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Can Gravity Distinguish between Dirac and Majorana Neutrinos?

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Abstract—The interaction of neutrinos with gravitational fields in the weak field regime at one loop to the leading order has been studied by Menon and Thalappilil. They deduced some theoretical differences between the Majorana and Dirac neutrinos. Then they proved that, in spite of these theoretical differences, as far as experiments are concerned, they would be virtually indistinguishable. We study the interaction of neutrinos with weak gravitational fields to the second order (at two loops). We show that there appear new neutrino gravitational form factors which were absent in the first-order calculations, so from a theoretical point of view there are more differences between the two kinds of neutrinos than in the first order, but we show that likewise they are indistinguishable experimentally.

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1. INTRODUCTION

The whole of the last century of physics is recognized by two main theories: Quantum Mechanics and Relativity. The physical phenomena in which gravitational and quantum effects appear simultaneously are very interesting from both theoretical and experimental points of view [1–13]. There has been an extensive research in theoretical physics which brought out an unexpected interplay between general relativity and quantum field theory. On the other hand, many attempts have been made to see whether there are some novel experimental or observational ways of studying quantized fields coupled to curved space-time. Interaction of quantum particles with gravitational fields is one the interesting subjects at the interface of quantum mechanics and general relativity. The neutrino is one of the most mysterious and interesting particles in the universe. Neutrino physics is one of the most important fields of research in high energy physics, astrophysics and cosmology, see [14, 15] for recent reviews. Interaction of neutrinos with gravitational fields and the distinguishability between Dirac and Majorana neutrinos are very exciting issues in neutrino physics.

The graviton-neutrino vertex to the first order (1 loop) has been studied in [16] using general symmetry principles. The authors tried to understand how the Majorana and Dirac neutrinos could be different as far as the gravitational interaction is concerned. They found that, in spite of theoretical differences, the Majorana and Dirac neutrinos cannot be experimentally distinguished by gravitational interaction. It is worth mentioning that neutrino form factors have also been studied in [17, 18]. In this work we study the graviton-neutrino vertex and gravitational neutrino transition form factors to the second order, i.e., in 2 loops. We show that, from a theoretical perspective, the second-order calculations reveal more differences between Dirac and Majorana neutrinos than the first order, but we also show that they are indistinguishable as far as experiments are concerned.

It has been shown in [19] that the spin-gravity interaction can distinguish between Dirac and Majorana neutrino wave packets propagating in a Lense-Thirring background, but it was pointed out in [20] that the treatment of the Majorana neutrino in [19] is not valid, so the claim stated in [19] does not follow, see also [21]. In [22–29] one can find some papers on the possibilities of distinguishing Dirac from Majorana neutrinos b ut not in the gravitational field.

2. TWO-LOOP CALCULATIONS

For Dirac neutrinos, there are eighteen 2-loop graviton-neutrino vertices which are shown in Figs. 1–18.

The Feynman amplitudes of these diagrams are presented below. The energy-momentum four-vector of the graviton propagator for diagrams 4 and 11 is k', and for the rest of the diagrams it is p''. The Feynman rules for the gravitational interactions with Standard Model (SM) fields that are relevant to our study are presented in Appendix A. We start from the Feynman amplitude of diagram 1:

$$\bar{u}_j(p')(i\Delta J^{(1)}_{AB})u_i(p) \sim \sum_{l=e,\mu,\tau} \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4p''}{(2\pi)^4}$$

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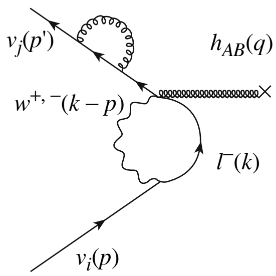


Fig. 1.

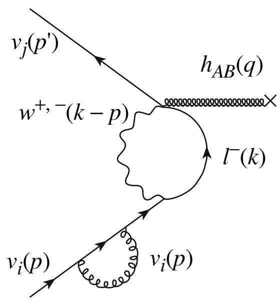


Fig. 2.

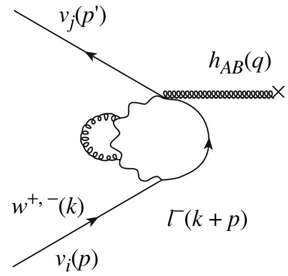


Fig. 3.

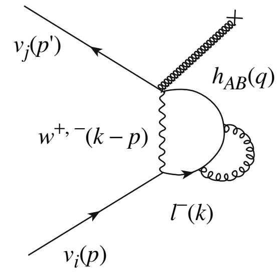


Fig. 4.

$$\times (\bar{u}_j(p'))\bigg(-\frac{i\mathrm{k}}{8}[\gamma_{\{\mu'}(2p'-p'')_{v'\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu'v'}[2\not{p}'-\not{p}''-2m_v]\bigg)$$
$$\times \bigg((\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{v'\mu}-\eta^{\mu'v'}\eta^{\mu v})\times\frac{i}{2p''^2}\bigg)$$
$$\times \bigg(i\frac{\not{p}'-\not{p}''+m_v}{(p'-p'')^2-m_v^2}\bigg)\bigg(-\frac{i\mathrm{k}}{8}[\gamma_{\{\mu}(2p'-p'')_{v\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu v}[2\not{p}'-\not{p}''-2m_v]\bigg)\bigg(i\frac{\not{p}'+m_v}{p'^2-m_v^2}\bigg)$$
$$\times \bigg(\frac{i\mathrm{k}g}{2\sqrt{2}}\Gamma_{AB\rho\beta}\gamma^{\beta}P_LU^*_{lj})\bigg)\bigg(i\frac{\not{k}+m_l}{k^2-m_l^2}\bigg)$$
$$\times\bigg(-i\frac{\eta^{\rho\sigma}}{(k-p)^2-m_w^2}\bigg)\bigg(\frac{ig}{\sqrt{2}}\gamma_\sigma P_L U_{li}\bigg)(u_i(p)),$$

where

$$\Gamma_{\mu v\rho\lambda} = \bigg[\eta_{\mu v}\eta_{\rho\lambda}-\frac{1}{2}(\eta_{\mu\rho}\eta_{v\lambda}+\eta_{\mu\lambda}\eta_{v\rho})\bigg],$$

the curly brackets $\{\}$ denote complete symmetrization of the indices (see Appendix C); the notation $\not{p}$ means $\not{p}=\gamma_\mu p^\mu = p_\mu\gamma^\mu$, where the Dirac γ matrices satisfy the anticommutation relation $\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu = 2g^{\mu\nu}$.

