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Developing Models of Turbine Thermal Processes in Low-Steam and Motor Modes

G. A. Pikina[a, b, *], E. K. Arakelyan[a, **], F. F. Pashchenko[b], and G. A. Filippov[a]

[a] National Research University Moscow Power Engineering Institute, Moscow, 111250 Russia
[b] Trapeznikov Institute for Management Problems, Russian Academy of Sciences, Moscow, 117997 Russia
*e-mail: PikinaGA@mail.ru
**e-mail: arakelianek@mpei.ru

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Abstract—The main purpose of the motor mode is to keep the turbine in hot standby (with the boiler off) without generating electricity to the grid. To do this, the generator is switched to the engine mode, which rotates the turbine rotor, and the cooling of the flow path is performed by supplying one or two intermediate steam extractions from the extractions of the neighboring turbine. The first stages of the high-pressure cylinder run on steam from the front seals. The noted specificity of the purpose and operating conditions of the turbine in low-steam modes imposes additional requirements on the mathematical model. When developing models of a turbine stage in a motor mode, the physical media included in the object were taken into account: the steam of the flow path passing through the nozzle and working discs, participating in the conversion of its thermodynamic and kinetic energy into mechanical energy of the rotor rotation; vapor leakage into the space between the discs; steam leakage through the rotor-guide vane seal; metal of rotating devices (working disks and turbine rotor); and metal of the stator and exhaust pipes. Multipoint models are obtained with lumped parameters distributed along the length of the flow channel with values that vary depending on the ordinal number of the stage. The main part of the step models are eight ordinary differential equations or partial differential equations of physical media. The developed models are necessary for computer modeling and investigating the temperature distribution of the turbine flow path. Based on the research results, it is proposed to create a real automatic control system that ensures the implementation of restrictions on metal heating and the generation of added power.

Keywords: turbine, low steam and motor modes, thermal processes, multipoint models, distributed parameter model, partial differential equations, lumped parameter model, ordinary differential equations

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When developing models of steam turbines for normal operation of power units, assumptions were usually made about the weak effect of heat exchange between steam and metal structural elements and about the absence of losses to the environment since the main energy component was the transformation of the thermodynamic work of steam into mechanical energy of rotation of the turbine generator rotor. However, when a turbine operates in a low-steam and, especially, in a motor mode [1–4], the characteristics of the processes occurring in it are determined precisely by the heat exchange between physical media.

With the help of the motor mode, the turbine is kept in a hot standby (with the boiler off) without generating electricity to the grid. The generator is transferred to the engine mode, which rotates the turbine rotor. Cooling of the flow path is performed by supplying a small amount of steam to intermediate extractions from the extractions of a neighboring turbine. The first stages of the high-pressure cylinder only operate on a pair of front rotor seals.

The noted specificity of the purpose and operating conditions of the turbine in low-steam modes imposes additional requirements on the mathematical model. The developed model should take into account:

1. supplying external power for rotation of the rotor from the generator;
2. heat release from friction between steam and rotating parts of the rotor;
3. energy consumption of the rotating parts of the rotor for ventilation of the interdisk space;
4. no consumption of fresh steam in the high-pressure cylinder (HPC) from its boiler;
5. entrance from outside of the cooling steam flows into the turbine extractions;
6. the presence of steam consumption through the outer seals of the turbine cylinders;
7. steam leakage through the stage guide vanes' seals;

8. steam leaks into the disk space;

9. heat loss to the environment through the stator insulation;

10. the occurrence, due to the action of the centrifugal force, of the pressure drop along the radius of the flow path, which can cause moisture suction from the turbine condenser in the last stages of the low-pressure cylinder.

Since a turbine is structurally and functionally a set of the same type of components—stages, it is quite natural to display the processes occurring in it using a multipoint model with parameters that vary depending on the ordinal number of the stage [5].

If at least one partial differential equation is used as differential equations, which takes into account the change in physical quantities along at least one spatial coordinate, then such a model belongs to the class of models with distributed parameters (DP-models).

If only ordinary differential equations are used as differential equations and the change in physical quantities only in time is taken into account, then such a model belongs to the class of models with lumped parameters (LP-models).

