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THE BEHAVIOR OF PLASMA WITH AN ARBITRARY DEGREE OF DEGENERACY OF ELECTRON GAS IN THE CONDUCTIVE LAYER

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We obtain an analytic solution of the boundary problem for the behavior (fluctuations) of an electron plasma with an arbitrary degree of degeneracy of the electron gas in the conductive layer in an external electric field. We use the kinetic Vlasov–Boltzmann equation with the Bhatnagar–Gross–Krook collision integral and the Maxwell equation for the electric field. We use the mirror boundary conditions for the reflections of electrons from the layer boundary. The boundary problem reduces to a one-dimensional problem with a single velocity. For this, we use the method of consecutive approximations, linearization of the equations with respect to the absolute distribution of the Fermi–Dirac electrons, and the conservation law for the number of particles. Separation of variables then helps reduce the problem equations to a characteristic system of equations. In the space of generalized functions, we find the eigensolutions of the initial system, which correspond to the continuous spectrum (Van Kampen mode). Solving the dispersion equation, we then find the eigensolutions corresponding to the adjoint and discrete spectra (Drude and Debye modes). We then construct the general solution of the boundary problem by decomposing it into the eigensolutions. The coefficients of the decomposition are given by the boundary conditions. This allows obtaining the decompositions of the distribution function and the electric field in explicit form.

Keywords: characteristic system, eigenfunction, Drude mode, Debye mode, Van Kampen mode, decomposition of the solution with eigenfunctions

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1. Introduction

Vlasov first considered the problem of plasma fluctuations [1]. Landau [2] analytically solved the problem of the behavior of a collisionless plasma in the half-space in an external electric field perpendicular to the surface where the electrons reflect from the boundary according to the mirror law. Hence, the problem of plasma fluctuations is naturally called the Vlasov–Landau problem.

The problem of fluctuations of a degenerate electron plasma in a metal layer was first analytically solved in [3]. Fluctuations of a Maxwellian plasma in the half-space were considered in [4]. This paper is a continuation of [3] and [4]. We generalize these results to the general case of a plasma with an arbitrary degree of degeneracy of the electron gas.

The problem with a diffusion boundary condition for a collision plasma in the half-space was first considered in [5], where general problems of the its solvability with diffusion boundary conditions and the structure of the discrete spectrum depending on the parameters of the problem were studied. Nevertheless, there was no detailed analysis of the general solution there because the solution is complicated.

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A complete solution of the problem of fluctuations of the degenerate plasma was given in [6], [7].

Some related problems appear in studying the reflection of an electromagnetic field. They were considered in [8], [9], where the method of integral transformations was used.

In [10], [11] a general asymptotic analysis of the behavior of an electric field far from the surface was performed. In [10], the special importance of analyzing the behavior of the field near the plasma resonance was emphasized. It was found in [11] that the behavior of the field in this case for the mirror and diffusion dissipation of electrons on the surface differs crucially.

This problem is important in plasma theory (see, e.g., [12], [13]) and currently remains to be studied from different standpoints [14].

Our aim here is to obtain an analytic solution. Reducing the problem to a one-dimensional problem with a single velocity is a crucial step toward an analytic solution. This idea is not innovative. It goes back to Landau [2], who introduced the main idea: to consider a lateral external electric field directed along the x axis. After this assumption, the problem becomes one-dimensional (along the x axis) with a single velocity. This velocity is the projection v_x of the electron velocity $\mathbf{v}$ on the x axis. The initial three-dimensional problem (with respect to time and velocities) with such a decomposition does not lose generality, because this decomposition is only given by the direction of the external electric field.

In the passage to dimensionless variables and parameters, a small parameter naturally appears in the equations, a fluctuation of the dimensionless (chemical) potential due to the presence of the external electric field. Using the small-parameter method, we can linearize the problem with respect to the absolute Fermi–Dirac electron distribution. Using the conservation law for the number of particles, we can finally solve the problem as a one-dimensional boundary value problem with a single velocity.

