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On Nonlinear Multidimensional Gravity and the Casimir Effect

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Abstract—We study the properties of an effective potential for the scale factor of extra dimensions in a Kaluza-Klein-type model with a spherical extra factor space, including a function of the scalar curvature and other quadratic curvature invariants, taking into account the Casimir energy of massless scalar fields. We demonstrate the existence of a minimum of the potential, able to induce a physically reasonable value of the effective cosmological constant in our space-time. Under the adopted assumptions, it is shown that the huge Casimir energy density can be compensated by the fine-tuned contribution of the curvature-nonlinear terms in the original action.

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1. INTRODUCTION

The idea of extra dimensions is firmly established in modern theoretical physics in many contexts, such as Kaluza—Klein theories [1, 2], supergravity, strings and M-theory [3, 4], brane-world theories [5—7], etc. This idea provides a powerful methodological framework for many crucial problems: geometric unification of physical interactions [2, 8, 9], the hierarchy problem [5, 7], possible variations of fundamental constants [10, 11] as well as searches for realistic cosmological scenarios including inflationary, string or brane backgrounds.

Extra dimensions present an elegant theoretical concept (as repeatedly emphasized since Kaluza's pioneer work [1]) but have a number of issues of their own to be clarified in any realistic higher-dimensional theory. One of them is the unobservable nature of extra dimensions at the accessible spatial or energy scales. A common explanation is that an extra space is compact and sufficiently small. (Another possibility which we do not consider here is considered in theories with large extra dimensions involving mechanisms of matter localization on a brane.) Realistic models of this type require a physically appropriate description of the compact extra dimensions and their stability (more precisely, the stability of ground state manifolds like $\mathbb{M}^4 \times \mathbb{V}^d$, where $\mathbb{M}^4$ is Minkowski space-time and $\mathbb{V}^d$ is a compact "internal" space). This problem for theories governed by a multidimensional action of the Einstein-Hilbert type was studied in detail in the 80s [12, 13]. One of the simple approaches to stabilize the extra dimensions is to consider effective scalar fields with suitable potentials, obtained from the extra-dimensional metric tensor components. A more accurate analysis takes into account that, beyond a classical level, there exist quantum vacuum corrections to the effective potential due to the compact topology of extra dimensions which create a nontrivial contribution to the energy density. Such a contribution is known as the cosmological Casimir effect [13—15].

The geometric scalar fields are in general coupled to gauge fields (as excitations over the ground state according to the Kaluza—Klein standard recipe). This also leads to possible space-time variations of the effective gravitational and gauge coupling constants (e.g., the fine-structure constant in the electromagnetic case) [16].

On the other hand, various inflationary scenarios and low-energy limits of superstring models applied to cosmology lead to gravitational models including nonlinear functions of the Ricci scalar ($F(R)$ theories) and high-order curvature invariants (such as the Einstein-Gauss-Bonnet gravity, see, e.g., [17]). The stability conditions for a specific class of such theories at a classical level were studied in [19], see also [20].

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In this paper, we study the properties of an effective potential of a scalar field in the framework of a Kaluza-Klein-type model including both $F(R)$ and quadratic curvature invariants. We also take into account the Casimir energy of massless fields. For simplicity we restrict ourselves to ground states of the model in the geometry $\mathbb{M}^4 \times \mathbb{S}^n$, where $\mathbb{S}^n$ is an n-dimensional sphere of sufficiently small radius while $\mathbb{M}^4$ is any 4D pseudo-Riemannian geometry whose curvature is extremely small as compared to that of $\mathbb{S}^n$. After dimensional reduction and a transition to the Einstein conformal frame, we demonstrate the existence of a set of parameters which provide a minimum of the effective potential at a physically reasonable length scale. However, this kind of models does not make easier the well-known Cosmological Constant Problem since, as in many other models, fine tuning of the initial parameters is necessary to achieve a realistic value of the effective cosmological constant ($\Lambda_{\rm eff}$).

