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Inflationary Scenario in Bianchi Type V Space-Time for a Barotropic Fluid Distribution with Variable Bulk Viscosity and Vacuum Energy Density

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Abstract—We examine an inflationary scenario in Bianchi Type V space-time for a barotropic fluid distribution with variable bulk viscosity and decaying vacuum energy density. We observe that the matter density ρ, the coefficient of bulk viscosity ζ and the expansion θ all diverge at $\tau = 0$. The spatial volume increases with time, representing an inflationary scenario. The deceleration parameter $q < 0$ for barotropic, dust and radiation dominated models representing an accelerated universe, while for a stiff fluid distribution $q > 0$ corresponding to a decelerated universe. The vacuum energy density Λ decreases with time. The entropy per unit volume is proportional to the absolute temperature. The energy conditions (weak, dominant and strong) are discussed for the model. The reality condition $\rho + p \geq 0$ is violated for the inflationary model due to the presence of a scalar field (ϕ). We also discuss the importance of Bianchi Type V model where the anisotropy dies away during the inflationary era. We also calculate the inflationary parameters and compare the results with the Planck data and discuss their compatibility with anisotropy and BAO estimates. The cosmological constant Λ is a function of time without break general covariance. We also discuss the bounds of the model, how the model isotropizes, where the fluid goes after inflation and how viscosity may realize a graceful exit from inflation to a radiation dominated era.

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1. INTRODUCTION

Spatially homogeneous anisotropic cosmological models (Bianchi I-IX) play a significant role in the description of the universe at its early stage of evolution. The process of isotropization is also studied. The existence of an anisotropic universe that approaches an isotropic phase was investigated by Land and Magueijo [1]. Bianchi type V models are interesting in this study because these models are an anisotropic generalization of open FRW (Friedmann-Robertson-Walker) models and allow for arbitrarily small anisotropy levels at any instant of cosmic time. As inflation helps in isotropization of the universe, Bianchi Type V models are the most suitable for inflation. Bianchi type V models are studied in detail by a number of authors [2–9] in different contexts.

In modern cosmology, inflation is an essential ingredient. During the inflationary epoch, the scale factor of the universe grows exponentially allowing a small casually coherent region to become big enough to be identified with the present observable universe. Therefore, the inflationary scenario is a satisfactory solution to some of the conceptional issues in cosmology, but it is not understood in the standard Big Bang theory. The inflationary scenario explains several mysteries of modern cosmology like homogeneity, isotropy, flatness of observed universe and the primordial monopole problem. Guth [10] introduced the concept of inflation while investigating the problem of why we do not see magnetic monopole today. By [10], the standard model of hot Big Bang cosmology requires initial conditions which are problematic in two ways (i) the early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); (ii) the initial value of the Hubble constant must be fine-tuned to extraordinary accuracy to produce a flat universe (near-critical mass density) seen today (the flatness problem). These problems disappear if at early phase the universe is supercooled to temperatures of 28 or more orders of magnitude below the critical temperatures for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary particle interactions.

Guth [10] has also suggested that a rapid expansion is due to false vacuum energy, and after inflation

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the universe is filled with bubbles. This inflationary scenario is also confirmed by CMB observations [11]. Also, as pointed out by Myrzakulov and Sebastiaini [12], after inflation the fluid turns out to acquire the equation-of-state parameter $\gamma = 1/3$, so that we recover the radiation/ultrarelativistic matter universe of the standard model without invoking the reheating, as the energy density of fluid itself is converted into radiation. The inflationary period is divided into the slow-roll and reheating phases. At the slow-roll epoch of inflation, the potential of the inflaton should remain large as compared to kinetic energy [13]. At the end of inflation, the inflaton starts oscillating about the minimum of its potential, while the potential and kinetic energies are comparable [14–16].

Inflation plays an important role in isotropization of the universe. Inflation does not start at the end of isotropization, on the contrary, isotropization starts at the end of inflation, as pointed out by Hervik et al. [17]. The inflationary scenario for homogeneous and isotropic (FRW) models has been studied by many authors [18–21]. Rothman and Ellis [22] pointed out that we can have a solution for an isotropic problem if we work with anisotropic space-times that isotropoizes in a special case. Keeping in mind these investigations, we have studied [23, 24] some inflationary models with a flat potential in different contexts in LRS Bianchi Type I and Bianchi Type I space-times.

In the recent years, introduction of viscosity in the cosmic fluid content has been found very useful in explaining many significant physical aspects of the dynamics of homogeneous cosmological models. The dissipative mechanism not only modifies the nature of the singularity but also successfully accounts for the large entropy per baryon in the present universe. The observed physical phenomena, such as large entropy per baryon and the remarkable degree of isotropy of cosmic microwave background radiation (CMBR) reveal the significance of dissipative effects in cosmology. Heller and Klimek [25] investigated a viscous fluid cosmological model without initial singularity, and it has been shown that introduction of bulk viscosity effectively removes the initial singularity. Chimento et al. [26] studied cosmological solutions with a nonlinear viscosity. Gron [27] studied a viscous inflationary universe in Bianchi Type I space-time. Zimdahl [28] showed that a sufficiently large viscous pressure leads to inflationary-type solution in a flat, homogeneous and isotropic universe. Isotropization of the cosmic fluid induced by viscosity is an important physical effect, as discussed by Brevik and Pettersen [29, 30] in the context of simple Kasner space. The effect of viscosity on cosmological models has been studied by many authors [31–36] in different contexts.