Figure 2: The amplitude of this diagram can be derived from that of diagram (1) by some minor changes.

Figure 3: The Feynman amplitude of this diagram is as follows :

$$\bar{u}_j(p')(i\Delta J^{(3)}_{AB})u_i(p) \sim \sum_{l=e,\mu,\tau}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times(\bar{u}_j(p'))\bigg(\frac{i\mathrm{k}g}{2\sqrt{2}}\Gamma_{AB\beta\lambda}\gamma^\lambda P_LU^*_{lj}\bigg)$$
$$\times\bigg(\frac{\not{k}+\not{p}+m_l}{(k+p)^2-m_l^2}\bigg)\bigg(-i\frac{\eta^{\beta\alpha'}}{k^2-m_w^2}\bigg)\times\Gamma'_{\mu'v'\alpha'\rho'}$$
$$\times\frac{i}{2p''^2}(\eta^{\mu'\mu}\eta^{vv'}+\eta^{\mu'v}\eta^{\mu v'}-\eta^{\mu'v'}\eta^{\mu v})$$
$$\times\bigg(-i\frac{\eta^{\rho'\alpha}}{(k-p'')^2-m_w^2}\bigg)\times\Gamma'_{\mu v\alpha\rho}$$
$$\times\bigg(-i\frac{\eta^{\sigma\rho}}{k^2-m_w^2}\bigg)\bigg(\frac{ig}{\sqrt{2}}\gamma_\sigma P_LU_{li}\bigg)(u_i(p)),$$

where

$$\Gamma'_{\mu'v'\alpha'\rho'} = \frac{i\mathrm{k}}{2}m_w^2[\eta_{\mu'v'}\eta_{\alpha'\rho'}-\eta_{\mu'\rho'}\eta_{\alpha'v'}-\eta_{v'\rho'}\eta_{\mu'\alpha'}]$$
$$-\frac{i\mathrm{k}}{2}[\eta_{\mu'v'}((k-p'')\cdot k\eta_{\alpha'\rho'}-k_{\alpha'}(k-p'')_{\rho'})$$
$$+(k-p'')_{\{\mu'}k_{\alpha'}\eta_{\rho'v'\}}+(k-p'')_{\rho'}k_{\{v'}\eta_{\mu'\}\alpha'}$$
$$-(k-p'')_{\{\mu'}k_{v'\}}\eta_{\rho'\alpha'}-k(k-p'')\eta_{\{\mu'\alpha'}\eta_{v'\}\rho'}].$$

Figure 4: For the Feynman amplitude of this dia-

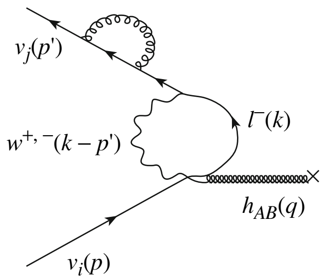


Fig. 5.

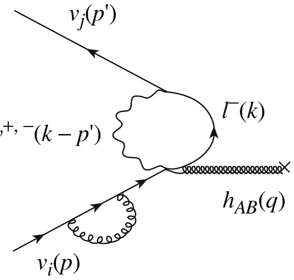


Fig. 6.

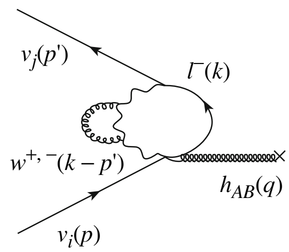


Fig. 7.

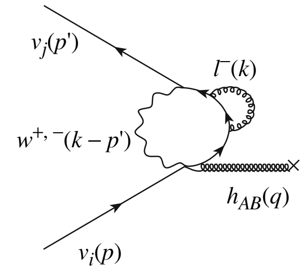


Fig. 8.

gram we have

$$\bar{u}_j(p')(i\Delta J^{(4)}_{AB})u_i(p) \sim \sum_{l=e,\mu,\tau}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4} \times(\bar{u}_j(p'))\left(\frac{i\mathrm{k}g}{2\sqrt{2}}\Gamma_{AB\rho\beta}\gamma^{\beta}P_L U^*_{lj}\right)\left(i\frac{\not{k}+m_l}{k^2-m_l^2}\right) \times\left(-\frac{i\mathrm{k}}{8}[\gamma_{\{\mu'}(2k-k')_{v'\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu'v'}[2\not{k}-\not{k}'-2m_v]\right) \times\left(\frac{1}{2}(\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{v'\mu}-\eta^{\mu'v'}\eta^{\mu v})\times\frac{i}{k'^2}\right) \times\left(i\frac{\not{k}-\not{k}'+m_l}{(k-k')^2-m_l^2}\right)\left(-i\frac{\mathrm{k}}{8}[\gamma_{\{\mu}(2k-k')_{v\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu v}[2\not{k}-\not{k}'-2m_v]\right)\left(i\frac{\not{k}+m_l}{k^2-m_l^2}\right) \times\left(-i\frac{\eta^{\rho\sigma}}{(k-p')^2-m_w^2}\right)\left(\frac{ig}{\sqrt{2}}\gamma_{\sigma}P_L U_{li}\right)(u_i(p)).$$

Figure 5: The Feynman amplitude of this diagram is given by

$$\bar{u}_j(p')(i\Delta J^{(5)}_{AB})u_i(p) \sim \sum_{l=e,\mu,\tau}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4} \times(\bar{u}_j(p'))\left(-\frac{i\mathrm{k}}{8}[\gamma_{\{\mu'}(2p'-p'')_{v'\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu'v'}[2\not{p}'-\not{p}''-2m_v]\right) \times\left((\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{v'\mu}-\eta^{\mu'v'}\eta^{\mu v})\times\frac{i}{2p''^2}\right) \times\left(i\frac{\not{p}'-\not{p}''+m_v}{(p'-p'')^2-m_v^2}\right)\left(-i\frac{\mathrm{k}}{8}[\gamma_{\{\mu}(2p'-p'')_{v\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu v}[2\not{p}'-\not{p}''-2m_v]\right)\left(i\frac{\not{p}'+m_v}{p'^2-m_v^2}\right) \times\left(\frac{ig}{\sqrt{2}}\gamma_{\sigma}P_L U^*_{li}\right)\left(i\frac{\not{k}+m_l}{k^2-m_l^2}\right) \times\left(-i\frac{\eta^{\rho\sigma}}{(k-p'^2)-m_w^2}\right)\left(\frac{i\mathrm{k}g}{2\sqrt{2}}(\eta_{AB}\eta_{\rho\beta} - \frac{1}{2}\eta_{A\beta}\eta_{B\rho}-\frac{1}{2}\eta_{A\rho}\eta_{B\beta})\gamma^{\beta}P_L U_{li}\right)(u_i(p)).$$

Figures 6, 7, 8: The amplitudes of these diagram can be derived from those of diagrams 5, 3, and 4, respectively, by some minor changes.

