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HYDROPHYSICAL PROCESSES

The Effect of the Horizontal Dimensions of Inland Water Bodies on the Thickness of the Upper Mixed Layer

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Abstract—The effect of the horizontal dimensions of inland water bodies (lakes and reservoirs) on the vertical mixing processes is studied. We consider a three-dimensional hydrostatic model and a one-dimensional LAKE model based on the averaging of three-dimensional equations over a horizontal section of the water body. The processes of vertical mixing in both models were simulated with the use of k–ε-closure. LAKE model takes into account the seiches through the parameterization of the pressure gradient and horizontal viscosity. The models were verified against the Kato-Phillips experiment data, and a series of numerical experiments was carried out to demonstrate the effect of the horizontal size of the water body on mixed-layer depth. It is shown that the horizontal dimensions of a water body has to be taken into account in the simulation of the vertical temperature distribution in lakes and reservoirs with the size much less than the internal Rossby deformation radius.

Keywords: mathematical modeling, inland water bodies, turbulence, seiches

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INTRODUCTION

Inland water bodies (lakes and reservoirs) occupy 1.3–1.8% of the total area of continents [16, 21] and play an important role in the socioeconomic development of the regions in which they are located; they are the focus of studies in many problems in hydrology, ecology, meteorology, and climatology. The thermodynamic characteristics of lakes and reservoirs have a considerable effect on several processes of regional atmospheric circulation. In addition, changes of temperature in lakes and reservoirs can intensify the processes of eutrophication [3, 6, 7], i.e., an increase in the biological production of water bodies, in particular, as the result of biomass growth of diatoms and deleterious blue-green algae, which can cause mass fish kill and water quality deterioration.

We also have to mention the role of inland water bodies in climate changes and the response of water bodies to such changes [1, 13, 28]. In regions with large numbers of lakes and reservoirs, a pronounced climate warming can be seen [12], resulting in the early ice breaking and a shorter under-ice period. To take into account the interaction between inland water bodies and the atmosphere, the climate models have to include the calculation of thermohydrodynamic and biological characteristics of continental waters. The thermohydrodynamics of lakes has to be correctly reproduced in the mesoscale atmospheric models, where the spatial resolution reaches several kilometers, which is less than the characteristic horizontal dimensions of large inland water bodies.

An important aspect in the simulation of the thermohydrodynamics of inland water bodies is the correct description of mixing processes, including those related with gravity (seiche) oscillations. The seiches are caused by the horizontal redistribution of mass and the effect of the hydrostatic pressure gradient; they are not incorporated in the majority of the existing one-dimensional (vertical) models. However, in the models of lakes and reservoirs with horizontal dimensions much lesser than the internal Rossby deformation radius L_R, the Coriolis force becomes negligible compared with the force of the horizontal pressure gradient [10], and the models not taking seiches into account fail to give a correct description of the velocity field in such water bodies; this can lead to a significant error in the estimate of the mixed layer thickness (this value can be overestimated, especially, in the period of summer stratification [27]). In the mid latitudes, where the value of L_R is 2–3 km, it should be expected that seiches will have an appreciable effect on the mixing processes in relatively small lakes, which account for the major portion of inland water bodies [5].

The objective of this study is to evaluate the effect of the horizontal dimensions of a lake or a reservoir on the processes of mixing in the water body, in particular, on the thickness of the mixed layer. To simplify the analysis, we consider water bodies of idealized shape under prescribed atmospheric conditions not changing over time.

THE METHODS OF DESCRIPTION OF THE PROCESSES OF THERMOHYDRODYNAMICS OF INLAND WATER BODIES

The mathematical models developed by now enable calculating the distributions of thermohydrodynamic characteristics in inland water bodies. The most detail description can be obtained with the use of three-dimensional models [4, 13, 18], which are based on the Reynolds-averaged system of thermohydrodynamic equations in Boussinesq approximation and hydrostatic equation [2]. Such system is used in this study to describe the circulation in a thermally stratified inland water body. The effect of short-wave radiation can be neglected for short time scales in nights in the warm seasons and in cold seasons, when there is practically no effect of wind on the hydrodynamics of the water body. Therefore, the formulation of the problem in this study is a relatively crude approximation to the conditions in the nature; however, it allows one to take into account the effect of the horizontal dimensions of the water body on the vertical distribution of temperature, which, obviously, exists in real objects. Under such conditions, the system of equations becomes:

