Evaluation of the Flow Measurement Performance of Compound Sharp-Crested Over-Fall Weirs and Optimized Discharge Measurement in Data Scarce Areas

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Abstract—The hydraulic relationships of flow through the weir of different common shapes were investigated in this study. The hydraulic performances of weirs were carried out experimentally, by using eight different types of weir models, and the effect of various shapes of the weir to specify different hydraulic parameters such as coefficient of discharge (*Cd*), weir geometric shape and size constants (*K*) and weir shape number (*n*), energy dissipation ratio (*E*%) for selecting best flow measuring weir type. The experimental work was carried out on a tilted rectangular flume with 7.5 m long, 0.30 m wide and 0.45 m deep with a discharge range 0.50— 20.95 L/s. The experimental results of this study showed that the dissipation energy corresponding to triangular notches is larger than the resulted from rectangular notches. For compound notches of rectangle and triangle, turbulence inflow increases as the angle of lower notch decreases. Thus, this type with an angle of ($\theta = 120^{\circ}$) is the most efficient between the studied models, which gives the average *Cd* of 0.82, the value of *K* is equal to 0.75 and *n* values of 1.96, and the dissipation of flow energy ratio of (35.86%) for strong hydraulic jump. Finally, depending on a statistical basis, this study derived empirical relationships to estimate the coefficient of discharge with acceptable values of the coefficient of determination.

Keywords: compound sharp-crested weir, discharge coefficient, open channel **DOI:** 10.1134/S0097807822030034

INTRODUCTION

The management of water resources has become a major challenge in recent years [15]. The efficient management of water supply, particularly in the arid regions of the world, is becoming more and more important as the water demand grows even greater with the world increasing population and as new sources of water become harder to find [18]. Therefore, water resources are one of our most vital commodities and they must be conserved by reducing the amounts of water lost through inefficient management. The world commission on Water (2000) estimates that water use will increase by 50% in the next 30 years. By 2025, about 4 billion people will live under the condition of severe water stress [7]. Therefore, the development of priority water measurement system and improvement of water management have essential and complementary roles in contributing to achieving sustainable growth and poverty reduction in developing countries like Ethiopia. One way of improving water management is through increasing accurate and efficient flow measurements [5]. Because essential part of water conservation is the accurate measurement and regulation of discharges by the use of appropriate measurement structures [10].

Optimum development of water resources for public use needs a high priority consideration for water resource management to the economic growth of the most developing countries [5]. The problem in developing countries is the management of water resources and measurement systems are also very poor and peoples may not give attention to the system [13]. Therefore, accurate flow measurement is very important for the proper and equitable distribution of water among multiple water users. According to some specialists, the water crisis in the 21st century is much more related to management than to a real crisis of scarcity and stress. Moreover, the management of water resources is a big challenge due to the lack of accurate flow measuring techniques and the use of appropriate flow measuring devices [6]. The ultimate goal of water measurement is to conserve water by improving the management of distribution and application [11]. Attention to measure, management, and maintenance will take advantage of the rural water demand and help to prevent reduced yields and other crop damage caused by under or overwatering [14].

Applications of the weir in the measurement of discharge through large and small open channels in the field or the laboratory remains a long-time practice [9,

17]. When measuring high flow rates by using the single sharp-crested weirs, backwater effects might affect the structures located upstream of the weir. Moreover, the bottom of the channel directly downstream of the weir crest is subject to high pressures due to the impact of the falling nappe, and channel protection is required in this area [2] otherwise, flow measurement accuracy is reduced. When the discharge is to be measured accurately with a reasonable sensitivity over a wide range of flows, the use of compound sharpcrested over fall weirs (CSOW) could be an appropriate solution. CSOW are more commonly used due to their simplicity, easy maintenance, and good flow measurement precision [16]. Therefore, to overcome this problem and also to measure flow rates accurately for a wide range of discharges, the use of CSOW having a linear combination of rectangular, triangular, and trapezoidal weirs with different types of notches has been proposed.