Taking into account the specifics of the turbine in the form of the same type of components, in the classification of models, the class of multipoint models can be divided into multipoint LP-models (MT-LP) and multipoint DP-models (MT-DP).

Each stage consists of a stationary disc with steam flow guiding nozzles and a working disc with blades.

The object includes the following physical media: steam of the flow path passing through the nozzle and working disks participating in the transformation of its thermodynamic and kinetic energy into mechanical energy of the rotor rotation, vapor leakage into the space between the discs, steam leakage through the rotor-guide vane seal, metal of rotating devices (working disks and turbine rotor), and metal of the stator and exhaust pipes. Based on the named media, it is possible to determine the volume of the differential part of the equations for the thermal process model of each stage: six equations of the laws of conservation of energy for four steam flows and two steam volumes between the disks and four equations of thermal conductivity for the metal of the disks, rotor, and stator.

In this work, it is proposed to use a one-dimensional dynamic model for steam flow in disks, which takes into account the temperature distribution only along the length of the flow channel. The assumption is made about the same temperature values in the section, made on the basis of two factors. This is a low steam consumption (i.e., a small mass of steam in the channel), which leads to noticeable overheating of the steam, and turbulent flow, accompanied by mixing of the substance and averaging its temperature.

To check the feasibility of taking into account the distribution of the steam temperature along the length of the channel, two models (DP and LP) are presented with the possibility of their subsequent comparison when carrying out numerical calculations of specific turbines.

The content of this article reflects the results of the first stage of the volumetric work, ensuring the creation of a real automatic control system capable of meeting the requirements for the temperature state of the turbine in the engine mode and generating an added power not exceeding the idle power.

EQUATIONS OF THE LAW OF ENERGY CONSERVATION OF THE STEAM FLOW

The specific form of the energy equation depends on the assumptions made in the derivation, which are determined by the features of the energy conversion processes in the object. Further, these features are considered in relation to heat exchangers.

Supplied (diverted) to the mass flow M heat $\mathrm{d}Q$ (i.e., the energy of the flow $\mathrm{d}E$) is spent on changing its internal energy $\mathrm{d}U$ and the work done by the thread (or on the thread) $\mathrm{d}L$:

$$\mathrm{d}Q = \mathrm{d}E = \mathrm{d}U + \mathrm{d}L.$$

Total heat $\mathrm{d}Q$ consists of heat supplied (removed) from the external medium with respect to the flow $\mathrm{d}Q_{\mathrm{ext}}$ and heat released (absorbed) inside the flow as a result of the work of friction forces $\mathrm{d}Q_{\mathrm{fr}}$.

Work $\mathrm{d}L$ can be decomposed into mechanical according to the rotation of the working disk of the rotor during the movement of steam in a curved channel $\mathrm{d}L_{\mathrm{mech}}$, kinetic (by acceleration or deceleration of the flow) $\mathrm{d}L_{\mathrm{kin}}$, potential (according to the flow displacement in the gravitational field) $\mathrm{d}L_{\mathrm{f}}$, technical $\mathrm{d}L_{\mathrm{t}}$ components of work flow and work of friction forces $\mathrm{d}L_{\mathrm{fr}}$.

It can be assumed that the work to overcome the frictional forces is used to generate heat $\mathrm{d}L_{\mathrm{fr}} = \mathrm{d}Q_{\mathrm{fr}}$. With regard to steam flows in a turbine, the change in potential energy can be neglected. In addition, the change in mechanical and kinetic energy occurs at the rate of flow of hydrodynamic processes, which is one to two orders of magnitude higher than the rate of change in temperatures of a large mass of turbine metal. Therefore, in what follows, we can assume that L_{fur} and L_{kin} change instantly and remain constant. Then

$$\mathrm{d}Q_{\mathrm{ext}} = \mathrm{d}U + \mathrm{d}L_{\mathrm{tech}} = \mathrm{d}U + \mathrm{d}(pV) = \mathrm{d}(mh),$$

where p is pressure, V is flow volume, m is steam mass, and h is its enthalpy.