$$\frac{\partial u}{\partial t} = -A(u) + D_H(u,\lambda_m) + D_z(u, K_m + \nu) - g\frac{\partial \eta}{\partial x} - \frac{g}{\rho_0}\frac{\partial}{\partial x}\int_z^{\eta}\rho dz' + fv, \quad (1)$$

$$\frac{\partial v}{\partial t} = -A(v) + D_H(v,\lambda_m) + D_z(v, K_m + \nu) - g\frac{\partial \eta}{\partial y} - \frac{g}{\rho_0}\frac{\partial}{\partial y}\int_z^{\eta}\rho dz' - fu, \quad (2)$$

$$\nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (3)$$

$$\frac{\partial T}{\partial t} = -A(T) + D_H(T,\lambda_h) + D_z(T, K_h + \chi'), \quad (4)$$

$$\rho = \rho(T), \quad (5)$$

$$\frac{\partial \eta}{\partial t} = w. \quad (6)$$

Here $\mathbf{u} = (u, v, w)$ is velocity vector; η is the deviation of the free surface from the equilibrium state; T is temperature; ρ is density; $K_m(\lambda_m)$ and $K_h(\lambda_h)$ are the coefficients of vertical (horizontal) turbulent viscosity and thermal diffusivity, respectively; ν, χ' are the coefficients of molecular viscosity and thermal diffusivity; f is Coriolis parameter (assumed constant); g is the acceleration due to gravity; z is the vertical coordinate passing from the bed of the water body $z = -H(x, y)$ to the surface; t is time.

In the system of equations (1)–(6), $A(q)$ is the advection operator:

$$A(q) = \frac{\partial uq}{\partial x} + \frac{\partial vq}{\partial y} + \frac{\partial wq}{\partial z},$$

while $D_H(q,\lambda)$ and $D_z(q,K)$ represent horizontal and vertical diffusion with coefficients λ and K, respectively:

$$D_H(q,\lambda) = \frac{\partial}{\partial x}\lambda\frac{\partial q}{\partial x} + \frac{\partial}{\partial y}\lambda\frac{\partial q}{\partial y},$$

$$D_z(q,K) = \frac{\partial}{\partial z}K\frac{\partial q}{\partial z}.$$

Currently, the hydrological and thermodynamic characteristics of inland water bodies at the seasonal, annual, and climate scale can be best studied by one-dimensional models due to their computational simplicity. The one-dimensional system of equations describing the vertical distribution of momentum and heat can be obtained by averaging the above three-dimensional equations (1)–(6) by the horizontal section of the water body [11, 26]:

$$\frac{\partial \overline{T}}{\partial t} = \frac{1}{A}\frac{\partial}{\partial z}\left(A(K_h + \chi')\frac{\partial \overline{T}}{\partial z}\right), \quad (7)$$

$$\frac{\partial \overline{u}}{\partial t} = -\overline{\left(\frac{1}{\rho}\frac{\partial p}{\partial x}\right)} + \frac{1}{A}\frac{\partial}{\partial z}\left(A(K_m + \nu)\frac{\partial \overline{u}}{\partial z}\right) - \frac{1}{A}\frac{dA}{dz}\left((K_m + \nu)\frac{\partial \overline{u}}{\partial z}\right)_{\text{bot}} + \overline{D_H(u,\lambda_m)} + f\overline{v}, \quad (8)$$

$$\frac{\partial \overline{v}}{\partial t} = -\overline{\left(\frac{1}{\rho}\frac{\partial p}{\partial y}\right)} + \frac{1}{A}\frac{\partial}{\partial z}\left(A(K_m + \nu)\frac{\partial \overline{v}}{\partial z}\right) - \frac{1}{A}\frac{dA}{dz}\left((K_m + \nu)\frac{\partial \overline{v}}{\partial z}\right)_{\text{bot}} + \overline{D_H(v,\lambda_m)} - f\overline{u}. \quad (9)$$

Here $A(z)$ is the area of the horizontal section of the water body, p is hydrostatic pressure, and the horizontal line means averaging over $A(z)$. Here, in accordance with the above reasoning, the heat flux on the bottom boundary is set to zero, and the flux of momentum is assumed constant on the boundary of each horizontal section (the values denoted by a subscript bot).

