ISSN 0040-6015, Thermal Engineering, 2015, Vol. 62, No. 6, pp. 423–433. © Pleiades Publishing, Inc., 2015.

Heat Transfer and Pressure Drop in Rectangular Channels with Crossing Fins (a Review)

N. P. Sokolov, V. G. Polishchuk, K. D. Andreev, V. A. Rassokhin, and N. A. Zabelin

St. Petersburg State Polytechnic University, Politekhnicheskaya ul. 29, St. Petersburg, 195251 Russia
e-mail: akostik@land.ru

Abstract—Channels with crossing finning find wide use in the cooling paths of high-temperature gas turbine blade systems. At different times, different institutions carried out experimental investigations of heat transfer and pressure drop in channels with coplanar finning of opposite walls for obtaining semiempirical dependences of Nusselt criteria (dimensionless heat-transfer coefficients) and pressure drop coefficients on the operating Reynolds number and relative geometrical parameters (or their complexes). The shape of experimental channels, the conditions of experiments, and the used variables were selected so that they would be most suited for solving particular practical tasks. Therefore, the results obtained in processing the experimental data have large scatter and limited use. This article considers the results from experimental investigations of different authors. In comparing the results, additional calculations were carried out for bringing the mathematical correlations to the form of dependences from the same variables. Generalization of the results is carried out. In the final analysis, universal correlations are obtained for determining the pressure drop coefficients and Nusselt number values for the flow of working medium in channels with coplanar finning.

Keywords: gas turbines, blade cooling systems, coplanar finning, heat transfer, pressure drop

DOI: 10.1134/S0040601515060105

GEOMETRICAL CHARACTEREISTICS OF PATHS WITH COPLANAR CHANNELS AND BASIC SIMILARITY CRITERIA

Cooling paths used in the high-temperature gas turbine blade systems are characterized by a set of geometrical parameters, from a certain combination of which it is possible to obtain, by applying simple mathematical correlations, the characteristic sizes used in criterion-type processng of experimental data.

At present, heat-transfer surfaces with fins placed on channel walls at an angle to the flow are finding increasingly growing use in different fields of thermal engineering. In the limiting case, when the fins on the opposite walls come in contact with one another, polyzone finning of a channel with an annular cross section or crossing finning of a channel with a rectangular cross section is obtained. Channels with such finning are widely used, e.g., in the blade cooling paths of high-temperature gas turbines [1]. If the fins on one wide wall of a rectangular channel with a large ratio of side lengths are intersecting with the fins on the opposite wall, a heat-transfer path with mutually intersecting and crossing finning is obtained (Fig. 1).

The heights H, h_f, and h_i are interlinked by the expression $2h_f = H + h_i$. For processing experimental data on heat transfer and hydraulics in paths having the above-mentioned geometrical configuration, the following expression for the path equivalent diameter is often used: $d_{eq} = 4V_a/F_\Sigma$. The air volume V_a occupied by coolant in the path with coplanar channels and the total heat-transfer area of the path surface F_Σ in the case of using rectangular finning can be determined from the expressions

$$V_a = LB\,[2h_f(1 - b/S) + h_i(b/S)^2]; \tag{1}$$

$$F_\Sigma = 2LB\big[1 + 2h_f/S + H(1 - b/S)/B - 2b(2h_i + b)/S^2\big], \tag{2}$$

where L is the length of the initial base channel.

Thus, in accordance with (1) and (2), in the limiting case for the basic channel without finning, we obtain the usual formula for determining the equivalent hydraulic diameter

$$d_h = 4f_0/\Pi_0, \tag{3}$$

where f_0 is the channel cross-section area, and Π_0 is the channel inner (wetted) perimeter.

By applying the dimensionality analysis method we come to the following similarity equations for describing the regularities pertinent to pressure drop and heat transfer [2]:

$$\zeta = f(\mathrm{Re}, L/B, B/H, S/B, S/b, h_f/S, \beta); \tag{4}$$

$$\mathrm{Nu} = f(\mathrm{Re}, L/B, B/H, S/B, S/b, h_f/S, \beta, \mathrm{Pr}), \tag{5}$$

where $\zeta = 2\Delta p/(\rho w^2)$ is the pressure drop coefficient in a path with coplanar channels, Re and Pr are the Reynolds and Prandtl numbers, Δp is the pressure dif-

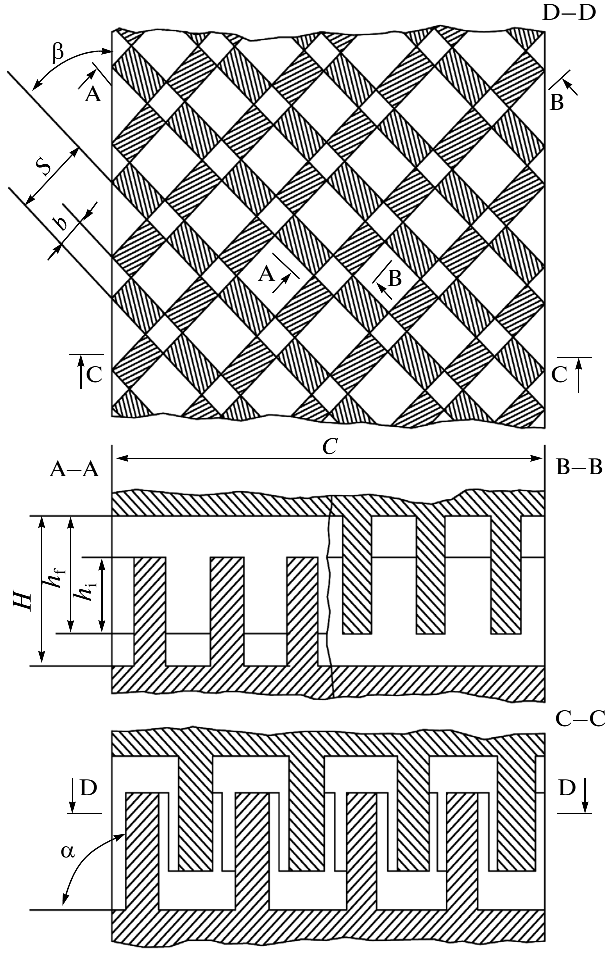

Fig. 1. Configuration of the path containing mutually intersecting and crossing finning (coplanar channels). H and B are the height and width of the initial basic channel, b and h_f are the thickness and height of fins with rectangular cross section, S is the placement pitch of rectangular fins, and h_i is the fin mutual intersection height.

ference, ρ is the average flow density, and w is the average flow velocity in the path with coplanar channels.