In many of the engineering projects, the culture of using experimental outputs supported by laboratory equipment and demo structure for the practical world is very rare. Depending on scientific evidence, different basic equations, and empirical formulae for different types of hydraulic structure types, sizes, and others may be selected to be utilized for any project. However, without evaluating the performance of the structures, selecting type and size both in the laboratory and the field; it is not cost-effective and suitable for the project site and decreases the confidence of the measurement accuracy. This study will focus on the evaluation of the flow measurement performance of CSOW having different notch types and optimized discharge measurement in data scarce areas. The specific aims of the present study are to (1) determine the coefficient of discharge for selected weir models (*Cd*); (2) validate the theoretical discharge equation using an experimental procedure; (3) select the most suitable flow measuring compound weir type; (4) evaluate the efficiency of the selected weir in dissipating flow energy.

MATERIALS AND METHOD

Discharge Equation Derivation

The equation governing flow through weirs is given by [20] as:

$$Q = KH^n, \tag{1}$$

where Q is the flow rate (L³/T), K is the parameter related to weir geometric shape and size, n is the weir shape number, H is the flow head (L).

According to Bagheri et al. [4] for the rectangular sharp-crested weir, the head-discharge equation is:

$$Q = \frac{2}{3}Cd\sqrt{2g}WH^{\frac{3}{2}},$$
 (2)

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where *W* the crest length of the weir (L), *Cd* is the coefficient of discharge, g is the gravity acceleration (L/T^2) , *H* is the water head above weir crest (L).

According to Yahya [1] the basic equation for a triangular sharp-crested weir is expressed as:

$$Q = \frac{8}{15}Cd\sqrt{2g}\tan\frac{\theta}{2}H^{\frac{5}{2}},$$
(3)

where θ = the notch angles. For trapezoidal weirs, the flow equation is as follows [12]:

$$Q = \frac{2}{3}Cd\sqrt{2g}\left(L_1 + \frac{4}{5}H\tan\theta\right),\tag{4}$$

where *b* is the bottom width of the cross-section (L), θ is the side slope angle.

In open channel flow because a large discharge requires more depth as compared to other weirs and flumes. Therefore, the measurement errors associated with the depth measurement have a significant effect on the discharge estimation. A compound weir and circular weirs both measure small discharges accurately while also having the capacity to measure the large discharge without large head requirements [3]. Experimentally studied four compound sharp-crested weirs (CSW) and confirmed that the linear combination method for developing the discharge equation of CSW is reasonable [21]. The cross-section of the studied compound weir consists of two areas, including areas $A_{\rm rec}$ and $A_{\rm tri}$, a theoretical discharge equation for the compound of regular weirs will be derived by the linear combination of basic head-discharge equations for triangular and rectangular weirs as shown in Fig. 1a below. When the head was above the lower part of the weir (H > Y), the discharge equation of compound rectangular and triangular weirs has been obtained as follows. The theoretical discharge equation of compound weir is:

$$Q_{1} = \sqrt{2g} \left[\frac{8}{15} \tan \frac{\theta}{2} Y^{\frac{5}{2}} + \frac{2}{3} B_{r} \left(H - Y \right) \frac{3}{2} \right], \qquad (5)$$

where B_r is total length of the horizontal crest of the rectangular weir (L). Equation (5) can be compound into a single (Cd_1) , the resulting actual discharge equation is:

$$Q_{1} = \sqrt{2g}Cd_{1}\left[\frac{8}{15}\tan\frac{\theta}{2}(Y)^{\frac{5}{2}} + \frac{2}{3}B_{r}(H-Y)^{\frac{3}{2}}\right].$$
 (6)

General definition, sketch for the proposed compound of regular weirs of rectangular and trapezoidal as shown in Fig. 1b. The cross-section of the weir consists of two areas (A_{rec} and A_{trap}) as shown in Fig. 1b below. When the head was above the lower part of the weir (H > Y), the theoretical discharge equation obtained by the discharge relations of one trapezoidal weir with side slope (θ) and a rectangular weir as follows:



Fig. 1. (a) Sketch for combined rectangle and triangle geometries; (b) rectangular and trapezoidal geometries.

$$Q_2 = \sqrt{2g} \left[\frac{2}{3} B_r \left(H - Y \right)^{\frac{3}{2}} + \frac{2}{3} Y^{\frac{3}{2}} \left(B_t + \frac{4}{5} Y \tan \theta \right) \right].$$
(7)

Equation (7) can be compound into a single (Cd_2) , and the resulting actual discharge equation is:

$$Q_2 = \sqrt{2g}Cd_2 \left[\frac{2}{3}B_r \left(H - Y\right)^{\frac{3}{2}} + \frac{2}{3}Y^{\frac{3}{2}} \left(B_t + \frac{4}{5}Y\tan\theta\right)\right], (8)$$

where B_t is width trapezoidal weirs respectively.