Figure 9: The Feynman amplitude of this diagram is

$$\bar{u}_j(p')(i\Delta J^{(9)}_{AB})u_i(p) \sim \sum_{l=e,\mu,\tau}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4} \times(\bar{u}_j(p'))\left(-\frac{i\mathrm{k}}{8}(\gamma_{\{\mu'}(2p'-p'')_{v'\}} - 2\eta_{\mu'v'}(2\not{p}'-\not{p}''-2m_v))\right)$$

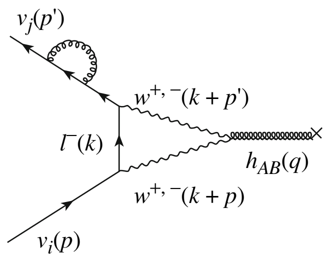


Fig. 9.

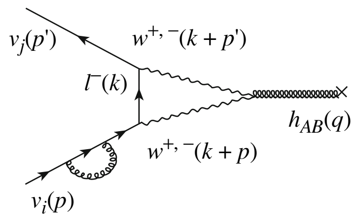


Fig. 10.

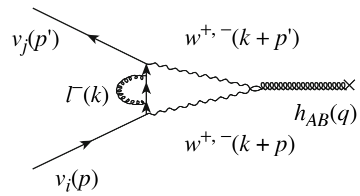


Fig. 11.

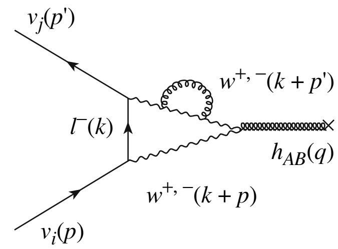


Fig. 12.

$$\times\left(\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{\mu v'}-\eta^{\mu'v'}\eta^{\mu v}\frac{i}{2p''^2}\right)$$

$$\times\left(i\frac{\not{p}'-\not{p}''+m_v}{(p'-p'')^2-m_v^2}\right)\left(-i\frac{\mathrm{k}}{8}(\gamma_{\{\mu}(2p'-p'')_{v\}}\right.$$

$$\left.-2\eta_{\mu v}(2\not{p}'-\not{p}''-2m_v))\right)\left(i\frac{\not{p}'+m_v}{p'^2-m_v^2}\right)$$

$$\times\left(\frac{ig}{\sqrt{2}}\gamma_\beta P_L U^*_{li}\right)\left(-i\frac{\eta^{\lambda\beta}}{(p'+k)^2-m_w^2}\right)$$

$$\times\,(\Gamma'_{AB\rho\lambda})\left(-i\frac{\eta^{\rho\alpha}}{(p+k)^2-m_w^2}\right)$$

$$\times\,\left(i\frac{\not{k}+m_l}{k^2-m_l^2}\right)\left(\frac{ig}{\sqrt{2}}\gamma_\alpha P_L U_{li}\right)(u_l(p)).$$

Figure 10: The amplitude of this diagram can be derived from that of diagram 9 by some minor changes.

Figure 11: The relevant Feynman amplitude is as follows:

$$\bar{u}_j(p')(i\Delta J^{(11)}_{AB})u_i(p)\sim\sum_{l=e,\mu,\tau}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k'}{(2\pi)^4}$$

$$\times\,(\bar{u}_j(p'))\left(\frac{ig}{\sqrt{2}}\lambda_\beta P_L U^*_{lj}\right)\left(-i\frac{\eta^{\lambda\beta}}{(k+p')^2-m_w^2}\right)$$

$$\times\,(\Gamma'_{AB\rho\lambda})\left(-i\frac{\eta^{\alpha\rho}}{(k+p)^2-m_w^2}\right)\left(i\frac{\not{k}+m_l}{k^2-m_l^2}\right)$$

$$\times\left(-i\frac{\mathrm{k}}{8}[\gamma_{\{\mu}(2k-k')_{v\}}]\right.$$

$$\left.-2\eta_{\mu v}[2\not{k}-\not{k}'-2m_l]\right)$$

$$\times\,\frac{i}{2k'^2}\left(\eta^{\mu\mu'}\eta^{vv'}+\eta^{\mu v'}\eta^{v\mu'}-\eta^{\mu v}\eta^{\mu'v'}\right)$$

$$\times\left(i\frac{\not{k}-\not{k}'+m_l}{(k-k')^2-m_l^2}\right)\left(-i\frac{\mathrm{k}}{8}([(2k-k')_{v'}]\right.$$

$$\left.\times-2\eta_{\mu'v'}[2\not{k}-\not{k}'-2m_l])\right)\left(i\frac{\not{k}+m_l}{k^2-m_l^2}\right)$$

$$\times\left(\frac{ig}{\sqrt{2}}\gamma_\alpha P_L U_{li}\right)(u_i(p)).$$

Figure 12: The corresponding Feynman amplitude is given by

$$\bar{u}_j(p')(i\Delta J^{(12)}_{AB})u_i(p)\sim\sum_{l=e,\mu,\tau}\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$

$$\times\,(\bar{u}_j(p'))\left(\frac{ig}{\sqrt{2}}\gamma_\beta P_L U^*_{lj}\right)\left(-i\frac{\eta^{\beta\beta'}}{(k+p')^2-m_w^2}\right)$$

$$\times\,(\Gamma'_{\mu'v'\sigma\beta'})\Big((\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{\mu v'}$$

$$-\,\eta^{\mu'v'}\eta^{\mu v})\frac{i}{2p''^2}\Big)$$

$$\times\left(-i\frac{\eta^{\sigma\sigma'}}{(p'+k-p'')^2-m_w^2}\right)(\Gamma'_{\mu v\sigma'\lambda'})$$

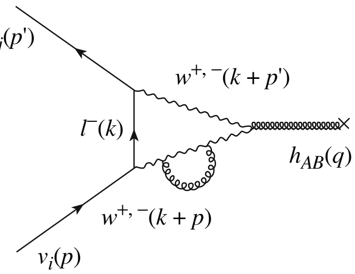


Fig. 13.

