Turbulent Diffusion and Eddy Scales in Rivers

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Abstract—The relationship between turbulent diffusion with Eulerian and Lagrangian scales of turbulence in natural flows is considered. An estimate of the depth-averaged Lagrangian time scale as a function of Eulerian scale is suggested. The vertical turbulent diffusion in open natural flows is studied. The mean velocity profile is described by a power law. On the assumption of flat flow under incomplete self-similarity of the global Reynolds number, a universal expression for the vertical transfer coefficient is obtained. This expression enables one to estimate the time and length of complete mixing using minimum experimental data. The coefficients of longitudinal and vertical diffusion in different rivers are compared with each other and the universal ratio of these coefficients is suggested.

INTRODUCTION

The intensity of turbulence plays a great role in impurity transfer in natural streams. The system of Reynolds equations, describing stream motion, includes Reynolds stress tensor, which is usually unknown. To close the system, the tensor of turbulent transfer coefficients ε is used which are related with the scales of eddies determined by the velocity and morphological characteristics of the flow.

Despite the small value of turbulent transfer in comparison with that along the stream direction, it counts for much significance in rivers and reservoirs. Turbulent diffusion is one of the mechanisms of oxygen transfer to the bottom regions of a flow. The estimate of the time of dilution of wastewaters till nondangerous concentrations requires the knowledge of the magnitude of ε and its distribution over the depth. Turbulent diffusion is also very important during spring ice melting in natural water bodies. In practice, the vertical turbulent transfer is often estimated by depth-averaged coefficient, as it is many times smaller than that in the horizontal transverse direction, but sometimes it is necessary to know the vertical distribution of the coefficient of turbulent diffusion in the vicinity of the source of impurity. For example, in case the distance between the impurity source and the water off take is not large enough.

In this work we study relationship between turbulent transfer coefficients and the mean characteristics of natural flow.

BACKGROUND

The description of water motion in a river is a complex problem of determining the structure of a three-dimensional flow above a movable boundary. Mixing processes and impurity propagation in the flow depend on the relationships between flow velocities in different points of the stream. The mathematical apparatus for the description of river flow is based on differential equations of the conservation of mass and momentum. In river flows, the terms responsible for molecular interaction (diffusion, viscosity, and heat conduction) can be neglected since the contribution of these processes to the mean transfer in developed turbulent flow is very small.

To close the set of equations describing the movement of a three-dimensional stream, one commonly uses the gradient form of turbulent diffusion. The tensor coefficient of eddy transfer ε, arising in this procedure, is to be determined as well. Different hypotheses were used to determine the coefficient ε, the most physical one is Prandtl's mixing-length hypothesis developed, for example, by Czernuszenko & Rylov (2000) [4]. In paper [3], the coefficient ε was estimated with the help of a depth distribution of the mixing length measured in a laboratory flume, but it is difficult to develop the same procedure for open flows in nature. It seems reasonable to define this coefficient as a function of Lagrangian time scale of turbulence T_i [13, 15, 22]:

$$\varepsilon_i = T_i \sigma_i^2, \tag{1}$$

where

$$T_i = \int\limits_0^\infty r_{Li}(t)dt, \qquad (1a)$$

$r_{Li}(t)$ is the normalized Lagrangian correlation function of velocity, σ_i^2 is the variance of velocity fluctuations, $i = 1, 2, 3$ are the longitudinal, vertical and horizontal transverse axes.

The Lagrangian scale of turbulence is a measure of time during which the individual character of the motion of a chosen fluid particle is conserved. Information about the Lagrangian correlation functions is of great interest in studies of transfer processes in rivers. The Lagrangian characteristics of the velocity field of a stream can be investigated with the help of visualization methods, whose realization in a river is a difficult problem yet.