The systems of equations (1)–(6) and (7)–(9) were written under the assumption that the gradient

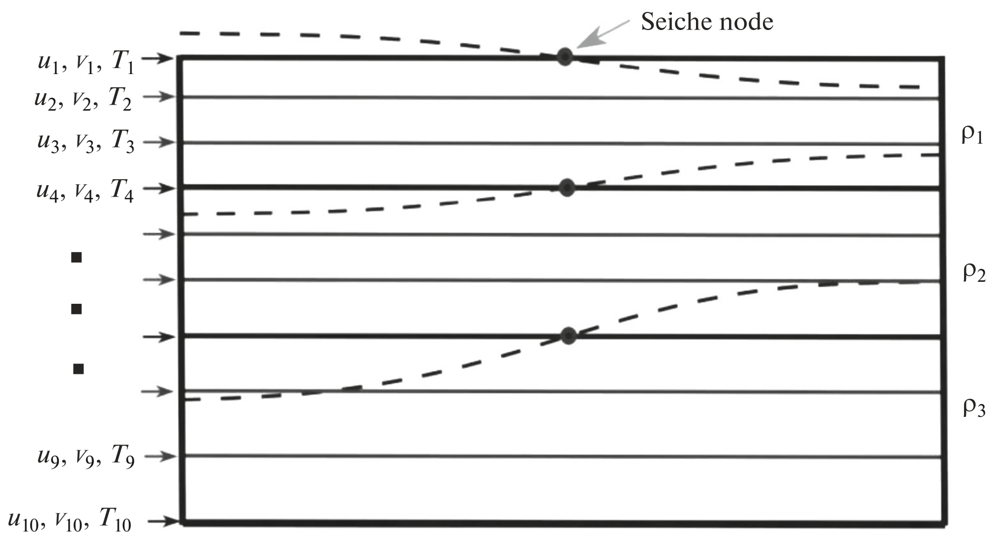

Fig. 1. Multilayer representation of fluid in seiche parameterization in one-dimensional LAKE model.

approximation is valid for the description of turbulent flows. An important condition for the problems considered in this study is a consistent description of the vertical mixing in the three-dimensional and one-dimensional models. Therefore, coefficients K_m and K_h in both models are hereafter calculated with the use of two-equation k–ε-closure in the standard formulation [8, 20]. It is based on prognostic equations for turbulent kinetic energy (TKE) and the rate of its dissipation ε:

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z}\left(\frac{K_m}{\delta_k} + \nu\right)\frac{\partial k}{\partial z} + P + B - \varepsilon, \quad (10)$$

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial z}\left(\frac{K_m}{\delta_\varepsilon} + \nu\right)\frac{\partial \varepsilon}{\partial z} + \frac{\varepsilon}{k}\left(C_{1\varepsilon}P - C_{2\varepsilon}\varepsilon + C_{3\varepsilon}B\right), \quad (11)$$

$$K_m = C_\varepsilon \frac{k^2}{\varepsilon}, \quad (12)$$

$$K_h = C_{\varepsilon T}\frac{k^2}{\varepsilon} = \frac{C_{\varepsilon T}}{C_\varepsilon}K_m. \quad (13)$$

Here, the term P corresponds to production of turbulence energy due to velocity shear; B accounts for the production or consumption of energy under the effect of buoyancy forces; $\delta_k, \delta_\varepsilon$ are turbulent Schmidt numbers for TKE and the dissipation rate, respectively; $C_{1\varepsilon}$, $C_{2\varepsilon}$, $C_{3\varepsilon}$ are empirical constants; C_ε and $C_{\varepsilon T}$ are stability functions for the momentum and scalar variables, assumed constant.

The system of equations of the one-dimensional model (7)–(13) is not closed; the parameterization is required for the first, third, and fourth terms in the right hand sides of (8)–(9). The three-dimensional and one-dimensional systems of equations are supplemented by the necessary boundary conditions.

THE PARAMETRIZATION OF THE GRADIENT OF PRESSURE AND HORIZONTAL VISCOSITY FOR ONE-DIMENSIONAL MODEL

We will derive the parametrization of the horizontal pressure gradient and the horizontal diffusion of a two-dimensional water body with Coriolis force not taken into account. The case of three-dimensional water body with Earth rotation taken into account and with no horizontal viscosity is considered in [10]. The seiches are incorporated by a method based on the explicit reproduction of the first horizontal mode. Suppose that the fluid consists of N layers (Fig. 1) with constant densities ρ_i ($\rho_{i+1} > \rho_i$). The thickness of each layer h_i is written in the form: $h_i = H_i + h_i'$ (H_i is the thickness of the i-th fluid layer at rest; h_i' is the deviation of the thickness, with $\left|h_i'/H_i\right| \ll 1$, H is water body depth). The problem is considered for the rectangular domain $[-L_x/2, L_x/2] \times [0, H]$. The solution of this problem is equivalent to the solution of a three-dimensional problem in the domain $[-L_x/2, L_x/2] \times [-L_y/2, L_y/2] \times [0, H]$, in which the momentum flux on the surface is directed along the x axis, and the velocity component along the y axis is zero.

