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Patterns of the Rotor-over-Stator Rolling under Change in the Damping Components

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Abstract—As experimental studies show, the rubbing of the rotor against the structure usually excites harmonics of different frequencies. In high-frequency regions, the power of the vibration signal appears to be considerable. The rotor–supports–stator system is in an unstable equilibrium state during the contact interaction between the rotor and the stator. The forces exerted on the rotor facilitate the excitation of the asynchronous rolling and its damping. The forces have been determined that facilitate the excitation of the progressive and retrograde rotor precession. The consideration of these forces in the algorithm for modeling the rotor-over-stator rolling development allows investigation of the impact of the components of the above forces on the behavior of the rotor system. The initial excitation—disturbance of the normal operation—of the rotor and subsequent unsteady oscillations of it result from sudden imbalance in the second span. The results of numerical modeling of the rubbing in the second span and the rotor-over-stator rolling upon change in the damping components of secondary (gyroscopic) components b_{ij} ($i \neq j$) of the damping matrix are presented for the rotor on three bearing-supports considering the synergetic effect of the forces of various types exerted on the rotor. It is shown that change in one of the parameters of the excitation forces leads to ambiguity of the pattern (manifestation form) of the asynchronous rotor-over-stator rolling and proves the existence of more than one states towards which the rotor–supports–stator system tends. In addition to the rolling with a constant rotor–stator contact, oscillations of the rotor develop in the direction perpendicular to the common trajectory of the precession motion of the rotor's center with transition to the vibro-impact motion mode. The oscillations of the rotor tend towards the symmetry center of the system (the stator bore center). The reason is the components of the stiffness and damping forces that act in the direction transverse to the rotor's motion trajectory. Recommendations are given for eliminating dangerous consequences of the development of the asynchronous rolling fraught with great financial losses.

Keywords: rotor, stator, asynchronous rolling, rolling-exciting forces, contact stiffness coefficients, gyroscopic forces, motion trajectory of the rotor section's center, angular velocity of the rotor precession, normal pressure force exerted on the stator

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In [1–3], potential patterns of the developing rotor-over-stator rolling—rolling manifestation forms—are considered, viz., synchronous rolling accompanied by slipping and subsequent loss of contact, asynchronous rolling[1] accompanied by slipping and transition into pure rolling without slipping, and vibro-impact motions of the rotor under the contact and bounce when the rotor imbalance and damping may cause a loss of contact between the rotor and stator or the development of rolling. What these modes have in common is that the development of the rolling is a sufficiently hazardous phenomenon capable of causing the forced stopping of the turbine plants and even their failure. In Fig. 1 and in [4–6], the consequences of the rolling development on a 300-MW turbine plant are shown. The authors of [7] consider that the rolling that occurred during the accident at Kashirskaya State District Power Plant in 2002 and the rolled blades of the intermediate pressure rotor (IPR) prove this (see Figs. 1b, 1c).

During modeling the oscillations of the rotor with rubs, especially in the systems on several supports with plain journal bearings, we observed the development of the rolling with transition to the vibro-impact mode. The obtained data were first viewed with some skepticism. It was considered that the results of modeling the vibrations of the rotor accompanied by the rubbing against the stator for multibearing systems might be affected by the calculation error. In further studies, definite regularity of the transition from the rolling to the vibro-impact rotor–stator interaction was noticed. In this article, the causes of such a transition are explained and the forces are indicated that are

[1] The notions "synchronous" and "asynchronous rolling" were first introduced by E.L. Poznyak in [1].

