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Gravitational Lensing of Twisted Electromagnetic Waves

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Abstract—The recent research in the field of classical and quantum optics has allowed for establishing that an electromagnetic wave possesses not only energy and momentum but also angular momentum. The wave front of an electromagnetic wave having an angular momentum is twisted with respect to the direction of motion. In the present paper, using the model of seven-dimensional space-time developed by the author, the motion of twisted electromagnetic waves in gravitational fields is considered. It is shown that the existence of an angular momentum of an electromagnetic wave leads to smaller values of the gravitational redshift than at zero angular momentum. The motion of electromagnetic waves near a massive object is also considered. It is shown that the gravitational deflection angle of an electromagnetic wave with a nonzero angular momentum is larger than in the case of zero angular momentum. The author believes that this approach will allow for explaining the gravitational lensing using only baryonic matter and also provide a new look at some problems of modern cosmology.

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1. INTRODUCTION

The gravitational lensing effect manifests itself in deformations of the images of background objects or in the emergence of multiple images of the same object [1, 2]. This effect, predicted by general relativity (GR) and confirmed by experiment in 1919, afterwards became a basis of modern methods of estimating the matter density of the sources of gravitational lensing [3]. As a rule, the gravitational lenses that substantially distort the image of a background object are very large mass concentrations, such as galaxies or clusters of galaxies [4]. Less massive objects, e.g., stars also bend the light beams but by small angles. Solving the inverse problem, that is, calculating the gravitational field necessary for obtaining such images, one can estimate the mass of the gravitational lens [5, 6]. As shown by calculations, in some clusters the resulting values of mass needed for explaining the gravitational lensing effect turn out to be much larger than the mass of visible matter [7–9]. Such a visible mass deficit is usually explained by the existence of invisible dark matter [10, 11] interacting with ambient matter only gravitationally.

But this explanation looks questionable. Thus, for instance, an investigation of motion of more than 400 stars located up to 13,000 light years from the Sun [12] did not reveal any indication of dark matter existence. Historically, the light beam bending in the gravitational field was already calculated in Newton's theory of gravity. However, the predicted deflection angle in the framework of Newton's theory has turned out to be smaller than the experimentally measured value by a factor of two. At the beginning of the 20th century, the scientists did not introduce any additional, invisible entities in order to explain the discrepancy between theory and experiment. Instead, the theory of GR was created, and in its framework a solution was found which completely explained the experimental data [13].

In the present paper we pose the problem of explaining the gravitational lensing (according to the observational data) without using the notion of dark matter, taking into account a twisted nature of light beams.

The paper by Allen et al. [14], published as early as in 1992, suggested schemes for creation and detection of twisted light and also expressed the idea that a light wave possesses an angular momentum. A twisted light wave differs from a usual one in that in a flat light wave all wave fronts follow each other, whereas in a twisted light wave the front is like a helix with the direction of wave propagation.

In 1995 [15] narrow twisted light beams were obtained experimentally. Technologically, this was realized with the aid of a special prism of variable thickness. This difference in thickness allows a certain part of the wave that passes through a thicker layer of the prism to lag behind with respect to the part of the wave that has passed through a thinner

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layer. Thus, at the output, different parts of the wave have different phases. Another method of obtaining twisted light waves [16] has been realized with the aid of diffraction gratings with dislocations.

The existence of an angular momentum of a light wave found a confirmation in the following experiment [17]: at the focus of a twisted laser beam, a microparticle was suspended, which, having absorbed the light, began to rotate. The direction of its rotation depended on the light twisting direction. At present, the development of light twisting technology has made it possible to obtain twisted waves [18] with photon energies up to 99 eV. Applications of twisted light are also broadly developing, for example, in quantum information theory, in micromachine control, in astrophysics and microscopy [19].

In 2010 Y. Thide and colleagues published a paper [20] describing a methodology able to determine the rotation characteristics of black holes by analyzing the orbital angular momentum of light passing close to the accretion disk.