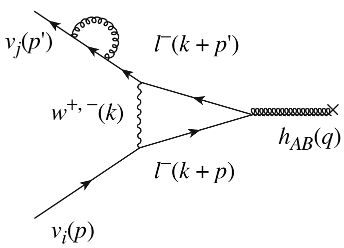


Fig. 14.

$$\times\Bigg(-i\frac{\eta^{\lambda\lambda'}}{(p'+k)^2-m_w^2}\Bigg)(\Gamma'_{AB\rho\lambda})$$
$$\times\Bigg(-i\frac{\eta^{\rho\alpha}}{(p+k)^2-m_w^2}\Bigg)\Bigg(i\frac{\not k+m_l}{k^2-m_l^2}\Bigg)$$
$$\times\Bigg(\frac{ig}{\sqrt{2}}\gamma_\alpha P_L U_{li}\Bigg)(u_i(p)).$$

Figure 13: The amplitude of this diagram can be derived from that of diagram 12 by some minor changes.

Figure 14: The Feynman amplitude of this diagram is given by the following expression:

$$\bar u_j(p')(i\Delta J^{(14)}_{AB})u_i(p)\sim\sum_{l=e,\mu,\tau}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times(\bar u_j(p'))\Bigg(\frac{i\mathrm{k}}{8}\gamma_{\{\mu'}(2p'-p'')_{v'\}}-2\eta_{\mu'v'}(2\not p'-\not p''$$
$$-2m_v)\Bigg)\Bigg((\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{v'\mu}-\eta^{\mu'v'}\eta^{\mu v})\frac{i}{2p''^2}\Bigg)$$
$$\times\Bigg(i\frac{\not p'-\not p''+m_v}{(p'-p'')^2-m_v^2}\Bigg)\Bigg(-\frac{i\mathrm{k}}{8}(\gamma_{\{\mu}(2p'-p'')_{v\}}$$
$$-2\eta_{\mu v}(2\not p'-\not p''-2m_v))\Bigg)\Bigg(-i\frac{\not p'+m_v}{p'^2-m_v^2}\Bigg)$$
$$\times\Bigg(\frac{ig}{\sqrt{2}}\gamma_\alpha P_L U^*_{lj}\Bigg)\Bigg(i\frac{\not k+\not p'+m_l}{(k+p'^2)-m_l^2}\Bigg)$$
$$\times\Bigg(-\frac{i\mathrm{k}}{8}(\gamma_{\{A}(2k+p+p')_{B\}})$$

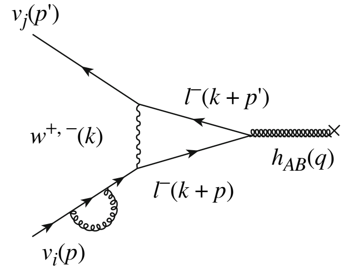


Fig. 15.

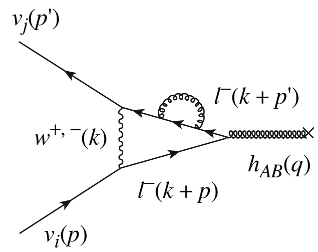


Fig. 16.

$$-2\eta_{AB}(2\not k+\not p+\not p'-2m_l)\Bigg)$$
$$\times\Bigg(i\frac{\not k+\not p+m_l}{(k+p)^2-m_l^2}\Bigg)\Bigg(-i\frac{\eta^{\rho\alpha}}{k^2-m_w^2}\Bigg)$$
$$\times\Bigg(\frac{ig}{\sqrt{2}}\gamma_\rho P_L U_{li}\Bigg)(u_i(p)).$$

Figure 15: The amplitude of this diagram can be derived from that of diagram 14 by some minor changes.

Figure 16: The Feynman amplitude of this diagram is

$$\bar u_j(p')(i\Delta J^{(16)}_{AB})u_i(p)\sim\sum_{l=e,\mu,\tau}\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times(\bar u_j(p'))\Bigg(\frac{ig}{\sqrt{2}}\gamma_\alpha P_L U^*_{lj}\Bigg(i\frac{\not k+\not p'+m_l}{(k+p'^2)-m_l^2}\Bigg)$$
$$\times\Bigg(-\frac{i\mathrm{k}}{8}(\gamma_{\{\mu'}(2p'+2k-p'')_{v'\}}$$
$$-2\eta_{\mu'v'}(2\not p'+2\not k-\not p''-2m_l))\Bigg)$$
$$\times(\eta^{\mu'\mu}\eta^{v'v}+\eta^{\mu'v}\eta^{\mu v'}-\eta^{\mu'v'}\eta^{\mu v})\Bigg)\Bigg(\frac{i}{2p''^2}\Bigg)$$
$$\times\Bigg(i\frac{\not p'+\not k-\not p''+m_l}{(p'+k-p'')^2-m^2}\Bigg)$$
$$\times\Bigg(-\frac{i\mathrm{k}}{8}(\gamma_{\{\mu}(2p'+2k-p'')_{v\}}$$

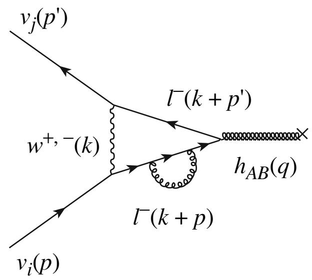


Fig. 17.

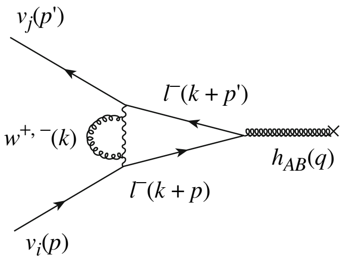


Fig. 18.

$$-2\eta_{\mu\nu}(2\not{p}'+2\not{k}-\not{p}''-2m_l))\Big)$$
$$\times\left(i\frac{\not{k}+\not{p}'+m_l}{(k+p')^2-m_l^2}\right)$$
$$\times\Big(-\frac{\mathrm{ik}}{8}(\gamma_{\{A}(p+p'+2k)_{B\}})$$
$$-2\eta_{AB}(\not{p}+\not{p}'+2\not{k}-2m_l)\Big)\left(i\frac{\not{k}+\not{p}+m_l}{(k+p)^2-m_l^2}\right)$$
$$\times\left(\frac{-i\eta^{\rho\alpha}}{k^2-m_w^2}\right)\times\left(\frac{ig}{\sqrt{2}}\gamma_\rho P_L U_{li}\right)(u_i(p).$$

Figure 17: The amplitude of this diagram can be derived from that of diagram 16 by some minor changes.