A REVIEW OF EXPERIMENTAL RESULTS AND CRITERION CORRELATIONS FOR HEAT TRANSFER AND PRESSURE DROP IN PATHS WITH COPLANAR CHANNELS

Only a few works have by now been published on experimental investigations of heat transfer. Their results are summarized in the table. It can be seen from the table that the authors of some investigations did not indicate the parameter variation ranges (e.g., [3–5]), or the authors did not generalize their results by final formulas [6].

The finning investigated at the Central Institute of Aviation Motors (TsIAM) [5] had a shape close to a trapezoid one (rather than rectangular) with rounded vertices of trapezoids (of radiuses r'' and r') and spans between them (Fig. 2). The main geometrical characteristics of the investigated channels with such finning were varied in the following ranges: $h_f = 1.43–1.8$ mm, $S = 2.54–4.40$ mm, $r' = 0.30–1.06$ mm, $r'' = 0.47–1.075$ mm, and $\alpha = 51°–68°$.

The work [7] carried out by the Chemical Engineering Research Group, Council for Scientific and Industrial Research (CSIR) (Pretoria, Republic of South Africa) published in the middle 1980s was devoted to studying the effect the crossing angle of two opposite sinusoidally corrugated walls has on the thermal–hydraulic characteristics of the channels formed by these walls in plate-type heat exchangers (Fig. 3).

All model channels, except with the channels with an annular cross section intended for investigation of the path thermal–hydraulic characteristics carried out at the Bauman Moscow Technical University (MVTU) [3], were made as flat ones and had the cross section either in the form of a rectangle with right [4, 6, 8] or rounded [9–11] angles or in the form of an elongated isosceles triangle with a rounded acute angle at the vertex [12]. The relative length L/B of the studied channels was in the range 1.0–5.5, but in the majority of works it was not varied and remained unchanged at a level of 2–3. The relative height H/h_f of channels with noncrossing finning had the trivial value equal to 2 and was decreased to 1.35 in only in the investigation carried out by specialists of the Leningrad State Technical University (LGTU) [8], in which mutually intersecting finning was used, which resulted in that the relative fin intersection depth was changed to $h_i/h_f = 0–0.665$. The relative finning pitch S/h_f in the entire group of investigations was varied in the range from 1 to 7, and the relative fin thickness b/h_f was varied in the range from 0.07 to 0.8.

All experiments were carried out on experimental setups with using air as coolant. In all investigations, except with the experiments carried out at the MVTU [4], the average heat transfer in the characteristic zones of the path heat-transfer surfaces was determined.

On the whole, in view of the variety and heterogeneity of the parameters used by different authors for describing the regularities of hydraulics and heat transfer in vortex paths, direct comparison of the obtained experimental results does not seem to be possible, and the available criterion correlations need certain additional transformations. First of all, this relates to the correlation between the hydraulic and d_h and equivalent d_{eq} diameters of channels with crossing rectangular finning used in different investigations. In so doing, the hydraulic diameter is calculated, as is usually done, through the flow pass section area f_c and wetted perimeter $\Pi_c = S - b + 2h_f$ of an individual interfin channel $d_h = 4f_c/\Pi_c$, and the equivalent diameter is calculated using expressions (1) and (2).

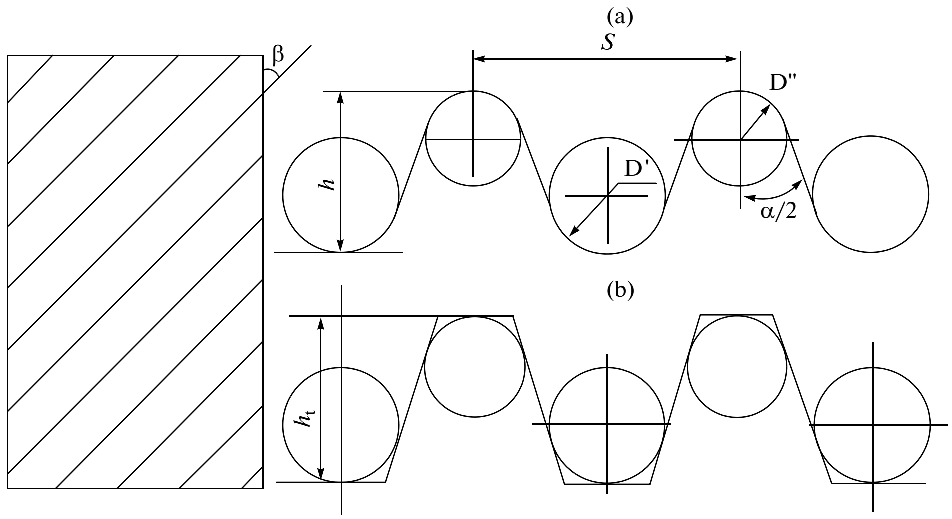

Fig. 2. Configuration of finning with undulate profile investigated at the TsIAM[5] (a) and finning with the trapezoid profile (b) to which the initial undulate finning was brought.

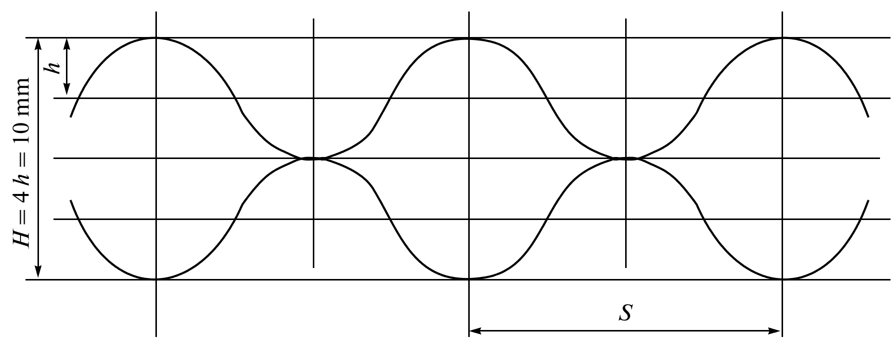

Fig. 3. Configuration of coplanar channels formed by sine-wave finning on the opposite sides of the flat channel the mass transfer characteristics of which at different fin crossing angles were investigated at the CSIR [7].