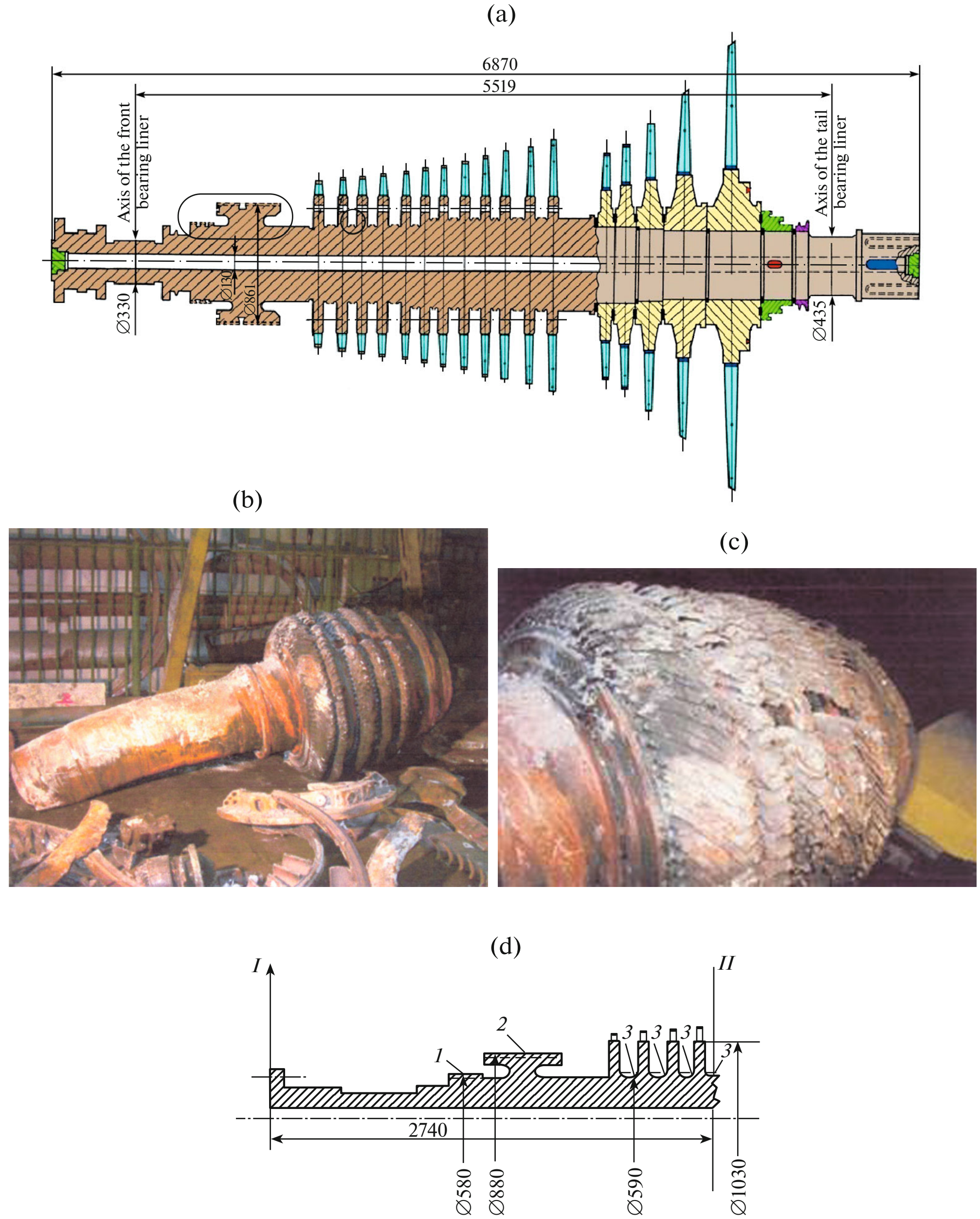

Fig. 1. Consequences of failure of a 300-MW turbine pant: (a) intermediate-pressure rotor, (b, c) rolled blades of a part of the rotor; (d) rotor fragment; *I* and *II*—points of destruction of coupling bolts and across the section between the fourth and fifth disks; *1*–*3*—seals.

capable of changing a seemingly usual rolling development pattern under the unstable state of the rotor–bearings system.

As is well known, the rotor-against-stator rubbing and the potential asynchronous rolling are the consequences of disturbance to the normal operation of power and power-generating plants. These processes can be facilitated by great vibration amplitudes of resonance or self-excited oscillations, nonstationary kinematic impacts, sudden and severe imbalance of the rotor, and other impacts. The self-excited oscillations under a contact rotor–stator interaction are maintained by the kinetic energy of the rotating rotor. Experimental studies show that harmonics of various frequencies are usually induced in this case with a considerably large portion of the vibration signal power being concentrated in the high-frequency region. The developing asynchronous rolling is a state of unstable equilibrium when the rotor is exposed to the forces of damping proportional—in general—to linear and angular movements of the rotor and of contact rotor–stator interaction. The components of the above forces maintain the unstable state of the rotor–bearings system.