The most theoretically appealing possibility for dark energy is the energy density stored in the vacuum state of all existing fields in the universe. However, a cosmological constant (Λ) does not explain the huge difference between Λ inferred from observations and the vacuum energy density resulting from quantum field theories. The observational relevance of the cosmological constant is described by Zel'dovich [37], Krauss and Turner [38]. Two independent groups led by Riess et al. [39] and Perlmutter et al. [40] used Type Ia supernovae to show that the universe is not only expanding but this expansion is also accelerating. This discovery provided the first direct evidence that Λ is nonzero being $\Lambda \sim 1.7 \times 10^{-121}$ in Planck units. Thus the present accelerating behavior of universe is due to the dominance Λ. Obviously, this extremely small value of Λ indicates that the vacuum energy density, or Λ, is not a strict constant but decays as the universe expands, i.e., Λ is time-dependent.

Isotropization does not belong to the Dark Energy era because, as pointed out by Narlikar et al. [41], with the popularity of inflationary models, the cosmological constant found a new life with its claimed origin in the force generated by transitions of the vacuum. It is tempting to suppose that the present (Λ) owes its origin to the inflationary phase. However, Weinberg [42] has commented on the extremely low observational value of Λ as compared to primordial inflationary scenarios: if the inflation took place at the grand unified epoch, the value of Λ is too low by a factor of $\sim 10^{-108}$. If the inflation took place at the quantum gravity epoch, the above factor is still $\sim 10^{-120}$. Now, it is easier to suppose that after inflation was over, the cosmological constant dropped to zero. Many authors [43–49] investigated cosmological models with a time-dependent vacuum energy density which obeys general covariance.

The manuscript is organized as follows. In Section 2 we present the cosmological equations for a bulk-viscous barotropic fluid distribution with vacuum energy density for an inflationary scenario in Bianchi Type V space-time. In Section 3, we determine the space-time for a flat potential under the condition $\rho = 3H^2$, $p = \gamma\rho$, $\zeta \propto \rho^{1/2}$, $\Lambda \sim 1/R^2$, where ρ is matter density, H the Hubble parameter, ζ the coefficient of bulk viscosity, R the scale factor, and p the isotropic pressure. In Section 4, we determine the characteristics of an inflationary scenario with a flat potential in Bianchi Type V space-time. We compare our results with recent astronomical observations. We also discuss the importance of Bianchi Type V model, the deceleration parameter for inflation, the era of isotropization and Dark Energy with bounds of the model, the time dependence of Λ which does not break general covariance, and how the anisotropy disappears and again appears after a long time, i.e., where the fluid goes after inflation, and how viscosity may realize a graceful exit from inflation to a radiation dominated era.

2. THE METRIC AND FIELD EQUATIONS

We consider Bianchi Type V space-time in the orthogonal form as

$$ds^2 = -dt^2 + A^2dx^2 + e^{2x}(B^2dy^2 + C^2dz^2), \quad (1)$$

where A, B, C are functions of t-alone. The cosmic fluid is assumed to be viscous, given by the energy-momentum tensor with a scalar field (ϕ) as

$$T_i^j = (\rho + p)v_iv^j + pg_i^j - \zeta\theta(g_i^j + v_i^j) + \partial_i\phi\,\partial^j\phi - \left\{\frac{1}{2}\partial_k\phi\,\partial^k\phi + V(\phi)\right\}g_i^j. \quad (2)$$

The Lagrangian in which gravity is minimally coupled to the scalar field (ϕ) is defined by Stein-Schabes [50] as

$$L = \int_M \sqrt{-g}R_i^i - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)d^4x. \quad (3)$$

The energy conservation law coincides with the equation of motion for ϕ, and we have

$$\frac{1}{\sqrt{-g}}\partial_i(\sqrt{-g}\partial^i\phi) = -\frac{dV}{d\phi},$$

which leads to

$$\phi_{44} + \phi_4\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = \frac{dV}{d\phi}, \quad (4)$$

where ρ is the matter density, p the isotropic pressure, ζ the coefficient of bulk viscosity, v^i the flow vector of the fluid satisfying $v_iv^i = -1$, ϕ the scalar field subject to the potential $V(\phi)$, R_i^i the Ricci scalar, and M the space-time manifold. The homogeneous scalar field ϕ which is identified with the inflaton is a function of cosmic time only, and $\phi_4 = d\phi/dt$, etc.

We see that the energy-momentum tensor (EMT) in the form (2) has contributions from a perfect fluid for which we assume the existence of an equation of state of the form $p = \gamma\rho$ ($0 \le \gamma \le 1$) and from a homogeneous massless scalar field (ϕ) with the potential $V(\phi)$. We have also introduced viscosity to have a graceful exit from inflation and seek a possibility to recover the reheating phase as production of matter particles or conversion of the fluid energy to the standard radiation-dominated phase.

We assume the coordinates to be comoving, so that $v^1 = v^2 = v^3 = 0$, $v^4 = 1$. Einstein's field equations (in gravitational units $8\pi G = c = 1$) with time-varying vacuum energy density (Λ) are

$$R_i^j - \frac{1}{2}R_k^k\,g_i^j + \Lambda(t) = -T_i^j. \quad (5)$$

Einstein's equations (5) for the space-time (1) with (2) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} - \frac{1}{A^2} + \Lambda = -\left[p - \zeta\theta + \frac{1}{2}\phi_4^2 - V(\phi)\right], \quad (6)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} - \frac{1}{A^2} + \Lambda = -\left[p - \zeta\theta + \frac{1}{2}\phi_4^2 - V(\phi)\right], \quad (7)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{1}{A^2} + \Lambda = -\left[p - \zeta\theta + \frac{1}{2}\phi_4^2 - V(\phi)\right], \quad (8)$$

$$\frac{A_4B_4}{AB} + \frac{B_4C_4}{BC} + \frac{A_4C_4}{AC} - \frac{3}{A^2} + \Lambda = -\left[-\rho - \frac{1}{2}\phi_4^2 - V(\phi)\right], \quad (9)$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0. \quad (10)$$

Equation (10) leads to

$$A^2 = BC, \quad (11)$$

where the constant of integration is assumed to be unity for simplicity, and the indices 4, 44 indicate partial derivatives w.r.t. t.