In general, each type of transfer (molecular, heat, momentum) in water stream is characterized by its own coefficient of transfer [2]. Many natural streams can be considered as 2D flows, in which case we introduce only longitudinal and vertical diffusivity. Let us consider turbulent flow with the mean width B and depth h in the coordinate system x, y, z, where the axis x corresponds to the stream direction and the longitudinal velocity component u, the axis y corresponds to the vertical direction and the vertical velocity component v, and the axis z corresponds to the transverse direction and the transverse velocity component w. For steady-state conditions, the longitudinal diffusion is small compared to the longitudinal advection, and the advective-diffusion equation in a flat stream ($h/B \ll 1$) looks like:

$$u\frac{\partial c}{\partial x} = \frac{\partial}{\partial y}\left(K_y\frac{\partial c}{\partial y} + \omega c\right) + \frac{\partial}{\partial z}\left(K_z\frac{\partial c}{\partial z}\right), \qquad (2)$$

where c is the concentration of impurity, K_y, K_z are the vertical and horizontal transverse components of turbulent mass transfer tensor coefficients, ω is fall velocity.

Thus the vertical transport of mass M in a wide stream can be described with the help of K_y, by the following expression:

$$M = -K_y\frac{\partial c}{\partial y}. \qquad (3)$$

Following Boussinesq hypothesis and introducing a coefficient of turbulent viscosity ε, one can present the right part of Reynolds equation for momentum transfer in a flat stationary flow as

$$\rho\frac{\partial}{\partial y}\left(v\frac{\partial u}{\partial y} - \overline{u'v'}\right) = \rho\frac{\partial}{\partial y}\left([v + \varepsilon]\frac{\partial u}{\partial y}\right), \qquad (4)$$

where ρ, v are the density and kinematic viscosity of water, u', v' are fluctuations of the longitudinal and vertical transverse components of stream velocity, respectively.

Then the vertical momentum transfer in an open flow can be described by the expression

$$\rho\varepsilon_y\frac{du}{dy} = \tau_{xy}, \qquad (5)$$

where $\tau_{xy} = -\rho\overline{u'v'}$ is shear stress tensor component.

In natural flows, the magnitude of ε_y is many times larger than the molecular viscosity v, excluding the layer near the bottom. As opposed to v, the turbulent transfer coefficient is a function of coordinates. If we know the depth distributions of $u(y)$ and τ_{xy} in a planar flow, the vertical transfer coefficient can be found using Eq. (5), which allows one to close the system of Reynolds equations.

It is known that, for impurity of neutral buoyancy, turbulent Schmidt number Sc = 1 and $K_y = \varepsilon_y$ [18, 19, 26], which allows one to compare the magnitudes of transfer coefficients described by Eqs. (3) and (5) obtained in experiments with dye and calculated from the data of velocity measurements. Equation (5) is the starting point for estimation of the vertical diffusivity in channel flow.

Under the assumptions of logarithmic mean velocity profile and linear depth distribution of the shear stress τ_{xy}, Elder (1959) [12] obtained the coefficient of vertical momentum transfer ε_y, in Eq. (5) as

$$\varepsilon_y = \frac{y}{h}\left(1 - \frac{y}{h}\right)\kappa h u_*, \qquad (6)$$

where $u_* = \sqrt{\tau_0/\rho}$ is the shear velocity, τ_0 is the wall shear stress, κ is the von Karman's constant.

From Eq. (6), the depth-averaged value of ε_y for a wide river flow equals to

$$\langle\varepsilon_y\rangle = 0.067 h u_*. \qquad (7)$$

THE COEFFICIENT OF LONGITUDINAL DIFFUSION AND THE SCALE OF TURBULENCE

If we explore the time–space structure of a turbulent flow in Eulerian coordinates, the quantitative characteristic of velocity connection at two points of the flow is the correlation tensor of second rank [20]:

$$B_{ik} = \langle(V_{2i} - V_{1i})(V_{2k} - V_{1k})\rangle, \qquad (8)$$

where $\vec{V_1}, \vec{V_2}$ are fluid velocities at points 1 and 2. The results of experimental studies of turbulent-stream structure are realizations of fluctuations of velocity components. The most accessible in experiments is the realization of the longitudinal component of velocity. Thus, we have the first diagonal term in