Thus, we can assume that a change in the amount of heat or work received (given off) from the external environment (disk metal) is converted into a change in the enthalpy of the flow

$$\mathrm{d}Q_{\mathrm{ext}} = \mathrm{d}(mh) = \mathrm{d}E.$$

When the flow moves in the channel, two options are possible:

$$Q_{\text{ext}} = \begin{cases} -qS \text{ flow movement with cooling;} \\ +qS \text{ flow movement with heating,} \end{cases}$$

where q is specific heat flux, kW/m^2; and S is channel surface area, m^2.

It should be noted that the variables included in the equations (flow energy E and specific heat flux q) are functions of two arguments: time t and spatial coordinates (lengths) z. Taking into account the smallness of the flow rates and the turbulence of the flow, it was assumed that there is no distribution of thermal parameters in the cross section.

FLOW MODEL WITH DISTRIBUTED PARAMETERS

Scratch disk channel. First, you need to reveal the total derivative of the flow energy:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial E}{\partial t} + \frac{\partial E}{\partial z}\frac{\partial z}{\partial t} = V\frac{\partial(\rho h)}{\partial t} + l\frac{\partial(f\rho v h)}{\partial z}$$
$$= +qS - N_{\text{mech}},$$

where $N_{\text{mech}} = \dfrac{\mathrm{d}L_{\text{mech}}}{\mathrm{d}t}$ is power of the rotor rotation during the movement of steam in a curved channel, l and f are length and cross-sectional area of the channel, and v and ρ are the flow rate and density.

Moving on to steam consumption $D = f\rho v$, available

$$V\frac{\partial(\rho h)}{\partial t} + l\frac{\partial(Dh)}{\partial z} = +qS - N_{\text{mech}}. \tag{1}$$

Along with the form (1) of writing the flow energy equation, another is often used, the obtainment of which is possible to represent the differential of the product of variable quantities by the sum

$$\partial(\rho h) = \rho\partial h + h\partial\rho,\quad \partial(Dh) = D\partial h + h\partial D.$$

After substituting these sums in (1) from the resulting equation, you need to subtract the equation of the law of conservation of matter, previously multiplied by the enthalpy h:

$$V\rho c_p\frac{\partial h}{\partial t} + Dl\frac{\partial h}{\partial z} = +qS - N_{\text{mech}}. \tag{2}$$

And, finally, one can give one more writing of the energy equation for a one-dimensional single-phase flow through its temperature θ, taking into account that the isobaric heat capacity c_p and enthalpy are related as $\partial h = c_p\partial\theta$:

$$V\rho c_p\frac{\partial\theta}{\partial t} + Dc_p l\frac{\partial\theta}{\partial z} = +qS - N_{\text{mech}}. \tag{2a}$$

Nozzle disc channel. Unlike the working channel, thermodynamic energy is converted into kinetic energy of the flow in the channel of the guide vane:

$$V\rho\frac{\partial h}{\partial t} + Dl\frac{\partial h}{\partial z} = +qS - N_{\text{kin}}; \tag{3}$$

$$V\rho c_p\frac{\partial\theta}{\partial t} + Dc_p l\frac{\partial\theta}{\partial z} = +qS - N_{\text{kin}}. \tag{3a}$$

Here, $N_{\text{kin}} = \dfrac{\mathrm{d}L_{\text{kin}}}{\mathrm{d}t}$ is the conversion power into kinetic energy.

FLOW MODEL WITH CONCENTRATED PARAMETERS

Scratch disk channel. Integration of equation (2) along the coordinate z ranging from 0 to l gives

$$V\int_0^l \rho\frac{\partial h}{\partial t}\partial z + lD\int_0^l \frac{\partial h}{\partial z}\partial z = S\int_0^l q\partial z - \int_0^l N_{\text{mech}}\partial z.$$

For the first integral, we can introduce the product of the mean integral over the length values of density and enthalpy

$$\int_0^l \rho\frac{\partial h}{\partial t}\partial z = \bar{\rho}\frac{\partial\bar{h}}{\partial t}.$$

In the second integral, the flow rate of the medium D can be considered constant along the length since hydrodynamic processes occur almost instantaneously in comparison with thermal ones. Then

$$D\int_0^l \partial h = D\left(h^{\text{fin}} - h^{\text{in}}\right),$$

where h^{fin} and h^{in} are the initial and final values of the enthalpy of the flow.