The existence of energy, momentum and angular momentum of a light wave leads to the idea that twisted electromagnetic waves have six instead of three degrees of freedom, like any rigid body in mechanics. Therefore, in the present paper, for a description of the motion of twisted electromagnetic waves we will use the model of 7-dimensional space-time instead of a 4-dimensional one.

As shown in [21–23], to explain the dynamics of not only translational but also rotational motion of bodies in gravitational fields, one can use a 7-dimensional space-time, in which, in addition to time and three spatial coordinates, there are three coordinates describing the body's orientation in space: $x^4 = \varphi$, $x^5 = \psi$, $x^6 = \theta$—the Euler angles. The geodesic equations in 7-dimensional space-time make it possible to obtain not only the equations of translational motion but also those describing the rotational motion of a gyroscope [23].

2. THE MAIN PART

In empty flat 7-dimensional space-time [21–23], the metric has the form

$$\begin{aligned} g_{00} &= -g_{aa} = 1, \\ g_{45} &= g_{54} = -\frac{J_\omega \cos(\theta)}{m}, \\ g_{44} &= g_{55} = g_{66} = -\frac{J_\omega}{m}, \end{aligned} \tag{1}$$

where J_ω is the moment of inertia of a test body with respect to the axes of rotation, precession and nutation, m is the test body mass, $a = 1, 2, 3$. The paradoxicality of writing the metric tensor [24] which, contrary to the 4-dimensional metric tensors of GR, depends on the parameters of a test body (the moment of inertia to mass ratio) allows one to suppose that the space is not absolute, and the gravitational field depends on a test body placed in it. However, to date the only available method of detecting gravity is to measure the curvature of geodesic lines which are paths of the test bodies. Therefore we can only speak of the presence or absence of the gravitational field from the position of a moving test body. Such a concept leads to a reconsideration of the notion of relativity: not only motion becomes relative but even the space-time itself depends on the test body we are dealing with.

In [22, 23, 25], the Gravity Probe B experiment has been considered, and it has been shown that in 7-dimensional space-time the equality between the angular velocity of geodesic precession and the angular velocity of the Lense-Thirring effect is achieved only if the gravitational equations are written in the form

$$R_{\mu\nu} = k\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) + \Lambda_{\mu\nu}, \tag{2}$$

$$R_{\mu\nu} - \frac{1}{5}g_{\mu\nu}R = kT_{\mu\nu} + \left(\Lambda_{\mu\nu} - \frac{1}{5}g_{\mu\nu}\Lambda\right), \tag{3}$$

where $\Lambda_{\mu\nu}$ is the additional tensor of zero-point energy. The introduction of $\Lambda_{\mu\nu}$ is necessary because not all components $R_{\mu\nu}$ for the metric (1) turn to zero in the absence of matter ($T_{\mu\nu} = 0$). In what follows, the Latin indices run over the values from 1 to 6 while the Greek ones from 0 to 6.

To describe twisted electromagnetic waves, let us extend Maxwell's 4-dimensional theory of the electromagnetic field to 7-dimensional space-time. The interaction of particles with each other is described with the aid of a force field whose properties, unlike those in classical theory, will be characterized by the 7-vector A_μ. It will be afterwards called the 7-potential, whose components are functions of the coordinates, time, and the orientation angles [21]. The three spatial components of the 7-potential A^λ form a 3-dimensional vector called the vector potential of the field, the temporal component will be called the scalar potential $A^0 = \Phi$, and the three orientational components of the 7-potential will form the rotational potential of the field. The index lowering of the 7-potential A^λ will be carried out using the metric (1):

$$A_\mu = g_{\mu\lambda}A^\lambda.$$

The Lagrange function of a charged body in an electromagnetic field in 7-dimensional space-time has the form

$$L = -mc^2\sqrt{1-\frac{V^2}{c^2}-R_i^2\frac{\omega^2}{c^2}} - q\Phi + \frac{q}{c}(A_iV^i) + \frac{q}{c}(A_j\omega^j). \quad (4)$$

It is necessary to note that due to homogeneity and isotropy of space it can be assumed that the rotational part of the electromagnetic field potential can only depend on the angular coordinates while the spatial part depends on the linear coordinates.

