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Multiple Capture in a Group Pursuit Problem with Fractional Derivatives

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Abstract—In a finite-dimensional Euclidean space, we consider a problem of pursuing one evader by a group of pursuers with equal capabilities of all participants. The dynamics of the problem is described by the system

$$D^{(\alpha)} z_i = a z_i + u_i - v, \quad u_i, v \in V,$$

where $D^{(\alpha)} f$ is the Caputo derivative of order $\alpha \in (1, 2)$ of a function f. The set of admissible controls V is compact and strictly convex, and a is a real number. The aim of the group of pursuers is to catch the evader by at least m different pursuers, possibly at different times. The terminal sets are the origin. The pursuers use quasi-strategies. We obtain sufficient conditions for the solvability of the pursuit problem in terms of the initial positions. The investigation is based on the method of resolving functions, which allows us to obtain sufficient conditions for the termination of the approach problem in some guaranteed time.

Keywords: differential game, group pursuit, multiple capture, pursuer, evader.

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INTRODUCTION

An important direction in the development of the modern theory of differential games is associated with the creation of solution methods for game problems of pursuit and evasion with several objects [1–4]. In this area, not only are the classical solution methods deepened, but also new problems are sought to which the existing methods are applicable. In particular, in [5–7], problems of pursuing two objects described by equations with fractional derivatives were considered and sufficient conditions of a capture were obtained.

In the present paper, we consider a problem on a multiple capture by a group of pursuers of one evader provided that all the participants have equal capabilities and the movements of the players are described by equations with Caputo fractional derivatives. Sufficient conditions of a capture are obtained. Grigorenko found [8] necessary and sufficient conditions of a multiple capture for the problem of simple pursuit. Conditions of a simultaneous multiple capture for the problem of simple pursuit with equal capabilities of the participants were obtained by Blagodatskikh [9]. The problem on a multiple capture of an evader in Pontryagin's example was presented in [10–13]. A multiple capture in linear differential games was considered in [2, 14–16]. The problem of group pursuit with state constraints and fractional derivatives of order $\alpha \in (0, 1)$ was studied in [17].

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1. PROBLEM STATEMENT

Definition 1 [18]. Suppose that f is a function from $[0,\infty)$ to $\mathbb{R}^k$, its derivative f' is absolutely continuous on $[0,\infty)$, and $\alpha \in (1,2)$. The Caputo derivative of f of order α is the function

$$\big(D^{(\alpha)} f\big)(t) = \frac{1}{\Gamma(2-\alpha)} \int\limits_0^t \frac{f''(s)}{(t-s)^{\alpha-1}}\, ds, \qquad \text{where} \quad \Gamma(\beta) = \int\limits_0^\infty e^{-s} s^{\beta-1}\, ds.$$

In the space $\mathbb{R}^k$ ($k \ge 2$), we consider an $(n+1)$-person differential game: there are n pursuers $P_1, \ldots, P_n$ and one evader E. Each pursuer P_i moves according to the law

$$D^{(\alpha)} x_i = a x_i + u_i, \quad x_i(0) = x_i^0, \quad \dot{x}_i(0) = x_i^1, \quad u_i \in V. \tag{1.1}$$

The motion law of the evader E has the form

$$D^{(\alpha)} y = a y + v, \quad y(0) = y^0, \quad \dot{y}(0) = y^1, \quad v \in V. \tag{1.2}$$

Here $\alpha \in (1,2)$, $x_i, y, u_i, v \in \mathbb{R}^k$, V is a strictly convex compact set in $\mathbb{R}^k$, and a is a real number. In addition, $x_i^0 \ne y^0$ for all i.

Instead of systems (1.1), (1.2), we consider the system

$$D^{(\alpha)} z_i = a z_i + u_i - v, \quad z_i(0) = z_i^0 = x_i^0 - y^0, \quad \dot{z}_i(0) = z_i^1 = x_i^1 - y^1, \quad u_i, v \in V. \tag{1.3}$$

Here and below, $i \in I = \{1, \ldots, n\}$. Denote by $z^0 = \{z_i^0, z_i^1\}$ the vector of initial positions. We assume that $z_i^1 \ne 0$ for all i.

Definition 2. A mapping U_i taking a vector of initial positions z^0, time t, and arbitrary prehistory of the evader's control $v_t(\cdot)$ to a measurable function $u_i(t)$ with values in V is called a quasi-strategy of the pursuer P_i.