The integral on the right-hand side of the equation is equal to the mean-integral value of the heat flux, and we obtain the following when we consider the formula for convective heat exchange between the flux and the wall:

$$\int_0^l q\partial z = \bar{q} = \alpha\left(\bar{\vartheta} - \bar{\theta}\right),$$

where α is the heat transfer coefficient and $\bar{\theta}$ and $\bar{\vartheta}$ are mean values of the temperature of the flow and the channel wall along the length.

Thus, the equation for the flow energy in the pointwise approximation has the form

$$V\bar{\rho}\frac{\partial\bar{h}}{\partial t} + Dh^{\text{fin}} = Dh^{\text{in}} + \alpha S\left(\bar{\vartheta} - \bar{\theta}\right) - \bar{N}_{\text{mech}},$$

where the line above all parameters indicates the mean integral value.

In point approximation models, abstracted from dimensions, there are only two values for any parameter—at the input and output. Therefore, the mean integral values of the parameters in the obtained equations have to be expressed in terms of their initial and final values.

In the practice of mathematical modeling, the method of replacing with final values has become widespread: $\overline{\rho} = \rho^{\text{fin}}$, $\overline{h} = h^{\text{fin}}$, $\overline{\theta} = \theta^{\text{fin}}$, then

$$V\rho^{\text{fin}}\frac{\partial h^{\text{fin}}}{\partial t} + Dh^{\text{fin}} = Dh^{\text{in}} + \alpha S\left(\overline{\vartheta} - \theta^{\text{fin}}\right) - \overline{N}_{\text{mech}}. \quad (2b)$$

It should especially be noted that the record $+\alpha S\left(\overline{\vartheta} - \theta^{\text{fin}}\right)$ will be valid both for heating the flow when moving in the channel (the temperature difference is positive), and when the steam gives off heat to the wall (the temperature difference is negative).

Nozzle disc channel. Integration of equation (3) along the coordinate z ranging from 0 to l results in

$$V\rho\frac{\partial h^{\text{fin}}}{\partial t} + Dh^{\text{fin}} = Dh^{\text{in}} + \alpha S\left(\overline{\vartheta} - \theta^{\text{fin}}\right) - \overline{N}_{\text{kin}}. \quad (3b)$$

DEVELOPMENT OF A MODEL OF THERMAL PROCESSES IN THE STEP

When developing a model of thermal processes in a step, it is necessary to take into account all media, the energy equations for which are included in the model of thermal processes in a step:

1. steam of the flow path passing through the nozzle and working discs;
2. steam in the interdisk space;
3. steam leakage into the space above the working disc;
4. steam leakage through the rotor-guide vane seal;
5. metal of rotating devices (working disk and parts of the turbine rotor within the stage);
6. fixed metal part of the stator and exhaust pipes.

To describe auxiliary media (steam in interdisk volumes, leaks, leaks and metal), it is optimal to use point (LP) models, while the choice of LP- (equations (2b) and (3b)) or DP-model (equations (2) and (3)) remains with the developer for the steam of the flow path.

Energy Equation of the Volume of Steam between the Steps (j–1) and (j)

If we refer the volume of steam between the previous and considered turbine stages to the composition of the considered stage, then the equation of the law of conservation of energy for it can be represented in the following form:

$$m_{\text{i-d1}}\frac{\mathrm{d}h_{\text{i-d1}}^{\text{fin}}}{\mathrm{d}t} + D_{\text{i-d1}}h_{\text{i-d1}}^{\text{fin}} = D_{\text{i-d1}}h_{\text{i-d1}}^{\text{in}} + \alpha S_{\text{i-d1}}\left(\overline{\vartheta} - \theta^{\text{fin}}\right)_{\text{i-d1}} + Q_{\text{i-d1}}^{fr}. \quad (4)$$