Due to the time dependence of Λ (vacuum energy density), Eq. (5) does not leads to breaking of general covariance because ρ (matter density) + Λ = Total density = $\rho_{(T)}$, $p - \zeta\theta$ = Effective pressure = $\bar{p}$, and $\bar{p} - \Lambda$ (Λ exerts negative pressure) = Total pressure = $\bar{p}_{(T)}$. Thus general covariance leads to

$$\rho_4 + (\rho_T + \bar{p}_T)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0, \Rightarrow$$

$$\rho_4 + \Lambda_4 + (\rho + \Lambda + \bar{p} - \Lambda)\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0.$$

Thus we have

$$\rho_4 + \Lambda_4 + (\rho + \bar{p})\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right) = 0.$$

We find that addition of a time dependence of Λ, does not break general covariance.

3. SOLUTION OF FIELD EQUATIONS

Equations (6), (7), and (8) lead to

$$\frac{A_{44}}{A} - \frac{B_{44}}{B} + \frac{C_4}{C}\left(\frac{A_4}{A} - \frac{B_4}{B}\right) = 0, \tag{12}$$

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} + \frac{A_4}{A}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = 0. \tag{13}$$

From (6) and (9) we have

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{2B_4C_4}{BC} + \frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} - \frac{4}{A^2} + 2\Lambda = -p + \rho + \zeta\theta + 2V. \tag{14}$$

To find the solution in terms of cosmic time t, we assume:

(i) the universe is filled with a barotropic fluid distribution, i.e.,

$$p = \gamma\rho, \quad 0 \le \gamma \le 1; \tag{15}$$

$$\text{(ii)} \quad \zeta \propto \rho^n \quad \text{and} \quad \rho = 3H^2, \tag{16}$$

as considered by Barrow [19],

$$\text{(iii)} \quad \Lambda = 2/R^2, \tag{17}$$

as considered by Chen and Wu [44], where R is the scale factor. Using (15), (16), and (17) in (14), we have

$$\frac{(BC)_{44}}{BC} + \frac{1}{2}\frac{(BC)_4^2}{(BC)^2} = (1-\gamma)\rho + \zeta\theta + 2K,$$
$$V(\phi) = K, \tag{18}$$

which leads to

$$\frac{\mu_{44}}{\mu} + \frac{1}{2}\frac{\mu_4^2}{\mu^2} = (1-\gamma)\frac{\theta^2}{3} + \frac{\theta^2}{3} + 2K, \tag{19}$$

where $BC = \mu$, $\zeta = \sqrt{\rho/3}$, $\rho = 3H^2$, $H = \theta/3$, $\theta = (3/2)(\mu_4/\mu)$.

For comparison with the corresponding anisotropic models, we consider flat FRW models where the coefficient of bulk viscosity is proportional to expansion, i.e., the Hubble factor, $\zeta = aH$, a is constant [27]. Also, for a flat RW universe, the density of the cosmic fluid is given by $\rho = 3H^2$. Thus $\zeta \propto H$ is equivalent to $\zeta \propto \rho^{1/2}$, and we have

$$\frac{\mu_{44}}{\mu} + \frac{1}{2}\frac{\mu_4^2}{\mu^2} = \frac{3(2-\gamma)}{4}\frac{\mu_4^2}{\mu^2} + 2K, \tag{20}$$

which leads to

$$2\mu_{44} + \left(\frac{3\gamma - 4}{2\mu}\right)\mu_4^2 = 4K\mu. \tag{21}$$

To solve Eq. (21), we assume that

$$\mu_4 = f(\mu).$$

Thus

$$\mu_{44} = ff', \quad f' = \frac{df}{d\mu}.$$

Now, Eq. (21) leads to

$$\frac{df^2}{d\mu} + \left(\frac{3\gamma - 4}{2\mu}\right)f^2 = 4K\mu. \tag{22}$$

Thus we have

$$\frac{\mu^{3\gamma/4-1}}{\sqrt{\mu^{(3\gamma/4)^2} + \beta^2}}d\mu = \sqrt{\frac{8K}{3\gamma}}dt, \tag{23}$$

where

$$\beta^2 = \frac{3\alpha_1\gamma}{8K}, \tag{24}$$

α_1 being a constant of integration. Equation (23) leads to

$$\mu^{3\gamma/4} = \beta\sinh(at + b), \tag{25}$$

where

$$0 < \gamma \le 1 \quad \text{and} \quad a = \sqrt{\frac{3K\gamma}{2}}, \tag{26}$$

i.e., the solution is valid for (i) a barotropic fluid distribution. (ii) a stiff fluid distribution ($\gamma = 1$), (iii) a radiation-dominated model ($\gamma = 1/3$), and the solution is not valid for $\gamma = 0$. To find the value of ν for the above fluid distributions, i.e., for $\gamma = 1$, $\gamma = 1/3$, and $\gamma = 1/2$, Eq. (13) leads to

$$\frac{\nu_4}{\nu} = \frac{L}{\mu^{3/2}}, \tag{27}$$

where $BC = \mu$, $B/C = \nu$, and L is constant of integration. From Eqs. (25) and (27) we have

$$\log\nu = \int \frac{L}{\beta^{4/(3\gamma)}\sinh^{4/(3\gamma)}(at + b)}dt. \tag{28}$$

Therefore the metric (1) leads to

$$ds^2 = -\frac{d\tau^2}{a^2} + \sinh^{4/(3\gamma)}\tau dX^2 + e^{2X}\sinh^{4/(3\gamma)}\tau(\nu dY^2 + \nu^{-1}dZ^2), \tag{29}$$

where $at + b = \tau$, $\beta^{2/(3\gamma)}x = X$, $\beta^{2/(3\gamma)}y = Y$, $\beta^{2/(3\gamma)}z = Z$, and ν is determined by Eq. (28).