The separation of variables reduces the equations of the problem to the characteristic system of equations containing the spectral parameter. In the space of generalized functions, there exists a continuous family of eigensolutions of the initial system corresponding to the continuous spectrum. These are the Van Kampen modes. This family corresponds to a single solution in the form of an integral over the continuous spectrum of the eigensolutions multiplied by an unknown function called the coefficient of the continuous spectrum.

Solving the dispersion equation, we then find the eigensolutions corresponding to the adjoint and discrete spectra; these are the respective Drude and Debye modes. We then compose the general solution of the boundary value problem as a decomposition into the eigensolutions corresponding to the continuous, adjoint, and discrete spectra. This decomposition is the sum of the integral over the continuous spectrum of the eigensolutions multiplied by some function of the spectral parameter and the eigensolutions corresponding to the adjoint and discrete spectra multiplied by unknown variables. These are the coefficients corresponding to adjoint and continuous spectra.

The decomposition coefficients of the solution of the continuous, adjoint, and discrete spectra can be found from the boundary conditions. This allows obtaining the decomposition of the distribution function and the electric field in explicit form.

2. Statement of the problem and the main equations

Let the nondegenerate Fermi–Dirac plasma be in a layer $|x| < L$ that is filled with a conductive medium. We consider the external field to be sufficiently weak such that we can use the linear approximation [12]. We use the τ-model Vlasov–Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{H}]\right)\frac{\partial f}{\partial \mathbf{p}} = \nu(f_{\text{eq}} - f) \tag{1}$$

and the Maxwell equation for the electric field

$$\operatorname{div}\mathbf{E} = 4\pi\rho, \qquad \rho = e\int (f - f_0)\, d\Omega, \qquad d\Omega = \frac{(2s+1)d^3p}{(2\pi\hbar)^3}. \tag{2}$$

Here, $f_{\rm eq}$ is the locally equilibrated Fermi–Dirac distribution function,

$$f_{\rm eq}(\mathbf{r}, v, t) = \left\{1 + e^{(\mathcal{E}-\mu(\mathbf{r},t))/kT}\right\}^{-1},$$

$f_0 = f_{\rm FD}$ is the nonperturbed (absolute) Fermi–Dirac distribution function,

$$f_0(v, \mu) = f_{\rm FD}(v, \mu) = \left\{1 + e^{(\mathcal{E}-\mu)/kT}\right\}^{-1},$$

$\mathbf{p} = m\mathbf{v}$ is the electron momentum, $\mathcal{E} = mv^2/2$ is the kinetic electron energy, μ and $\mu(\mathbf{r}, t)$ are the respective nonperturbed and perturbed chemical potentials, e and m are charge and mass of the electron, ρ is the charge density, $\hbar$ is the Planck constant, ν is the effective frequency of the electron dissipation, s is the particle spin ($s = 1/2$ for the electron), k is the Boltzmann constant, T is the plasma temperature (assumed to be constant in this problem), and $\mathbf{E}(\mathbf{r}, t)$ and $\mathbf{H}(\mathbf{r}, t)$ are electric and magnetic fields inside the plasma.

We can reduce Eqs. (1) and (2) to one-dimensional with a single velocity. This step is necessary for obtaining an analytic solution. The idea, as previously mentioned, is due to Landau [2], who proposed considering a longitudinal external electric field directed along the x axis.

Therefore, we consider a longitudinal external electric field outside the plasma perpendicular to the plasma boundary and changing by the law $\mathbf{E}_{\rm ext}(t) = E_0 e^{-i\omega t}(1, 0, 0)$. The self-consistent electric field inside the plasma is denoted by $\mathbf{E}(x, t) = E(x)e^{-i\omega t}(1, 0, 0)$. It is easy to verify that $\mathbf{H} = -(ic/\omega)\operatorname{curl}\mathbf{E} = 0$ for a chosen configuration of the external electric field. Therefore, the magnetic field has no role in Eq. (1). Because the external field has one x component, the distribution function f has the form $f = f(x, v_x, t)$, where v_x is the projection of the electron velocity on the x axis. The external electric field causes changes of the chemical potential $\mu(x, t) = \mu + \delta\mu(x)e^{-i\omega t}$, where $\mu = \text{const}$ is the value of the chemical potential in the absence of an external electric field on the plasma boundary. We introduce a dimensionless (adjusted) chemical potential of the electron gas $\alpha(x, t) = \mu(x, t)/kT$. For the adjusted chemical potential, the last equality has the form $\alpha(x, t) = \alpha + \delta\alpha(x)e^{-i\omega t}$, $\alpha = \text{const}$.