The expression for the equivalent diameter at $h_i = 0$ is

$$d_{eq} = \frac{4V_a}{F_\Sigma} = \frac{2H(1-b/S)}{1 + H/S + (H/B)(1-b/S) - 2(b/S)^2}, \quad (6)$$

i.e., it is determined by the length ratio between its cross section sides B/H and the relative fin thickness b/S.

The hydraulic diameter of an individual interfin channel at $h_i = 0$ will be

$$d_h = \frac{4f_{c\Sigma}}{\Pi_\Sigma} = \frac{2H}{1 + H/S}\left(1 - \frac{b}{S}\right), \quad (7)$$

where Π_Σ is the total perimeter of sections, and $f_{c\Sigma}$ is the total flow pass area of the sections.

With $h_i = 0$ we have

$$f_{c\Sigma} = 2(S-b)h_f B\cos\beta/S = HB(1-b/S)\cos\beta. \quad (8)$$

The cross section equivalent area is given by

$$f_{eq} = V_a/L = BH(1-b/S). \quad (9)$$

The correlations between the diameters d_h and d_{eq} and between the areas $f_{c\Sigma}$ and f_{eq} allow us to recalculate the Re criteria, Nu numbers, and flow friction coefficients ξ from one characteristic sizes to other ones, which makes it possible to carry out a comparison between the results of different investigations presented in the form of criterion correlations.

AN ANALYSIS AND COMPARISON OF THE RESULTS OBTAINED FROM EXPERIMENTAL INVESTIGATIONS OF PRESSURE DROP IN PATHS CONTAINING COPLANAR CHANNELS

Since each and every criterion correlation obtained by different researchers for determining flow friction coefficients ξ contain their dependence on the Re

Main results from the experimental investigations into the hydraulic characteristics and cooling heat transfer of a path containing mutually crossing and intersecting fins

Authors, year, organization, references	Structure of similarity criteria			Correlation criteria for λ and ζ			Criteria for heat transfer correlation		
	Nu	Re	ζ and λ	on the main surface	on the lateral surface	parameter variation range	on the main surface	on the lateral surface	parameter variation range
V.M. Kudryavtsev, 1983, MVTU, [3, 4]		$\frac{Gd_h}{\mu f_{fl}}$	$\lambda = \frac{\Delta p d_h / x}{\rho w^2 / 2}$	$\lambda = e^{1.47(\beta' + \beta'') + 5.24/Re^{1.32}} + e^{1.73(\beta' + \beta'') + 4.7}$		$0.75 < \beta' + \beta'' < 1.815$; $10^3 < Re < 3 \times 10^4$	–	–	–
M.N. Galkin, 1984, MATI, [9, 10]	$\frac{\alpha d_{eq}}{\lambda}$	$\frac{Gd_{eq}}{\mu f_{eq}}$	$\lambda = \frac{\Delta p \left(\frac{d_{eq}}{L}\right)}{\frac{\rho w_{eq}^2}{2}}$	$\lambda = 1370 \frac{\bar{\beta} \times 2.75}{Re^{0.23}} \times \left(\bar{\beta}^2 - 0.78\bar{\beta} + 0.19\right)$		$0.17 < \bar{\beta} < 0.68$; $500 < Re < 2 \times 10^4$	$Nu = 4.04 Re^{0.42} \bar{\beta}^{0.65}$	$Nu = 2.1 Re^{0.5} \bar{\beta}^{0.58}$	$0.17 < \bar{\beta} < 0.6$; $2 \times 10 < Re < 2 \times 10^4$
M.N. Galkin, 1985, 1987, MATI, [12, 13]							$Nu = 0.95 \frac{Re^{0.8} \bar{\beta}^{0.46}}{(h_f/s) 0.05 (L/d_{eq}) 0.91}$	$Nu = 1.146 \times \frac{Re^{0.8} \bar{\beta}^{0.42}}{(h_A/s)^{0.6} (L/d_y)^{1.15}}$	$0.30 < \frac{h_f}{S} < 0.84$; $2.65 < \frac{L}{d_{eq}} < 37$
G.P. Nagoga, 1986–1988, KNPO Trud, [11]	$Nu_x = \frac{\alpha x}{\lambda}$; $Nu_d = \frac{\alpha d_h}{\lambda}$	$Re_x = \frac{\rho w}{\mu}$; $Re_d = \frac{\rho w d_h}{\mu}$	$\zeta = \frac{2\Delta p}{cw^2}$ $\lambda = \zeta \frac{d_h}{x}$	$\lambda = 0.43 \times [1 + 28(\sin 2\beta)^3] Re_x^{n-1}$; $n = 0.8 + 0.16 \times [(1.27\beta - 1)^2 - 1]$	$\zeta = 125 \bar{f}_w^{0.5} \bar{f}_e^{2} = 5hp\sin 2\beta / [(s-b)z^{0.5}]$	$0.03 < \bar{f}_w < 1.0$; $0.3 < \bar{f}_e < 2.6$	$Nu_x = AmRe_x^m Pr^{0.44}/\psi^{0.55}$; $\overline{Nu} = ARe_x^m Pr^{0.44}/\psi^{0.55}$; $A = 0.455(\sin 2\beta)^2 + 0.043 = 0.637 + 0.066(2\beta - \pi/2)^2$	$Nu_d = 0.165 \bar{f}_e^{0.64} \bar{f}_w^{0.11} \times Re_d^{0.715} Pr^{0.43}/D^{0.21} \times \psi^{0.55} = 0.22 \times \left(\frac{h}{9-6} \sin 2\beta \sin \beta\right)^{0.4} \times Re_d^{0.715} / \psi^{0.55} m^{0.11}$	$6 < m < 38$; $5 \times 10^3 < Re_d < 7 \times 10^4$; $0.14 < D < 1.0$; $0 < 2\beta < 180$

Table (Contd.)