The equations of motion of a charge in a given electromagnetic field are the Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial u^i} - \frac{\partial L}{\partial x^i} = 0, \quad (5)$$

where L is given by Eq. (4). The derivative with respect to velocity is the generalized momentum of the body,

$$p_i = \partial L/\partial u^i.$$

Thus the equations of motion can be written as

$$\frac{d}{dt}\left(p_k + \frac{q}{c}A_k\right) = \frac{q}{c}\partial_k\left(-c\Phi + (A_nu^n)\right), \quad (6)$$

Calculating the full time derivative of the 7-potential, after some transformation we obtain:

$$\frac{dp_k}{dt} = -\frac{q}{c}\frac{\partial A_k}{\partial t} - q\partial_k\Phi + \frac{q}{c}\left(u^n\varepsilon_{knl}\varepsilon_{sdh}g^{ls}g^{dm}g^{kf}\partial_mA_f\right), \quad (7)$$

where ε_{knl} are Levi-Civita-like symbols [26] in which the indices run over the values from 1 to 6. Since in the space under consideration one cannot multiply the spatial and angular coordinates, the structure of the Levi-Civita symbols looks as follows:

$$\varepsilon_{ikl} = \left\{\begin{matrix} -1 & (1,2,3);(2,3,1);(3,1,2); \\ & (4,5,6);(5,6,4);(6,4,5) \\ +1 & (3,2,1);(1,3,2);(2,1,3); \\ & (6,5,4);(4,6,5);(5,4,6) \\ 0 & \end{matrix}\right\} \quad (8)$$

The left-hand side of Eqs. (7) contains a time derivative of the body's momentum. Consequently, the right-hand side contains the force acting on the body. By analogy with the theory of electromagnetism, let us divide this force into two parts, the velocity-dependent one and the velocity-independent one.

The force of the first kind is the electric field strength:

$$E_k = -\frac{1}{c}\frac{\partial A_k}{\partial t} - \partial_k\Phi. \quad (9)$$

The force of the second kind is the magnetic field strength:

$$H_r = \varepsilon_{rdh}g^{dm}g^{hf}\partial_mA_f. \quad (10)$$

Unifying the parametric equations (9) and (10), we can obtain the Maxwell equations modified to 7 dimensions:

$$\varepsilon_{rdh}g^{dm}g^{hf}\partial_mE_f = -\frac{1}{c}\frac{\partial}{\partial t}H_r. \quad (11)$$

Multiplying (10) scalarly by the 7-dimensional nabla operator $\nabla = \partial_kdx^k$, we can obtain the second modified Maxwell equation:

$$g^{kh}\partial_kH_h = 0. \quad (12)$$

A product of the body's charge density ε by the velocity 7-vector u^λ [21] will be called the current density 7-vector:

$$j^\lambda = \rho u^\lambda. \quad (13)$$

Its three spatial components form the 3-dimensional current density while its rotational components form the charge rotation density.

Let us introduce the 7-dimensional electromagnetic field tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (14)$$

Finding the equations of motion from the least action principle, we obtain the equation

$$\partial_\mu F^{\nu\mu} = -\frac{4\pi}{c}j^\nu, \quad (15)$$

which is the second pair of Maxwell's equations written in a 7-dimensional form. Substituting different values of i and components of the tensor (14), we arrive at the equations

$$g^{km}\partial_kE_m = 4\pi\rho, \quad (16)$$

$$\varepsilon_{klm}g^{lh}g^{mf}\partial_hH_f = \frac{1}{c}\frac{\partial E_k}{\partial t} + \frac{4\pi}{c}j_k, \quad (17)$$

Eqs. (12) and (11) together with (16) and (17) determine the electromagnetic field in 7-dimensional space. In what follows we call them the 7-dimensional Maxwell equations.