Since we are dealing with stationary states of the extra dimensions, all fundamental physical constants remain true constants in these models, unlike models with evolving extra dimensions many of which have been considered previously [11, 17, 18].

2. BASIC EQUATIONS. THE EFFECTIVE POTENTIAL

We consider a $(D = 4 + n)$-dimensional manifold with the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu - \mathrm{e}^{2\beta(x^\mu)}d\Omega_n^2 \tag{1}$$

where x^μ are the observable four space-time coordinates, and $d\Omega_n^2$ is the metric on a sphere $\mathbb{S}^n$ of fixed radius. In this space-time, we consider a curvature-nonlinear theory of gravity with the action

$$S = \frac{1}{2}m_D^{D-2}\int \sqrt{g_D}\, d^D x\, (L_g + L_m),$$
$$L_g = F(R) + c_1 R^{AB}R_{AB} + c_2 R^{ABCD}R_{ABCD}, \tag{2}$$

where capital Latin indices cover all D coordinates, $g_D = |\det(g_{MN})|$, $F(R)$ is a certain smooth function of the D-dimensional scalar curvature R, c_1 and c_2 are constants, L_m is a matter Lagrangian, and $m_D = 1/r_0$ is the D-dimensional Planck mass, which is, in general, different from the 4D Planck mass m_4.

We use the system of units with the speed of light c and the Planck constant $\hbar$ equal to unity. As L_m, we will consider the Casimir energy density corresponding to the geometry (1). The constant $r_0 = 1/m_D$ is the D-dimensional Planck length, actually playing the role of a fundamental length in the present theory. Our goal is to find stable stationary states of the extra dimensions, that is, possible solutions to the field equations with $r(x^\mu) \equiv r_0 \mathrm{e}^\beta = \mathrm{const}$ corresponding to a minimum of a certain effective potential. To this end, following the methodology of [20, 21], we simplify the problem as follows:

(a) Integrate over the sphere $\mathbb{S}^n$ thus reducing all quantities to 4D variables and $\beta(x^\mu)$; we have, in particular,

$$R = R_4 + \phi + f_1,$$
$$f_1 = 2n\Box\beta + n(n+1)(\partial\beta)^2, \tag{3}$$

where R_4 is the 4D scalar curvature corresponding to $g_{\mu\nu}$, $\Box = \nabla^\mu\nabla_\mu$ is the 4D d'Alembert operator, $(\partial\beta)^2 = g^{\mu\nu}\partial_\mu\beta\partial_\nu\beta$, and we have introduced the effective scalar field equal to the Ricci scalar of S^n,

$$\phi(x^\mu) = m_D^2\, n(n-1)\, \mathrm{e}^{-2\beta(x^\mu)}. \tag{4}$$

(b) Suppose that all quantities are slowly varying as compared with the D-dimensional Planck scale, i.e., consider each derivative ∂_μ as an expression with a small parameter ε and neglect all quantities of orders higher than $O(\varepsilon^2)$ (see [20, 21]). This approximation is well justified in almost all thinkable situations.

(c) The 4D formulation of the theory has the form of a scalar-tensor theory in a Jordan conformal frame. We perform a transition to the Einstein frame, more suitable for studying the dynamics of the scalar field ϕ since in this frame it is minimally coupled to the 4D curvature.