Authors, year, organization, references	Structure of similarity criteria			Correlation criteria for λ and ζ			Criteria for heat transfer correlation		
	Nu	Re	ζ and λ	on the main surface	on the lateral surface	parameter variation range	on the main surface	on the lateral surface	parameter variation range
L.V. Arsen'ev, 1991, LGTU, [8, 14, 15]	$\frac{\alpha d_{eq}}{\lambda}$	$\frac{Gd_{eq}}{\mu f_{eq}}$	$\zeta = \frac{2\Delta p}{\rho w_{eq}^2}$	$4 \times 10^3 < \mathrm{Re} < \mathrm{Re}_b = (29.5 - 18\bar{\beta} - 0.6\bar{h}_i) \times 10^3$; $\zeta = 1.15\bar{\beta}^2 - 985\bar{\beta} + 193 + (162\bar{\beta}^2 - 90b + 21.5)/(0.8 - \bar{h}_i)\mathrm{Re} > \mathrm{Re}_b$; $\zeta = A\mathrm{Re}^{0.15 + 0.525h_i}$; $A = f(\bar{h}_i, 2\beta)$		$4 \times 10^3 < \mathrm{Re} < 4 \times 10^4$; $0.33 < \bar{\beta} < 0.667$; $0 < \bar{h}_f < 0.667$; $L/B = 2.35$	$\mathrm{Nu} = (22-30\bar{\beta}) \times \mathrm{Re}^{0.45 + 0.63\bar{\beta}}$	–	The same as for ζ
A.N. Antonov, 1997, TsIAM, [5]	$\frac{\alpha d}{\lambda}$	$\frac{Gd}{\mu f_{fl}}$	$\zeta = \frac{2\Delta p}{\rho w^2} \times \frac{d}{L}$	$\zeta = \frac{4.85(2\beta/\pi)^{3.71}(d/L)^{2.39}}{(h_f/s)^{3.2}[\alpha/(2\pi)]^{8.91}} \times \frac{1}{\mathrm{Re}^{4.26}}$		$200 < \mathrm{Re} < 5 \times 10^3$; $50° < 2\beta < 130°$; $51° < \alpha < 68°$	$\mathrm{Nu} = 0.0396\mathrm{Re}^{0.723} \times \frac{(2\beta/p)^{1.05}}{(h/s)^{1.394}}\frac{(d/L)^{0.982}}{(\alpha/\pi)^{3.747}}$	–	–
CSIR, RSA, 1985 [7]	$2Ha/\lambda$	$\frac{4G}{\mu B}$	$\frac{2\Delta p}{\rho w'^2} \times \frac{2H}{L}$	$\zeta' = 1.21 + 3.67/\mathrm{Re}$; $\zeta' = 5.84/\mathrm{Re}^{0.177}$		$\mathrm{Re} = 150–1800$; $\mathrm{Re} = (1.8–30) \times 10^3$	In tabular form	–	–

number in a fairly wide range, it seems advisable to compare them in the form $\xi = f(\mathrm{Re})$. In what follows, we will carry out comparison for nonintersecting finning ($h_i = 0$) with the crossing angle $2\beta = 90°$ (the relative crossing angle $\overline{\beta} = 0.5$) because such angle was set in all experiments and was close to the average value from its variation range in different experiments $\overline{\beta} = 0–1.0$. In all of the compared correlations $\xi_{eq} = f(\mathrm{Re}_{eq})$, the equivalent hydraulic diameter d_{eq} determined from (1) and (2) was adopted as the determining size, and the equivalent velocity w_{eq} was adopted as the characteristic velocity:

$$\xi_{eq} = \frac{2\Delta p}{\rho w_{eq}^2}\frac{d_{eq}}{L} = \zeta_{eq}\frac{d_{eq}}{L}; \quad \mathrm{Re}_{eq} = \frac{Gd_{eq}}{\mu f_{eq}} = \frac{\rho w_{eq} d_{eq}}{\mu}, \quad (10)$$

where ζ_{eq} is the total equivalent pressure drop coefficient; G and ρ are the flow rate, kg/s, and average density, kg/m^3, of the current; and μ is the dynamic viscosity coefficient.

For the fixed value of crossing angle $2\beta = 90°$, the ratio H/S was selected as the determining parameter due to the following reasons. According to the results of works carried out at the KNPO Trud [11] and GSIR [7], at $2\beta = 90°$, cross motion of flows is mainly realized along individual interfin channels belonging to opposite matrices with interaction of these crossing flows in the open and facing each other sides of the channels. In view of this circumstance, it seems quite plausible that the hydraulic resistance offered to the liquid flow will in this case be determined by two components. As is usually the case, the first one of these components will be determined by the friction of liquid against three solid walls of single interfin channels, whereas the second component will be solely due to interaction of individual flows moving in mutually perpendicular directions over the surfaces of their interaction. The first of the above-mentioned effects, which is stemming from friction of the flow against the wall, causes the flow velocity to slow down in the wall boundary layer to its zero value right on the wall, which requires certain expenditure of the flow motion kinetic energy. The second of the above-mentioned effects, which is due to interaction of crossing flows, gives rise to secondary vortex flows in the zone of their interaction, which also require some fraction of the flow motion kinetic energy to be spent for maintaining them. Hence, the total resistance offered to the flow of liquid will be a combination of these two components, which will depend on the ratio between these two components, i.e., between the pressure loss for friction against the solid walls and the pressure loss for formation of additional vortices during interaction of the flows. Obviously, the ratio between these two components will be mainly determined by the ratio of the corresponding fractions in the total wetted perimeter of the individual interfin channel, i.e., by the ratio of the interfin distance width $S - b$ within which the secondary vortex flow is realized to the remaining part of the perimeter $S - b + H$ within which the flow drags against the wall. Thus, with the same thickness of fins b, the fraction of the perimeter open part $S - b$ decreases monotonically with increasing the parameter H/S, i.e., with a growth of the relative depth of individual interfin channels. Thus, with the values of geometrical parameters $S/b = 4$ and $H/b = 2$, at $H/S = 1/2$ this fraction is $(S - b)/[2(S - b) + H] = 3/8$, and with the values of geometrical parameters $S/b = 2$ and $H/b = 4$, at $H/S = 2$ this fraction is equal to 1/5.

Now, if we succeed in determining which of the loss components per unit length of the total perimeter of a single interfin channel is the most significant one, we would be able to estimate the effect the geometrical ratio H/S has on the pressure drop in the path with coplanar channels. If we adopt the assumption that the specific loss for vortex formation resulting from interaction of crossing flows exceed the loss due to friction against solid walls, which is quite plausible one, there is nothing to do but state the following: with all other things being equal, increasing the relative height of crossing fins H/S should lead to a smaller relative fraction of loss for vortex formation and, consequently, to a smaller overall pressure drop.

In view of the above-mentioned assumption, it can be hoped that the use of the geometrical parameter H/S for generalizing experimental data on the pressure drops in channels with finning crossing at the angle $2\beta = 90°$ will make it possible to approach to a certain extent to a universal correlation between the pressure loss (the pressure drop coefficient ζ) and flow velocity (the Re criteria).