Consider the 7-dimensional Maxwell equations in a space-time without charges, $\rho = 0$, and without currents, $j_k = 0$, then

$$g^{kh}\partial_kH_h = 0, \quad (18)$$

$$\varepsilon_{rdh}g^{dm}g^{hf}\partial_mE_f = -\frac{1}{c}\frac{\partial}{\partial t}H_r, \quad (19)$$

$$g^{km}\partial_k E_m = 0, \tag{20}$$

$$\varepsilon_{klm} g^{lh} g^{mf} \partial_h H_f = \frac{1}{c}\frac{\partial E_k}{\partial t}. \tag{21}$$

Let us take the 7-dimensional curl of Eq. (19):

$$\varepsilon_{eru} g^{rb} g^{us} \partial_b \varepsilon_{sdh} g^{dm} g^{hf} \partial_m E_f = -\frac{1}{c}\frac{\partial}{\partial t}\varepsilon_{eru} g^{rb} g^{us} \partial_b H_s;$$

using Eq. (21), one can write the wave equation

$$\varepsilon_{eru} g^{rb} g^{us} \partial_b \varepsilon_{sdh} g^{dm} g^{hf} \partial_m E_f = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2}. \tag{22}$$

We can transform this wave equation by raising the indices in the Levi-Civita symbols, which results in

$$\varepsilon_{eru}\varepsilon^{mfu} g^{rb} \partial_b \partial_m E_f = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2}.$$

The product $\varepsilon_{eru}\varepsilon^{mfu}$ forms a true sixth-rank tensor [27], which can be expressed as a combination of products of components of the unit tensor δ^i_k as follows:

$$\varepsilon_{eru}\varepsilon^{mfu} = \delta^f_e \delta^m_r - \delta^m_e \delta^f_r.$$

Using this transformation, we can bring the wave equation to the form

$$(\delta^f_e \delta^m_r - \delta^m_e \delta^f_r) g^{rb} \partial_b \partial_m E_f = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2},$$

and simplifying it, we obtain

$$g^{mb}\partial_m \partial_b E_e - \partial_e g^{mb} \partial_b E_m = -\frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2}.$$

The second term in the left-hand side of this equation is equal to zero due to the third Maxwell equation (20): $g^{mb}\partial_b E_m = 0$, and then the final form of the wave equation is

$$g^{mb}\partial_m \partial_b E_e + \frac{1}{c^2}\frac{\partial^2 E_e}{\partial t^2} = 0. \tag{23}$$

We will suppose that a component of the electric induction vector is a function of time and the spatial and rotational coordinates:

$$E = E_2(t, x^1, x^4).$$

In space-time with the Schwarzschild metric, Eq. (23) takes the form

$$-\left[\left(1 - \frac{2kM}{c^2 x^1}\right)\partial_1^2 + \frac{1}{R_i^2}\partial_4^2\right] E + \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0, \tag{24}$$

where R_i is the inertia radius of the electromagnetic wave, M is the mass of a body that creates the gravitational field, and k is Newton's gravitational constant. A solution of Eq. (24) in the case of a wave moving along the $x^1 = x$ axis and rotation around this axis with a phase change by the angle φ has the form

$$E(t, x, \varphi) = A \times \exp\left[-i\left(\omega t - \frac{2\pi}{\lambda}x - \frac{2\pi}{\mu}\varphi + \sigma\right)\right], \tag{25}$$

where ω is the cyclic frequency of the oscillations; $\lambda = cT$ is the wavelength; T is the oscillation period; $\mu = \Omega T$ is the wave rotation angle, i.e., the angle by which the phase of the wave turns for a single period of time; Ω is the angular velocity of phase rotation in the wave; σ is the initial phase.

Substituting (25) into (24), we obtain a relation connecting the wavelength, the oscillation frequency and the rotation frequency:

$$\frac{1}{\lambda^2}\left(1 + \frac{2kM}{c^2 x^1}\right) + \frac{1}{\mu^2 R_i^2} - \frac{\nu^2}{c^2} = 0.$$

Expressing the wave frequency from this relation, we obtain the dependence of the frequency on the wavelength and the rotation angle in the gravitational field:

$$\nu = \frac{c}{\lambda}\sqrt{1 - \frac{2kM}{c^2 x^1} + \frac{\lambda^2}{R_i^2 \mu^2}}. \tag{26}$$

As is evident from (26), the wave frequency becomes smaller if the wave moves in the gravitational field.