Here, $h_{\text{i-d1}}^{\text{fin}}$ and $h_{\text{i-d1}}^{\text{in}}$ are final and initial enthalpy of steam in the volume between the steps; $Q_{\text{i-d1}}^{\text{fr}}$ is the heat flux from the friction of the working disk of the previous stage on the steam in the interdisk space, the subscript 1 denotes the first interdisk volume of the stage; $m_{\text{i-d1}}$ is mass of steam in volume; $D_{\text{i-d1}}$ is steam flow from the previous stage, equal to the sum of flow rates from the flow path $D(j-1)$ and leaks D_{leak}:

$$D_{\text{i-d1}}(j) = D(j-1) + D_{\text{leak}}(j-1).$$

The convective heat transfer component in equation (4) reflects the transfer of heat from steam to the face of the nozzle disk (guide vanes) and to the contact part of the turbine stator.

The initial enthalpy can be found from the mixing equation of the two streams:

$$D_{\text{i-d1}}(j)\,h_{\text{i-d1}}^{\text{in}}(j) = D(j-1)\,h^{\text{fin}}(j-1) + D_{\text{leak}}(j-1)\,h_{\text{leak}}^{\text{fin}}(j-1),$$

from which we get

$$h_{\text{i-d1}}^{\text{in}}(j) = \frac{D(j-1)}{D_{\text{i-d1}}(j)}h^{\text{fin}}(j-1) + \frac{D_{\text{leak}}(j-1)}{D_{\text{i-d1}}(j)}h_{\text{leak}}^{\text{fin}}(j-1),$$

where $h_{\text{leak}}^{\text{fin}}$ is the final enthalpy of the leakage vapor from the previous stage.

Energy Equation of the Volume of Steam in the Channel of the Guide Vane

The equation for the energy conservation law of the LP model for steam in the channel of the guide vanes can be written in the following form

$$m_{\text{g.v}}\frac{\partial h_{\text{g.v}}^{\text{fin}}}{\partial t} + D_{\text{g.v}}h_{\text{g.v}}^{\text{fin}} = D_{\text{g.v}}h_{\text{g.v}}^{\text{in}} + \alpha S_{\text{g.v}}\left(\overline{\vartheta} - \theta_{\text{g.v}}^{\text{fin}}\right) - \overline{N}_{\text{kin}}. \quad (5)$$

Here, $m_{\text{g.v}}$ is mass of steam in the channel of the guide disc and $D_{\text{g.v}}$ is steam flow rate entering the channel, which is equal to the steam flow rate in the interdisk volume minus the leakage rate $D_{\text{g.v.l}}$ through the rotor seal:

$$D_{\text{g.v}}(j) = D_{\text{i-d1}}(j) - D_{\text{g.v.l}}(j).$$

Initial enthalpy of flow at the entrance to the channel of the guide vane $h_{\text{g.v}}^{\text{in}}$ is equal to the final $h_{\text{i-d1}}^{\text{fin}}$.

Equation of the Energy of Vapor Leakage through a Seal with a Rotor

The inertialess model can be adopted for leakage, neglecting the vapor mass in the seal. Due to the small flow rate of the leak, it can be assumed that it will not affect the pressure drop across the nozzle disc. Then it is fair to assume that the steam flow through the channel of the guide vane and the seal will be proportional to the average areas of their flow sections:

$$D_{\mathrm{g.v.l}} = \frac{f_{\mathrm{g.v.l}}}{f_{\mathrm{g.v}}} D_{\mathrm{g.v}}.$$

In addition, the passage of steam through the seal can be viewed as a throttling process for which the enthalpy remains constant, i.e.,

$$h_{\mathrm{g.v.l}}^{\mathrm{fin}} = h_{\mathrm{g.v.l}}^{\mathrm{in}} = h_{\mathrm{i\text{-}d1}}^{\mathrm{fin}}. \quad (6)$$