In using the pressure drop coefficients ζ per unit dimensionless length of the channel for carrying out a comparison, the experimental data of LGTU [8] had to be additionally processed, because the results of those experiments were processed as a dependence of the channel's total pressure drop coefficient ζ_{eq} on the Re_{eq} criterion. As a result, for calculating the pressure drop coefficient ζ_{eq} for a channel with $2\beta = 90°$ and $h_i = 0$ from the experimental data of LGTU, formula (2.15) (see [8]) was used for the self-similarity region, yielding, as should be expected, a constant value of the pressure drop coefficient

$$\zeta_{eq} = (d_{eq}/L)\Big[1150\overline{\beta}^2 - 985\overline{\beta} + 193 + \left(162\overline{\beta}^2 - 90\overline{\beta} + 21.5\right)\big/0.8\Big] = 0.425. \quad (11)$$

The boundary value of the criterion $\mathrm{Re}_{eq.b}$ for this self-similarity region was found to be $\mathrm{Re}_{eq.b}^{max} = 20.5 \times 10^3$.

Starting from this value, the pressure drop coefficient ζ_{eq} was calculared as follows:

$$\zeta_{eq} = \frac{1.8735}{Re^{0.15}}. \tag{12}$$

The dependence $\zeta_{eq} = f(Re_{eq})$ constructed from expressions (11) and (12) that generalizes the LGTU data [8] is shown in Fig. 4 (curve *1*). For carrying out a sufficiently grounded comparison of the results from the experimental investigations considered above, all of them were recalculated for relative geometrical sizes of the path with coplanar channels investigated at the LGTU [8], i.e., to $L/B = 210/90 = 2.35$, $B/H = 90/12 = 7.5$, etc.

The curve $\zeta_{eq} = f(Re_{eq})$ generalizing the MVTU data [3] was constructed so that the following was valid for the considered particular case with $2\beta = \pi/2$:

$$\zeta_h = 1900/Re_h^{1.32} + 0.137. \tag{13}$$

Based on this, the following final dependence was obtained using the correlations between Re_h and Re_{eq} and between ζ_h and ζ_{eq}:

$$\zeta_{eq} = 2593/Re_{eq}^{1.32} + 0.283. \tag{14}$$

Curve *2* in Fig. 4 obtained by calculation using formula (14) testifies that the pressure drop coefficients ζ obtained from the results of experiments carried out at the MVTU [3] are approximately by 25% smaller than those obtained from the experiments carried out at the LGTU [8] in the range of their overlapping according to the criteria $Re_{eq} = (4 - 20) \times 10^3$. This is quite explainable because in the MVTU model with an annular cross section and multizone finning the flow does make turns at the lateral walls.

Some notes should be made regarding the very high values of pressure drop coefficient reported in [13] that were obtained for the model channel investigated at the Tsiolkovskii Moscow Institute of Aviation Technology (MATI). One possible explanation to this result is that the flow in this channel underwent a large number of turns near the lateral walls, which gave rise to a significant loss of pressure. As a consequence, the equation generalizing the data of [13] led to the following simple power dependence for the finning crossing angle $2\beta = 90°$ selected for making the comparison:

$$\zeta_{eq} = 10.8/Re^{0.23}. \tag{15}$$

The results of calculations carried out using this formula, which are represented by curve *3* in Fig. 4, testify that the pressure drop in the channels with coplanar finning studied at the MATI [13] is approximately a factor of 3 higher than its level obtained in the channels studied at the LGTU [8].

Now, coming to an analysis of the generalizing criterion correlations for the pressure drop coefficients obtained at the KNPO [11], it should be pointed out that the path with coplanar channels dealt with in that work was considered as a system composed of $k = 2B\cos\beta/S$ parallel channels, each of which has the total extension $l = L/\cos\beta$ and the number of U-turns $n = L/(l\cos\beta)$ at the lateral walls and includes the following: the segment $x_{in} \le B/\sin\beta$ of the channel initial section, the channel segment $n - 1$ between its turns of length $B/\sin\beta$, and the channel segment x_e after the last turn before its outlet of length $x_e \le B/\sin\beta$. In view of this, the total pressure drop coefficient for the entire individual channel is composed of the pressure drop coefficients in its individual sections

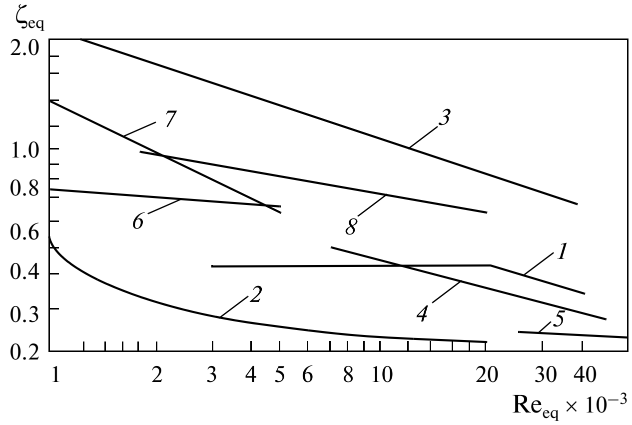

Fig. 4. Empirical criterion correlations for calculating the pressure drop coefficients in channels with finning crossing at the angle $2\beta = 90°$. (*1*) LGTU [8], (*2*) MVTU [3], (*3*) MATI [13], (*4*) KNPO [11], (*5*) LNPO [6], (*6*) and (*7*) TsIAM [5], and (*8*) CSIR [7].

$$\zeta = \zeta_{in} + \zeta_m + \zeta_{end}. \tag{16}$$

In turn, the pressure drop coefficients in each of the above-mentioned sections (the initial ζ_{in}, main ζ_m, and end ζ_{end}) are combined of the hydraulic loss coefficients at the section inlet ζ_{inl}, loss due to flow friction ζ_{fr}, loss due to flow turning near the path lateral wall ζ_{trn}, and loss at the section outlet ζ_{out}.

The total pressure drop coefficients over all path sections were determined in accordance with formula (16). Recalculation of these coefficients for the pressure drop coefficients per unit channel length and its equivalent diameter was carried out using correlation (15), and the criteria Re_{eq} corresponding to the criteria Re_h were recalculated using correlation (14).

The dependence $\zeta_{eq} = f(Re_{eq})$ generalizing the results of experiments carried out at the KNPO [11] that was obtained from this recalculation is represented by curve *4* in Fig. 4. It can be seen that this dependence is in quite satisfactory agreement with curve *1* that represents the experimental data of the LGTU [8].