Now consider a dimensionless quantity, the gravitational redshift:

$$z_G = \frac{\nu_0 - \nu}{\nu},$$

where ν is the measured wave frequency and ν_0 is the laboratory wave frequency. Substituting the dependence (26) into z_G, we obtain the redshift value

$$z'_G = \frac{1}{\sqrt{1 - \frac{2kM}{c^2 x^1} + \frac{\lambda^2}{R_i^2 \mu^2}}} - 1. \tag{27}$$

In the Newtonian limit, in which

$$\frac{2kM}{c^2 x^1} - \frac{\lambda^2}{R_i^2 \mu^2} \ll 1,$$

the redshift can be calculated from the relation

$$z'_G \approx \frac{kM}{c^2 x^1} - \frac{\lambda^2}{2R_i^2 \mu^2}. \tag{28}$$

As is seen from Eq. (28), if the electromagnetic wave is not twisted, $\mu \to \infty$, assuming that $x^1 = R^*$, we obtain for the approximate value of the redshift the standard expression

$$z_G \approx \frac{kM}{c^2 R^*},$$

where R^* is the distance between the center of mass of the attracting body and the point at which the photon is emitted. If the electromagnetic wave is rotating, the redshift is always smaller than in the absence of rotation, $z'_G < z_G$.

As shown in [21], the Lagrange function of a free body in the metric (1) has the form

$$L = -mc^2 \sqrt{1 - \frac{V^2}{c^2} - R_i^2 \frac{\omega^2}{c^2}}.$$

We traditionally call the momentum and angular momentum of a rigid body the vectors whose components are the derivatives of the Lagrange function with respect to the corresponding velocity components. The momentum of a rigid body is

$$p^k = \frac{mV^k}{\sqrt{1 - V^2/c^2 - R_i^2 \omega^2/c^2}},$$

while its angular momentum is

$$p_\omega^n = \frac{J_\omega \omega^n}{\sqrt{1 - V^2/c^2 - R_i^2 \omega^2/c^2}},$$

where $J_\omega = m \cdot R_i^2$ is the body's moment of inertia. The energy of the body is calculated by the formula

$$E = \frac{mc^2}{\sqrt{1 - V^2/c^2 - R_i^2 \omega^2/c^2}}.$$

The full relativistic energy, momentum and angular momentum can be unified in the 7-dimensional momentum:

$$P_7 = \left(E/c,\ p^k,\ p_\omega^n / R_i^2 \right).$$

Let us find a relationship between the energy, momentum and angular momentum, and to do so, let us find the square of the 7-momentum of a rigid body. Since the 7-velocity squared is the velocity of light squared, the relationship of energy, momentum and angular momentum is

$$E^2 = m^2 c^4 + p^2 c^2 + p_\omega^2 c^2 / R_i^2.$$

For the description of a photon, the relationship of energy, momentum and angular momentum has the form

$$E^2 = p^2 c^2 + p_\omega^2 c^2 / R_i^2.$$

Substituting the photon energy in the form $E = h\nu$ and using Eq. (26), we obtain

$$p^2 + \frac{p_\omega^2}{R_i^2} = \frac{h^2}{\lambda^2} \left[\left(1 - \frac{2kM}{c^2 x^1} \right) + \frac{\lambda^2}{R_i^2 \mu^2} \right].$$

Simplifying the terms in the right-hand side, we obtain

$$p^2 + \frac{p_\omega^2}{R_i^2} = \frac{h^2}{\lambda^2} \left(1 - \frac{2kM}{c^2 x^1} \right) + \frac{h^2}{R_i^2 \mu^2}.$$

The first term in the right-hand side is the photon momentum squared:

$$p = \frac{h}{\lambda} \sqrt{1 - \frac{2kM}{c^2 x^1}}, \qquad (29)$$

which, as is evident from this relation, depends on the mass creating the gravitational field. The second term is the photon angular momentum squared:

$$p_\omega = \frac{h}{\mu}, \qquad (30)$$

which does not depend on the gravitational field.