The *Cd* calculated as the ratio of actual discharge to the theoretical discharge, thus:

$$Cd = \frac{Qac}{Qth}.$$
 (9)

The *Cd* is which compensates for all the effects not taken into consideration in the derivation of discharge-head relation. Some of those effects are viscous effects; streamline curvature due to weir contraction, weir angle, three-dimensional flow structures behind the weir plate, and surface tension. From dimensional analysis arguments, it is found that the *Cd* is a function of several other parameters:

$$Cd = f\left(Fr_{1}, \theta, \text{Re}, \text{We}, \frac{h_{w}}{b}, \frac{h_{w}}{B}, \frac{h_{w}}{P_{w}}, g, \rho, \mu\right).$$
(10)

The Efficiency of dissipating flow energy (E %):

$$E_1 = H + \frac{V_1}{2g},$$
 (11)

$$E_2 = Y_2 + \frac{V_2}{2g},$$
 (12)

where E_1 , E_2 are the flow energy at the upstream weir and at the downstream of hydraulic jump respectively (L), H is the water head above weir crest (L), Y_2 is water depth downstream hydraulic jump (L), V_1 , V_2 are flow velocities in upstream and downstream respectively (L/T).

$$\% E = \frac{E_2 - E_1}{E_1} \times 100.$$
(13)

Experimental Setup

The experiments with proposed sharp-crested over fall weirs were performed in the laboratory. The experimental set-up mainly consists of a rectangular circulating open channel flume of 7.5 m long, 0.3 m wide and 0.45 m deep, and a collecting ground reservoir from the downstream end of the channel. The channel consisted of toughened glass walls of 10mm thick and a stainless-steel floor. The CSW are made of metal plates and attached at the end of the open channel with the help of G-clamps and water sealant to prevent leakages. Water was supplied from the ground reservoir into the uplifted reservoir by the centrifugal pump with a maximum capacity of 40 L/s. Moreover, the Water is also supplied to the main channel from an uplifted reservoir by the gravity system. The actual discharge through the channel was measured using the CSOW attached to the downstream end of the main channel. For this purpose, initially, it was calibrated accurately by using a volumetric method. Eight weir models were used to carry out the experimental work. These models were prepared from a 10 mm thick metal sheet of the dimensions $(30 \times 40 \text{ cm})$, which include a standard rectangular notch and triangle notch with three different angles. Also, the compound notch models were used with different angles. All model dimensions and their designation are shown in Table 1.

Design Procedure

Water was filled into the ground reservoir up to a reasonable limit. The weir plate was fixed in the downstream end of the main channel. The point gauge was assembled with the point fitted. The crest of the weir

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Weir Model	Model no.	Dimensions, cm		Weir angle (A)
wen wouer		height (P)	width	wen angle (0)
Rectangular	1	25	20	_
	1	8.5	21.5	45
Triangular	2	8.5	30.7	60
	3	8.5	30	90
Destangle and Triangle	1	8.5	24.8	120
Rectangle and Triangle	2	8.5	20	130
Destangle and Transcidel	1	8.5	20	60
Rectangle and Trapezoluar	2	8.5	20.7	90

Table 1. Main characteristic of standard and compound notches

plate opening was set as the datum or reference point. Start pumping water from the ground reservoir into the uplifted reservoir until overflow the constant head pipe inside the reservoir. Open the control valve and adjust the valve to produce the flow over the weir. Ensure that the flow is large enough to prevent the outflow from the notch clinging to the weir plate. Determine the volumetric flow rate using a rounded plastic container overfall weir by using the time (stopwatch). When the container full measure the mass by using a balance (scale). Read the water level (H) using the point gauge by equal interval or by incremental of flow depth over the weir plate equally. Repeat the process for different trials for each weir type. The calibration was done before every run of the experiment to prevent instrumental errors. The hook and point gauge were adjusted accurately to the surface of the water to get the value of H while measuring H it was ensured that the flow in the channel was stable and constant. The Discharge was maintained for an individual run. Weir plates made up of metal sheets were used for experimental study. This weir was made by using the Standardization of flow measuring structures. The width of the main channel (B) is 0.30 m was measured. The crest height (Pw) is varied as 0.085 and 0.25 m. Thus, for each crest height, there will be the width of weir opening (b) like 0.2, 0.22, 0.25, 0.30 m.