Coming to an analysis of the experimental data obtained at the LNPO [6] on the pressure drop coefficients in paths containing coplanar channels, it is worthwhile to note that these data were not represented by a generalizing criterion correlation, due to which we had to use the dependences of pressure drop coefficients on the Reynolds criteria plotted on the graphs. For processing purposes, we took the experimental data for two versions of the path containing coplanar channels with the crossing angle $2\beta = 90°$ without and with installing a thin-walled pad between the finned plates. The pressure drop coefficients were calculated using the following correlation:

$$\zeta_{\rm eq} = \frac{2\Delta p}{\rho w_{\rm eq}^2} = 2.17\times10^{-12}\frac{\Delta p\times p}{G^2},$$
$$\xi_{\rm eq} = \zeta_{\rm eq}\frac{d_{\rm eq}}{L} = \zeta_{\rm eq}\frac{4.9}{164}. \tag{17}$$

Based on the results of calculations by these formulas, curve *5* averaging them is plotted in Fig. 4, which lies noticeably (by 25–35%) lower than curves *1* and *4* that generalize the experimental data of LGTU and KNPO, respectively, and only slightly below the continuation of curve *2* that reflects the experimental data of MATI. It should be noted that curve *5* generalizes the results of experiments carried out both in the path containing coplanar channels without the pad and with the pad inserted in the path.

With this analysis done, the comparison of the generalizing criterion correlations for pressure drop coefficients in paths with coplanar rectangular finning crossing at the angle $2\beta = 90°$ could be regarded to have been finished. However, it is of certain interest to compare these data with the data on pressure drop coefficients in paths with undulate finning that were investigated at the TsIAM [5]. The following dependence of ζ on the criterion $\mathrm{Re_h}$ was obtained after substituting the values of parameters adopted for the calculation:

$$\zeta_{\rm h} = 10.45/\mathrm{Re}_{\rm h}^{0.426}. \tag{18}$$

The only thing that remained to do was to transform the parameters $\zeta_{\rm h}$ and $\mathrm{Re_h}$ into $\zeta_{\rm eq}$ and $\mathrm{Re_{eq}}$, which was carried out using formulas (14) and (15):

$$\zeta_{\rm eq} = \frac{\zeta_{\rm h}}{\cos^2\beta}\frac{d_{\rm eq}}{d_{\rm h}} = 2.674\zeta_{\rm h};$$
$$\mathrm{Re_{eq}} = \mathrm{Re_h}\frac{d_{\rm eq}}{d_{\rm h}}\frac{f_{\rm h}}{f_{\rm eq}} = \mathrm{Re_h}\frac{d_{\rm eq}/d_{\rm h}}{\cos\beta} = 0.945\,\mathrm{Re_h}. \tag{19}$$

The results of calculations using formula (19) after having been modified using formulas (14) and (15) are represented by curve *6* in Fig. 4.

A conclusion can be drawn from an analysis of Fig. 4 that the pressure drop coefficients in paths with coplanar channels formed by rectangular finning crossing at the angle $2\beta = 90°$ can be calculated using the empirical correlations of LGTU [8] or KNPO [11]. The values of pressure drop coefficients calculated from these correlations are almost identical with each other and lie in the middle of the scatter band of the pressure drop coefficients obtained from the data of different investigations adopted for comparison.

At the same time, in order to reveal the reasons causing the discrepancies between the dependences of pressure drop coefficients in paths containing coplanar channels at their crossing angle $2\beta = 90°$ on the criterion Re, for each of the considered dependences we analyzed the main geometrical parameters of the paths from which the considered criterion correlations were obtained.

The obvious essential differences we revealed in the course of the performed analysis prompted us to find a universal dependence with which so different results could be generalized. To this end, we approximated the experimental data presented in Fig. 4 by power dependences of the form $\zeta = C/\mathrm{Re}^n$. After that, we found simple linear approximations for the coefficient C and the exponent n from the relative geometrical parameter H/S, namely:

$$C = 3.65\,H/S - 1.35;\quad n = H/(6S), \tag{20}$$

suitable for the considered particular case of finning crossing at the angle $2\beta = 90°$.

To obtain a more universal criterion correlation for pressure drop coefficients, formulas (20) were further modified. The relative crossing angle $\bar\beta = 2\beta/\pi$ was introduced into their structure, and the empirical dependences obtained at the CSIR [7] were used as the basic ones. (It should be noted that the work [7] was specially devoted to studying the effect the crossing angle 2β has on the thermal–hydraulic characteristics of paths containing coplanar channels formed by sine-wave finning). In the considered work, empirical criterion correlations of the form $\zeta = C/\mathrm{Re}^n$ were obtained at $H/S = 1$ for the relative crossing angles $\bar\beta$ varying in the range $\bar\beta = 1/6$–1.0 with the exponents n and with the coefficients C varying with changing the relative angle $\bar\beta$. Applying the superposition principle to the effect the parameters H/S and $\bar\beta$ have on the pressure drop coefficient ζ and assuming that the influence of the parameter H/S on the coefficients ζ is retained at the relative crossing angle $\bar\beta$ differing from 1/2, and that so is the intensity of the effect the parameter $\bar\beta$ has on the pressure drop coefficients at any relative height H/S, we can obtain the following dependences:

$$\zeta = C/\mathrm{Re}^n;\quad C = 3.65\,H/S + 150\bar\beta^2 - 120\bar\beta + 21.15;$$
$$n = H/(6S) + 1.80\bar\beta^2 - 2.0\beta + 0.55. \tag{21}$$

The obtained correlation was checked for adequacy to the available experimental data, and it was shown from that check that this correlation is valid in the variation range of the criteria Re = (4–40) × 10^3, relative heights $H/S = 0.6$–1.5, and relative crossing angles $\bar{\beta} = 1/6$–1.0.

The results from additional statistical calculations showed that the average error with which the experimental data obtained from different investigations are generalized by criterion correlation (21) at $2\beta = 90°$ is equal to ±16.7% at Re = 4 × 10^3, ±13.6% at Re = 10 × 10^3, and ±13.0% at Re = 20 × 10^3, although the maximal deviations can be significantly larger. The latter relates mainly to the results of the LNPO work [6], for which the largest errors of generalization are typically observed for all of the above-mentioned values of the Re criteria. In all likelihood, these data need additional check for gross error (miss).