Eq. (8)—Eulerian autocorrelation function and correlation coefficient

$$r_x = \frac{\langle u'(t)u'(t+t')\rangle}{\langle (u')^2 \rangle}. \tag{9}$$

Thus, the experimental data enable us to estimate Eulerian time scale:

$$\theta_x = \int_0^\infty r_x(t')\,dt'. \tag{10}$$

The upper limit in this integral is commonly taken to be the time t_0', at which the autocorrelation function crosses the abscissa for the first time. Eulerian integral time scale of the turbulence presents "a memory of the flow about itself." The space integral scale L can be calculated with the help of space correlation function $R(x_i, x_i + l)$ as

$$L = \int R(x_i, x_i + l)\,dl. \tag{11}$$

To obtain space correlation function in a river, it is necessary to measure velocity fluctuations by identical probes simultaneously. Because of this fact, Taylor's hypothesis is usually used to estimate the Eulerian space scale of turbulence:

$$L_x = u\int_0^\infty r(t')\,dt'. \tag{12}$$

The value of θ can be easily obtained from experiments in laboratory flumes as well as in rivers. The estimation of L_x by Eq. (12) is also possible, because there are many current meters for accurate measurements of the mean velocity profile in nature. Thus, every time we obtain an autocorrelation function of velocity, and then the integral Eulerian scale of turbulence, we estimate the characteristic time during which the flow "remembers" its structure. Different measurement methods and results of experiments reviewed by Grinvald & Nikora (1988) [17] show that the largest longitudinal dimension of vortices in a turbulent stream is about $10h$, the maximum magnitude being measured in laboratory by visualization. It is practically impossible to measure the largest dimension of turbulent vortices in a river as it has been obtained by visualization in a laboratory flume. Thus, the estimate of L_x in nature can give different values up to $10h$.

No theoretical relationship between Lagrangian and Eulerian scales (Eqs. (1a) and (10)) is known. Since the measurements of Lagrangian characteristics in rivers are quite rare, of great interest is the relationship between Eulerian and Lagrangian scales obtained in laboratory flume [13] which reads:

$$T = 4\theta. \tag{13}$$

A relationship between T and θ may be found from the semi-empirical relationships for longitudinal velocity component derived in [5]

$$r_E(\beta_x\tau) = r_L(\tau), \quad \theta\beta_x = T, \quad \beta_x = \frac{u}{p\sqrt{\sigma^2}}, \tag{14}$$

where $p = 2.5$ for water.

For depth distribution of the turbulence intensity we use the following expression

$$\frac{\sigma_i}{u_*} = a_i + b_i\sqrt{\frac{y}{h}}, \tag{15}$$

where coefficients $a_1 = 2.1$, $b_1 = -1.2$, $a_2 = 1.3$, $b_2 = -0.6$, $a_3 = 1.6$, $b_3 = -1.0$ were obtained by Dolgopolova (1995) [9], $i = 1, 2, 3$ correspond to the axes x, y, z, respectively.

Averaging Eq. (15) through the depth and using for shear velocity the expression $u_* = \kappa n\langle u\rangle$ obtained in [11], where n is the power exponent in the power velocity profile Eq. (20), we derive

$$\langle\sigma_x\rangle = 1.3u_* = 1.3\kappa n\langle u\rangle. \tag{16}$$

Then one obtains for β_x

$$\beta_x = 1/1.3n. \tag{17}$$

Equations (14), (16), and (17) give us the relationship between depth-averaged values of T_x and θ_x in a planar flow:

$$T_x = \theta_x/1.3n. \tag{18}$$

To estimate the longitudinal Lagrangian scale for rivers, we use n in the range $0.1-0.3$ as obtained in different rivers [10], which gives $T_x = (2.6-7.1)\theta_x$. The constant 4 for the longitudinal component in Eq. (13) obtained by visualization of the flow in a flume is within this range. Results of calculations of T_x by Eq. (18) in different natural flows are presented in [7].

VERTICAL TRANSFER COEFFICIENT IN OPEN CHANNEL FLOW

To obtain ε_y as a function of mean characteristics of an open channel flow by Eq. (5), it is necessary to find parametric expressions for du/dy and τ_{xy}. The depth distribution of the mean velocity of a boundary layer can be adequately described by the universal logarithmic law and that an in open channel, by a power law with the exponent $1/7$ [23]. Detailed analysis of experimental data shows a systematic dependence of Karman's constant κ in logarithmic law on the Reynolds number Re [18]. In a general case, the mean velocity profile can be described with the help of the theory of dimensions [1]