Once the calibration process was completed the accuracy of discharge measurement depends on the measurement of water level (H) on the upstream side of the weir. The point gauge was fixed at an upstream distance four times the maximum head over the weir [19]. After the entrance, water passes through a rectangular channel and exits over the weir down into the

 Table 2.
 Summary of experimental data range

No <i>R</i>	Parameter	Range
1	Main channel discharge, L/s	0.50-20.95
2	Upstream depth of flow, mm	10-120
3	Weir height, <i>P</i> cm	8.5-25

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ground reservoir and this circulation continues. The water head was measured at a distance of 1.52 m upstream of the weir section. In literature, it was recommended that 3–4 times the maximum water head will be far enough to get rid of the water drawdown while approaching the exit section [19]. The point gauge was used to measure the water head at the centerline of the approach channel. Its accuracy is ± 0.05 mm. Weir plate was made from a metal sheet mounted with the main channel at the downstream end. By adjusting the distance of the opening gap between the plates, the desired contraction width was obtained and the surrounding of the plate's weir insulated against the unwanted leakage. The summary of experimental data range was shown in Table 2.

RESULTS AND DISCUSSION

Determination of the Coefficient of Discharge Cd

The *Cd* value is a parameter which mainly governs the weir applicability to various hydraulic references. The Cd was calculated with experimental data as per the standard procedure explained in the previous section. It is a function of the total flow head over the weir (hw), weir crest height (Pw), and upstream Froude number (Fr_1). Therefore, the relation between the Cd and Froude number (Fr_1) is shown in Figs. 2a, 2b for both regular and compound weirs. From these figures, it can be seen that each triangular notch angle (θ) , Cd increases as (Fr_1) increases. Moreover, when the triangular notch is acute a maximum value of Cd can be reached for the acute angle of (45°) and decrease as a triangular notch angle increase as for ($\theta = 90^{\circ}$). From Fig. 2b when compound rectangular and the triangular notch is used, the lower triangular notch is obtuse a maximum Cd can be reached for an obtuse angle (120°) . The rapid increase in the coefficient of the discharge as a triangular notch angle ($\theta = 130^{\circ}$) for the low value of primary Froude number ($Fr_1 < 6.3$). For the compound notch of rectangular and trapezoidal, the notch angle is acute $(60^\circ, 90^\circ)$ a slight increase of Cd as (Fr₁) increase.

The relation between Cd and parameter (hw/pw)was plotted in Figs. 3a, 3b; Fig. 3a shows that Cd increases as the ratio of upstream water depth to weir height (hw/pw) increase, and maximum Cd can be reached at (hw/pw = 0.71) for triangular notch angle of (90°) , but Cd is decreased for rectangular notch compared with a both acute and obtuse triangular notch. From Fig. 3b when the parameter (hw/pw) increases, the Cd decreases for both angles of the triangular notch of compound regular notches, a minimum Cd can be recognized at more détente of triangular notch angle. For the compound notches of rectangular and trapezoidal, the maximum Cd can be reached approximately at (hw/pw = 0.82) for both angle of $(60^\circ, 90^\circ)$, beyond this value Cd decreases slightly as the parameter (hw/pw) increased.

Validation of the Proposed Discharge Equation

The 'Standard' measuring structures presented in these studies are those which have been calibrated in the laboratory. The discharge equation so established is therefore used to check the accuracy of discharge by adjusting the theoretical discharge values to obtain the actual discharge. If it is not constant *Cd* values, try to find out the functional relationship. Thus, adjustments were done by the determination of geometric size and weir shape parameters (K and n). Experimental determination of weir constant is carried out by using Eq. (1) on the above general consideration section. Experiments are performed to determine the constant for a particular weir. For the different heads over the crest, the corresponding discharges are determined by measuring the quantity of liquid discharged over the weir.

By taking the logarithm of both sides of Eq. (1):

$$\log Q = \log K + n \log H, \tag{14}$$

when H = 1, $\log H = 0$ and hence $\log Q = \log K$.

