

$$m_i\vec{a} = e_i \sum_{k\neq i} \left(\vec{E}(i,k) + \vec{v}_i \times \vec{B}(i,k)\right) + \vec{f}^{\,\mathrm{rad}} - qe_i^2 \sum_{n} \left(\vec{E}_g(n,i) + \vec{v}_i \times \vec{B}_g(n,i)\right), \qquad (31)$$

where $\vec{E}(i,k)$ and $\vec{B}(i,k)$ are strengths of the electric and magnetic "fields," created by particle k at the location of particle i, while $\vec{E}_g(n,i)$ and $\vec{B}_g(n,i)$ are similar quantities that do not include the charge of particle i, being a source of these "fields" (we remember that the "fields" are understood as auxiliary constructions describing the direct interaction of particles). Equation (31) is similar to the equation of motion in gravitoelectromagnetism (the well-known approximation in GR). In the case of electrostatics, we have the following expressions for field strengths:

$$\vec{E}(i,k) = -\vec{E}(k,i) = k_e \frac{e_k}{R_{ki}^3}\vec{R}_{ki},$$

$$\vec{E}_g(i,n) = -\vec{E}_g(n,i) = k_e \frac{1}{R_{ni}^3}\vec{R}_{ni}, \qquad (32)$$

where k_e is a proportionality factor in Coulomb's law, and $\vec{R}_{ik}$ is the difference of particles' radius vectors. To obtain the Newtonian force of universal gravity in the right-hand side of (31), q must satisfy the relation

$$qe_i^2 k_e = Gm_i m_n, \qquad (33)$$

where G is the gravitational constant. Let us estimate the dimensionless constant q. To do that, we take into account one more well-known cosmological coincidence: the ratio of electromagnetic and gravitational interaction forces of the electron and the proton in the classical theory is approximately

$$\frac{e_i^2 k_e}{Gm_i m_n} \approx \sqrt{N}. \qquad (34)$$

Hence it is evident that the constant q, as well as the previously introduced constant k, are approximately (up to about two orders of magnitude)

$$q \approx k \approx 1/\sqrt{N}. \qquad (35)$$

Since Eq. (31) is linear in the "field" strengths, it is also applicable to macroscopic bodies, and additivity of the charge and mass takes place. Thus we have shown that under the assumption of a nonzero advanced electromagnetic interaction and a few additional assumptions, *the emergence of particle masses and gravity are consequences of electromagnetism.* The main additional assumptions of this theory are a restriction to linear terms in the Taylor expansions (18) and (26) of the potentials, the cosmological coincidences (24) and (34), the existence of a characteristic distance scale R, and the assumption of different values of the constants q and k for the electron, proton and neutron.

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