$$\frac{y}{u_*}\frac{\partial u}{\partial y} = \left(\frac{u_* y}{\nu}\right)^n f(\mathrm{Re}_*), \tag{19}$$
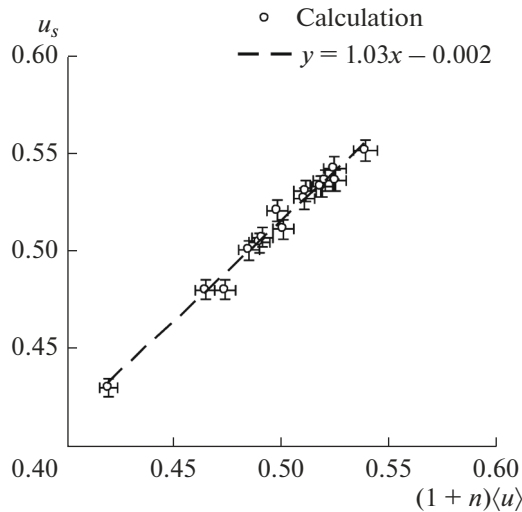
Fig. 1. Magnitudes of u_s obtained from the mean velocity profiles measured in the Kirzhach River versus the magnitudes calculated through the depth-averaged velocity and power exponent.

where $\mathrm{Re}_* = u_* h / \nu$ is the global Reynolds number, n is the power exponent depending on Re_*.

The universal logarithmic velocity profile is found by integrating Eq. (19) under the assumption of complete self-similarity of the flow. If this assumption is not satisfied, that is $n \neq 0, f \neq \mathrm{const}$, one gets a power velocity profile with $n = f(\mathrm{Re}_*)$. The application of power law with variable exponent to the description of profile of mean velocity of different open streams [11, 16, 27] allows one to consider it as accurate and theoretically-founded as the logarithmic one. The hypothesis of incomplete self-similarity of shearing layer with respect to Re was also confirmed in paper [28], where the author found that the power law with exponent depending on Re must be used for description of depth velocity distribution in an open flow.

The depth distribution of mean velocity can be described as:

$$u = u_s \left(\frac{y}{h}\right)^n. \tag{20}$$

Integration of Eq. (20) over the depth gives the parameter as $u_s = (1 + n)\langle u \rangle$, where $\langle u \rangle$ is the depth-averaged velocity. Then Eq. (20) can be rewritten as:

$$u = (1 + n)\langle u \rangle (y/h)^n. \tag{21}$$

The validity of expression for u_s is confirmed by data of measurements of velocity profiles at the River Kirzhach (Table 1) presented in Fig. 1. The power exponent in Eq. (21) is a function of Re and, for plain rivers, varies in the range $0.1 < n < 0.3$ [8]. Later the

range of variation of n for rivers was extended to $0.08 < n < 0.32$ by Dolgopolova (2007) [6].

To calculate ε_y by Eq. (5), we find the derivative of the mean velocity from Eq. (21)

$$\frac{du}{dy} = (1 + n)n\langle u \rangle \left(\frac{1}{h}\right)\left(\frac{y}{h}\right)^{n-1}. \tag{22}$$

Assuming the depth distribution of shear stress τ_{xy} linear, we have

$$\tau_{xy} = u_*^2(1 - y/h), \tag{23}$$

where $u_* = \sqrt{\tau_0/\rho}$, τ_0 is the shear stress at the bottom.

Substituting Eqs. (22) and (23) into Eq. (5) and using $u_* = \kappa n \langle u \rangle$ we obtain ε_y

$$\varepsilon_y = \frac{\kappa^2 \langle u \rangle hn(1 - y/h)}{(1 + n)(y/h)^{n-1}}. \tag{24}$$

The dimensionless coefficient of transfer ε_y' can be derived from Eq. (24) by dividing it by $\langle u \rangle h$,

$$\varepsilon_y' = \frac{\kappa^2 n(1 - y/h)}{(1 + n)(y/h)^{n-1}}. \tag{25}$$

Division of Eq. (25) by κn results in the expression

$$\varepsilon_y'' = \frac{\kappa(1 - y/h)}{(1 + n)(y/h)^{n-1}}. \tag{26}$$

The distributions of ε_y' over the depth for $n = 0.08$, 0.143, 0.32 presented in Fig. 2, show the dependence of ε_y' on the power exponent. The power exponent $n = 0.143 = 1/7$ is considered by Schlichting (2000) [22] as universal for river flows.