The damping forces (proportional to the speed) are [8–10]:

$$-\bar{q} = B\dot{\bar{u}}, \tag{1}$$

where $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ is the matrix of the damping coefficients, e.g., in the base lubricant film of the slipping bearings, and $\dot{\bar{u}} = \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix}$ is the velocity vector of the geometric rotor center; subscripts 1 and 2 correspond to the horizontal and vertical oscillation directions.

The components $\begin{bmatrix} 0 & b_{12} \\ b_{21} & 0 \end{bmatrix}$ determine the gyroscopic forces [9] and components $\begin{bmatrix} b_{11} & 0 \\ 0 & b_{22} \end{bmatrix}$ determine the dissipative forces. The gyroscopic forces are proportional to the velocity components $\dot{u}_1$, and $\dot{u}_2$ and perpendicular to the direction of the latter. These forces may include the Goriolis forces, the Lorentz forces, etc.

The forces proportional to the linear movements of the rotor are [8, 11]:

$$-\bar{q}_1 = K^{(M)}\bar{u}, \tag{2}$$

where $\bar{q}_1 = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}^{(M)}$; $K^{(M)} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}^{(M)}$ is the matrix of the stiffness coefficients of the lubricant film in the bearings; $\bar{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ is the vector of the rotor's linear movements; u_1, u_2 are the projections of the radial displacement of the rotor onto the horizontal and vertical oscillation directions.

The components $\begin{bmatrix} 0 & k_{12} \\ k_{21} & 0 \end{bmatrix}^{(M)}$ determine the circulation or quasi-gyroscopic force [9] caused by the properties of the lubricant film formed between the moving and fixed surfaces. These forces are perpendicular to the vector components of the rotor's displacements in horizontal (1) and vertical (2) oscillation directions and facilitate excitation of the progressive precession of the rotor. They were discovered first by Sommerfeld. The components $\begin{bmatrix} k_{11} & 0 \\ 0 & k_{22} \end{bmatrix}^{(M)}$ determine the elastic forces. Fundamental studies of stiffness and dissipative characteristics of the oil film of the bearings of various types were conducted by E.L. Poznyak [11].

The forces proportional to linear and angular movements of the rotor, e.g., the aerodynamic flow forces in the gas path and seals of the turbine are [8, 12]:

$$-\bar{q}_2 = K^{(t)}\bar{u} + K^{(u)}\bar{\varphi}, \tag{3}$$

where $\bar{q}_2 = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$; $K^{(t)} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}^{(t)}$, $K^{(u)} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}^{(u)}$ is the matrix of the stiffness coefficients; and $\bar{\varphi} = \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$ is the vector of the rotor's angular displacements.

The subscripts and superscripts in Eqs. (2) and (3) refer, respectively, to the lubricant film in the bearings (M), aerodynamic forces in the gas path (t), and the seals of the turbine (u). The action of forces (3) is similar to that of forces (2).

The physical processes and the algorithms for calculation of the forces in Eqs. (1)–(3) are set forth in detail in [8, 9, 11, 12].

The forces of the contact rotor–stator interaction under rubbing are [4, 5, 7]:

$$-\bar{q}_3 = A^{(k)}\bar{u}, \tag{4}$$

where $\bar{q}_3 = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$; $A^{(k)} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^{(k)}$ is the matrix of the contact stiffness coefficients;

$$a_{11} = a_{22} = k\left(1 - \frac{\delta}{u}\right), \quad a_{12} = -k\chi\left(1 - \frac{\delta}{u}\right), \quad a_{21} = k\chi\left(1 - \frac{\delta}{u}\right); \tag{5}$$

$u = \sqrt{u_1^2 + u_2^2}$ is the radial displacement of the rotor; a_{ij} is the contact stiffness coefficient; k is the stiffness of the stator; δ is the gap between the rotor and stator under concentric position of the rotor in the stator bore; and χ is the sliding friction coefficient at the rotor–stator contact point under rubbing.

In the absence of the rotor–stator contact or upon the loss of the contact, i.e., at $u < \delta$, $a_{ij} = 0$.