For each layer, the following linearized equations of motion, continuity, and hydrostatic can be written:

$$\frac{\partial u_i}{\partial t} = -\frac{1}{\rho_i}\frac{\partial p_i'}{\partial x} + \lambda_m \frac{\partial^2 u_i}{\partial x^2}, \quad (14)$$

$$\frac{\partial h_i'}{\partial t} + H_i \frac{\partial u_i}{\partial x} = 0, \quad (15)$$

$$p_i^{'} = g\sum_{k=1}^{N}\rho_{\min(i,k)}h_k^{'}, \quad i = \overline{1,N}. \tag{16}$$

In the inland water bodies, the energy of modes with the first horizontal wave number commonly dominates in the spectrum of internal oscillations [19, 25]; therefore, we represent the solution of the system of equations (14)–(16) in the form of a Fourier series up to the first harmonic and average the results over a horizontal section, which, in this case, is $[-L_x/2, L_x/2]$. We obtain:

$$\frac{\partial \overline{u}_j}{\partial t} = -\frac{\pi g}{2L_x p_i}\sum_{k=1}^{N}\rho_{\min(i,k)}\Delta_x\overline{h_k^{'}} - \frac{\pi\nu\overline{u}_i}{L_x^2}, \tag{17}$$

$$\frac{d\Delta_x\overline{h_i^{'}}}{dt} = \frac{2\pi H_i}{L_x}\overline{u}_i, \quad i = \overline{1,N}, \tag{18}$$

where $\Delta_x\overline{h_i^{'}}$ denotes the difference between values $h_i^{'}$, averaged over the right ($[0, L_x/2]$) and left ($[-L_x/2, 0]$) halves of the section [17]. This system relates the horizontally averaged velocity components with the average pressure gradient. In the system of equations (17)–(18), the vertical distribution of the variables is piecewise constant, unlike the system of equations (7)–(9), which is formulated for functions differentiable with respect to z. We use equations (17)–(19) to close the system of equations (7)–(9); to do this, we divide the vertical profile of water density $\rho = \rho[\overline{T}(z,t)]$ into N layers with thicknesses $H_i = z_{i+1} - z_i$, such that the density in each layer varies over depth insignificantly. In any such layer, the horizontal gradient of pressure can be calculated by (17)–(18) with the use of $\overline{u_i}$ as the vertically averaged horizontal velocity from the one-dimensional model $\overline{u}(z,t)$; this gradient will be constant within each semi-interval $[z_i, z_{i+1})$. The system of equations of the one-dimensional model supplemented in such a manner can be written as:

$$\frac{\partial\overline{u}}{\partial t} - \frac{\partial}{\partial z}(K_m + \nu)\frac{\partial\overline{u}}{\partial z}$$

$$= -\frac{\pi g}{2L_x\rho_i}\sum_{k=1}^{N}\rho_{\min(i,k)}\Delta_x\overline{h_k^{'}} - \frac{\pi\nu\overline{u_i}}{L_x^2}, \quad i\colon z \in [z_i, z_{i+1}),$$

$$\frac{d\Delta_x\overline{h_i^{'}}}{dt} = \frac{2\pi H_i}{L_x}\widehat{u}_i, \quad i = \overline{1,N},$$

$$\hat{u}_i = H_i^{-1}\int_{z_i}^{z_{i+1}}\overline{u}dz, \quad i = \overline{1,N}.$$

The obtained model can be classified as 1.5-dimensional, because it partially takes into account the effects of horizontal heterogeneity of a specified form; mathematically, this formulation includes the elements of partial differential equations, ordinary differential equations, and integral equations. Hereafter, the presented closure of a one-dimensional system will be also referred to as seiche parameterization, as it allows us to explicitly reproduce seiches with a horizontal wave number 1.