In the expression (3), only ϕ has the order $O(1)$ whereas both terms f_1 and R_4 are $O(\varepsilon^2)$. It is natural to use a Taylor decomposition of the function $F(R) = F(\phi + R_4 + f_1)$:

$$F(R) = F(\phi + R_4 + f_1)$$
$$\simeq F(\phi) + F'(\phi)\cdot(R_4 + f_1) + ..., \tag{5}$$

where the prime denotes $d/d\phi$. Thus the 4D (Jordan-frame) action obtained from (2) takes the form

$$S = \frac{1}{2}\mathcal{V}m_D^2\int \sqrt{g_4}d^4x\Big[\mathrm{e}^{n\beta}F'(\phi)R_4$$
$$+ [\mathrm{Kin}]_J - 2(V_J + V_{\rm Cas})\Big], \tag{6}$$

where $F'(\phi) \equiv dF/d\phi$, $\mathcal{V} = 2\pi^{(n+1)/2}/\Gamma(\frac{1}{2}(n+1))$ is the volume of a unit sphere $\mathbb{S}^n$, and

$$[\mathrm{Kin}]_\mathrm{J} = (\partial\beta)^2\mathrm{e}^{n\beta}\big[n(n-1)(4\phi F'' - F')$$
$$+ 4(c_1 + c_2)\phi\big],$$
$$V_J(\phi) = -\frac{1}{2}\mathrm{e}^{n\beta}[F(\phi) + c_J\mathrm{e}^{-4\beta}],$$
$$V_{\rm Cas} = C_n r_0^{-2}F'\mathrm{e}^{(n-4)\beta}. \tag{7}$$

The dimensionless constants C_n are factors characterizing the Casimir energy density [12],[1] and the constant c_J is defined as

$$c_J = n(n-1)r_0^{-4}[(n-1)c_1 + 2c_2] \qquad (8)$$

(recall that $c_{1,2}$ are the coefficients in the original action (2)). The expression for $V_{\rm Cas}$ is written in such a way that the corresponding energy density contributes to the T_0^0 component of the total stress-energy tensor of matter in the theory with the action (6).

A transition to the Einstein frame is carried out using the conformal mapping

$$g_{\mu\nu} \mapsto \widetilde{g}_{\mu\nu} = |f(\phi)|g_{\mu\nu}, \quad f(\phi) = e^{n\beta}F'(\phi). \qquad (9)$$

In what follows we suppose $F'(\phi) > 0$. The resulting action in the Einstein frame has the form [11, 20]

$$S = \frac{1}{2}\mathcal{V}[n]m_D^2 \times \int \sqrt{\widetilde{g}}\left[\widetilde{R}_4 + [\mathrm{Kin}]_{\rm E} - 2V_{\rm E(tot)}(\phi)\right], \qquad (10)$$

$$[\mathrm{Kin}]_{\rm E} = K_{\rm E}(\partial\beta)^2 = (\partial\beta)^2\left[6\phi^2\left(\frac{F''}{F'}\right)^2 - 2n\phi\frac{F''}{F'} + \frac{1}{2}n(n+2) + \frac{4(c_1+c_2)\phi}{F'}\right], \qquad (11)$$

$$V_{\rm E(tot)}(\phi) = \frac{e^{-n\beta}}{2F'^2}\Big[-F(\phi) - c_J e^{-4\beta} + C_n r_0^{-2}\mathcal{V}^{-1}e^{-(n+4)\beta}\Big], \qquad (12)$$

where the tilde marks quantities obtained from or with $\widetilde{g}_{\mu\nu}$; the indices are raised and lowered with $\widetilde{g}_{\mu\nu}$; everywhere $F = F(\phi)$ and $F' = dF/d\phi$, etc.; the quantities e^{β} and ϕ are related by (4).

The expression (12) is the total effective scalar field potential, including the Casimir contribution, and its minimum, if any, corresponds to a stable equilibrium equilibrium of the extra dimensions even if the metric $g_{\mu\nu}$ and matter in our 4D space-time are changing with time.[2]

Let us enumerate the properties of such a minimum required if we wish it to be consistent with observations in the present-day Universe.

1. We describe our space-time classically, therefore the size $r = r_0 e^{\beta}$ of the extra dimensions should exceed the fundamental length scale $r_0 = 1/m_D$, i.e., $e^{\beta} \gg 1$.