The components $\begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix}^{(k)}$ determine the elastic forces that prevent the displacements of the rotor in the horizontal and vertical oscillation directions during the contacts, and the components $\begin{bmatrix} 0 & a_{12} \\ a_{21} & 0 \end{bmatrix}^{(k)}$ characterize the circulation forces that induce the asynchronous rolling. The nature of forces (5) is determined by the occurrence of the sliding friction during the rotor–stator contact. Coefficients a_{12} and a_{21}

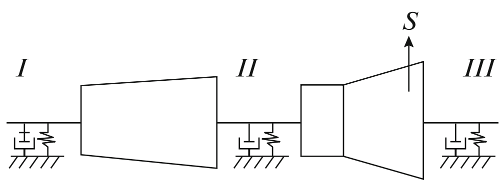

Fig. 2. Schematic of the system of the high-pressure and intermediate-pressure rotors of a 300-MW turbine plant: *S*—point of sudden imbalance; *I*, *II*, and *III*—bearing-supports of the rotor.

change according the hyperbolic law depending on the rotor's displacements. The signs of these forces determine the direction of the action on the rotor of the nonconservative component of the contact interaction forces reverse—excitation of the retrograde rotor precession—compared with the action on the rotor of the forces in the plain bearing film or the aerodynamic forces in the gas path and seals of the turbine. It is shown in [4–6] that the secondary matrix terms of the contact stiffness coefficients characterize the forces that excite the asynchronous rolling under rubbing of the rotor against the stator.

In Eqs. (1)–(5), k_{ij}, b_{ij}, and a_{ij} ($i \neq j$) depend on the properties of the environment in which the rotor runs, the structural features of the bearings, seals, damping devices, and the character of the contact rotor–stator interaction. In general, the secondary components of the forces—Eqs. (1)–(5)—act in the direction transverse to the components of velocity vector (1) or the components of the rotor's displacement vector—Eqs. (2)–(4). The projections of the forces onto the direction of the tangent and the normal at the contact point periodically change depending on the position of the rotor center on the trajectory of its motion. Thus, the rotor–bearings–stator system is exposed to the forces that prevent and facilitate the development of the rolling. The synergetic effect of the forces that act on the rotor upon change in individual parameters, including the frequency parameters of the rotor–bearings system, is difficult to predict without numerical simulation. However, it obeys the common laws of complicated nonequilibrium systems with ambiguity of their behavior. The behavior ambiguity is also demonstrated by the results of studies of the impact of the gyroscopic forces on the rolling pattern at the moment of the unstable state of the rotor–bearings system under contact interaction with the stator.

The rolling development was modeled for a three-bearing rotor (see Fig. 2) with plain oil bearings whose dynamic stiffness and damping coefficients are known. It is assumed that the linearity of the characteristics of the oil film is preserved under displacements comparable with the gap in the bearing. The angular rotation speed of the rotor is supposed to be constant, i.e., in the time interval τ = 0–0.09 s, the stop valves are not closed, and the generator is not disconnected from the network; in other words, the closing time of the stop valves is longer than the time of the development of the rolling with oscillation amplitudes dangerous for the strength. The equations of motion for a finite-element rotor model in accordance with [5] will have the following form:

(i) Under the motion of the rotor within the gap between the rotor and stator

$$[M]\ddot{\overline{w}} + [B]\dot{\overline{w}} + [C]\overline{w} = \overline{q}, \tag{6}$$

where $\overline{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$; $\dot{\overline{w}}$ and $\ddot{\overline{w}}$ are the vectors of velocity displacements and accelerations in the sections of the rotor; $[M]$, $[B]$, and $[C]$ are the global matrices of inertia, damping, and stiffness of the rotor–bearings–seals system; and $\overline{q}$ is the vector of the external forces caused by sudden imbalance in section *j* of the rotor; in this case, only two components of vector $\overline{q}$ will be other than zero.

In the finite element nodes—sections of the rotor—the vectors of displacement $\overline{w}_i$ and force factors $\overline{Q}_i$ will have the following form:

$$\overline{w}_i = \begin{bmatrix} u_1 \\ \varphi_1 \\ u_2 \\ \varphi_2 \end{bmatrix}_i; \quad \overline{Q}_i = \begin{bmatrix} Q_1 \\ m_1 \\ Q_2 \\ m_2 \end{bmatrix}_i.$$

The expression of the global matrix of the rotor stiffness as follows is of the main interest:

$$[C] = C_w + \left(K^{(M)}\right) + \left(K^{(t)}\right) + \left(K^{(u)}\right), \tag{7}$$

where C_w is the matrix of the rotor stiffness and $K^{(M)}$, $K^{(t)}, K^{(u)}$ are matrices of the bearing-support stiffness and the aerodynamic flow; the parentheses mean that the summation is performed only over coinciding sections (nodes) of the rotor, points of application of the forces from the bearings' side, and the aerodynamic flow.