Thus, from the intercept of the line with the vertical axis, the values of $\log K$ and hence K may be obtained. When the value of $\log Q$ is plotted against the corresponding values of $\log H$, a straight line is obtained Eq. (1) maybe written as:

$$n = \frac{\log Q - \log K}{\log H}.$$
 (15)

The values of K and n as determined by the above procedure and substituted in Eq. (1) to get the rating equation of the proposed weir. Therefore, the results of each parameter and the respective rating equation were tabulated as shown below in Table 3. The calibration process entails establishing the Cd, which adjusts the theoretical discharge equation to obtain the actual discharge. However, it may be necessary to check the calibration of a measuring structure in the field should any suspicion arise as to its reliability. Since the laboratory calibration normally has a reliable rating, and



Fig. 2. Relation between *Cd* and Froude number (Fr_1) for (a) sharp-crested over fall Standard weirs; (b) compound sharp-crested over fall weirs.

the coefficient of discharge so established are generally of the order of, typically 2-5%, it is necessary to use special equipment and methods in a check calibration to make field measurements of comparable accuracy.

Selection of Suitable Flow Measuring Compound Sharp-Crested Weir Type

As a typical result, the variation in measured discharge with depth for CSW type is shown in Figs. 4a, 4b. As can be noted, there is the discontinuity of flow shown at the transition region of the two-section part of the weirs. This is not the case with regular sharpcrested weirs having the single weir plate, where the discharge versus the head relationship is continuous as a result show in the Figs. 4a, 4b. When the head is near the transition region of CSW the flow measurements are not very accurate.

The discontinuity of flow over the CSOW for variation of the head was checked using experimental data. The variation of the head with discharge for single standard sharp-crested overfall weir was checked as shown below in Fig. 4a. The discontinuity of flow did not occur for high and low discharge measurement regular weirs. If there is the discontinuity of flow, there



Fig. 3. Relation between Cd and (hw/pw) for (a) sharp-crested over fall standard weirs; (b) compound sharp-crested over fall weirs.

is the change in discharge values over the measuring head.

Moreover, in Fig. 4b, the graph as shown above for three types of the CSW with notch angle of $(60^\circ, 90^\circ)$, and 130°) was the change in the flow pattern of the plot shows discontinuity of flow has occurred. It shows that the variation of the discharge over the head. The rest one has the notch angle of (120°) was well-defined flow pattern and recommended for accurate flow measurement.

> Energy Dissipation Ratio (E%) for the Compound of Regular Weirs

Figures 5a, 5b show that the effect of weir angle (θ) and Froude number (Fr₁) on energy dissipation ratio E% for standard weirs. In Fig. 5a E% approximately

No R	Weir types	Weir R
1	Rectangular	Q = 0.281 H1.42
-		

Table 3. Calibrated head-discharge rating equation for all proposed weir types

No R	Weir types	Weir Rating Equations
1	Rectangular	Q = 0.281 H1.4243
2	Triangular with a 45-degree angle	Q = 0.083 H1.499
3	Triangular with a 60-degree angle	Q = 0.106 H1.616
4	Triangular with a 90-degree angle	Q = 0.173 H1.686
5	Triangular + Rectangular with a 120-degree angle	Q = 0.747 H1.963
6	Triangular + Rectangular with a 130-degree angle	Q = 0.569 H1.858
7	Trapezoidal + Rectangular with a 60-degree angle	Q = 0.137 H1.156
8	Trapezoidal + Rectangular with a 60-degree angle	Q = 0.396 H1.471



Fig. 4. Variation of measured discharge for (a) single sharp-crested over fall weirs; (b) compound sharp-crested over fall weirs.

increase linearly when (Fr_1) increases for the rectangular weir, While for *V*-notch models (*E* %) increase rapidly with a small difference with the increase of (Fr_1) for acute notches (45° and 60°), whereas acute notch (90°), *E*% decrease significantly when (Fr_1) was increased. Figure 5b shows that when the (Fr_1) increases, as the *E*% increases linearly for both lower triangles of the compound notches. A higher value of (*E*%) can be obtained at the small *V*-notch angle of (120°), while for the compound weirs of rectangular and trapezoidal, (*E*%) changes are not valuable for a small range of Froude numbers ($6.5 \le Fr_1 \le 8.0$) for both studied angles. From Figs. 5a and 6b it is observed that relative energy loss increases with an increase in approach Froude number.