THE IMPLEMENTATION OF NUMERICAL EXPERIMENTS

The numerical experiments were carried out with the use of one-dimensional LAKE model (a detailed description is given in [9]), supplemented by the parameterization of seiche. The model is being developed in the Moscow State University, and it is incorporated in the most recent version of the land surface model being developed in the Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences [2] and Moscow State University. The one-dimensional model was verified with the use of a three-dimensional hydrostatic model being developed in the Research Computer Center, Moscow State University, and in the Institute of Numerical Mathematics, Russian Academy of Sciences, based on a universal hydrodynamic code, combining the DNS (Direct Numerical Simulation), LES (Large-Eddy Simulation), and RANS (Reynolds-Averaged Navier-Stokes) approaches for calculation of geophysical turbulent flows [22, 23]. The numerical method used to solve the system of equations (1)–(6) is based on conservative finite-difference methods with discretization on rectangular grids and the use of a semi-implicit method for approximation over time, in which the advective transport and horizontal diffusion are described by explicit schemes.

The values of empirical constants in k–ε-closure of the one-dimensional and three-dimensional models are prescribed according to those given in [27]; and their choice is substantiated, for example, in [14, 15]. Note that the turbulent Prandtl number was assumed constant $\mathrm{Pr}_t = K_m/K_h = 1.25$, and the constant $C_{3\varepsilon}$, which determines the changes in the dissipation rate under the effect of buoyancy forces, was set equal to 1.14 for unstable conditions $B > 0$ and –0.4 when $B < 0$.

The one-dimensional and three-dimensional models were used to carry out the following experiments: the verification by numerical implementation of the classical laboratory experiment of Kato–Phillips [17], the results of which are used as the main data for calibration of turbulent closures for shear flows in a stratified fluid, and calculations with idealized water bodies with rectangular vertical cross-sections at different horizontal size (10 and 1000 m).

In the experiment of Kato–Phillips, a horizontally homogeneous stratified fluid is considered in the absence of vertical boundaries. The initial temperature profile is linear, and the only source of turbulence is the wind shear, which provides a constant momentum flux on the surface. In the classical formulation described in [17], a ring-shaped reservoir was considered, on the surface of which a frictional stress was created in the circumferential direction. The internal and external diameters were 152.4 and 106.7 cm, respectively; therefore, the channel was 22.8 cm in width. The depth of the reservoir was 28 cm.

The results of this experiment can be well described by the theoretical formula for variations of the thickness of the mixed layer over time [24]:

$$h_{ML}(t) = \frac{Cu^*\sqrt{t}}{\sqrt{N(t=0)}} \quad (C \sim 1.05), \tag{19}$$

where h_{ML} is the thickness of the mixed layer, u^* is the friction velocity on the surface ($u^* = \sqrt{\tau/\rho_0}$).

For the numerical implementation of the Kato–Phillips experiment, the equations of the one-dimensional and three-dimensional models were supplemented by the following boundary conditions on the bed:

$$\frac{\partial T}{\partial z} = 0,$$
$$\frac{\partial v}{\partial z} = 0,$$
$$w = 0,$$

and on the surface:

$$\frac{\partial T}{\partial z} = 0,$$
$$\frac{\partial v}{\partial z} = 0,$$
$$-\rho(K_m + \nu)\frac{\partial u}{\partial z} = \tau.$$

Periodic boundary conditions in the horizontal directions were used in the three-dimensional model.

In the series of experiments with the presence of vertical walls, the three-dimensional model was supplemented by lateral boundary conditions:

$$\left.\frac{\partial T}{\partial x}\right|_{x=-L_x/2,L_x/2} = 0, \quad \left.\frac{\partial T}{\partial y}\right|_{y=-L_y/2,L_y/2} = 0,$$
$$u\big|_{x=-L_x/2,L_x/2} = 0, \quad v\big|_{y=-L_y/2,L_y/2} = 0,$$
$$\left.\frac{\partial \eta}{\partial x}\right|_{x=-L_x/2,L_x/2} = 0, \quad \left.\frac{\partial \eta}{\partial y}\right|_{y=-L_y/2,L_y/2} = 0.$$

Note, that the lateral boundary conditions are not specified in the one-dimensional model; however, the first horizontal mode of the velocity and pressure fields, for which the parameterization of seiches has been obtained, satisfy the above conditions.

In all experiments, the following parameters were specified:

water body depth of 10 m;

the calculation time of 7 or 30 days;

the initial temperature gradient $\partial T/\partial z = 1.5°$ C/m, which corresponds to the Brunt–Väisälä frequency (buoyancy frequency) $N = 4 \times 10^{-2}$ s^{-1};

the momentum flux on the surface $\tau = 10^{-2}$ N/m^2,

the Coriolis force is not taken into account.