(ii) Under the rotor-against-stator rubbing, i.e., at the moment of preserving the rotor-stator contact up to the loss of the contact

$$[M]\ddot{\overline{w}} + [B]\dot{\overline{w}} + [C_k]\overline{w} = \overline{q}, \tag{8}$$

where $[C_k]$ is the global stiffness matrix into the *j*th element of which the matrix of the stator's contact stiffness coefficients $K_J^{(k)}$ is built in, which considers the parameters of the stator and the sliding friction coefficient at the rotor–stator contact point.

Thus, the system of motion equations (8) for the rotor that contacts the stator differs from the system of motion equations (6) for the rotor that does not contact the stator—the motion in the gap—in the stiffness matrix owing to an additional coupling between the rotor and the stator at the point of rubbing. Then,

$$[C_k] = [C] + K_J^{(k)}. \tag{9}$$

The elements of matrix $K_J^{(k)}$ change in the course of the contact according to the change in the stator's deformation. For the finite-element analysis, matrix $K_J^{(k)}$ differs somewhat in the representation form from its representation in Eq. (4); however, this does not change the character of the rotor–stator interaction. The forces that facilitate the excitation of the retrograde precession (asynchronous rolling) of the rotor are set forth in detail in [5, 6]. The rubbing may occur at several points against both the casing and the bearings. This approach was implemented by an algorithm and a software package in [4, 5].

Global damping matrix $[B]$ comprises damping $B^{(M)}$in the journal bearings of the rotor. The matrices of the bearing support stiffness and the aerodynamic forces are generated in the same way.

It is supposed for the three-bearing rotor that the sudden imbalance of 0.08% of the rotor' second span weight at a radius of 1 m occurs at the time point $\tau = 0$ in section S (see Fig. 2). Then,

$$\overline{q} = m_j\omega^2 \begin{bmatrix} \varepsilon_1 & -\varepsilon_2 \\ \varepsilon_2 & \varepsilon_1 \end{bmatrix} \begin{bmatrix} \cos(\omega t) \\ \sin(\omega t) \end{bmatrix}, \tag{10}$$

where ω is the angular speed of the rotor at the moment of sudden imbalance ($\varepsilon_1 \neq 0$, $\varepsilon_2 = 0$ corresponds to the imbalance on the horizontal plane and $\varepsilon_1 = 0$, $\varepsilon_2 \neq 0$ corresponds to that on the vertical plane). The dimensionality of vector $\overline{q}$ is $n \times 4$. The order of the global matrices of the system of Eqs. (6) and (8) is $n \times 4$.

The spectrum of the frequencies and oscillation forms of a rotor on the journal bearings is preserved under oscillations within the limits of the gap [4, 5]. Under the rotor–stator contact, the dynamic characteristics of the rotor–bearings–stator system change depending on the stiffness properties of the stator as an additional coupling that occurs during the rubbings.

The nonlinearity of the system is determined by the presence of a gap in section S and the nonlinearity of the contact rotor–stator interaction. The effect of the change in the secondary terms of the damping matrix (gyroscopic forces) manifests itself in the qualitative change in the pattern of the asynchronous rolling development. Against the contact interaction forces that act on the rotor, the variable damping parameter is a parameter of relative small magnitude. In equilibrium systems, such fluctuations—random deviations of the parameters—are usually neutralized by back couplings and the state of the system close to the equilibrium. In nonequilibrium systems, the development of the oscillations is determined by the fluctuations in the parameters of the system and its tendency towards definite potential and undeveloped states. In [4, 5, 7], the trajectory of the center of section S is shown under the development of the asynchronous rolling with a constant contact with the stator when there are no secondary damping terms—the gyroscopic forces—and it is shown in Fig. 3 under the action of the secondary damping terms in the lubricant film of the cylindrical bearings. The dashed line denotes a circumference that represents the gap between the rotor and stator in section S of the second span. The arrows denote the positive directions of the coordinate axes; the gap and the displacements of the rotor, u_1 and u_2, are given in terms of relative units with respect to the gap in section S.

At the initial moment of the rolling development at $\tau = 0$–0.06 s, the process does not greatly depend on transverse force components (Eqs. (1)–(5)). The motion of the rotor is determined by the inertia components of the sudden imbalance, while the influence of the gyroscopic forces manifests itself weakly. Further, the asynchronous rolling develops under gradual excitation of the oscillations in the direction transverse to the motion trajectory, damping of transverse oscillation components, subsequent excitation of the latter with even greater amplitude, and, finally, transition to vibro-impact oscillations of the rotor against stator (Figs. 3–5).