We assume that the quantity $\delta\alpha(x, t) = \delta\alpha(x)e^{-i\omega t}$, which is the perturbation of the adjusted chemical potential, is a small parameter, i.e., $|\delta\alpha(x, t)| = |\delta\alpha(x)| \ll 1$. Physically, this inequality means that the perturbation of the chemical potential is much smaller than the electron thermal energy: $|\delta\mu(x, t)| \ll \mathcal{E}_T$, $\mathcal{E}_T = mv_T^2/2$. We use the method of consecutive approximations, taking $|\delta\alpha(x)| \ll 1$.

We linearize Eqs. (1) and (2) with respect to the absolute Fermi–Dirac distribution function f_0. We also pass to dimensionless variables and parameters.

We introduce the dimensionless electron momentum (velocity) $\mathbf{P} = \mathbf{p}/p_T = \mathbf{v}/v_T$, where $v_T = \sqrt{2kT/m}$ is the electron thermal velocity and $\alpha = \mu/kT$ is the dimensionless (adjusted) chemical potential. We linearize the locally equilibrated distribution function: $f_{\rm eq}(x, P, t) = f_0(P, \alpha) + g(P, \alpha)\delta\alpha(x)e^{-i\omega t}$, where

$$f_0(P, \alpha) = f_{\rm FD}(P, \alpha) = \frac{1}{1 + e^{P^2-\alpha}}, \qquad g(P, \alpha) = \frac{e^{P^2-\alpha}}{(1 + e^{P^2-\alpha})^2}.$$

We linearize the electron distribution function:

$$f(x, P_x, t) = f_0(P, \alpha) + g(P, \alpha)h(x, P_x)e^{-i\omega t}, \tag{3}$$

where $h(x, P_x)$ is a new unknown function.

We note that in the linear approximation, we have

$$e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = e\mathbf{E}\frac{\partial f_0}{\partial \mathbf{p}} = \frac{e}{p_T}\mathbf{E}\frac{\partial f_0}{\partial \mathbf{P}} = -E(x)e^{-i\omega t}\frac{2eP_x}{p_T}g(P,\alpha).$$

Using (3) and other relations instead of Eqs. (1) and (2), we obtain

$$-i\omega h(x,P_x) + v_T P_x\frac{\partial h}{\partial x} + \nu h(x,P_x) = eE(x)\frac{2P_x}{p_T} + \nu\delta\alpha(x), \tag{4}$$

$$\frac{dE(x)}{dx} = \frac{8\pi e p_T^3}{(2\pi\hbar)^3}\int h(x,P_x)g(P,\alpha)\,d^3P. \tag{5}$$

The value $\delta\alpha(x)$ can be found from the conservation law for the number of particles:

$$\int f_{\text{eq}}\,d\Omega = \int f\,d\Omega.$$

From this equation, we find that

$$\delta\alpha(x) = \int h(x,P_x)\,g(P,\alpha)\,d\Omega\left[\int g(P,\alpha)\,d\Omega\right]^{-1}.$$

Here,

$$\int g(P,\alpha)\,d^3P = 4\pi g_2(\alpha), \qquad g_2(\alpha) = \int_0^\infty g(P,\alpha)P^2\,dP = \frac{1}{2}s_0(\alpha),$$

where

$$s_0(\alpha) = \int_0^\infty \frac{dP}{1+e^{P^2-\alpha}} = \int_0^\infty f_0(P,\alpha)\,dP.$$

It follows that

$$\delta\alpha(x) = \frac{1}{4\pi g_2(\alpha)}\int h(x,P_x)\,g(P,\alpha)\,d^3P.$$

Calculating the inner double integral in the plane (P_y,P_z), we obtain

$$\delta\alpha(x) = \frac{1}{2s_0(\alpha)}\int_{-\infty}^{\infty} f_0(P_x,\alpha)h(x,P_x)\,dP_x.$$