For n in the range $0.08-0.32$, the coefficient $\varepsilon_y'' = \varepsilon_y'/\kappa n$ Eq. (26) remains practically constant (Fig. 3), which allows one to regard Eq. (26) as the universal expression for open river flows. The comparison of ε_y'' in Eq. (26) with the dimensionless coefficient $\varepsilon_y/u_* h$ in Eq. (6), obtained under the assumption of logarithmic velocity profile [12], shows that these distributions are close to each other (Fig. 4). The distribution $\varepsilon_y''(y/h)$ calculated by Eq. (26) agrees with the results of measurements in river presented in [25], which confirms the adequacy of such description of turbulent transfer coefficient in open flow. Integration of Eq. (26) over the depth gives $\langle \varepsilon_y'' \rangle = 0.066$, and the depth-averaged coefficient of turbulent transfer can be written as:

$$\langle \varepsilon_y \rangle = 0.066 \kappa n \langle u \rangle h. \tag{27}$$
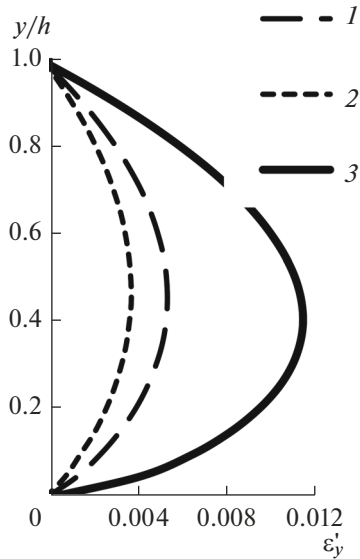
Fig. 2. Vertical distribution of the vertical transfer coefficient for different n: (*1*) $n = 0.143$, (*2*) $n = 0.08$, (*3*) $n = 0.32$.

Fig. 3. Depth distribution of the vertical transfer coefficient calculated by Eq. (26). Confidence intervals are shown for the range of power exponents $n = 0.08-032$.

DISCUSSION AND APPLICATIONS

The Eq. (27) obtained above shows the dependence of the depth-averaged coefficient of transfer on the power exponent n which is a function of Re and hydraulic resistance of a flow. Parameters n and $\langle u \rangle$ can be more readily measured in natural stream than shear velocity. Moreover, the depth-averaged velocity can be estimated by measuring the velocity of the flow at one point located at the distance $0.4\,h$ from a bottom with the help of the well-known expression $\langle u \rangle = u_{0.4}$.

Comparison of Eqs. (27) and (7) obtained in [12] shows that numerical coefficients in both formulas are very close to each other. Equation (27) contains parameters easily measured in a natural flow. Two numerical coefficients in Eq. (27) were obtained $\langle \varepsilon_y \rangle = 0.0021 \langle u \rangle h$ and $\langle \varepsilon_y \rangle = 0.0084 \langle u \rangle h$ for the boundary values of power exponent in mean velocity profile $n = 0.08$ and $n = 0.32$ respectively.

Eulerian time scales, diffusion coefficients, and characteristics of the rivers of Volga, Kirzhach, Polomet, Missouri, Mississippi, as well as Atrisco and Rio Grande channels [21], are presented in Table 1. In all the streams, there were sand dunes on the bottom. In the Polomet River, the ratio of dune height to the depth is about 0.25, resulting in the maximum power exponent n. The Eulerian longitudinal scale of turbulence L_x calculated by Eq. (12) for the rivers of Volga and Missouri and the Atrisco channel shows a significant dependence of L_x on the distance from the bank, varying from $\sim 2h$ in the middle of the flow to $0.7-1\,h$ near the bank [7]. The same tendency was revealed by Sukhodolov et al. (1998) [25] at the measurements in the Spree River. This can be explained by the destruc-
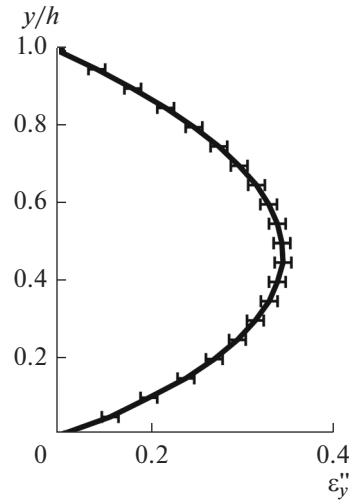
tion of the flow structure as the distance from the bank decreases and the role of bottom roughness increases.