In Fig. 3, the transition from the constant contact with the stator to vibro-impact oscillations is shown (see Fig. 3f). At $\tau = 0.06$–0.08 s, the component of the rotor's motion with an alternating contact with the stator and the development of the oscillations in the direction transverse to the motion trajectory is the determining component (see Fig. 4). At $\tau > 0.07$ s, vibro-impact oscillations with increasing amplitudes develop. The oscillations of the rotor (see Fig. 3f) tend to the center of symmetry (the center of the stator bore). The modeling of the rotor oscillations with the rubbing against the stator and gyroscopic forces allowed considering another potential pattern of the rolling development.

In Fig. 5, change with time in the basic characteristics of the motion of the center of the rotor's section S for the time interval $\tau = 0$–0.09 s is shown. The pressure force exerted on the stator (see Fig. 5c) exceeds the weight of the second rotor span by 8500 times. The stresses in the fixing elements of the upper and lower IPR housings of a turbine plant can be calculated by the algorithm set forth in [7]. To establish the main reason for change in the behavior of the rotor–bearings–stator system under unstable equilibrium, interference of the forces (Eqs. (1)–(5)), and change in the dynamic characteristics of the system during the contact and under the loss of contact, additional studies are required. The performed modeling of the rotor-over-stator rolling suggest the existence of more than one modes to which the rotor–bearings–stator system tends under unstable conditions and changes in the