Figure 18: And finally the Feynman amplitude of diagram 18 is given by

$$\bar{u}_j(p')(i\Delta J_{AB}^{(18)})u_i(p)$$
$$\sim\sum_{l=e,\mu,\tau}\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}(\bar{u}_j(p'))\left(\frac{ig}{\sqrt{2}}\gamma_\alpha P_L U_{lj}^*\right)$$
$$\times\left(i\frac{\not{k}+\not{p}'+m_l}{(k+p'^2)-m_l^2}\right)\Big(-\frac{\mathrm{ik}}{8}(\gamma_{\{A}(p+p'+2k)_{B\}})$$
$$-2\eta_{AB}(\not{p}+\not{p}'+2\not{k}-2m_l)\Big)\left(i\frac{\not{k}+\not{p}+m_l}{(k+p)^2-m_l^2}\right)$$
$$\times\left(-i\frac{\eta^{\alpha\alpha'}}{k^2-m_w^2}\right)(\Gamma'_{\alpha'\sigma'\mu'\nu'})(\eta^{\mu'\mu}\eta^{\nu'\nu}+\eta^{\mu'\nu}\eta^{\mu\nu'}$$
$$-\eta^{\mu'\nu'}\eta^{\mu\nu})(\frac{i}{2p''^2})\left(-i\frac{\eta^{\sigma\sigma'}}{(k-p'')^2-m_w^2}\right)(\Gamma'_{\mu\nu\rho\sigma})$$
$$\times\left(-i\frac{\eta^{\rho\rho'}}{k^2-m_w^2}\right)\left(\frac{ig}{\sqrt{2}}\gamma_{\rho'}P_L U_{li}\right)(u_i(p)).$$

We have calculated the invariant Feynman amplitude for all diagrams, but due to a large number of calculations we do not present their details for all diagrams, but instead we present some calculations of two typical diagrams and write down only the final results for the rest of them. Our purpose is to find what form factors are produced by individual diagrams. Let us consider the Feynman amplitude of diagram 1. Multiplication of the numerator of the second and third parentheses regardless of the constant coefficients gives:

$$(4\gamma^\mu p'^\nu-2\gamma^\mu p''^\nu+4\gamma^\nu p'^\mu-2\gamma^\nu p''^\mu$$
$$-8\eta^{\mu\nu}m_\nu+4\not{p}'\eta^{\mu\nu}-2\not{p}''\eta^{\mu\nu}),\qquad(1)$$

and for the case of the fourth and fifth parentheses we have

$$(2\not{p}'\gamma_\mu p'_\nu-2\not{p}''\gamma_\mu p'_\nu+2m_\nu\gamma_\mu p'_\nu+2\not{p}'\gamma_\nu p'_\mu$$
$$-2\not{p}''\gamma_\nu p'_\mu+2m_\nu\gamma_\nu p'_\mu-\not{p}'\gamma_\mu p''_\nu+\not{p}''\gamma_\mu p''_\nu$$
$$-m_\nu\gamma_\mu p''_\nu-\not{p}'\gamma_\nu p''_\mu+\not{p}''\gamma_\nu p''_\mu-m_\nu\gamma_\nu p''_\mu$$
$$-4\not{p}'\eta_{\mu\nu}\not{p}'+4\not{p}''\eta_{\mu\nu}\not{p}'+2\not{p}'\not{p}''\eta_{\mu\nu}$$
$$-2\not{p}''\not{p}''\eta_{\mu\nu}+2\not{p}''\eta_{\mu\nu}m_\nu$$
$$-4\eta_{\mu\nu}m_\nu\not{p}''+4\eta_{\mu\nu}m_\nu^2).\qquad(2)$$

Multiplying the numerators of the parentheses 7–10 in the same amplitude, regardless of the constant coefficients, we have :

$$(\Gamma_{AB\rho\beta}\gamma^\beta P_L)(\not{k}+m_l)(\eta^{\rho\sigma})(\gamma_\sigma P_L)$$
$$=\left(\eta_{AB}\eta_{\rho\beta}-\frac{1}{2}\eta_{A\beta}\eta_{B\rho}-\frac{1}{2}\eta_{A\rho}\eta_{B\beta}\right)\gamma^\beta$$
$$\times P_L(\not{k}+m_l)(\eta^{\rho\sigma})(\gamma_\sigma P_L)$$
$$=\left(\eta_{AB}\gamma_\rho-\frac{1}{2}\gamma_A\eta_{B\rho}-\frac{1}{2}\eta_{A\rho}\gamma_B\right)(P_L)$$
$$\times(\not{k}+m_l)(\eta^{\rho\sigma})(\gamma_\sigma P_L)$$
$$=\left(\eta_{AB}\gamma^\sigma-\frac{1}{2}\gamma_A\eta_B^\sigma-\frac{1}{2}\gamma_B\eta_A^\sigma\right)(\not{k})\gamma_\sigma P_L.\qquad(3)$$

But:

$$\gamma_A\not{k}=\gamma_A k^\alpha\gamma_\alpha=k^\alpha(2\eta_{A\alpha}-\gamma_\alpha\gamma_A)=2k_A-\not{k}\gamma_A.$$

So Eq. (3) takes the following form:

$$-2\eta_{AB}\not{k}P_L-\frac{1}{2}(2k_A-\not{k}\gamma_A)\gamma_B P_L$$
$$-\frac{1}{2}(2k_B-\not{k}\gamma_B)\gamma_A P_L$$
$$=-2\eta_{AB}\not{k}P_L-(k_A\gamma_B+k_B\gamma_A)P_L$$

$$+\frac{1}{2}\,\not{k}(\gamma_A\gamma_B+\gamma_B\gamma_A)P_L$$
$$=-2\eta_{AB}\,\not{k}P_L-(k_A\gamma_B+k_B\gamma_A)P_L+\not{k}(\eta_{AB})P_L$$
$$=-\eta_{AB}\,\not{k}P_L-(\gamma_A k_B+\gamma_B k_A)P_L. \quad (4)$$