Special Model (Dust Distribution, $\gamma = 0$)

Equation (21) for $\gamma = 0$ leads to

$$2\mu_{44} - 2\frac{\mu_4^2}{\mu} = 4K\mu. \tag{30}$$

From (30) we have

$$\frac{d\mu}{dt} = \sqrt{4K\log(\alpha_2\mu)}, \tag{31}$$

which leads to

$$\frac{d\mu}{\mu\sqrt{\log(\alpha_2\mu)}} = \sqrt{4K}dt.$$

Thus we have

$$\mu = \frac{1}{\alpha_2}e^{(\sqrt{K}t+K_1)^2}, \tag{32}$$

where K, α_2, and K_1 are integration constants. Equation (13) leads to

$$\frac{d\nu}{\nu} = \frac{L}{\mu^{3/2}}dt = L(\alpha_2)^{3/2}\frac{dt}{e^{\frac{3}{2}(\sqrt{K}t+K_1)^2}}. \tag{33}$$

Thus we have

$$\log\nu = \int L(\alpha_2)^{3/2}\, e^{-\frac{3}{2}(\sqrt{K}t+K_1)^2}dt. \tag{34}$$

Therefore the metric (1) leads to

$$ds^2 = -Kdt^2 + e^{(\sqrt{K}t+K_1)^2}dX^2 + e^{2X+(\sqrt{K}t+K_1)^2}(\nu dY^2 + \nu^{-1}dZ^2), \tag{35}$$

where ν is determined by Eq. (34).

4. PHYSICAL AND GEOMETRIC ASPECTS

For the model (29) the expansion (θ), the Hubble parameter (H), the matter density (ρ), the isotropic pressure (p), the spatial volume (V^3), the deceleration parameter (q), and the vacuum energy density (Λ) are given by

$$\theta = \frac{2a}{\gamma}\coth(\tau), \tag{36}$$

$$H = \frac{\theta}{3} = \frac{2a}{3}\coth(\tau), \tag{37}$$

$$\rho = 3H^2 = \frac{4a^2}{3}\coth^2(\tau), \tag{38}$$

$$p = \gamma\rho = \frac{4a^2\gamma}{3}\coth^2(\tau), \tag{39}$$

$$\sigma = \frac{L}{2\mu^{3/2}} = \frac{L}{2\beta^{2/\gamma}\sinh^{2/\gamma}\tau}, \tag{40}$$

$$V^3 = \beta^{2/\gamma}\sinh^{2/\gamma}\tau, \tag{41}$$

$$q = -\frac{V_{44}/V}{V_4^2/V^2} = -\frac{(4-6\gamma)\coth^2\tau + 6\gamma}{4\coth^2\tau}. \tag{42}$$

Therefore,

$$q = \begin{cases} -1 & < 0 \text{ for } \gamma = 0;, \\ -\dfrac{\coth^2\tau + 3}{4\coth^2\tau} & < 0 \text{ for } \gamma = 1/2; \\ -\dfrac{\coth^2\tau + 1}{2\coth^2\tau} & < 0 \text{ for } \gamma = 1/3; \\ \dfrac{\coth^2\tau - 3}{2\coth^2\tau} & > 0 \text{ for } \gamma = 1; \end{cases} \tag{43}$$

$$\Lambda = \frac{2}{\beta^{4/(3\gamma)}\sinh^{4/(3\gamma)}\tau}. \tag{44}$$

The quantities θ, H, σ, ρ, p, V^3, q, and Λ for the model (35) are given by:

$$\theta = \frac{3\mu_4}{2\mu} = 3\sqrt{K}(\sqrt{K}t + K_1), \tag{45}$$

$$H = \frac{\theta}{3} = \sqrt{K}(\sqrt{K}t + K_1), \tag{46}$$

$$\sigma = \frac{\alpha_2 L^{3/2}}{2}e^{-\frac{3}{2}(\sqrt{K}t+K_1)^2}, \tag{47}$$

$$\rho = 3H^2 = 3K(\sqrt{K}t + K_1)^2, \tag{48}$$

$$V^3 = ABC = \mu^{3/2} = \alpha_2^{-3/2}e^{\frac{3}{2}(\sqrt{K}t+K_1)^2}, \tag{49}$$

$$q = -\frac{V_{44}/V}{V_4^2/V^2} = -1 - \frac{1}{(\sqrt{K}t + K_1)^2} < 0, \tag{50}$$

$$\Lambda = \frac{2}{R^2} = \alpha_2 e^{-(\sqrt{K}t+K_1)^2}. \tag{51}$$

5. ENTROPY

To study the entropy, we apply the combined form of the first and second laws of thermodynamics for the system of comoving volume V as

$$Tds = d(\rho V) + pdV. \tag{52}$$

Equation (52) can be written as

$$Tds = d((\rho + p)V) - Vdp. \tag{53}$$

Following [51, 52], $S = S(T, V)$, and we have

$$\frac{\partial^2 S}{\partial T\partial V} = \frac{\partial^2 S}{\partial V\partial T}, \tag{54}$$

which leads to a relation between the pressure p and the temperature T

$$dp = (\rho + p)\frac{dT}{T}. \tag{55}$$

Using the barotropic condition $p = \gamma\rho$ in (55), we have

$$\rho^{\gamma/(\gamma+1)} = T, \tag{56}$$

which leads to

$$\rho = T^{(1+\gamma)/\gamma}. \tag{57}$$

Equations (55) and (53) lead to

$$ds = \frac{1}{T}d[(\rho + p)V] - \frac{(\rho + p)V}{T^2}dT, \tag{58}$$

whence

$$dS = d\left[\frac{(1+\gamma)\rho V}{T} + k\right]. \tag{59}$$

Thus the entropy S is given by

$$S = \frac{(1+\gamma)\rho V}{T}. \qquad (60)$$

The entropy density s is

$$s = \frac{S}{V} = \frac{(1+\gamma)\rho}{T} = (1+\gamma)T^{1/\gamma}, \qquad (61)$$

using (57). We observe that the entropy per unit volume is proportional to absolute temperature.