Under the above boundary conditions, experimental parameters, and the choice of initial conditions homogeneous over y direction, the three-dimensional problem becomes two-dimensional, which corresponds to the assumptions used above for the case of a one-dimensional model.

THE DYNAMICS OF MIXED-LAYER THICKNESS

In the numerical implementation of the classical Kato–Phillips experiment, both models show a good agreement with the analytical solution. As to the experiments with water bodies of finite size, it was demonstrated that an increase in the longitudinal size of the water body is accompanied by an increase in the thickness of the mixed layer (Fig. 2).

It is worth mentioning that, to the authors' knowledge, no laboratory experiments have been made for such conditions, and no empirical estimates similar to (19) are available; therefore, the authors think it possible in this case to take the result obtained using the three-dimensional hydrostatic model as a reference sample. The one-dimensional model shows a good agreement with the three-dimensional model: the parameterization of the horizontal gradient of pressure and viscosity makes it possible to adequately reproduce the thickness of the mixed layer. The depth of the mixed layer h_{ML} significantly depends on the horizontal dimensions of the water body, and the greater the water body, the closer this depth to the result of the classical Kato–Philips experiment, where there are no vertical walls. The restriction on h_{ML} in the presence of vertical walls is due to the action of the hydrostatic pressure gradient, which forms in the water body in such case and acts oppositely to the flux of momentum from the atmosphere. This leads to the formation of a quasi-steady-state circulation in the mixed layer. In the case of water layer unrestricted in the horizontal directions (Kato–Phillips experiment), the horizontal pressure gradient will not form and the flux of momentum from the atmosphere will lead to a monotonous and unlimited increase in the maximal velocity in the mixed layer, which, in turn, contributes to a rapid increase of h_{ML}.

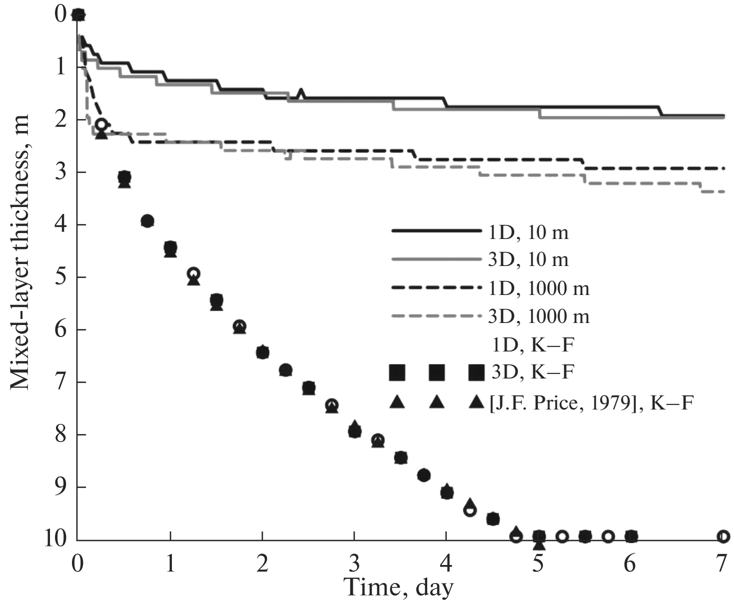

Fig. 2. Variations of mixed-layer thickness over time in calculations for finite water bodies and in the classic experiment of Kato–Phillips.