U is the average river velocity $U = Q/A$ where Q is water discharge and A is the cross-section area, h is the mean depth of a river reach. The data of measurements at the rivers Missouri and Mississippi and Atrisco and Rio Grande channels were taken from the report [21].

The longitudinal scale of turbulence L_x was estimated from simultaneous measurements of velocity by two identical current meters at the River Kirzhach. Two current meters were located at the same horizon $z = 0.2h$, one being fixed and the other moving streamwise in the middle of the river, which allowed us
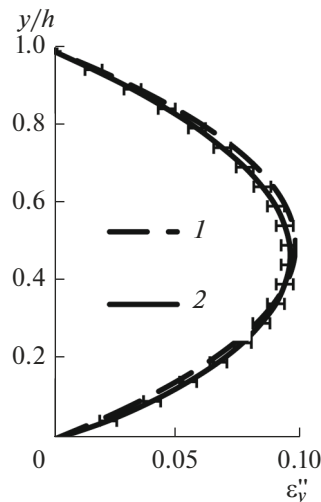

Fig. 4. Depth distribution of the vertical transfer coefficient (*1*) obtained by Elder (1959) and (*2*) from Eq. (26).

Table 1. Mean characteristics of natural streams

Natural stream	B, m	h, m	Re	U, m/s	$\langle\theta_x\rangle$, s	n	$\langle\varepsilon_y\rangle$, m²/s
Polomet	25	0.4	1.7×10^5	0.42	1.52	0.27	1.04×10^{-3}
Kirzhach	17	0.5	2×10^5	0.41	–	0.2	1.08×10^{-3}
Volga	700	12	1×10^7	0.81	12.0	0.15	2.87×10^{-2}
Missouri	213	4.4	5.3×10^6	1.22	3.4	0.14	1.95×10^{-2}
Atrisco Canal	17.1	0.52	3.3×10^5	0.64	0.75	0.17	1.64×10^{-3}
Rio Grande	20.1	0.85	7.0×10^5	0.82	0.64	0.18	3.31×10^{-3}
Mississippi	723	16.5	1.7×10^7	1.0	6.66	0.16	6.97×10^{-2}

to calculate L_x without using Taylor's hypothesis. As a result, the space correlation function was measured and the longitudinal scale of turbulence was found to be $L_x = 1.0$ m which is $2h$ for the Kirzhach River at the elevation of $0.2h$ from the bottom. The same result with the help of Taylor's hypothesis was derived from the experiment in the middle of the Missouri River by McQuivey (1973) [21].

The vertical space scale of turbulence L_y was obtained from the measurements of fluctuations of the longitudinal velocity component by two current meters located at the same vertical, one being fixed and the other moving along the vertical axis y from the bottom to the surface (the Kirzhach River [7]). These results of measurements confirm the existence of vortices occupying the whole depth of the flow and the validity of assumption $L_y = h$.

For different rivers, the depth-averaged longitudinal diffusivity coefficients $\langle\varepsilon_x\rangle$ can be estimated by Eq. (1) with the help of model relationships (16), (18) through the depth-averaged characteristics $\langle\theta_x\rangle$,

$\langle u\rangle$, and n. Then, using Eq. (27), one can derive the ratio of $\langle\varepsilon_x\rangle/\langle\varepsilon_y\rangle$:

$$\frac{\langle\varepsilon_x\rangle}{\langle\varepsilon_y\rangle} = 7.88\frac{\langle\theta_x\rangle\langle u\rangle}{h}. \qquad (28)$$

In the approximation $L_x = h$, the ratio $\langle\varepsilon_x\rangle/\langle\varepsilon_y\rangle = 7.88$ is constant. If the magnitudes of $\langle\theta_x\rangle$ are available from measurements, one can check the validity of the assumption $L_x = h$. The results of this verification are presented in Fig. 5.