6. ENERGY CONDITIONS

Following Kolassis et al. [53], Chatterjee and Banerjee [54], we briefly discuss weak, dominant and strong energy conditions in the context of inflationary universe with bulk viscosity for our model. We have

$$T_{44} = \rho + \frac{\phi_4^2}{2} + V(\phi);$$
$$T_{11} = p - \zeta\theta + \frac{\phi_4^2}{2} - V(\phi) = T_{22} = T_{33}. \qquad (62)$$

In the locally Minkowskian frame, the roots of matrix equations

$$|T_{ij} - rg_{ij}| = \mathrm{diag}\big(T_{00} - r,\ T_{11} + r,\ T_{22} + r,\ T_{33} + r\big) \qquad (63)$$

give the eigenvalues r for our EMT

$$r_0 = \rho + V + \frac{\phi_4^2}{2};$$
$$r_1 = -p + \zeta\theta + V - \frac{\phi_4^2}{2} = r_2 = r_3. \qquad (64)$$

The energy conditions for our model are:

The weak energy condition

$$r_0 \geq 0 \quad \text{leads to} \quad \rho + \phi_4^2/2 + V \geq 0,$$
$$r_0 - r_i \geq 0 \quad \text{leads to} \quad \rho + p + \phi_4^2 \geq \zeta\theta. \qquad (65)$$

The dominant energy condition

$$r_0 \geq 0 \quad \text{leads to} \quad \rho + \phi_4^2/2 + V \geq 0,$$
$$-r_0 \leq -r_i \leq r_0 \quad \text{leads to} \quad \rho \geq p - 2r - \zeta\theta. \qquad (66)$$

The strong energy condition

$$r_0 - \sum r_i \geq 0 \ \Rightarrow\ \rho + 3p + 2\phi_4^2 \geq \zeta\theta + 2V. \qquad (67)$$

If we group (65) and (67), we have

$$\rho + 3p + 2\phi_4^2 \geq \zeta\theta + 2V.$$

We find that the reality condition $\rho + p \geq 0$ is violated due to the scalar field (ϕ), which is in agreement with the inflationary model.

7. CALCULATION OF INFLATIONARY PARAMETERS

We calculate the inflationary parameters, i.e., the slow roll parameters, the scalar spectral index, the tensor-to-scalar ratio and the non-Gaussianity parameter f_{NL}^{equil} for the model (35) to examine whether these parameters are in excellent agreement with the Planck results for a canonical or non-canonical scalar field. We first examine our model (35) obtained for a canonical scalar field with $V(\phi) = \text{const}$.

To the first approximation, for the model (35), the scale factor R is given by

$$R = \beta^{2/(3\gamma)} \sinh^{2/(3\gamma)} T \sim \beta^{2/(3\gamma)} \tau^{2/(3\gamma)}. \qquad (68)$$

The Hubble parameter H is

$$H = R_4/R \simeq 2/(3\gamma\tau) \sim \alpha/\tau, \qquad (69)$$

where

$$\alpha = 2/(3\gamma). \qquad (70)$$

The slow-roll parameters ϵ and δ are defined as [55]

$$\epsilon = -H_4/H^2 = \alpha^{-1}, \qquad (71)$$
$$\delta = \epsilon - \frac{\epsilon_4}{2H\epsilon} = \epsilon = \alpha^{-1}. \qquad (72)$$

The slow-roll PLI (power-law inflation) corresponds to $\epsilon \ll 1$ which occurs when $\alpha \gg 1$.

In this paper, we discuss a new PLI model in which inflation is driven by a canonical scalar field ϕ with the Lagrangian

$$L(\phi, X) = X - V(\phi), \qquad (73)$$

where

$$X = \phi_4^2/2. \qquad (74)$$

For a generic $L(\phi, X)$, it is convenient to introduce a third slow-roll parameter S as given by [56]

$$S = \frac{(C_s)_4}{HC_s}, \qquad (75)$$

where C_s is the speed of sound of the scalar field as given by [57]:

$$C_s{}^2 = \frac{\partial L/\partial X}{\partial L/\partial X + 2X\partial^2 L/\partial X^2} = 1. \qquad (76)$$

Thus $S = 0$ slow-roll inflation requires not only $\epsilon \ll 1$ and $|\delta| \ll 1$ but also $|S| \ll 1$. For a canonical scalar field, the value of S is identically zero, and this is also the case for kinetically driven and non-canonical models [58].

The power spectrum of scalar curvature perturbation R_k is defined as

$$P_s(k) = \frac{k^3}{2\pi^2}|R_k|^2, \qquad (77)$$

while the tensor power spectrum is given by

$$P_T(k) = \frac{k^3}{\pi^2}|h_k|^2, \tag{78}$$

where h is the amplitude of tensor perturbation. The scalar spectral index n_s and the spectral index of tensor perturbations n_T are defined as

$$n_s - 1 = \frac{d\ell_n P_s}{d\ell_n k}, \tag{79}$$

$$n_T = \frac{d\ell_n P_T}{d\ell_n k}. \tag{80}$$

The tensor-to-scalar ratio is defined as

$$\gamma = P_T/P_s. \tag{81}$$

For the canonical PLI model (35), in the slow-roll limit, we have

$$n_T = n_s - 1 \sim -2/\alpha, \tag{82}$$

$$\gamma \simeq 16/\alpha. \tag{83}$$

The power-law solution $R \propto T^\alpha$ requires that

$$38 \leq \alpha \leq 101, \tag{84}$$

which suggests for $\gamma = 16/\alpha$ to lie in the range

$$0.16 < \gamma < 0.43. \tag{85}$$

This is well above the limit set in the Planck data which indicate $\gamma < 0.12$ at 95% CL when BAO data are also included [59]. Thus we find that the PLI based on a canonical scalar field is in tension with the Planck data. The model (29) exists during the span of time $0 < T \leq \infty$, while the model (35) exists for $0 < t \leq \infty$.