Definition 3. An m-multiple capture (for $m=1$, a capture) occurs in the game if there exist a time $T(z^0)$ and quasi-strategies $U_1, \ldots, U_n$ of the pursuers $P_1, \ldots, P_n$ such that, for any measurable function $v(\cdot)$ with values $v(t) \in V$ for $t \in [0, T(z^0)]$, there exist times $\tau_1, \ldots \tau_m \in [0, T(z^0)]$ and pairwise different indices $i_1, \ldots, i_m \in I$ such that $z_{i_s}(\tau_s) = 0$ for $s = 1, \ldots, m$.

We introduce the following notation:

$$E_\rho(z,\mu) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\rho^{-1} + \mu)} \quad \text{is the generalized Mittag-Leffler function [20, p. 17],}$$

$$f_i(t) = \begin{cases} t^{\alpha-1} E_{1/\alpha}(a t^\alpha, 1) z_i^0 + t^\alpha E_{1/\alpha}(a t^\alpha, 2) z_i^1 & \text{if } a < 0, \\ \dfrac{z_i^0}{t} + z_i^1 & \text{if } a = 0, \end{cases}$$

$$\lambda(z,v) = \sup\{\lambda \ge 0\colon -\lambda z \in V - v\}, \quad \gamma = -a\Gamma(2-\alpha),$$

$$\Omega(l) = \{(i_1, \ldots, i_l)\colon\ i_1, \ldots, i_l \in I \text{ and are pariwise different}\},$$

$$\delta_0^+ = \min_{v \in V} \max_{\Lambda \in \Omega(m)} \min_{j \in \Lambda} \lambda(z_j^1, v), \qquad \delta_0^- = \min_{v \in V} \max_{\Lambda \in \Omega(m)} \min_{j \in \Lambda} \lambda(-z_j^1, v),$$

$$\delta_t^+ = \min_{v \in V} \max_{\Lambda \in \Omega(m)} \min_{j \in \Lambda} \lambda(f_j(t), v), \qquad \delta_t^- = \min_{v \in V} \max_{\Lambda \in \Omega(m)} \min_{j \in \Lambda} \lambda(-f_j(t), v),$$

$$\delta_0 = \min\{\delta_0^+, \delta_0^-\}, \qquad \delta_t = \min\{\delta_t^+, \delta_t^-\},$$

$$r(t,s) = \begin{cases} 1 & \text{if } E_{1/\alpha}(a(t-s)^\alpha, \alpha) \ge 0, \\ -1 & \text{if } E_{1/\alpha}(a(t-s)^\alpha, \alpha) < 0, \end{cases} \qquad \overline{E}(t,s) = (t-s)^{\alpha-1} E_{1/\alpha}\left(a(t-s)^\alpha, \alpha\right).$$

2. SUFFICIENT CONDITIONS OF THE CAPTURE

2.1. Sufficient conditions of the capture for $a < 0$.

Lemma 1. *Let $a < 0$ and $\delta_0 > 0$. Then there exists a time $T > 0$ such that the inequality $\delta_t > 0.5\gamma\delta_0$ holds for all $t > T$.*

Proof. We have the following asymptotic estimates as $t \to +\infty$ [19, formula (1.2.4)]:

$$E_{1/\alpha}(at^\alpha, 1) = -\frac{1}{at^\alpha \Gamma(1-\alpha)} + O\Big(\frac{1}{t^{2\alpha}}\Big), \qquad E_{1/\alpha}(at^\alpha, 2) = -\frac{1}{at^\alpha \Gamma(2-\alpha)} + O\Big(\frac{1}{t^{2\alpha}}\Big),$$

where $O(g)$ as $t \to +\infty$ is a specific function G such that the function G/g is bounded on $(A, +\infty)$ for some $A > 0$. Hence, the function f_i can be represented in the form

$$f_i(t) = -\frac{z_i^0}{at\Gamma(1-\alpha)} + \frac{z_i^1}{\gamma} + O\Big(\frac{1}{t^\alpha}\Big),$$

which yields $\lim\limits_{t\to+\infty} f_i(t) = \dfrac{z_i^1}{\gamma}$. Since the function λ is continuous [2, Lemma 1.3.13], we have $\lim\limits_{t\to\infty} \lambda(f_i(t), v) = \lambda\Big(\dfrac{z_i^1}{\gamma}, v\Big)$ for all $v \in V$. Consequently,

$$\lim_{t\to+\infty} \delta_t^+ = \min_{v\in V} \max_{\Lambda\in\Omega(m)} \min_{j\in\Lambda} \lambda\Big(\frac{z_j^1}{\gamma}, v\Big) = \gamma\delta_0^+.$$