2. The extra dimensions should not be directly observable, which means that $r = r_0 e^{\beta} \lesssim 10^{-17}$ cm, associated with the TeV energy scale.

3. The effective cosmological constant $\Lambda_{\rm eff}$, corresponding to the value of $V_{\rm E(tot)}$ at its minimum, should conform to observations, which means that

$$\Lambda_{\rm eff} > 0 \quad \text{but} \quad \Lambda_{\rm eff}/m_4^2 \sim 10^{-120}, \qquad (13)$$

where $m_4 \sim 10^{-5}$ g is the familiar 4D Planck mass.

The third requirement actually comprises the well-known Cosmological Constant Problem, and one of the questions to be answered in multidimensional models is whether or not it can be solved or at least partly reduced.

3. A POSSIBLE STATIONARY STATE

Let us specify the function $F(R)$ in the general quadratic form

$$F(R) = -2\Lambda_D + F_1 R + F_2 R^2, \quad \Lambda_D,\ F_1,\ F_2 = \text{const}. \qquad (14)$$

Denoting $x = e^{-\beta}$, we can rewrite the potential (12) as

$$V_{\rm E(tot)}(\phi) \equiv \frac{W(x)}{r_0^2} = \frac{1}{r_0^2[F_1 + 2n(n-1)r_0^{-2}F_2]^2} \times \left[\lambda x^n - k_1 x^{n+2} - k_2 x^{n+4} + k_2 x^{2n+4}\right], \qquad (15)$$

where $W(x)$ is dimensionless as well as the constants

$$\lambda = r_0^2\Lambda_D, \quad 2k_1 = n(n-1),$$
$$2k_2 = n^2(n-1)^2 r_0^{-2}F_2 + r_0^2 c_J,$$
$$k_3 = C_n/\mathcal{V}. \qquad (16)$$

For specific calculations let us put $F_1 = 1$ (so that at small R the theory looks as D-dimensional general relativity) and $n = 3$, so that

$$W(x) = \frac{\lambda x^3 - 3x^5 - k_2 x^7 + k_3 x^{10}}{(1 + 12r_0^{-2}F_2 x^2)^2}. \qquad (17)$$

Moreover, since k_2 includes a contribution from c_1 and c_2, we can further simplify $W(x)$ by putting $F_2 = 0$ (so that $f(R)$ is linear, and nonlinearity of the theory in the curvature components only owes to the Ricci and Riemann tensors squared in (2)), still leaving k_2 arbitrary.

The evident minimum of $W(x)$ at $x = 0$ (provided $\lambda > 0$) does not correspond to a stationary state: near

[1] This expression for the Casimir energy density is only valid for odd n, while for even n the results are not so confident because of an additional logarithmic divergence [14]. We therefore consider only odd n.

[2] One more necessary condition for a stable equilibrium at a minimum of the potential is that the scalar field ϕ, or equivalently β, is of canonical, non-phantom nature. It is the case if and only if $K_{\rm E} > 0$ in the expression (11). If, on the contrary, $K_{\rm E} < 0$, then a stable equilibrium corresponds to a maximum of the potential.