The Non-Gaussianity Parameter f_{NL}^{equil}

We first carry out a simple estimate of non-Gaussianity for a non-canonical model with the Lagrangian

$$L(X, \phi) = X\left(\frac{X}{M^4}\right)^{\alpha-1} - V(\phi), \tag{86}$$

where α is dimensionless, while M has the dimension of mass. For a canonical model $\alpha = 1$, the non-Gaussianity parameter $f_{NL}^{\rm equil}$ is given by [60]

$$f_{NL}^{\rm equil} = \frac{5}{81}\Big(\frac{1}{C_s^2} - 1 - \frac{2\lambda}{N}\Big) - \frac{35}{108}\Big(\frac{1}{C_s{}^2} - 1\Big), \tag{87}$$

where

$$\lambda = X^2\frac{\partial^2 L}{\partial X^2} + \frac{2}{3}X^3\frac{\partial^3 L}{\partial X^3},$$

$$N = X\frac{\partial L}{\partial X} + 2X^2\frac{\partial^2 L}{\partial X^2}. \tag{88}$$

We find that

$$\lambda/N = (\alpha - 1)/3. \tag{89}$$

Using (88) and $C_s = 1/\sqrt{2\alpha - 1}$ in (87), we obtain

$$f_{NL}^{\rm equil} \simeq -0.57(\alpha - 1) \simeq -0.28(C_s^{-2} - 1), \tag{90}$$

as given by Unnikrishnan and Sahni [55]. For a canonical model $C_s = 1$ and $\alpha = 1$, which leads to $f_{NL}^{\rm equil} \simeq 0$. For $\alpha = 6$, Eq. (90) gives $f_{NL}^{\rm equil} \simeq -28$, which is in excellent agreement with the Planck result $f_{NL}^{\rm equil} = -42 \pm 75$, as given in [59].

We consider a viscous fluid whose equation of state (EOS) assumes the general form [61])

$$p = \gamma(\rho)\rho - 3H\zeta(R, H, H_4, H_{44}, \ldots), \tag{91}$$

where $\zeta(R, H, \ldots)$ is the bulk viscosity and depends on scale factor R, the Hubble parameter H and its derivative. We take $\gamma(\rho) = \gamma = \text{const}$ to get a graceful exit from inflation. By introducing this fluid in the background of GR, the field equations for flat FRW metric

$$ds^2 = dt^2 - R^2(t)\{dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\} \tag{92}$$

gives

$$\rho = 3H^2, \quad p = -(2H_4 + 3H^2), \tag{93}$$

and the energy conservation law for the fluid takes the form

$$\rho_4 + 3H\rho(1 + \gamma) = (3H)^2(R, H, H_4, H_{44}, \ldots).$$

Inflation is realized when the viscosity is negligible, and we take it in following form [12]):

$$\zeta(H) = e^{-H/H_0}f(H), \tag{94}$$

where H_0 is the constant Hubble parameter at the end of inflation, and $f(H)$ is a suitable function to be determined. We have two cases:

$$\zeta\left(1 \ll \frac{H}{H_0}\right) \simeq 0, \quad \zeta\left(\frac{H}{H_0} \ll 1\right) \simeq f(H).$$

We analyze the model in two asymptotic limits.

When $1 \ll H/H_0$, the solution of the Friedmann equation (93) is given by

$$H(t) = \frac{2}{3(1 + \gamma)t}, \quad \rho = \rho_0 R^{-3(1+\gamma)}; \quad -1 < \gamma,$$

where ρ_0 is constant. We exclude the case $\gamma < -1$ for which, in order to maintain the positivity of the Hubble parameter, we introduce the integration constant so that

$$H(t) = -\frac{2}{3(1 + \gamma)(t_0 - t)}; \quad \gamma < -1, \tag{95}$$

where $0 < t_0$ is a fixed time, and $t < t_0$. However, in such a case $0 < H_4$ and the Hubble parameter increases, making it impossible to exit from inflation and viscosity.

The solution (94) shows an initial singularity at $t = t_0$, which can be identified with the Big Bang. The acceleration is realized in the quintessence region

$$-1 < \gamma < -1/3; \quad 0 < \frac{R_{44}}{R} = H^2 + H_4 = \frac{4 - 6(1+\gamma)}{9(1+\gamma^2) + 2}. \tag{96}$$

To reproduce inflation, the solution must be close to the de Sitter one (i.e., γ close to -1). Secondly, in the limit $H/H_0 \ll 1$, the viscous term grows up and by combining the first Friedmann equation with a continuity equation. We have

$$\rho_4 = 3H\rho\left\{1 + \gamma - \frac{f(H)}{3H}\right\}. \tag{97}$$

If we choose $f(H)$ as

$$f(H) = 3H(\gamma - \gamma_{\rm rad}),$$

then at the end of inflation we may recover the radiation/ultrarelativistic matter universe of the standard model without invoking the reheating since the energy density of fluid itself is converted to radiation matter.