If we multiply parentheses 6–10, we get:

$$-\not{p}'\eta_{AB}\not{k}P_L-m_v\eta_{AB}\not{k}P_L-\not{p}'(\gamma_A k_B+\gamma_B k_A)P_L$$
$$-m_v(\gamma_A k_B+\gamma_B k_A)P_L. \quad (5)$$

We now calculate some typical integrals appearing in the calculations of diagram 1. By multiplying the first term of (1) by the first term of (2) and then multiplying the result by (5) and integrating over p'' and k, we have:

$$(4\gamma^\mu p'^v)\times(2\not{p}'\gamma_\mu p'_v)=-16\not{p}'p'^2, \quad (6)$$

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\not{p}'p'^2)\times(-\not{p}'\not{k}P_L\eta_{AB})}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p)^2-m_w^2)}$$
$$=16\not{p}p'^4B_0B_1P_L\eta_{AB}, \quad (7)$$

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\frac{(-16\not{p}'p'^2)\times(-m_v\not{k}P_L\eta_{AB})}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p)^2-m_w^2)}$$
$$=16m_vp'^2\,\not{p}'\,\not{p}P_L\eta_{AB}B_0B_1. \quad (8)$$

The coefficients B_0 and B_1 are defined in the Appendix B. These integrals generate no form factors. The integral

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\not{p}'p'^2)\times(-\not{p}'(\gamma_A k_B+\gamma_B k_A)P_L)}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p)^2-m_w^2)}$$
$$=16p'^4\times B_0B_1(\gamma_A p_B+\gamma_B p_A)P_L, \quad (9)$$

and the corresponding integral of diagram 5 generate the form factors E_3 and D_3. The definitions of the form factors are presented in Appendix C.

The following integral:

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\not{p}'p'^2)\times(-m_v(\gamma_A k_B+\gamma_B k_A)P_L)}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p)^2-m_w^2)}$$
$$=16p'^2m_v\not{p}'\times B_0B_1(\gamma_A p_B+\gamma_B p_A)P_L \quad (10)$$

also produces no form factors.

Now we study the Feynman amplitude of diagram 5 which we may call a corresponding diagram to diagram 1. Comparing the invariant amplitudes of diagrams 1 and 5, we observe that the parentheses 1–6 are the same in two amplitudes. Multiplying the numerators of parentheses 6–10, regardless of the constant coefficient, we have

$$-\not{p}'\not{k}P_L\eta_{AB}-m_v\not{k}P_L\eta_{AB}$$
$$-\not{p}'(\gamma_A k_B+\gamma_B k_A)P_L$$
$$-m_v(\gamma_A k_B+\gamma_B k_A)P_L. \quad (11)$$

Now let us calculate some typical integrals appearing in the diagram 5. Multiplying the first term of (1) by the first term of (2) and then multiplying the result by (11) and integrating over p'' and k, we have:

$$(4\gamma^\mu p'^v)\times(2\,\not{p}'\gamma_\mu p'_v)=-16\not{p}'p'^2, \quad (12)$$

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\not{p}'p'^2)\times(-\not{p}'\not{k}P_L\eta_{AB})}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p')^2-m_w^2)}$$
$$=16\not{p}'p'^4B_0B_1P_L\eta_{AB}. \quad (13)$$

This integral produces no form factors;

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\not{p}'p'^2)\times(-m_v\not{k}P_L\eta_{AB})}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p')^2-m_w^2)}$$
$$=16m'_vp'^4P_L\eta_{AB}B_0B_1. \quad (14)$$

This integral gives us the E_1 and D_1 form factors:

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\not{p}'p'^2)\times(-\not{p}'(\gamma_A k_B+\gamma_B k_A)P_L)}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p')^2-m_w^2)}$$
$$=16p'^4\times B_0B_1(\gamma_A p'_B+\gamma_B p'_A)P_L. \quad (15)$$

This integral and the corresponding integral of diagram 1 generate the form factors E_3 and D_3. Finally, the integral

$$\int\frac{dk^4}{(2\pi)^4}\int\frac{d^4p''}{(2\pi)^4}$$
$$\times\frac{(-16\,\not{p}'p'^2)\times(-m_v(\gamma_A k_B+\gamma_B k_A)P_L)}{(p''^2)((p''-p')^2-m_v^2)(k^2-m_l^2)((k-p')^2-m_w^2)}$$
$$=16p'^2m_v\not{p}'\times B_0B_1(\gamma_A p_B+\gamma_B p_A)P_L \quad (16)$$

produces no form factors.

We have also calculated the Feynman amplitudes of all other diagrams, and the results are as follows:

All diagrams generate the form factors E_3 and D_3, in addition, diagrams 2, 5, 9, 10, 14, and 15 generate the form factors E_1 and D_1. Interaction of neutrinos with weak gravitational fields to the first order (1 loop) involves four diagrams, which produce the form factors E_3 and D_3. From a theoretical point of view, these form factors can be used to distinguish between Dirac and Majorana neutrinos, but, as shown in [16], they satisfy the following relations:

$$E_3^M(q^2) = E_3^D(q^2) + D_3^D(q^2), \quad D_3^M(q^2) = 0. \quad (17)$$

Interaction of neutrinos with gravitational fields to the second order (2 loops) involves 18 diagrams. We have shown that they produce form factors E_1, D_1, E_3, and D_3. The form factors E_1 and D_1 are new, i.e., they were absent in the first-order calculations, so from a theoretical point of view there are more differences between Dirac and Majorana neutrinos than in the first order. But corresponding to each Feynman diagram of Dirac neutrinos, there is an additional "charge conjugate" diagram for Majorana neutrinos, with $P_L \to P_R$ and $l^- \to l^+$. For the electromagnetic interaction, the coupling "e" (electron charge) changes its sign under charge conjugation, but in this case (gravitational interaction), we have [16]:

$$g_{\mu v} \cong \eta_{\mu v} + kh_{\mu v} + 0(h^2), \quad k = \sqrt{32\pi G},$$
$$\eta_{\mu v} = (1, -1, -1, -1), \quad (18)$$

where $h_{\mu v}$ is the spin-2 graviton, the couling k does not change its sign under charge conjugation, so it is the same for Majorana and Dirac neutrinos. On the other hand, noting that $P_L = \frac{1}{2}(1-\gamma_5)$ and $P_R = \frac{1}{2}(1+\gamma_5)$, one can easily check that the axial vector parts γ_5 and $-\gamma_5$ cancel each other, so in the Majorana case the vertex factor does not have any terms proportional to γ_5, and therefore we do not have D form factors (for more information about form factors see Appendix C). On the other hand, the first terms in P_L and P_R do not cancel each other, so we have an E form factor for the Majorana neutrinos which is twice the same form factor for Dirac neutrinos. To be more clear, suppose in the Feynman amplitude we have a term proportional to P_L, like AP_L, where A is a cofficient (resulting from integration, see the calculations after diagram 18). As stated before, for Majorana neutrinos there is also a term AP_R in the amplitude, but for Dirac case there is no term proportional to P_R, so mathematically the above argument can be summarized as follows :