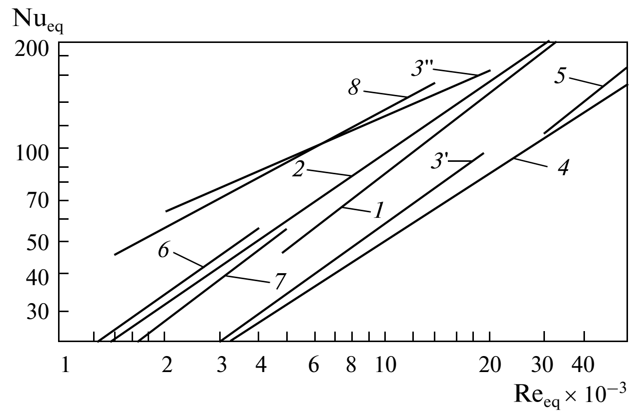

Fig. 5. Empirical criterion correlations for calculating average Nusselt numbers on the main flat surfaces in paths containing coplanar channels crossing at the angle $2\beta = 90°$. (*1*) LGTU [8, 14], (*2*) MVTU [4], (*3'*) and (*3"*) MATI [10, 12], (*4*) KNPO [11], (*5*) LNPO [6]. (*6*) and (*7*) TsIAM [5], and (*8*) CSIR [7].

ANALYSIS AND COMPARISON OF THE RESULTS FROM EXPERIMENTAL INVESTIGATIONS OF HEAT TRANSFER IN PATHS CONTAINING COPLANAR CHANNELS

Now, coming to a comparison of the results obtained from experimental investigations of heat transfer in paths containing coplanar channels, we should first of all point out that they also relate to finning crossing at the angle $2\beta = 90°$ and nonintersecting one ($h_i = 0$). In what follows, we consider heat transfer on the path's main wide surfaces without taking into account heat transfer on its lateral narrow surfaces on which the coolant flow makes a turn.

For these conditions, the criterion correlation of LGTU [8] becomes

$$\mathrm{Nu}_{eq} = 0.07\,\mathrm{Re}_{eq}^{0.765}, \qquad (22)$$

and the curve corresponding to this correlation is denoted in Fig. 5 by *1*.

The experimental data of MVTU [4] were not generalized by any empirical criterion correlation; nonetheless, the graph with experimental points reflecting the dependence of Stanton number St on the criterion Re that was given in [4] allowed us to draw a certain curve averaging these points, after which this curve was redrawn using the transformation formulas into the generalizing variables used in the present work

$$\mathrm{Nu}_{eq} = \mathrm{St}_h\,\mathrm{Pr}\,\mathrm{Re}_h\,d_{eq}/d_h = 0.723\,\mathrm{St}_h\,\mathrm{Re}_{eq}. \qquad (23)$$

Curve *2* drawn in this way in Figure 5 reflects the results from the experimental investigation of heat transfer in a path containing coplanar channels that was carried out at the MVTU [4].

The results from transforming the experimental data on heat transfer in paths containing coplanar channels obtained at the MATI [13] are represented in Fig. 5 by two curves. Curve *3"* corresponds to calculations carried out for the considered particular case $2\beta = 90°$:

$$\mathrm{Nu}_{eq} = 2.57\,\mathrm{Re}_{eq}^{0.42}. \qquad (24)$$

Curve *3'* characterizes the criterion correlation, which, after substituting in it the values of the parameters $h/S = 6/14$ and $L/d_{eq} = 210/9.15$ adopted for carrying out the present comparison, takes the form

$$\mathrm{Nu}_{eq} = 0.0417\,\mathrm{Re}_{eq}^{0.8}. \qquad (25)$$

For comparing the data on heat transfer in paths containing coplanar channels obtained at the KNPO [11] with the results of other researchers, we used the dependence describing the average heat transfer in the path main section, which, after making a shift from the characteristic size $x = l$ to the characteristic size d_h, took the following form for the considered case $2\beta = 90°$:

$$\mathrm{Nu}_h = A\,\mathrm{Re}_h^n\,(l/d_h)^{n-1}\,\mathrm{Pr}^{0.4}/\psi^{0.55},$$

where $A = 0.498$ and $n = 0.637$.

Taking into account the conditions under which the experiments were carried out (Pr = 0.7, and $\psi = T_w/T_a = 1.034$, where T_w and T_a are the average temperatures of path walls and air), this criterion correlation was transformed into the final form in the coordinate axes $\mathrm{Nu}_{eq} = 0.148\,\mathrm{Re}_{eq}^{0.637}$. The results of calculations by this criterion correlation are represented in Fig. 5 by curve *4*, which yields essentially smaller values of Nu_{eq} at a fixed value of Re_{eq} as compared with those obtained using the correlations of LGTU (curve *1*), MVTU (curve *2*), and MATI (curves *3'* and *3"*).

The duly processed experimental data on heat transfer in a path containing coplanar channels obtained at the LNPO [6] are represented in Fig. 5 by curve *5*. This curve correlates best with curve *4* that reflects the results from the experiments carried out at the KNPO [11] and, hence, gives underestimated values of Nu numbers as compared with those recommended by the LGTU [8] (curve *1*) or MVTU [4] (curve *2*).

With the information presented above, the data on heat transfer in paths containing coplanar channels available in open-access publications are exhausted, and we could make a shift to analyzing and generalizing them. However, from a purely methodological point of view, is also of interest to compare them with the data on heat transfer in paths with undulate finning [5] obtained at the TsIAM.

The formula for calculating average heat transfer in a path containing undulate finning with a geometry fixed for the comparison conditions has the form

$$\mathrm{Nu}_{\mathrm{eq}} = 1.337 \times 0.08755(\mathrm{Re}_{\mathrm{eq}}/0.945)^{0.723} = 0.122\,\mathrm{Re}_{\mathrm{eq}}^{\;0.723}.$$

The results of calculations by this formula are represented in Fig. 5 by formula *6*, which is also in good correlation with curve *2* generalizing the experimental data of MVTU [4].

On the whole, all dependences $\mathrm{Nu}_{\mathrm{eq}} = f(\mathrm{Re}_{\mathrm{eq}})$ shown in Fig. 5 except with only (24) represented by curve *3″* fit within the band characterized by the scatter from its certain mean value, e.g., at $\mathrm{Re}_{\mathrm{eq}} = 10^4$, at a level of ±55%. That is, if there are no doubts in the trustworthiness of all initial data and the performed transformations, the data of different authors on average heat transfer in paths containing coplanar channels crossing at the angle $2\beta = 90°$ may differ from each other by almost a factor of 2.