8. CALCULATION OF THE ANISOTROPY PARAMETER

For the model (29), we have

$$H = \frac{R_4}{R} = \frac{2a}{3\gamma}\coth T, \tag{98}$$

where R is scale the factor and H the Hubble parameter. If $\hat{A}$ is the anisotropy parameter then, $\hat{A}$ is defined as

$$\hat{A} = \frac{1}{3}\left[\left(\frac{H_1}{H} - 1\right)^2 + \left(\frac{H_2}{H} - 1\right)^2 + \left(\frac{H_3}{H} - 1\right)^2\right], \tag{99}$$

where $H_1 = A_4/A$, $H_2 = B_4/B$, $H_3 = C_4/C$ are directional Hubble parameters in the directions of x, y, z, respectively. Equation (99) leads to

$$\hat{A} = \frac{3L^2\gamma^2}{8\beta^{4/\gamma}\sinh^{4/\gamma}\tau\coth^2\tau}. \tag{100}$$

The anisotropy parameter $\hat{A}$ is initially large but decreases with time, and at large values of τ, $\hat{A} = 0$, i.e., the anisotropy disappears—the model isotropizes at late times, which matches with the results of astronomical observations. Similar is the case for the model (35). Thus it is possible for the model which undergoes sufficient inflation to become isotropic. Furthermore, the time it takes to become anisotropic after inflation is very long, i.e., the time scale is at least of the order $\simeq e^{2N\sqrt{\lambda}}$, where N is the number of e-folds of the universe expansion during the inflationary phase, and Λ is the cosmological constant [62]. N is defined by Remmen and Carroll [63] as $N = \int_{t_i}^{t_f} H dt$, where t_i and t_f are the proper times at the beginning and end of inflation.

Estimation of the Amount of Anisotropy

If μ is the anisotropy parameter, then it lies within the range $0 < \mu < 0.6$ to attain physical values of $\gamma < 0.2$ and $n_s = 0.96$, respectively. These make the model compatible with Bicep2 and WMAP7 data. For the isotropic case, all results lead to those obtained by Myrzakulov and Sebastiani [12].

The Role of Anisotropy in Cosmological Perturbations

Anisotropy leads to cosmological perturbations, as pointed out by Thorne [64] and Collins et al. [65]. It is also well known that the CMB temperature anisotropies are at the 10^{-5} level relative to the mean temperature. This remarkable level of isotropy suggests that the density fluctuation and cosmological perturbations were in a linear regime at the epoch in which the CMB radiation decoupled from the rest of the system. If we impose the BBN bound $T_{\rm dec} > MeV$ and the constant $m < H$, then we find $H \geq 10^7$ GeV. Another bound comes from the fact that the curvaton decay rate will be at least of the order $m^3/(Mp^2)$, corresponding to gravitational-strength interactions, as pointed out by Liddle and Lyth [66].

9. CONCLUSION

We observe that the matter density ρ, the expansion θ, the coefficient of bulk viscosity ζ, and the Hubble parameter H all diverge at $\tau = 0$ for the model (29). This model exists in the period $0 < \tau \leq \infty$.

It is a realistic inflationary universe because for the model (29) and $\lim_{\tau\to\infty}(\sigma/\theta) \simeq 0$ for large values of τ. Thus the anisotropy is small and vanishes at late times, which matches with the results of astronomical observations [39]. Also, Thorne [64] has pointed out that the velocity–redshift relation for extragalactic sourcess suggest that the Hubble expansion of the Universe is isotropic today to within 30 per cent, and suggests $\lim_{\tau\to\infty}(\sigma/H) \leq 0.30$, where σ is the shear. For a spatially homogenous metric, Collins et al. [65] also pointed out that σ/θ is small. The anisotropy of Bianchi metrics necessarily dies away during the inflationary era [67].

The deceleration parameter (q) is so important for inflation because, as pointed by Berman and Trevisan [68], the equation of state of the present universe is very near $p = -\rho$, which suggests that an inflationary scenario with an exponential scale factor could not only be of importance in the early universe but could also for the present accelerating universe because the deceleration parameter is negative, as required by modern developments in the observational data: Riess et al. [39] found evidence for an accelerating universe with an observed deceleration parameter near (-1) by means of Supernovae observation.

The spatial volume increases with time, representing an inflationary scenario. Since $\sigma/\theta \neq 0$, an anisotropy is maintained, however, the model isotropizes at late times. The vacuum energy density (Λ) decreases with time for a barotropic fluid distribution, a stiff fluid distribution and a radiation dominated model. The deceleration parameter $q < 0$ for a barotropic fluid, a dust distribution, a radiation dominated model, representing an accelerated universe, but for a stiff fluid distribution $q > 0$, which represents a decelerating universe. The scalar field decreases with time as the universe expands. The bulk viscosity prevents the matter density to vanish, it remains finite as $T \to \infty$. For a dust-filled universe, the model (35) not only represents an expanding universe, also an accelerating universe which matches with observations [69] since the deceleration parameter is $q < 0$. The model represents exponential expansion, it is initially anisotropic but becomes isotropic at late times. The vacuum energy density (Λ) decreases with time and it vanishes asymptotically. The results are valid as per astronomical observations. The entropy per unit volume is proportional to absolute temperature, and the energy conditions (weak, dominant and strong) have been discussed. The reality condition $\rho + p \geq 0$ is violated due to the scalar field ϕ, in agreement with an inflationary model. We have also discussed the importance of Bianchi Type V model for inflation, where the anisotropy dies away during the inflationary era. It happens not in the Dark energy era, it is at the end of inflation that isotropization starts. We have also calculated the inflationary parameters and made a comparison with the Planck data, and how the exit is compatible with anisotropy, and BAO estimation was also discussed. The questions of why the cosmological constant is a function of time and why the time dependence of Λ does not break general covariance are discussed. We have also calculated the anisotropy parameter, and discussed how the model isotropizes, where fluid goes after inflation and how viscosity may realize a graceful exit from inflation to a radiation dominated era.