For Majorana neutrinos :

$$AP_L + AP_R = \frac{1}{2}A(1-\gamma_5) + \frac{1}{2}A(1+\gamma_5)$$
$$= \left[2 \times \frac{1}{2} + 0 \times \gamma_5\right]A.$$

For Dirac neutrinos : $P_R = 0$, so

$$AP_L + AP_R = \left[\frac{1}{2} + \frac{1}{2}\gamma_5\right]A.$$

D_i and E_i, $i = 1,3$ are the coefficients of the axial and non-axial parts of the amplitude, respectively (see Appendix C), so we have :

$$E_i^M = 2A, \quad E_i^D = A \to E_i^M = 2E_i^D,$$

$$D_i^M = 0, \quad i = 1,3$$

and

$$E_i^D = D_i^D = A.$$

So we arrive at the following relation:

$$E_i^M = E_i^D + D_i^D, \quad i = 1,3. \quad (19)$$

This is an important result that shows that, despite theoretical differences, we are not able to distinguish experimentally Majorana neutrinos from Dirac neutrinos interacting with gravity. Let us explain it more clearly by a simple example. Suppose we get the value 4 in an experiment for the matrix element $\bar{u}_i(p')(i\Delta J_{AB})u_i(p)$. But we know $4 = 2+2$, so experimentally one cannot distinguish whether we have obtained 4 (the left-hand side) or $2+2$ (the right-hand side). 4 and $(2+2)$ correspond to Majorana and Dirac neutrinos, respectively. Therefore, in spite of theoretical differences in the graviton vertex of the two cases, one would not be able to distinguish Majorana and Dirac neutrinos experimentally.

3. DISCUSSION

The equivalence principle (EP) is one of the cornerstones of general relativity. Considerable efforts have been made and are still being made to test the EP for antimatter.

There are some direct experiments and observations which indicate that the EP holds also for antimatter, and the interactions between matter and antimatter are the same as those between matter and itself, so matter and antimatter behave identically in the gravitational fields. The famous worldwide experiments to test the equivalence principle for antimatter (independent of its composition or structure) with very high precision are as follows:

(1) ALPHA—Antihydrogen Laser Physics Apparatus [30].

(2) AEGIS—Antihydrogen Experiment : Gravity, Interferometry, Spectroscopy [31].

(3) GBAR—Gravitational Behavior of Antihydrogen at Rest [32].

All these three facilities rely on the Antiproton Decelerator (AD) at CERN, but AEGIS and GBAR use beams of antihydrogen rather than trapped antihydrogen.

(4) AGE—Antimatter Gravity Experiment at Fermilab [33].

They made measurements directly testing both the EP and that matter and antimatter behave identically in the gravitational field of the Earth. Using the gravitationally coupled Dirac equation, it is shown in [34] that particles and antiparticles experience the same coupling to the gravitational field, including all relativistic quantum corrections of motion. Their investigations demonstrate the consistency of quantum mechanics with general relativity and suggest that any conceivable differences of the gravitational coupling of particles and antiparticles should be assigned to a "fifth force," not to any conceivable "modifications of the gravitational mass" of antiparticles versus particles.

On the other hand, in particular, there is an observational confirmation for neutrinos and antineutrinos. Based on data from the supernova SN 1987A, it is confirmed that the Einstein equivalence principle is valid for electronic neutrinos and their antiparticles [35].

So the experimental and observational data show that gravity cannot distinguish between matter and antimatter. We know that for Majorana neutrinos, the neutrinos and antineutrinos are the same but in the Dirac case they are different. If gravity cannot distinguish matter from antimatter, it cannot distinguish Majorana and Dirac neutrinos.

4. CONCLUSIONS

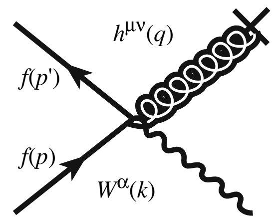
We have studied the graviton-neutrino vertex and gravitational neutrinos transition form factors to the second order, i.e., in 2 loops. We have shown that from a theoretical point of view the second-order calculations reveal more differences between Dirac and Majorana neutrinos than the first order. but we have also shown that they are indistinguishable as long as experiments are considered. As is well known, the most sensitive way to distinguish Majorana from Dirac neutrinos is the neutrinoless double beta decay, which is too far from the realm of gravity.

Appendix A

Consider some Feynman rules for gravitational interactions with SM fields that are relevant to the processes considered in this paper.

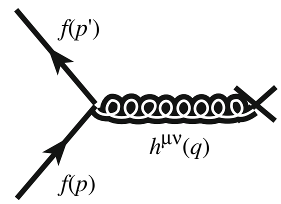
The Feynman rule for the graviton propagator is given by

$$P^{\mu v\alpha\beta}(q^2) = \frac{1}{2q^2}(\eta^{\mu\alpha}\eta^{v\beta} + \eta^{v\alpha}\eta^{\mu\beta} - \eta^{\mu v}\eta^{\alpha\beta}),$$



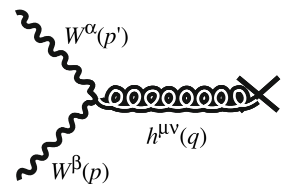
For the four-point gravition coupling, the Feynman rule is

$$i\frac{kg}{2\sqrt{2}}[\eta_{\mu v}\eta_{\alpha\beta} - \frac{1}{2}\eta_{\mu\beta}\eta_{v\alpha} - \frac{1}{2}\eta_{\mu\alpha}\eta_{v\beta}]\gamma^{\beta}P_L.$$