The comparison of the magnitudes of L_x/h obtained experimentally in the Rivers Polomet and Missouri shows that they are close to each other. There is no information on the height of sand dunes in the Missouri River, but, presumably, the closeness of L_x/h for these two rivers could be explained by the large height of the sand dunes compared to their depths. For the Missouri River, the mean value of $\langle\theta_x\rangle\langle u\rangle/h$ is 1.5. As the depth becomes larger, as is the case in the rivers of Volga and Mississippi, the ratio L_x/h becomes smaller than 1, and for the Mississippi River, the mean
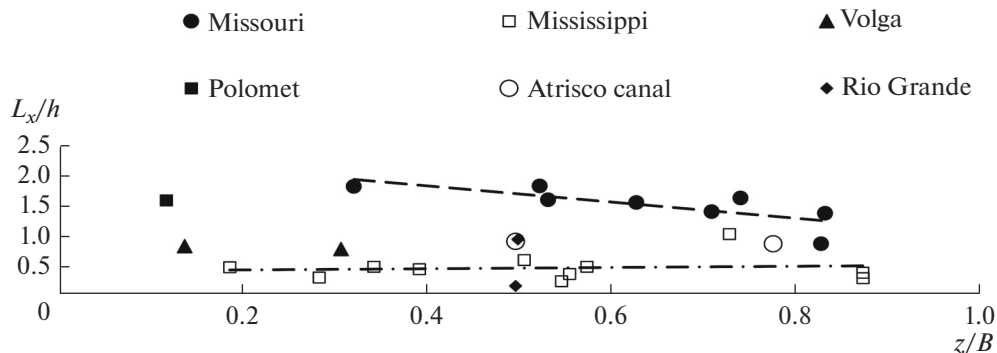
Fig. 5. Dependence of longitudinal space scale of turbulence on the distance from the bank.

value of L_x/h is 0.47, and that for the River Volga is 0.81. As the ratio L_x/h varies in the range 0.5–1.5, one can obtain from Eq. (28) the range for $\langle\varepsilon_x\rangle/\langle\varepsilon_y\rangle$ = 3.9–11.8, the lower boundary corresponding to deeper flows. It follows that a discharge of impurity from a point source propagates in a shallow stream farther than in a deep stream.

The comparison of the magnitudes of L_x and L_y obtained experimentally with those used in [4] for numerical modelling of 3D open channel flows shows that a simplifying assumption of the equality of the longitudinal and vertical scales of turbulence everywhere in the channel $L_x = L_y = h$ can be taken as a first approximation.

To show the application of the results of this study, let us estimate the ratio of the time required for an effluent from a point source to mix transversely across the river t_z to that required to mix vertically t_y, following [22]:

$$\frac{t_z}{t_y} = \frac{(B/h)^2}{\varepsilon_z/\langle\varepsilon_y\rangle}. \tag{29}$$

In our experiment in the Kirzhach River with floats of neutral buoyancy used as an impurity we obtained the coefficient of transverse diffusion in the surface layer of water $\varepsilon_z = 0.21u_*h$ which agrees with the value $\varepsilon_z = 0.23u_*h$ obtained in [12]. Reviewing the data of different authors, Roberts & Webster (2002) [22] show that in an ideal (straight, wide, fully roughed channel) flow for $B/h \geq 8$ and friction factor $f > 0.055$, the transverse diffusion coefficient is independent of the channel width and f and can be estimated as:

$$\frac{\varepsilon_z}{u_*h} = 0.13. \tag{30}$$

In natural rivers, their depth and width vary irregularly and there are many reaches with bends of different radius of curvature R_b. The bends increase the transverse diffusion, and ε_z/u_*h changes in the range 0.4–0.8 for weakly meandering rivers and even more for sharp bends [14]. This increase is due to helical cell motion in a cross section of a river. However, ε_z changes along the bend, being low at the upstream of the bend and increasing to the downstream reach [24]. So, the effect of the secondary flows in the bend on impurity propagation is minimizing. The transverse motion in river bends considerably impacts the dispersion only in sufficiently long and steep bends ($R_b/B \sim$ 5). The main cases when transverse diffusion is the single mechanism of high concentration of impurity dilution and must be taken into account are sudden spills and shoreline discharge into a wide river.