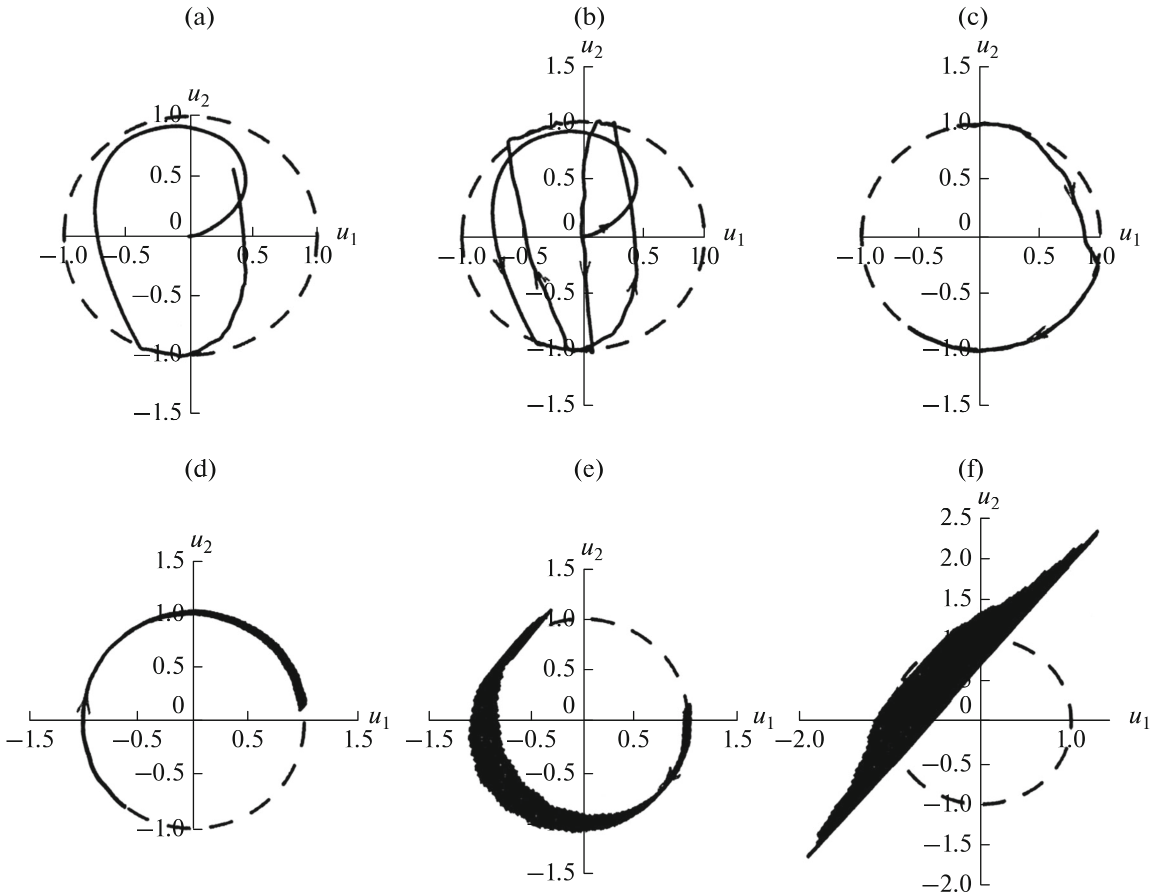

Fig. 3. Development of the transient oscillations of a three-bearing rotor upon sudden imbalance of 0.08% of the weight of the second rotor's span per 1 m in section S with transition to vibro-impact oscillations in the stator bore at $k = 3.62 \times 10^8$ kN/m, b_{ij} $(i \neq j) \neq 0$; τ, s: (a) 0–0.03; (b) 0–0.05; (c) 0.05–0.06; (d) 0.06–0.07; (e) 0.07–0.08; and (f) 0.08–0.09.

excitation forces, i.e., the existence of at least two different patterns of the development of the asynchronous rolling, viz.,

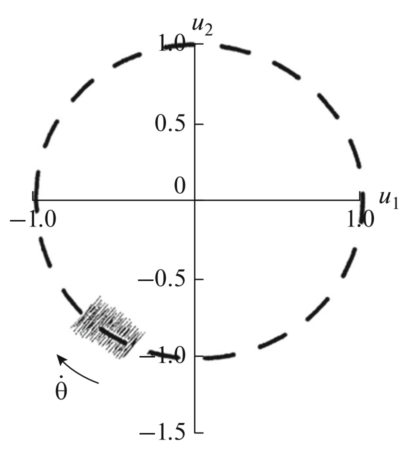

Fig. 4. Beginning of the development of the rotor-on-stator vibro-impact oscillations at $\tau = 0.07538$–0.07637 s; frequency of 35 kHz; and the direction of the angular rotor precession velocity, $\dot{\theta}$.

(i) a constant rotor–stator contact—the rotor-over-stator rolling (or "roll" as the phenomenon is termed in [2]) (see the figures in [4, 6, 8])—with increasing amplitudes of self-excited oscillations; in this case, the gyroscopic forces are equal to zero; and

(ii) gradual development of the oscillations in the direction transverse to the rotor's motion trajectory and transition to vibro-impact oscillations, i.e., the asynchronous rolling with a constant contact with the stator changes into a vibro-impact mode (see Figs. 3–5) in the presence of the gyroscopic forces in the system.

The pattern of the rotor-over-stator rolling development depends on the fluctuations in the parameters of the rotor–bearings–stator system. In the case in question, such parameters were the gyroscopic forces determined by secondary damping force coefficients in the rotor bearings. Both patterns of the rotor-over-stator rolling development are equally hazardous for the resistance of the structure to the impacts on the stator that occur during the rolling development.

Under real operating conditions of the plant, the possibility of any disturbance to the normal operation of the plant, for example, as a result of accumulation