Determination of Coefficient of Discharge for Compound Sharp-Crested over Fall Weirs Cd

Based on the open channel flow theory, the important variables among many factors the coefficient of discharge Cd relating the main flow parameters may be expressed as follows:

$$Cd = f_1\left(fr_1, \theta, \operatorname{Re}, \operatorname{We}, \frac{h_w}{b}, \frac{h_w}{B}, \frac{h_w}{P_w}, g, \rho, \mu\right).$$
(16)

According to Buckingham's π -Theorem and considering repeating Y_1 , ρ , variables [8], the following dimensionless groups developed:

$$\vartheta\left(\frac{Y_2}{Y_1}, \frac{V_1}{V_2}, \frac{E_1}{Y_1}, \frac{E_2}{Y_2}, \frac{V_1^2}{2g}, \frac{\rho V_1 Y_1}{\mu}, \tan\frac{\theta}{2}\right) = 0.$$
(17)

From the present study and based on the dimensional analysis it is found that almost all the hydraulic characteristics are the function of Froude number (Fr_1), Reynolds number (Re_1), and notch angle (θ). The effect of Reynolds number and Weber number is

assumed to be negligible for compound sharp-crested weir except at very low head.

$$Cd = f_2\left(\frac{Y_1}{P}, \operatorname{Fr}_1, \tan\frac{\theta}{2}\right),$$
 (18)

the relationship was developed by multiple non-linear regression analyses for experimental data using (IBM SPSS Statistics 20.0 Package), the performance of the model developed in this work was evaluated based on statistical criteria. The statistical measures adopted in this study where the multiple coefficients of determination and standard error of estimate denoted R^2 and SEE respectively. From the basis of nonlinear fitting between the hydraulic characteristics and dimensionless group developed herein, the following models were obtained for:

Standard *V*-notches form $(25^\circ \le \theta \le 90^\circ)$:

$$Cd = 0.654 \left(\mathrm{Fr}_{\mathrm{l}} \right)^{1.2} \left(\frac{Y_{\mathrm{l}}}{P_{w}} \right)^{1.12} \left(\tan \frac{\theta}{2} \right), \tag{19}$$
$$[R^{2} = 0.975; \, \mathrm{SEE} = 0.0683],$$

$$E\% = 25.519 \operatorname{Fr}_{1}^{0.659} (\operatorname{Re}_{1})^{-0.184} \left(\frac{Y_{2}}{Y_{1}}\right)^{0.739} \left(\tan\frac{\theta}{2}\right)^{-0.468},$$

$$[R^{2} = 0.987; \text{ SEE} = 1.299].$$

Compound Notches (Rectangle and Triangle) for $(\theta = 120^{\circ}, 130^{\circ})$:

$$Cd = 1.569 (\mathrm{Fr}_{1})^{0.17} \left(\frac{Y_{1}}{P_{w}}\right)^{1.869} \left(\tan\frac{\theta}{2}\right), \qquad (21)$$
$$[R^{2} = 0.995; \, \mathrm{SEE} = 0.012],$$

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Fig. 5. Relation between energy dissipation ratio (E %) and Froude number (Fr_1) for (a) standard weirs; (b) compound weirs.

$$E \% = 3.047 \times 10^{-8} (\mathrm{Fr}_{\mathrm{l}})^{2.056} (\mathrm{Re}_{\mathrm{l}})^{2.180} \times \left(\frac{Y_2}{Y_1}\right)^{-0.420} (\tan \frac{\theta}{2}), \qquad (22)$$
$$[R^2 = 0.984; \mathrm{SEE} = 5.188].$$

Validation for the proposed Empirical Models

The present model Eqs. (19) and (21) for the coefficient of discharge are validated using the plot of the predicted value of *Cd* with Experimental values of *Cd* for both *V*-notch and compound of rectangle and triangle, as shown in Figs. 6a, 6b respectively. The proposed equation was found to produce the results with a maximum of $\pm 10\%$ for almost 100% of the total data. The top curve $\pm 10\%$ ($P = E \pm 10\%$) shows the curve of predicted values 10% above the coefficient of discharge. The bottom curve (P = E - 10%) shows the curve of the predicted values 10% below the experi-

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mental coefficient of discharge. The area between the curves shows the range of the experimental coefficient of discharge plus or minus 10%. The present model Eqs. (20) and (22) for energy dissipation ratio (E%) are validated using the plot of the predicted value of (E%) with experimental values for both V-notch and compound of rectangle and triangle, as shown in Figs. 6a and 6b respectively.