Energy Equation for the Metal of the Guide Vanes

Heat exchange through the lateral surfaces of the guide disk occurs due to contact with the vapor of the first (index "md1") and the second (index "md2") interdisk volumes and through the inner surface area of the flow channel of the guide vane (index "n.A"):

$$\begin{aligned} m_{\mathrm{g.v}} c_{\mathrm{m}} \frac{\mathrm{d}\vartheta_{\mathrm{g.v}}}{\mathrm{d}t} &= \alpha S_{\mathrm{i\text{-}d1}} \left(\theta_{\mathrm{i\text{-}d1}}^{\mathrm{fin}} - \vartheta_{\mathrm{g.v}}\right) \\ &+ \alpha S_{\mathrm{i\text{-}d2}} \left(\theta_{\mathrm{i\text{-}d2}}^{\mathrm{in}} - \vartheta_{\mathrm{g.v}}\right) + \alpha S_{\mathrm{g.v}} \left(\theta_{\mathrm{g.v}}^{\mathrm{fin}} - \vartheta_{\mathrm{g.v}}\right). \end{aligned} \quad (7)$$

Here, $m_{\mathrm{g.v}}$, c_{m}, and $\vartheta_{\mathrm{g.v}}$ are mass, heat capacity, and temperature of the guide vane disk metal and α is the coefficient of heat transfer from steam to metal.

If the temperature difference between the steam and the metal is negative, then the metal will cool and, conversely, a positive temperature difference indicates the heating of the metal.

Energy Equation for the Volume of Steam between the Nozzle and Working Discs

For the volume of vapor between the disks of a stage, the energy equation of the SC model can be written as follows:

$$\begin{aligned} m_{\mathrm{i\text{-}d2}} \frac{\mathrm{d}h_{\mathrm{i\text{-}d2}}^{\mathrm{fin}}}{\mathrm{d}t} + D_{\mathrm{i\text{-}d2}} h_{\mathrm{i\text{-}d2}}^{\mathrm{fin}} &= D_{\mathrm{i\text{-}d2}} h_{\mathrm{i\text{-}d2}}^{\mathrm{in}} \\ &+ \alpha S_{\mathrm{i\text{-}d2}} \left(\vartheta - \theta^{\mathrm{fin}}\right)_{\mathrm{i\text{-}d2}} + Q_{\mathrm{i\text{-}d2}}^{\mathrm{fr}}. \end{aligned} \quad (8)$$

Here, $Q_{\mathrm{i\text{-}d2}}^{\mathrm{fr}}$ is heat flux from friction of the working disk of the stage on the steam in the interdisk space, the subscript 2 denotes the second interdisk volume of the stage; $m_{\mathrm{i\text{-}d2}}$ is mass of steam in the interdisk volume; $D_{\mathrm{i\text{-}d2}}$ is steam flow rate entering the interdisk volume from the guide vane and equal to the flow rate from the flow path $D_{\mathrm{g.v}}$ and leaks $D_{\mathrm{g.v.l}}$ guide vane:

$$D_{\mathrm{i\text{-}d2}} = D_{\mathrm{g.v}} + D_{\mathrm{g.v.l}}.$$

The convective heat transfer component in equation (4) reflects the transfer of heat from steam to the face of the nozzle disk (guide vanes) and to the contact part of the turbine stator.

The initial enthalpy can be found from the equation for mixing two streams

$$D_{\mathrm{i\text{-}d2}} h_{\mathrm{i\text{-}d2}}^{\mathrm{in}} = D_{\mathrm{g.v}} h_{\mathrm{g.v}}^{\mathrm{fin}} + D_{\mathrm{g.v.l}} h_{\mathrm{g.v.l}}^{\mathrm{fin}} = D_{\mathrm{g.v}} h_{\mathrm{g.v}}^{\mathrm{fin}} + D_{\mathrm{g.v.l}} h_{\mathrm{i\text{-}d1}}^{\mathrm{fin}},$$

from which we get

$$h_{\mathrm{i\text{-}d2}}^{\mathrm{in}} = \frac{D_{\mathrm{g.v}}}{D_{\mathrm{i\text{-}d2}}} h_{\mathrm{g.v}}^{\mathrm{fin}} + \frac{D_{\mathrm{g.v.l}}}{D_{\mathrm{i\text{-}d2}}} h_{\mathrm{i\text{-}d1}}^{\mathrm{fin}}.$$