Such a considerable difference (provided that the data being analyzed are absolutely trustworthy) immediately prompts us to make a conjecture that there is some parameter characteristic for each of the considered regularities of heat transfer and causing the obtained stratification of the curves reflecting these regularities in Fig. 5. Indeed, even a simplest analysis shows that the channel height related to the finning pitch H/S, the quantity we already used in processing the data on pressure drop, may serve as such parameter. Clearly, there is certain regularity, the essence of which is that the average values of Nu numbers in the path with coplanar channels crossing at the angle $2\beta = 90°$ tend to decrease with increasing the channel relative height H/S.

For analytically describing the above-mentioned regularity pertinent to the effect the parameter H/S has on the average level of heat transfer we had to screen the influence of the criterion Re, which was achieved by relating the Nu number to the corresponding to it Re criterion raised to the power m corresponding to each of the curves drawn in Fig. 5. However, here again we found that the exponent depends on the considered parameter H/S. In turn, it was found that this dependence can be approximated by the following linear function:

$$m = 0.16(H/S) + 0.62, \tag{26}$$

the relative deviation of which from the experimental values of m does not exceed 3.5%.

After constructing the graph $\mathrm{Nu}/\mathrm{Re}^m = f(H/S)$ and performing its subsequent analysis for selecting the suitable form of an approximating function we determined that the following linear function would be the best approximation:

$$\mathrm{Nu}_{\mathrm{eq}}/\mathrm{Re}_{\mathrm{eq}}^m = 0.24 - 0.185(H/S), \tag{27}$$

which gives the maximal relative deviation from the experimental points in the middle part of the H/S parameter variation range at a level of 7.0%. As a result, the empirical formula for determining the average heat transfer coefficients on the main heat-transfer surface of paths containing coplanar channels crossing at an angle of 90° determined taking expressions (26) and (27) into account has the following form:

$$\mathrm{Nu}_{\mathrm{eq}} = [0.24 - 0.185(H/S)]\,\mathrm{Re}_{\mathrm{eq}}^{0.16(H/S)+0.62}. \tag{28}$$

The calculations carried out using this formula have shown that its maximal deviation from the corresponding empirical dependences shown in Fig. 5 determined for the ends of the corresponding variation ranges of the Re criteria does not go beyond 11.0%. The obtained criterion correlation (28) is valid for paths containing coplanar channels formed by nonintersecting rectangular fins crossing at the angle $2\beta = 90°$ in the variation range of the Reynolds criteria $\mathrm{Re}_{\mathrm{eq}} = (5–50) \times 10^3$ and path relative heights $H/S = 0.5–1.15$.

REFERENCES

1. S. Z. Kopelev and A. F. Slitenko, *The Design and Calculation of Gas Turbine Engine Cooling Systems* (Osnova, Kharkiv, 1994) [in Russian].
2. V. P. Isachenko, V. A. Osipova, and A. S. Sukomel, *Heat Transfer* (Energiya, Moscow, 1975) [in Russian].
3. V. M. Kudryavtsev, S. A. Orlin, and S. A. Posnov, “An experimental investigation of pressure drop in paths containing coplanar channels,” Izv. Vyssh. Uchebn. Zaved., Mashinostroenie, No. 4, 54–58 (1983).
4. V. M. Kudryavtsev, S. A. Orlin, and S. A. Posnov, “An experimental investigation of heat transfer in a path containing coplanar channels,” in *Thermal Processes in the Engines and Power Installations of Aircrafts* (KAI, Kazan, 1985), pp. 56–62.
5. A. N. Antonov, Yu. F. Baranov, and A. Yu. Klochko, “An experimental investigation of heat-transfer surfaces

containing coplanar channels (similar to the Frenkel model)," Vestnik MEI, No. 1, 21–23 (1997).

6. A. S. Lebedev, "An experimental investigation of heat transfer in model turbine blade cooling channels," Izv. Vyssh. Uchebn. Zaved., Energetika, No. 9, 92–95 (1986).

7. W. W. Focke, J. Zachariades, and I. Olivier, "The effect of the corrugation inclination on the thermohydraulic performance of plate heat exchangers," Int. J. Heat Mass Transfer **28** (8), 1469–1479 (1985).

8. K. D. Andreev, L. V. Arsen'ev, V. G. Polishchuk, and N. P. Sokolov, "Investigation of heat transfer and pressure drops in a rectangular channel with mutually intersecting and crossing finning," Prom. Teplotekhn. **20** (3), 70–75 (1998).

9. M. N. Galkin, A. N. Boiko, V. G. Popov, and N. L. Yaroslavtsev, "Calculation and analysis of internal heat transfer in cooled structures with cross flow of coolant," in *Proceedings of the All-Union Conference "Methods and Means for Machine-Aided Diagnostics of Gas-Turbine Engines and Their Elements," KhAI, Kharkiv, 1983*, pp. 105–106.

10. M. N. Galkin, A. N. Boiko, V. G. Popov, and N. L. Yaroslavtsev, "Internal heat transfer in finned channels with crossing jets of cooling air," Izv. Vyssh. Uchebn. Zaved., Mashinostroenie, No. 5, 56–60 (1984).

11. G. P. Nagoga, I. S. Kopylov, and M. V. Rukin, "Heat transfer and pressure drop in paths composed of coplanar crossing channels," in *Working Processes in the Cooled Turbine Machines of Gas Turbine Engines* (KAI, Kazan, 1989), pp. 35–41.

12. M. N. Galkin, V. P. Litvinkov, and V. A. Mal'kov, "Enhancement of heat transfer in finned paths with ultimately large angles of crossing channels," Izv. Vyssh. Uchebn. Zaved., Mashinostroenie, No. 9, 60–63 (1987).

13. M. N. Galkin, V. G. Popov, and N. L. Yaroslavtsev, "Investigation and calculation of hydraulic and thermal characteristics of cooled structures with coplanar channels," Izv. Vyssh. Uchebn. Zaved., Mashinostroenie, No. 3, 73–76 (1985).

14. N. P. Sokolov, V. G. Polishcuk, and K. D. Andreev, "Pressure drops in rectangular channels with crossing finning," Nauchno-Tekhn. Vedom. SPbGPU, No. 171, 85–94 (2013).

15. N. P. Sokolov, V. G. Poloshchuk, K. D. Andreev, V. A. Rassokhin, and N. A. Zabelin, *Working Processes in Turbines at Thermal Power Plants and Gas Pumping Stations. Hydraulics and Heat Transfer in Channels with Coplanar Finning: A Handbook* (Izd. Politekhn. Univ, St. Petersburg, 2013) [in Russian].

Translated by V. Filatov