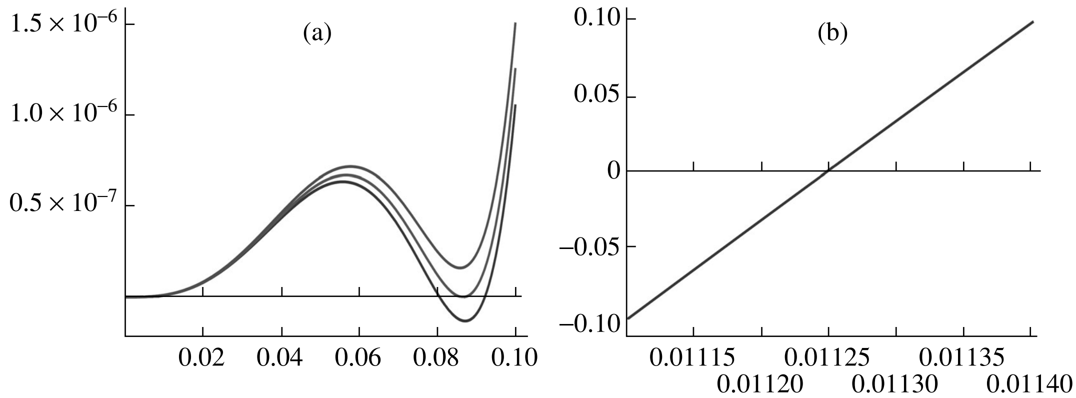

Fig. 1. (a) Plots of $W(x)$ for $n = 3$, $F_2 = 0$, $k_2 = -200$, $k_3 = 10^{-4}$, and $\lambda = 0.01105,\ 0.01125,\ 0.0115$ (bottom-up); (b) Plot of $W(x_{\rm min})$ as a function of λ for the same values of n, F_2, k_2, k_3.

it, the size $r \sim x^{-1} \to \infty$, but the particular parameters of the model may be chosen so that it grows very slowly and is still small enough at present; such cosmological models have been considered in [11, 18] and turned out to be quite viable. Now we are trying to find other viable minima corresponding to $r = \text{const}$, at which we could expect an influence of the Casimir energy.

The above "classicality" requirement 1 means that such a minimum should occur at $x = x_{\rm min} \ll 1$, for instance, $x_{\rm min} \approx 0.1$. From the third requirement it follows that $W(x_{\rm min})$ should be very close to zero; we postpone more precise estimates till the next section.

It turns out that such sets of parameters do exist. Let us first estimate k_3. According to [14], $C_3 = 7.5687046 \times 10^{-5}$ for the vacuum density of a single massless scalar field. Furthermore, $\mathcal{V}(3) = 2\pi^2$, and one should take into account that there is a certain number of degrees of freedom in 7D space-time, which may increase the Casimir energy density by a factor of $\lesssim 100$. Therefore it seems reasonable to take $k_3 = 10^{-4}$ for our qualitative estimates. Recalling that the expected values of $x_{\rm min}$ are about 0.1 or smaller, we can conclude that the Casimir term with k_3 in (17) very weakly affects our search for a minimum. Next, neglecting this term, it is easily shown that there can be only a maximum of $W(x)$ at $x > 0$ if $k_2 > 0$, hence we take $k_2 < 0$ in our search. Examples of such minima for $k_2 = -200$ are shown in the figure, where the right panel shows the dependence of $W(x_{\rm min})$ on the parameter λ. One can also verify that the necessary condition of a stable equilibrium, that $K_{\rm E} > 0$ in the expression (11), holds for $F_2 = 0$, $k_2 = -200$ and $x \lesssim 0.1$. It is clear that one can easily obtain an arbitrarily small positive value of $W(x_{\rm min})$ by properly choosing the value of λ.

4. SOME ESTIMATES

To obtain estimates characterizing the possible relevance of our models, it is necessary to decide which conformal frame corresponds to observations, and this in turn depends on how fermions should be described in a (so far unknown) fundamental unification theory of all interactions. We therefore consider two opportunities, the Jordan frame with the action (6), directly obtained from the D-dimensional theory, and the Einstein frame with the action (10).

4.1. The Einstein Frame

According to (10), the 4D Planck mass is $m_4 = \sqrt{\mathcal{V}(n)}m_D$, and $r_0 = \sqrt{\mathcal{V}(n)}/m_4$. Therefore, since $\sqrt{\mathcal{V}(n)}$ is only slightly larger than unity, the size of extra dimensions $r(x_{\rm min}) = r_0/x_{\rm min}$ for $x_{\rm min} \sim 0.1$ is of the order 10^{-31} cm, only an order of magnitude larger than the Planck length $1/m_D \approx 8 \times 10^{-33}$ cm.