The proposed equation was found to produce the results with a maximum of $\pm 10\%$ for almost 100% of the total data. The top curve +10% (P = E + 10%) shows the curve of predicted values 10% above the energy dissipation ratio. The bottom curve (P = E - 10%) shows the curve of the predicted values 10% below the experimental energy dissipation ratio. The area between the curves shows the range of the experimental energy dissipation ratio plus or minus 10%. From the Figs. 7a, 7b it is observed that validation for the proposed empirical models which is presented in Eqs. (19) and (21) for experimental and predicted values of coefficient of discharge and in Eqs. (20) and



Fig. 6. Validation of predicted and experimental values of coefficient of (a) discharge for standard *V*-notches; (b) compound rectangle and triangle notches.

(22) for experimental and predicted values of energy dissipation ratio plotted in the below figure respectively. A similar trend has been presented by [8] for the variation of relative energy loss with approach Froude number from their experimental studies. It is evident from the figure that approximately 80% of experimental data are laying within $\pm 10\%$ of the best fit curve with R^2 value of 0.959. In this study, a few data points deviate from the best fit curve which may be due to inaccuracy in the measurement of the depth of flow and discharge.

CONCLUSIONS

In this study, a CSW composed of a triangle, rectangle, and trapezoidal weirs has been proposed. A series of laboratory experiments were conducted to evaluate flow measurement performance and optimized discharge measurement in data scarce areas a wide range of flows without discontinuity. Furthermore, the energy dissipation ratio and the theoretical discharge equation for these types of the weir has been derived and experimentally validated. From the analysis of the experimental result, the following conclusion can be stated:

For a single sharp-crested weir, Coefficient of discharge increases as Fr₁ increases. Moreover, when the triangular notch is acute, a maximum value of Cd can be reached for an acute angle of (45°) and decrease as a triangular notch angle increase. For compound rectangular and the triangular notch is used, the lower triangular notch is obtuse a maximum Cd can be reached for an obtuse angle of (120°) . The rapid increase in the *Cd* is recorded for the low value of the Froude number. For rectangular and trapezoidal compound notch a slight increase of Cd as (Fr_1) increases. Cd increases as hw/pw increases and the maximum Cd can be reached at hw/pw = 0.71 for single sharp-crested standard weirs. The coefficient of discharge Cd decreases for both angles of the triangular notch of compound weirs a minimum Cd value can be recognized at more détente of triangular notch angle. For rectangular and trapezoidal the maximum Cd reached at hw/pw = 0.82for both angles of $(60^\circ, 90^\circ)$. Beyond this value Cd decreases slightly as the ratio of upstream water head to weir height increased.

The energy dissipation percentage (E %) increases linearly when (Fr_1) increases for the rectangular weir. While for triangular weir (E%) increase rapidly with a small difference with the increase of (Fr_1) for an acute notch of $(45^\circ, 60^\circ)$ whereas acute notch (90°) , energy dissipation percentage decrease significantly when (Fr_1) was increased. It has been validated the theoretical equation by using an experimental procedure provides a good estimation of the discharge rating equation by determining the value of K and n for each weir type. The values of K and n for 120° notch higher than the rest of the proposed weirs. The percentage of energy dissipation ratio decreased with increasing angle of V-notch for all (Fr_1) . The V-notch model has given a higher percentage to dissipate the flow energy than the rectangle notch. The proposed equation was found to produce the results with a maximum of $\pm 10\%$ almost 100% of the total data. A multiple non-linear regression model was predicated from the measured hydraulic and geometric parameter with excellent values of coefficient of multiple determinations and the value of the standard error of the estimate.

CONFLICT OF INTEREST

The authors declare no conflict of interest.



Fig. 7. Validation of predicted and experimental values of energy dissipation ratio for (a) standard V-notch; (b) compound rectangle and triangle notches.

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