Similarly, $\lim\limits_{t\to+\infty} \delta_t^- = \gamma\delta_0^-$. Therefore, there exists $T > 0$ such that $\delta_t > 0.5\gamma\delta_0$ for all $t > T$. $\square$

Lemma 2. *Let $a < 0$ and $\delta_0 > 0$. Then there exists $T_0 > 0$ such that, for any measurable function $v(\cdot)$ with values in V, there exists a set $\Lambda \in \Omega(m)$ such that, for all $j \in \Lambda$,*

$$T_0^{\alpha-1} \int_0^{T_0} \big|\overline{E}(T_0, s)\big| \lambda(f_j(T_0) r(T_0, s), v(s))\, ds \geq 1.$$

Proof. By Lemma 1 there exists $T_1 > 0$ such that $\delta_t > 0.5\gamma\delta_0$ for all $t > T_1$. Let $T > T_1$. Consider the functions

$$h_i(t) = t^{\alpha-1} \int_0^t \big|\overline{E}(t, s)\big| \lambda(f_i(T) r(T, s), v(s))\, ds, \qquad t \in [0, T].$$

Then

$$\max_{\Lambda\in\Omega(m)} \min_{j\in\Lambda} h_j(t) \geq \max_{\Lambda\in\Omega(m)} t^{\alpha-1} \int_0^t \big|\overline{E}(t, s)\big| \min_{j\in\Lambda} \lambda(f_j(T) r(T, s), v(s)) ds. \tag{2.1}$$

Since

$$\max_{\Lambda\in\Omega(m)} a_\Lambda \geq \frac{1}{C_n^m} \sum_{\Lambda\in\Omega(m)} a_\Lambda$$

for any nonnegative numbers $\{a_\Lambda\}_{\Lambda\in\Omega(m)}$, it follows from (2.1) that

$$\max_{\Lambda\in\Omega(m)}\min_{j\in\Lambda} h_j(t) \geq \frac{t^{\alpha-1}}{C_n^m}\int_0^t \left|\overline{E}(t,s)\right| \sum_{\Lambda\in\Omega(m)} \min_{j\in\Lambda} \lambda(f_j(T)r(T,s), v(s))ds$$

$$\geq \frac{t^{\alpha-1}}{C_n^m}\int_0^t \left|\overline{E}(t,s)\right| \max_{\Lambda\in\Omega(m)}\min_{j\in\Lambda} \lambda(f_j(T)r(T,s), v(s))\,ds$$

$$\geq \frac{\delta_0\gamma t^{\alpha-1}}{2C_n^m}\int_0^t \left|\overline{E}(t,s)\right| ds \geq \frac{\delta_0\gamma}{2C_n^m} t^{\alpha-1}\int_0^t \overline{E}(t,s)\,ds. \tag{2.2}$$

By [20, formula (1.15)], we have

$$\int_0^t \overline{E}(t,s)\,ds = t^\alpha E_{1/\alpha}\left(at^\alpha, \alpha+1\right).$$

Therefore, from (2.2), we obtain

$$\max_{\Lambda\in\Omega(m)}\min_{j\in\Lambda} h_j(T) \geq \frac{\delta_0\gamma}{2C_n^m} T^{\alpha-1}T^\alpha E_{1/\alpha}(aT^\alpha, \alpha+1) = \frac{\delta_0\gamma}{2C_n^m} T^{2\alpha-1}E_{1/\alpha}(aT^\alpha, \alpha+1).$$

In view of [19, formula (1.2.4)], the following asymptotic representation holds as $t\to+\infty$:

$$E_{1/\alpha}(at^\alpha, \alpha+1) = -\frac{1}{at^\alpha} + O\Big(\frac{1}{t^{2\alpha}}\Big).$$

Therefore,

$$\max_{\Lambda\in\Omega(m)}\min_{j\in\Lambda} h_j(T) \geq \frac{\delta_0\gamma}{2C_n^m}\Big(-\frac{T^{\alpha-1}}{a} + O\Big(\frac{1}{T}\Big)\Big), \quad T\to+\infty.$$

Since $a<0$ and $\alpha-1>0$, there exists $T_0>T_1$ such that

$$\frac{\delta_0\gamma}{2C_n^m}\Big(-\frac{T^{\alpha-1}}{a} + O\Big(\frac{1}{T}\Big)\Big) \geq 1.$$

Thus, there exists $T_0>0$ such that $\max_{\Lambda\in\Omega(m)}\min_{j\in\Lambda} h_j(T_0)\geq 1$. Consequently, there exists $\Lambda_0\in\Omega(m)$ such that $h_j(T_0)\geq 1$ for all $j\in\Lambda_0$, which completes the proof. □

Define the number

$$\hat{T} = \inf\Big\{t \;\Big|\; \inf_{v(\cdot)}\max_{\Lambda\in\Omega(m)}\min_{j\in\Lambda} t^{\alpha-1}\int_0^t |\overline{E}(t,s)|\lambda(f_j(t)r(t,s), v(s))\,ds \geq 1\Big\}.$$

By Lemma 2, $\hat{T}<\infty$.