For gravition-fermion coupling

$$-\frac{i\mathrm{k}}{8}[\gamma_{\{\mu}(p+p')_{v\}}] + \frac{i\mathrm{k}}{4}\eta_{\mu v}[\not{p} + \not{p}' - 2m_f].$$



The Feynman rule is

$$i\mathrm{k}\frac{M_w^2}{2}[\eta_{\mu v}\eta_{\alpha\beta} - \eta_{\mu\alpha}\eta_{v\beta} - \eta_{v\alpha}\eta_{\mu\beta}]$$
$$- i\frac{\mathrm{k}}{2}[\eta_{\mu v}(p'\cdot p\eta_{\alpha\beta} - p_\alpha p'_\beta) + p'_{\{\mu}p_\alpha\eta_{\beta v\}}$$
$$+ p'_\beta p_{\{v}\eta_{\mu\}\alpha} - p'_{\{\mu}p_{v\}}\eta_{\alpha\beta} - (p'\cdot p)\eta_{\{\mu\alpha}\eta_{v\}\beta}].$$

Appendix B

The definition of B_0 is

$$B_0 = \int \frac{1}{p''^2((p''-p')^2 - m_v^2)}d^4p''$$
$$\times \int_0^1 dx \int \frac{1}{(p''^2-\alpha)^2}d^4p'' = \int_0^1 dx I_2(\alpha),$$

where

$$\alpha = x^2p'^2 - x(p'^2 - m_v^2),$$
$$I_n(\alpha) = \int d^Dq\frac{1}{(q^2-\alpha)^n}$$

$$= i(-1)^n \pi^{D/2} \frac{\Gamma(n - D/2)}{\Gamma(n)} \alpha^{D/2-n-\epsilon}.$$

Now we define B^ρ as

$$B^\rho = \int \frac{k^\rho}{(k^2 - m_\lambda^2)((k-p)^2 - m_w^2)} d^4k.$$

But

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{(a(1-x) + bx)^2},$$

$$a = k^2 - m_l^2, \quad b = (k-p)^2 - m_w^2$$

$$\Rightarrow \quad B^\rho = \int_0^1 dx \int_k k^\rho[(k^2 - m_l^2)(1-x) + ((k-p)^2 - m_w^2)x]^{-2}.$$

Introducing the new variables A and k' as

$$A = x^2p^2 - x(p^2 - m_w^2 + m_\lambda^2) + m_\lambda^2,$$
$$k' = k - xp,$$

B^ρ can be rewritten as follows:

$$B^\rho = \int_0^1 dx \int (k' + xp)^\rho (k'^2 - A)^{-2} d^4k'.$$

So,

$$B^\rho = \int_0^1 dx \int \frac{(k' + xp)^\rho}{(k'^2 - A)^2} d^4k' = \int_0^1 dx \int \frac{xp^\rho}{(k'^2 - A)^2} d^4k'.$$

Therefore we have

$$B^\rho = p^\rho \times B_1,$$

where

$$B_1 = \int_0^1 dx \int \frac{x}{(k'^2 - A)^2} d^4k' = \int_0^1 dx\, xI_2(A).$$

Appendix C

Here we present the definitions of the form factors. We denote by $\Gamma_{\mu v}(p,q)$ the gravitational vertex function of the neutrino which is symmetric in its indices. The gravitational gauge invariance implies that it satisfies the condition

$$q^\mu \bar{u}(p')\Gamma_{\mu v} u(p) = 0.$$

To write down the general form of the matrix element, we introduce its tensor and pseudotensor components as follows:

$$\bar{u}(p')\Gamma_{\mu v} u(p) = \bar{u}(p')[\Gamma'_{\mu v} + \Gamma''_{\mu v}\gamma^5]u(p). \qquad \text{(C.1)}$$

Lorentz invariance implies that the vertex function can in general have the following components:

$$g_{\mu v},\ p_\mu p_v,\ q_\mu q_v,\ \{pq\}_{\mu v},\ \{p\gamma\}_{\mu v},\ \{q\gamma\}_{\mu v},$$

where $q = p - p'$ and $\{pq\}_{\mu v} \equiv (p_{\{\mu} q_{v\}}) = p_\mu q_v + q_\mu p_v$. Therefore the tensor and pseudotensor (parity conserving and parity violating) components of the neutrino gravitational vertex in have general the following forms:

$$\bar{u}(p')_j \Gamma'_{\mu v} u(p)_i \cong \bar{u}(p')_j [E_1(q^2)(q^2 g_{\mu v} - q_\mu q_v) + E_2(q^2)(p_\mu p_v) + E_3(q^2)(q^2\{\gamma p\}_{\mu v} - \Delta_{ij} m_v \{pq\}_{\mu v})]u(p)_i + O(\Delta_{ij} m_v^2), \qquad \text{(C.2)}$$

$$\bar{u}(p')_j \Gamma''_{\mu v}\gamma^5 u(p)_i \cong \bar{u}(p')_j [D_1(q^2)\gamma^5(q^2 g_{\mu v} - q_\mu q_v) + D_2(q^2)\gamma^5(p_\mu p_v) + D_3(q^2)\gamma^5(q^2\{\gamma p\}_{\mu v} - \Sigma_{ij} m_v \{pq\}_{\mu v})]u(p)_i + O(\Delta_{ij} m_v^2), \qquad \text{(C.3)}$$

where

$$\Delta_{ij} m_v = m_{vj} - m_{vi},$$
$$\sum_{ij} m_v = m_{vj} + m_{vi},$$
$$\Delta_{ij} m_v^2 = m_{vj}^2 - m_{vi}^2.$$

According to Eq. (C.2), the coefficients of $(q^2 g_{\mu v} - q_\mu q_v)$, $(p_\mu p_v)$ and $(q^2\{\gamma p\}_{\mu v} - \Delta_{ij} m_v\{pq\}_{\mu v})$ are $E_1(q^2)$, $E_2(q^2)$, and $E_3(q^2)$, respectively.

It is also seen from Eq. (C.3) that the coefficients of $\gamma^5(q^2 g_{\mu v} - q_\mu q_v)$, $(p_\mu p_v)\gamma^5$ and $\gamma^5(q^2\{\gamma p\}_{\mu v} - \Sigma_{ij}\mu_v\{pq\}_{\mu v})$ are $D_1(q^2)$, $D_2(q^2)$, and $D_3(q^2)$, respectively.

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