The motion of a photon depends on the space-time curvature, therefore it deflects when passing near a massive object. The deflection angle of an equation wave passing near a nonrotating source of gravity, as follows from the equations of GR [13], is calculated by the formula

$$\alpha = \frac{4kM}{c^2 r_0}, \qquad (31)$$

where r_0 is the impact parameter of the photon trajectory, corresponding to the closest point of approach to the gravitating body.

On the other hand, the gravitational field of a nonrotating body affects the direction of the photon's angular momentum as it moves along a geodesic. This influence can be described by the equation

$$\frac{d\vec{p}_\omega}{dt_0} \approx -\frac{3}{2} (\vec{c} \times \vec{\nabla}\phi) \times \vec{p}_\omega - (\vec{c} \cdot \vec{\nabla}\phi) \cdot \vec{p}_\omega, \qquad (32)$$

where $\vec{c}$ is the velocity vector of the light beam which is tangent to the geodesic line. Consider the simplest case where the geodesic line $\vec{c}$ is perpendicular to the gradient of the gravitational potential $\vec{\nabla}\phi$. In this case Eq. (32) is simplified to give

$$\frac{d\vec{p}_\omega}{dt_0} \approx \left(-\frac{3}{2} (\vec{c} \times \vec{\nabla}\phi) \right) \times \vec{p}_\omega. \qquad (33)$$

This enables us to say that the angular momentum of the photon $\vec{p}_\omega$, remaining invariable by absolute value, changes its direction towards the center of the gravitational field source with an angular velocity equal to

$$\vec{\Omega} = -\frac{3}{2} (\vec{c} \times \vec{\nabla}\phi). \qquad (34)$$

Using the angular velocity (34), one can obtain that the light deflection angle from straight motion caused by the angular momentum is

$$\vec{\beta} = -\frac{3}{2}(\vec{c} \times \vec{\nabla}\phi) \cdot dt_0. \quad (35)$$

Since the direction of the angular momentum of a photon always coincides with the direction of its motion, it happens that a photon possessing a moment of inertia experiences a double deflection: the one caused by the space-time curvature due to the gravitational field (31) and the one caused by the deflection of the photon's angular momentum vector in the gravitational field (35). We obtain that the full deflection angle of light is equal to

$$\zeta = \alpha + \beta. \quad (36)$$

As follows from Eq. (35), the effect will be larger if the time of motion near the gravitational field source is larger. At small size of the source of gravity, e.g., at light propagation near the Sun, this effect is insignificant, $\zeta \approx \alpha$, due to a small time while light is moving near the source of gravity, $dt_0 \approx 0$. At motion near stellar clusters, light is subject to gravity for a longer time, and, as a result, the deflection effect for a rotating light beam is much larger, $\zeta > \alpha$.

3. CONCLUSION

Consider the difference between the electromagnetic wave deflection angle calculated by Eq. (36) and the classical deflection angle according to (31). We obtain the ratio

$$\frac{\triangle\alpha}{\alpha} \approx \frac{3}{8}\frac{c \cdot dt_0}{r_0}.$$

Assuming that $c \cdot dt_0 = l_0$ is the linear size of the gravitating body near which the electromagnetic wave is moving, we obtain

$$\frac{\triangle\alpha}{\alpha} \approx \frac{3}{8}\frac{l_0}{r_0}. \quad (37)$$

From Eq. (37) it follows that if the linear size of the body is much larger than the impact parameter, $l_0/r_0 \gg 1$, then the gravitational lensing will contain a significant addition to the usual electromagnetic wave deflection angle.

Thus if we assume that the beams passing through gravitational lenses possess a nonzero angular momentum, then they experience the additional deflection (37). This explanation can solve the missing mass problem in gravitational lensing using only baryonic matter as the source of gravity. And it should be noted that the value of the angular momentum p_ω of the electromagnetic wave, see (35), is insignificant, it can be arbitrarily small.

One of the reasons by which an electromagnetic wave can acquire an angular momentum is the inhomogeneous interstellar gas: while passing through it, different parts of an electromagnetic wave acquire different phases.

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