Theorem 1. *Let $a<0$ and $\delta_0>0$. Then an m-multiple capture occurs in the game.*

Proof. Let $v(s)$ for $s \in [0, \hat{T}]$ be an arbitrary control of the evader. Consider the function

$$H(t) = 1 - \max_{\Lambda \in \Omega(m)} \min_{j \in \Lambda} \hat{T}^{\alpha-1} \int\limits_0^t |\overline{E}(\hat{T}, s)| \lambda(f_j(\hat{T}) r(\hat{T}, s), v(s))\, ds$$

and denote by $T_0 > 0$ its first root. Note that T_0 exists in view of Lemma 2 and the definition of $\hat{T}$. In addition, there exists a set $\Lambda_0 \in \Omega(m)$ such that, for all $j \in \Lambda_0$,

$$1 - \hat{T}^{\alpha-1} \int\limits_0^{T_0} |\overline{E}(\hat{T}, s)| \lambda(f_j(\hat{T}) r(\hat{T}, s), v(s))\, ds \le 0.$$

Therefore, there exist times $t_j \le T_0$, $j \in \Lambda_0$, for which

$$1 - \hat{T}^{\alpha-1} \int\limits_0^{t_j} |\overline{E}(\hat{T}, s)| \lambda(f_j(\hat{T}) r(\hat{T}, s), v(s))\, ds = 0. \tag{2.3}$$

For $j \notin \Lambda_0$, denote by t_j times for which condition (2.3) holds if such times exist. We define the pursuers' controls by setting

$$u_i(s) = \begin{cases} v(s) - \lambda(f_i(\hat{T}) r(\hat{T}, s), v(s)) f_i(\hat{T}) r(\hat{T}, s), & s \in [0, \min\{t_i, \hat{T}\}], \\ v(s), \quad s \in [\min\{t_i, \hat{T}\}, \hat{T}]. \end{cases}$$

Then the solution of system (1.3) can be represented in the form [21, formula (19)]

$$z_i(t) = E_{1/\alpha}(at^\alpha, 1) z_i^0 + t E_{1/\alpha}(at^\alpha, 2) z_i^1 + \int\limits_0^t \overline{E}(t, s)(u_i(s) - v(s)) ds.$$

Hence,

$$\hat{T}^{\alpha-1} z_i(\hat{T}) = f_i(\hat{T}) + \hat{T}^{\alpha-1} \int\limits_0^{\hat{T}} \overline{E}(\hat{T}, s)(u_i(s) - v(s))\, ds$$

$$= f_i(\hat{T}) - \hat{T}^{\alpha-1} \int\limits_0^{\hat{T}} |\overline{E}(\hat{T}, s)| \lambda(f_i(\hat{T}) r(\hat{T}, s), v(s)) f_i(\hat{T})\, ds$$

$$= f_i(\hat{T}) \Big(1 - \hat{T}^{\alpha-1} \int\limits_0^{t_i} |\overline{E}(\hat{T}, s)| \lambda(f_i(\hat{T}) r(\hat{T}, s), v(s)) \Big)\, ds = 0$$

for all $i \in \Lambda_0$. Consequently, $z_i(\hat{T}) = 0$ for all $i \in \Lambda_0$. □

2.2. Sufficient conditions of the capture for $a = 0$.

Lemma 3. *Let $a = 0$ and $\delta_0^+ > 0$. Then there exists a time $T > 0$ such that $\delta_t^+ > 0.5\delta_0^+$ for all $t > T$.*

Proof. Since $\lim\limits_{t\to+\infty} f_i(t) = z_i^1$ and the function λ is continuous [2, Lemma 1.3.13], we have $\lim\limits_{t\to+\infty} \lambda(f_i(t), v) = \lambda(z_i^1, v)$ for all $v \in V$. Therefore, $\lim\limits_{t\to+\infty} \delta_t^+ = \delta_0^+$, which yields the required inequality. □