Energy Equation for the Volume of Steam in the Channel of the Working Disk

In the model with lumped parameters, the equation of the law of conservation of energy for steam in the channel of the working disk will have the following form:

$$\begin{aligned} m_{\mathrm{r.d}} \frac{\mathrm{d}h_{\mathrm{r.d}}^{\mathrm{fin}}}{\mathrm{d}t} + D_{\mathrm{r.d}} h_{\mathrm{r.d}}^{\mathrm{fin}} &= D_{\mathrm{r.d}} h_{\mathrm{r.d}}^{\mathrm{in}} \\ &+ \alpha S_{\mathrm{r.d}} \left(\overline{\vartheta} - \theta_{\mathrm{r.d}}^{\mathrm{fin}}\right) - \overline{N}_{\mathrm{mech}}. \end{aligned} \quad (9)$$

Here, $m_{\mathrm{r.d}}$ is mass of steam in the channel of the working disk; $h_{\mathrm{r.d}}^{\mathrm{fin}}$ and $h_{\mathrm{r.d}}^{\mathrm{in}}$ are final and initial enthalpy of steam in the channel of the working disk; $S_{\mathrm{r.d}}$ is the area of the working disk; $D_{\mathrm{r.d}}$ is steam flow entering the channel of the working disk, equal to the steam flow from the interdisk volume minus the leakage through the annular gap above the disk:

$$D_{\mathrm{r.d}} = D_{\mathrm{i\text{-}d2}} - D_{\mathrm{leak}}.$$

Initial enthalpy of steam flow at the channel inlet $h_{\mathrm{i\text{-}d}}^{\mathrm{in}}$ is equal to the final enthalpy in the second disk volume $h_{\mathrm{i\text{-}d2}}^{\mathrm{fin}}$.

Energy Equation for Steam Leakage through the Annular Gap above the Working Disk

Due to the low leakage rate, it can be assumed that it will not affect the pressure drop across the working disk. Then it is fair to assume that the steam flow through the channel of the working disk and the annular gap between the disk and the stator will be proportional to the average areas of their flow sections:

$$D_{\mathrm{leak}} = \frac{f_{\mathrm{leak}}}{f_{\mathrm{r.d}}} D_{\mathrm{r.d}}.$$

In addition, the passage of steam through the seal can be viewed as a throttling process for which the enthalpy remains constant, i.e.,

$$m_{\text{leak}} \frac{dh_{\text{leak}}^{\text{fin}}}{dt} + D_{\text{leak}} h_{\text{leak}}^{\text{fin}} = D_{\text{leak}} h_{\text{leak}}^{\text{in}} - \alpha S_{\text{st}} \left(\theta_{\text{leak}}^{\text{fin}} - \vartheta_{\text{st}}\right) + \alpha S_{\text{r.d}} \left(\vartheta_{\text{r.d}} - \theta_{\text{leak}}^{\text{fin}}\right), \tag{10}$$

where S_{st} is the surface area of the stator, streamlined by vapor leakage, ϑ_{st} is stator temperature, $S_{\text{r.d}}$ is surface area of the crown of the working disk, and $\vartheta_{\text{r.d}}$ is its temperature.

Rotating Metal Energy Equation

To reduce the number of differential equations, a single system of rotating metal should be considered: the metal of the working disk and the part of the rotor shaft related to the stage under consideration.

In the motor mode, the rotor is rotated by the generator of the unit by transferring a stage of mechanical energy per unit of time N_{gen}. In addition, the rotation energy is transferred by the steam flow in the channel of the working disk, $\bar{N}_{\text{mech}}$.

Heat flow $Q_{\text{r.d}}$ is supplied or removed (depending on the sign of the temperature difference between the metal and steam) through the inner surface area of the flow channel of the working disk. Due to friction with the vapor of the interdisk volumes on the lateral surfaces of the rotating disk, a heat flux is formed $2Q_{\text{fr}}$, causing an increase in the temperature of both the metal of the disk and the vapor in contact with it. Heat exchange from friction with parts of the rotor shaft occurs in a similar way; Q_{rot} in the interdisk volumes of the step.