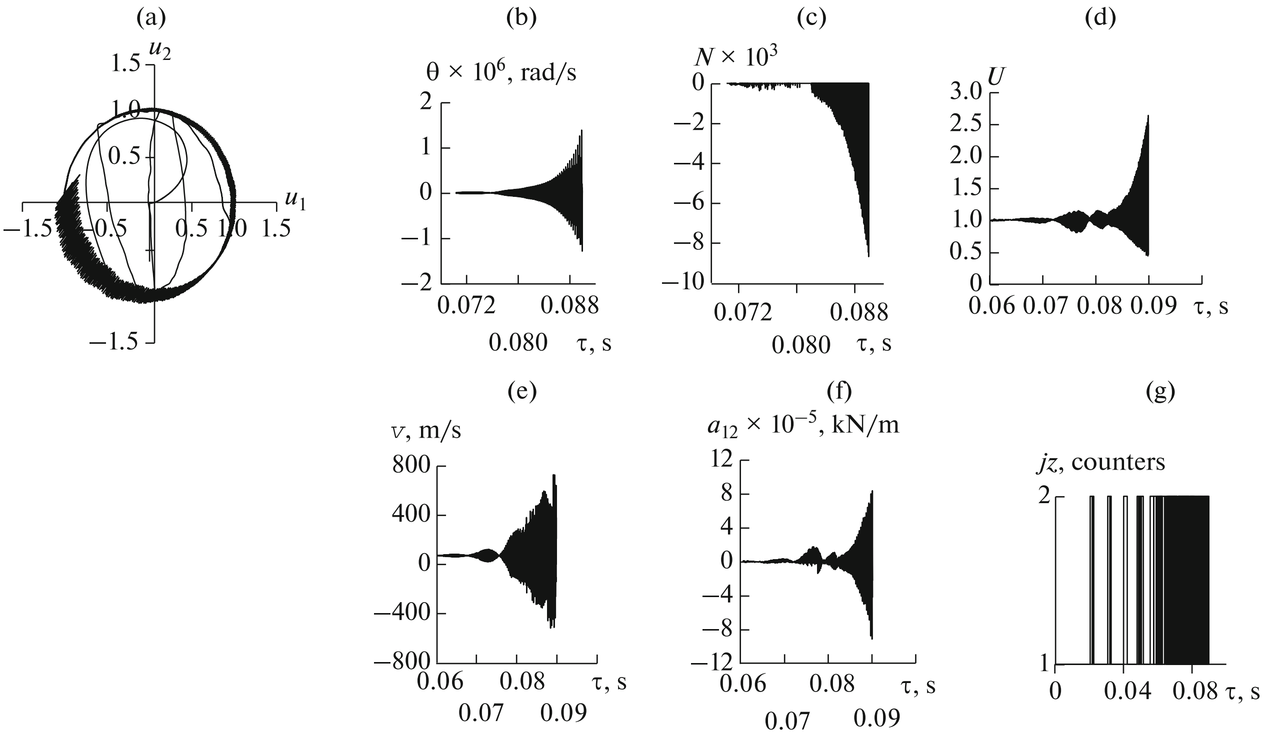

Fig. 5. Change in time of the basic characteristics of the motion of section S center of the three-bearing rotor upon sudden imbalance in the second span under rotor-against-stator rubbing at ω = const: (a) motion trajectory of the center of the rotor's section S in the time interval τ = 0–0.075 s; (b) angular rotor precession velocity; (c) dimensionless normal pressure force N exerted on the stator (with respect to the weight of the second rotor's span); (d) dimensionless displacement of the center of the rotor's section U (with respect to the gap in section S); (e) velocity of the rotor v with respect to the stator at the contact point; (f) the stator's contact stiffness coefficient a_{12}; and (g) counter that records the contact time and the time of the motion in the gap ($jz = 1$ corresponds to the motion in the gap).

of damage and some factors indicated in [5, 6] that can cause the rubbing and the rolling development, cannot be completely eliminated. The study of the rolling development should be aimed at minimization of adverse consequences of the development of this threatening and dangerous phenomenon capable of causing a complete breakdown of the plant and great financial losses.

CONCLUSIONS

(1) A complicated nonequilibrium rotor–bearings–stator system—under the conditions of the contact rotor–stator interaction and potential development of the asynchronous rolling—has at least two patterns of the dangerous development towards which the system may tend under change in its parameters and excitation forces, viz., the asynchronous rolling with a constant rotor–stator contact and the asynchronous rolling with transition to vibro-impact oscillations.

Both modes of the contact interaction are equally dangerous for the strength of the structure due to considerable pressure forces exerted on the stator.

(2) The dialectics of the manifestation of the rotor-over-stator rolling is represented by the duality of the damping forces. They act as the forces that prevent oscillations—facilitate the damping of the rotor-over-stator rolling development—and the forces, in particular, their gyroscopic components, that facilitate the excitation of one of the rotor-over-stator rolling development modes.

(3) Recommendations for prevention of hazardous consequences of the asynchronous rolling development are as follows:

(i) offset from the resonance of the rotor–bearings system by at least 10%;

(ii) reduction in the sliding friction between the rotor and stator surfaces at the points of potential contacts determined by the oscillation mode antinodes of the rotor–bearings system;

(iii) decrease in the stator stiffness at the point of a potential contact;

(iv) reduction in the energy losses in the bearings and the stator;

(v) increase in the gap between the rotor and the stiff components of the stator at the points of potential contacts by using elastic seals; and

(vi) increase in the response speed of the protection systems of the turbine plant; closing of the stop valves and disconnection of the generator from the network facilitate the braking of the rotor upon its contact interaction with the stator and departure from the resonance region.

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