Lemma 4. *Let $a = 0$ and $\delta_0^+ > 0$. Then there exists $T_0 > 0$ such that, for any measurable function $v(\cdot)$ with values in V, there exists a set $\Lambda \in \Omega(m)$ such that, for all $j \in \Lambda$,*

$$\frac{1}{T_0}\int_0^{T_0} \overline{E}(T_0, s)\lambda(f_j(T_0), v(s))ds \geq 1.$$

Proof. The lemma is proved similarly to Lemma 2 with the use of Lemma 3. □

Define the number

$$\hat{T} = \inf\left\{t > 0 \mid \inf_{v(\cdot)} \max_{\Lambda\in\Omega(m)} \min_{j\in\Lambda} \frac{1}{t}\int_0^t \overline{E}(t, s)\lambda(f_j(t), v(s))ds \geq 1\right\}.$$

By Lemma 4, $\hat{T} < +\infty$.

Theorem 2. *Let $a = 0$ and $\delta_0^+ > 0$. Then an m-multiple capture occurs in the game.*

Proof. Consider the function

$$H(t) = 1 - \max_{\Lambda\in\Omega(m)} \min_{j\in\Lambda} \frac{1}{\hat{T}}\int_0^t \overline{E}(\hat{T}, s)\lambda(f_j(\hat{T}), v(s))ds$$

and denote by T_0 its first root. Then there exists a set $\Lambda_0 \in \Omega(m)$ such that, for all $j \in \Lambda_0$,

$$1 - \frac{1}{\hat{T}}\int_0^{T_0} \overline{E}(\hat{T}, s)\lambda(f_j(\hat{T}), v(s))ds \leq 0.$$

Therefore, there exist times $t_j \leq T_0$, $j \in \Lambda_0$, for which

$$1 - \frac{1}{\hat{T}}\int_0^{t_j} \overline{E}(\hat{T}, s)\lambda(f_j(\hat{T}), v(s))\, ds = 0. \tag{2.4}$$

For $j \notin \Lambda_0$, denote by t_j times for which condition (2.4) holds if such times exist. We define the pursuers' controls by setting

$$u_i(s) = \begin{cases} v(s) - \lambda(f_i(\hat{T}), v(s))f_i(\hat{T}), & s \in [0, \min\{t_i, \hat{T}\}], \\ v(s), & s \in [\min\{t_i, \hat{T}\}, \hat{T}]. \end{cases}$$

Then the solution of system (1.3) can be presented in the form [21, formula (19)]

$$z_i(t) = z_i^0 + tz_i^1 + \int_0^t \overline{E}(t,s)(u_i(s) - v(s))ds.$$

Hence,

$$\frac{z_i(\hat{T})}{\hat{T}} = f_i(\hat{T}) + \frac{1}{\hat{T}}\int_0^{\hat{T}} \overline{E}(\hat{T},s)(u_i(s) - v(s))ds = f_i(\hat{T}) - \frac{1}{\hat{T}}\int_0^{\hat{T}} \overline{E}(\hat{T},s)\lambda(f_i(\hat{T}), v(s))ds \cdot f_i(\hat{T})$$

$$= f_i(\hat{T})\Big(1 - \frac{1}{\hat{T}}\int_0^{t_i} \overline{E}(\hat{T},s)\lambda(f_i(\hat{T}), v(s))ds\Big) = 0$$

for all $i \in \Lambda_0$. Consequently, $z_i(\hat{T}) = 0$ for all $i \in \Lambda_0$. □

Denote by $\operatorname{Int} A$ and $\operatorname{co} A$ the interior and the convex hull of a set A.

Lemma 5 [3, Assertion 1.3]**.** *Let V be a strictly convex compact set with smooth boundary, and let*

$$0 \in \bigcap_{\Lambda \in \Omega(n-m+1)} \operatorname{Int} \operatorname{co} \{z_j^1, j \in \Lambda\}. \tag{2.5}$$

Then $\delta_0 > 0$.

Theorem 3. *Suppose that $a \le 0$, V is a strictly convex compact set with smooth boundary, and condition* (2.5) *is satisfied. Then an m-multiple capture occurs in the game.*

Proof. The validity of this theorem follows from Lemma 5 and Theorems 1 and 2. □

Corollary. *Suppose that $a \le 0$, V is a strictly convex compact set with smooth boundary, and*

$$0 \in \operatorname{Int} \operatorname{co} \{z_1^1, \ldots, z_n^1\}.$$

Then there is a capture in the game.

Proof. The corollary is proved by setting $m = 1$ in (2.5). □

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