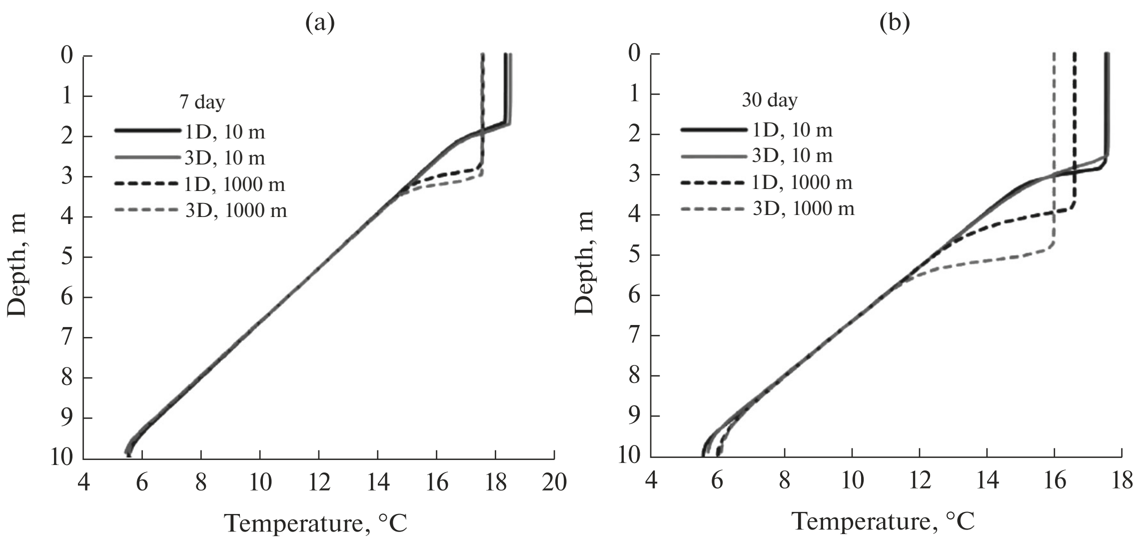

Fig. 3. The vertical distribution of temperature on the (a) 7th and (b) 30th days of calculations by the one-dimensional and three-dimensional models.

THE VERTICAL STRUCTURE OF FLOW

Numerical experiments were carried out for rectangular water bodies with different horizontal dimensions to analyze the vertical distributions of temperature, flow velocity, and the coefficient of turbulent viscosity. The duration of the numerical experiments was 30 days; other calculation parameters are given above. The comparison of the one- and three-dimensional models were carried out for the 7th and 30th days of calculation.

The profiles of the vertical temperature distribution were compared (Fig. 3).

The models on the time scales of several days show a good agreement. The difference between the results obtained with the use of models considered in this study for long time intervals (≥30 days) for a water body 1000 m in length may be due to the specific features of the models: note that the one-dimensional model is based on the averaging of three-dimensional equations, and the parameterization of pressure gradient takes into account only the first horizontal mode

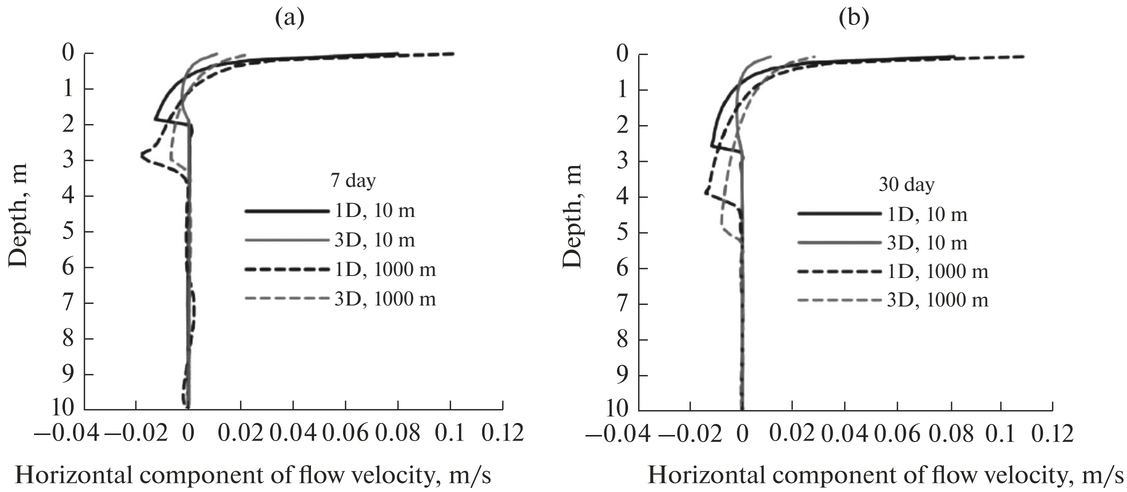

Fig. 4. The vertical distribution of horizontal velocity on the (a) 7th and (b) 30th days of calculations.

of the secihes. It should be stressed that in the case of a water body 10 m in length for the 30th day, both models give almost identical temperature profiles.

This suggests the assumption that, in the three-dimensional model, the share of the kinetic energy of harmonics with wave numbers >1 in the total kinetic energy is greater in a water body 1000 m in length than in the water body of 10 m in length.

Therefore, the first horizontal mode parameterized in the one-dimensional model gives a poorer description of the velocity field for a water body 1000 m in length than for one 10 m in length, resulting in a poorer reproduction of the vertical turbulent exchange and the deepening rate of the mixed layer.