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REFERENCES

1. K. Land and J. Magueijo, Phys. Rev. Lett. **95**, 071301 (2005).
2. C. B. Collins, Comm. Math. Phys. **39**, 131 (1974).
3. R. Maartens and S. D. Nel, Commun. Math. Phys. **59**, 273 (1978).
4. S. R. Roy and J. P. Singh, Astrophys. Space Sci. **96**, 303 (1983).
5. A. Banerjee and A. K. Sanyal, Gen. Rel. Grav. **20**, 103 (1988).
6. S. Ram, Int. J. Theor. Phys. **29**, 901 (1990).
7. A. A. Coley, Gen. Rel. Grav. **22**, 3 (1990).
8. R. Bali and D. K. Singh, Astrophys. Space Sci. **300**, 387 (2005).
9. R. Bali and P. Kumawat, Phys. Lett. B **665**, 332 (2008).
10. A. H. Guth, Phys. Rev. D **23**, 347 (1981).
11. B.A. Bassett, S. Tsujikawa and D. Wands, Rev. Mod. Phys. **78**, 537 (2006).
12. R. Myrzakulov and L. Sebastiani, Astrophys. Space Sci. **356**, 205 (2015).
13. A. Linde, Phys. Lett. B **108**, 389 (1982).
14. A. Albrecht and P. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
15. F. C. Adams, J. R. Bond, K. Freese, J. A. Friedman, and A. V. Orinto, Phys. Rev. D **47**, 426 (1993).
16. M. Sharif and S. Mohsaneen, Astrophys. Space Sci. **357**, 117 (2015).
17. S. Hervik, D. F. Mota, and M. Thorsrud, JHEP **146**, 11 (2011).
18. R. M. Wald, Phys. Rev. D **28**, 2118 (1983).
19. J. D. Barrow, Phys. Lett. B **187**, 12 (1987).
20. A. B. Burd and J. D. Barrow, Nucl. Phys. B **308**, 929 (1998).
21. D. La and P. J. Steinhardt, Phys. Rev. Lett. **62**, 367 (1989).
22. T. Rothman and G. F. R. Ellis, Phys. Lett. B **180**, 19 (1986).
23. R. Bali and V. C. Jain, Pramana J. Phys. **59**, 1 (2002).
24. R. Bali, Int. J. Theor. Phys. **50**, 3043 (2011).
25. M. Heller and Z. Klimek, Astrophys. Space Sci. **33**, PL 37 (1975).
26. L. P. Chimento et al., Class Quant. Gravity **14**, 3363 (1997).
27. O. Gron, Astrophys. Space Sci. **173**, 191 (1990).
28. W. Zimdahl, Phys. Rev. D **53**, 54831 (1996).
29. I. Brevik and S. V. Petterson, Phys. Rev. D **56**, 3322 (1997).
30. I. Brevik and S. V. Petterson, Phys. Rev. D **61**, 127305 (2000).

31. C. P. Singh, S. Kumar, and A. Pradhan, Int. J. Mod. Phys. D **9**, 393 (2000).
32. V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000).
33. B. Saha, Mod. Phys. Lett. A **20**, 2117 (2005).
34. R. Bali and A. Pradhan, Chin. Phys. Lett. **24**, 585 (2007).
35. R. Bali, P. Singh and J. P. Singh, Astrophys. and Space-Science, **341**, 701 (2012).
36. I. Brevik and O. Gron, in *Recent Advances in Cosmology* (Nova Science, New York 2013), p. 97.
37. Ya. B. Zel'dovich, Sov. Phys. Usp. **11**, 381 (1968).
38. L. M. Krauss and M. S. Turner, Gen. Rel. Grav. **27**, 1137 (1995).
39. A. Riess et al., Astron. J. **116**, 1009 (1998).
40. S. Perlmutter et al., Astroph. J. **517**, 565 (1999).
41. J. V. Narlikar, R. G. Vishwakarma, and G. Burbidge, J. Astron. Soc. Pacific **114**, 1092 (2002).
42. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
43. O. Bertolami, Nuovo Cim. B **93**, 36 (1986).
44. W. Chen and Y. S. Wu, Phys. Rev. D **41**, 695 (1990).
45. I. A. Arbab, Gen. Rel. Grav. **29**, 61 (1997).
46. P. Wang and X. H. Meng, Class. Quantum Grav. **22**, 285 (2005).
47. S. Ram and M. K. Verma, Astroph. Space Sci. **330**, 151 (2010).
48. J. D. Barrow and D. J. Shaw, Gen Rel. Grav. **27**, 1137 (2011).
49. R. Bali and S. Singh, Canad J. Phys. **92**, 1 (2014).
50. J. A. Stein-Schabes, Phys. Rev D **35**, 2345 (1987).
51. Y. Gong, B. Wang, and A. Wang, JCAP **0701**, 024 (2007).
52. Y. S. Myung, Phys. Lett. B **671**, 216 (2009).
53. A. Kolassis, N. O. Santos, and S. D. Odinstov, Phys. Rev. D **84**, 103508 (2011).
54. S. Chatterjee and A. Banerjee, Gen. Rel. Grav. **36**, 303 (2004).
55. S. Unnikrishnan and V. Sahni, arXiv: 1305.5260.
56. W. Hu, Phys. Rev. D **84**, 027303 (2011).
57. J. Garriga and V. F. Mukhanov, Phys. Lett. B **458**, 219 (1999).
58. S. Unnikrishnan, V. Sahni, and A. Toporensky, JCAP **1208**, 018 (2012).
59. P. A. R. Ade et al., "Planck 2013 results," arXiv: 1303.5082.
60. W. Chen et al., JCAA 0701 (2007).
61. S. Nojiri and S. D. Odintsov, Phys. Rev. D **72**, 023003 (2005).
62. L. G. Jensen and J. A. Stein-Schabes, Phys. Rev. D **34**, 931 (1986).
63. G. N. Remmen and S. M. Carroll, Phys. Rev. D **90**, 063517 (2014).
64. K. S. Thorne, Astrophys. J. **148**, 51 (1967).
65. C. B. Collins, E. N. Glass, and D. A. Wilkinson, Gen. Rel. Grav. **12**, 805 (1980).
66. A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large Scale Structure* (Cambridge Univ. Press, 2000).
67. K. Maeda and N. Ohta, Phys. Rev. D **71**, 063520 (2005).
68. M. S. Berman and L. A. Trevisan, gr-qc/0207051.
69. A. G. Riess et al., Astrophys. J. **607**, 665 (2004).