The power supplied from the generator is spent on the rotation of the metal mass of the working disk and the rotor shaft N_{rtn} to overcome rotational friction N_{fr} and ventilation N_{win}, giving additional speed of rotation to steam flows.

Taking into account the above, it is possible to write down the energy equation for the metal of the stage rotor:

$$m_{\text{rot}} c_{\text{m}} \frac{d\vartheta}{dt} = 2Q_{\text{fr}} + Q_{\text{r.d}} + Q_{\text{rot}} + N_{\text{gen}} + \bar{N}_{\text{mech}} - N_{\text{rtn}} - N_{\text{fr}} - N_{\text{win}}. \tag{11}$$

Here, m_{rot} is the mass of the metal of the rotor (working disk and shaft) in the stage; ϑ is metal temperature; $Q_{\text{r.d}} = -\alpha S_{\text{r.d}} \left(\vartheta - \theta_{\text{r.d}}^{\text{fin}}\right)$, which follows from equation (9); $\bar{N}_{\text{mech}} = D v_x \left(v_x^{\text{out}} - v_x^{\text{in}}\right) = D v_x \left(v^{\text{out}} \cos\beta^{\text{out}} - v^{\text{in}} \cos\beta^{\text{in}}\right)$; D is steam mass flow; v and β are the velocities and angles of the velocity triangles at the inlet and outlet of the channel.

The rest of the components are either specified as input actions or are calculated using empirical formulas [4, 6].

Stator Metal Energy Equation

Heat transfer through the stator surfaces occurs due to contact with the vapor of the first (index "md1") and the second (index "id2") interdisk volumes and through the surface area of the leakage channel (index "lk"). To simplify the calculations, you can determine the average stator temperature in the region of this stage using the differential equation

$$m_{\text{st}} c_{\text{m}} \frac{d\bar{\vartheta}}{dt} = \alpha S_{\text{i-d1}} \left(\theta_{\text{i-d1}}^{\text{fin}} - \bar{\vartheta}\right) + \alpha S_{\text{i-d2}} \left(\theta_{\text{i-d2}}^{\text{fin}} - \bar{\vartheta}\right) + \alpha S_{\text{leak}} \left(\theta_{\text{leak}}^{\text{fin}} - \bar{\vartheta}\right) - Q_{\text{loss}}, \tag{12}$$

where $Q_{\text{loss}} = \alpha_{\text{ins}} \left(S_{\text{i-d1}} + S_{\text{i-d2}} + S_{\text{leak}}\right)\left(\bar{\vartheta} - \theta_{\text{o.a}}\right)$ is heat loss (index "loss") through insulation (index "ins") into the environment with a known outdoor temperature (index "o.a"); α_{ins} is the coefficient of heat transfer from the thermal insulation of the stator to the environment or calculate the stator temperature for each part of the stage separately, dividing equation (12) into three separate components.

CONCLUSIONS

(1) The system of equations (1)–(12) presented in this paper is a nonlinear dynamic model with distributed (or lumped) parameters for a steam turbine in motor mode. It is valid both for an individual channel of the flow path and for the entire flow path with a full steam flow per stage and with full geometric dimensions of the stage elements.

(2) Stages with the same steam flow rate in the flow path can be combined into one group when calculating.

(i) The first group operates at a steam flow from the front seal of the HPC with an initial enthalpy equal to the enthalpy of steam at the outlet of the seal.

(ii) The second group of stages starts after extraction, which is supplied with cooling steam from a neighboring turbine. The consumption of the total steam of the second group increases by the volume of the consumption of the cooling steam entering the selection, with a change in its enthalpy to the weighted average (see formulas for steam in interdisk volumes).

(iii) Finally, two more steam flows are added to the flow rate to the first stage of the medium pressure cylinder (MPC) (or low pressure cylinder): from the end seal of the previous cylinder (HPC or MPC) and its front seal. The enthalpy of the total flow is also calculated as a weighted average.

(3) The developed models make it possible to carry out computer modeling and comprehensive analysis of the dependence of the metal temperature distribution on various influencing factors. The research results

will be used in the implementation of an automatic control system that guarantees the fulfillment of the requirements for the temperature state of the flow path of a steam turbine and does not exceed the permissible level of added energy.

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