For rivers with bends which increase the transverse mixing the review paper [22] proposes the expression:

$$\frac{\varepsilon_z}{u_*h} = 0.6 \pm 50\%. \tag{31}$$

The experiment at the Kirzhach River gives $\varepsilon_z/u_*h = 0.55$ which agrees with Eq. (31) and the range presented in [14]. Thus, for the Kirzhach River with its trapezoidal cross-section and quasistable flow, the coefficient in the formula for ε_z lies between that for an ideal channel and a natural river. The shear velocity $u_* = 0.039$ m/s was obtained from the long-term series of data on stream surface slope. The depth-averaged coefficient of vertical diffusion was calculated by Eq. (27). The estimate of the ratio t_z/t_y in Eq. (29) gives $t_z/t_y \cong 370$, i.e. $t_z \gg t_y$, which is typical of lowland rivers.

For the bottom discharge, the distance L_s required for the effluent to be mixed over the depth can be approximately estimated as [22]:

$$L_s = 0.4\frac{Uh^2}{\langle\varepsilon_y\rangle}. \tag{32}$$

We calculated the value of U for the Kirzhach River averaging measured velocity profiles over a cross sections, and then the average velocity U was obtained from 10 cross-sections spaced 10 meters apart. The calculated estimate $L_s = 31$ m is in a good agreement with the data of measurements with salt solute carried out for economic purposes, which give the range $L_s =$ 27–32 m.

CONCLUSIONS

The application of a power law for the description of mean velocity profile in natural streams is considered. Under the condition of incomplete self-similarity of shear flow with respect to the global Reynolds number, the depth distribution of the coefficient of vertical turbulent diffusion ε_y' Eq. (24) is obtained.

The dependence of dimensionless ε_y'' on the power exponent n in the power law for velocity profile does not contradict the universal law $\varepsilon_y'' = f(y/h)$ Eq. (26) in the range $n = 0.08$–0.32, which is typical of lowland rivers. Expressions (26) and (27) enable one to estimate the depth distribution and depth-averaged magnitude of vertical transfer coefficient with the minimum of necessary experimental data.

On the basis of semi-empirical expressions, a relationship between Eulerian and Lagrangian time scales of turbulence is found. The model expression for the ratio of the coefficients of longitudinal and vertical turbulent diffusion $\langle\varepsilon_x\rangle/\langle\varepsilon_y\rangle$ is obtained. It is found

that this ratio $\langle\varepsilon_x\rangle/\langle\varepsilon_y\rangle$ decreases as the depth of a flow increases.

Most results discussed in the paper have been obtained for nearly straight reaches without taking into account the processes in river meanders. For the diffusion at meandering reaches, it must be noted, that secondary flows in river bend play a great role in the dilution of impurity discharged along shoreline into a wide river.

LIST OF SYMBOLS

A	cross-sectional area of a flow
B	width of a flow
B_{ik}	correlation tensor of velocity at two points of the flow
a_i, b_i	empirical coefficients in the model expression for intensity of turbulence
K_y, K_z	vertical and horizontal transverse components of turbulent mass transfer tensor
L_x, L_y	Eulerian space integral scales
L_s	the distance, required for complete mixing of the discharge of impurity at the bottom
n	power exponent in the mean velocity profile
$R(x_i, x_i + l)$	space correlation function $R(x_i, x_i + l)$
$r_{Li}(t)$	normalized Lagrangian correlation function of velocity
r_x	Eulerian correlation coefficient
Re	Reynolds number
$\mathrm{Re}_* = u_* h/\nu$	global Reynolds number
R_b	radius of curvature of a bend
Sc	turbulent Schmidt number
T_i	Lagrangian time scale of turbulence
u, v, w	components of mean velocity corresponding to x, y, and z directions
u', v', w'	fluctuations of velocity components
u_*	shear velocity
β_x	empirical coefficient relating Eulerian and Lagrangian time scales of turbulence
ε_i	coefficient of turbulent viscosity with indices corresponding to the axes
θ	Eulerian time scale
ν	kinematic viscosity of water
ρ	density of water
σ_i	standard deviation of velocity fluctuations, $i = 1, 2, 3$ corresponds to longitudinal, vertical and transverse direction
τ_{ij}	shear stress tensor

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