Note that the results of the discussed numerical experiments for the time scale beyond 7–10 days are of purely theoretical interest. The thing is that, in the problem formulation, the flux of momentum from the atmosphere, which is determined by wind, has the same magnitude and direction throughout the calculation period. Such situation is impossible in the nature. First, the daily behavior of wind is statistically significant: wind is commonly stronger during day and weakens during night. Second, variations of wind at the synoptic time scale are associated with the passage of cyclones and anticyclones, the lifetime of which is commonly 7–10 days; therefore, at short time scales (not greater than several days), the wind conditions can be similar to those specified in the experiment.

Along with water temperature, the vertical distribution of flow velocity was also analyzed (Fig. 4).

In the profiles of flow velocity in the lower part of the mixed layer, oppositely directed pressure gradients and wind velocity can be seen, and the structure itself remains quasi-stationary. The thermocline features gravitational oscillations, the amplitude of which is suppressed by horizontal viscosity.

The profiles of the coefficient of turbulent viscosity (Figs. 5) show that its values in the mixed layer in one-dimensional model are much greater than they are in the three-dimensional model. This can be explained by the following mechanism: in both models, the friction between the oppositely directed upper and lower flows exists because of the vertical turbulent viscosity; however, the three-dimensional model also reproduces the vertical branches of the circulation cell, the role of which consists in the transfer of momentum between the upper and lower boundaries of the mixed layer. This contributes to a decrease in the velocity difference between the flows in the three-dimensional model, resulting in that the velocity shear in the one-dimensional model is greater. In accordance with (10), the production of turbulent kinetic energy in one-dimensional model is also greater; this is the cause of the overestimation of the eddy viscosity coefficient value. Because of this, in the description, for example, of the vertical turbulent transport of phytoplankton and greenhouse gases in the mixed layer, the results of the one-dimensional model can carry appreciable errors, and this aspect requires further studies.

CONCLUSIONS

The results of this study suggest that the horizontal dimensions of water bodies have a considerable effect on the depth of the top mixed layer and an increase in the size of the water body increases the thermocline deepening rate. It is confirmed that the seiche oscillations are to be taken into account for correct description of mixing processes in water bodies with horizontal dimensions much less than the Rossby deformation radius. The parameterization of the pressure gradient and the horizontal viscosity for the one-dimensional model LAKE, allows to reproduce the thickness of mixed layer with acceptable accuracy. It should be mentioned, however, that the one-dimensional model

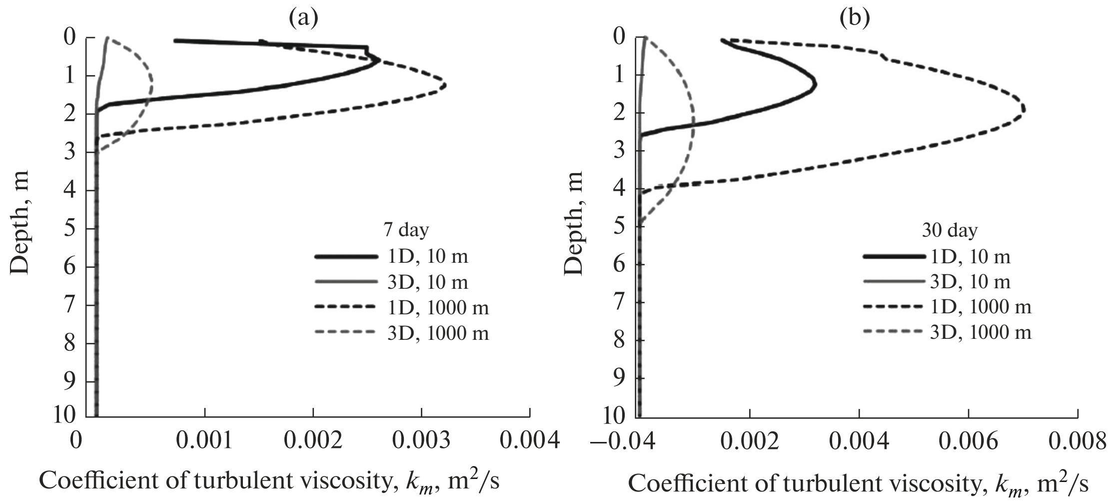

Fig. 5. The vertical distribution of the turbulent viscosity coefficient on (a) the 7th and (b) the 30th days of calculation.

overestimates the current velocity and, hence, the values of the turbulent viscosity coefficient. The authors plan to study the importance of this effects on the correctness of modeling, in particular, the processes of vertical transport of phytoplankton and greenhouse gases in real water bodies.

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