The effective cosmological constant is

$$\Lambda_{\rm eff} = W(x_{\rm min})/r_0^2 = W(x_{\rm min}) m_4^2/\mathcal{V}(n), \qquad (18)$$

therefore, to satisfy the requirement (13), the dimensionless quantity $W(x_{\rm min})$ may exceed the tiny number 10^{-120} only by about an order: for example, $\mathcal{V}(3) = 2\pi^2 \approx 20$. This goal is achieved by a fine tuning of the parameters, for example, at $\lambda \approx 0.01125$ in the figure above.

The Casimir contribution to $W(x)$ is certainly in general very large as compared to 10^{-120}: for example, at the parameter values used in the figure, this contribution is $k_3 x_{\rm min}^{10} \approx 1.4 \times 10^{-15}$. This comparatively large value is compensated by fine-tuned values of other parameters of the theory.

4.2. The Jordan Frame

Now the 4D Planck mass is related to m_4 by

$$m_D^2 = 1/r_0^2 = m_4^2[F'\mathcal{V}(n)]^{-1}x^n, \quad x = x_{\min}. \quad (19)$$

Since $x \ll 1$ and we may suppose $F' \sim 1$, r_0 is in general a few orders of magnitude larger than the Planck length, which at large enough n may be in tension with the invisibility of extra dimensions.

The effective cosmological constant in the Jordan frame is obtained if we present the integrand in (6) as

$$\sqrt{g_4}d^4x e^{n\beta}F'(\phi)[R_4 - 2\Lambda_{\text{eff}} + \text{kinetic term}],$$

which leads to

$$\Lambda_{\text{eff}} = F'x^{-n}W(x)/r_0^2. \quad (20)$$

However, expressing r_0 in terms of m_4 using Eq. (19), we arrive again at the second expression for Λ_{eff} in (18), $\Lambda_{\text{eff}} = W(x_{\min})m_4^2/\mathcal{V}(n)$. Thus we need the same fine tuning of the model parameters as in the Einstein frame and have the same estimate of the Casimir contribution to $W(x)$, despite another value of the fundamental length $1/m_D$.

5. CONCLUDING REMARKS

In the framework of curvature-nonlinear multidimensional gravity with spherical extra dimensions, we have proved the existence of viable minima of the effective potential, corresponding to a stable stationary size of the extra dimensions and able to induce small positive values of the effective cosmological constant in our 4D space-time. The existence of these minima is provided by a Casimir contribution to the potential combined with terms quadratic in the components of the curvature tensor; such states are absent in pure multidimensional general relativity.

Our estimates show that in these models the Casimir energy density is much larger than a realistic cosmological energy density, and viable models can only be obtained due to fine-tuned compensation of the Casimir energy by nonlinear curvature terms. It should be mentioned that the expression for the Casimir effect that we used have been obtained for a massless nongravitational field. One might use more precise expressions for the graviton contribution taking into account the specific gravitational model [13] as well as massive fields, but very probably this will not strongly change the magnitude of the effect.

Among possible interesting extensions of this study one can consider small perturbations of the stationary states considered here, which can lead to variations of fundamental constants depending on the size of extra dimensions, such as the gravitational constant G and the electromagnetic coupling constant α. The electromagnetic field in this case can be introduced in the Kaluza manner as a gauge field corresponding to the U(1) subgroup of the group of isometries of the extra space.

Our estimates concerned the modern epoch with the corresponding value of Λ_{eff}; however, it could be of interest to consider a possible application of such models to the inflationary epoch in the early Universe with maybe quantum tunneling between a minimum of the potential at $\phi > 0$ with finite r and its closely located minimum at $\phi = 0$ corresponding to infinitely growing r [22]. And, last but not least, it is highly desirable to consider other geometries of the extra dimensions, e.g., in the form of products of spherical, toroidal and/or hyperbolic factor spaces.

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