Hereafter, we assume that $E(x) = E_0 e(x)$. System of equations (4), (5) can be transformed into the form

$$v_T P_x\frac{\partial h}{\partial x} + (\nu - i\omega)h(x,P_x) = \frac{2eE_0}{p_T}P_x e(x) + \frac{\nu}{2s_0(\alpha)}\int_{-\infty}^{\infty} f_0(P_x,\alpha)h(x,P_x)\,dP_x, \tag{6}$$

$$E_0\frac{de(x)}{dx} = \frac{8\pi^2 e p_T^3}{(2\pi\hbar)^3}\int_{-\infty}^{\infty} f_0(P_x,\alpha)h(x,P_x)\,dP_x. \tag{7}$$

In these equations, we pass to the dimensionless variables and functions

$$x_1 = \frac{x}{\lambda}, \qquad \lambda = \tau v_T, \qquad P_x = \mu, \qquad H(x_1,\mu) = \frac{\nu p_T}{2eE_0}h(x,\mu),$$

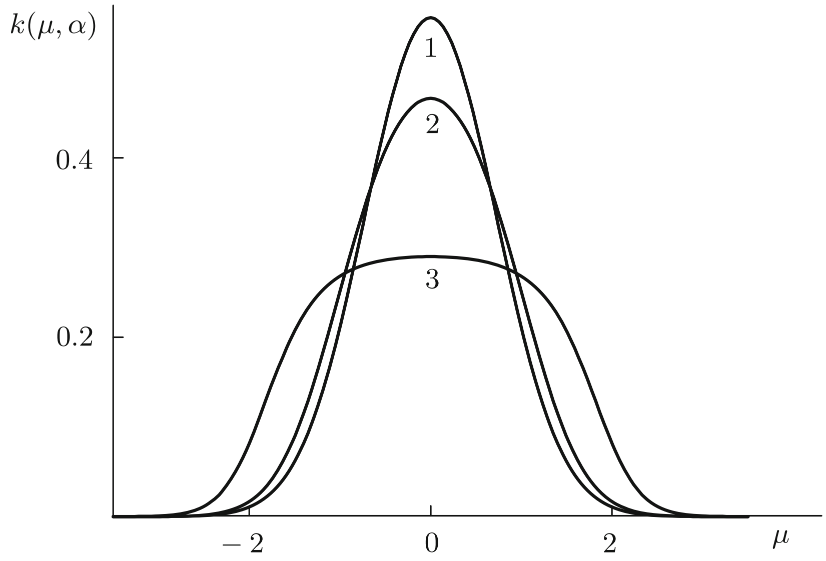

Fig. 1. Plots of the function $k(\mu, \alpha)$ for $\alpha = -3, 0, 3$ (curves 1, 2, and 3).

where λ is the electron mean free path (between two consecutive collisions). As a result of passing to dimensionless parameters and functions, we obtain the system of equations

$$\mu \frac{\partial H}{\partial x_1} + w_0 H(x_1, \mu) = \mu e(x_1) + \int_{-\infty}^{\infty} k(\mu', \alpha) H(x_1, \mu')\, d\mu', \tag{8}$$

$$\frac{de(x_1)}{dx_1} = \varkappa^2(\alpha) \int_{-\infty}^{\infty} k(\mu', \alpha) H(x_1, \mu')\, d\mu', \tag{9}$$

where

$$\varkappa^2(\alpha) = \frac{32\pi^2 e^2 p_T^3 s_0(\alpha)}{(2\pi\hbar)^3 m\nu^2}$$

and the new function $k(\mu, \alpha) = f_0(\mu, \alpha)/2s_0(\alpha)$ with the property

$$\int_{-\infty}^{\infty} k(\mu, \alpha)\, d\mu = 1$$

is introduced. Here, $w_0 = 1 - i\omega/\nu = 1 - i\omega\tau = 1 - i\Omega/\varepsilon$, where $\Omega = \omega/\omega_{\rm p}$, $\varepsilon = \nu/\omega_{\rm p}$, $\omega_{\rm p}$ is the plasma (Langmuir) frequency, $\omega_{\rm p} = \sqrt{4\pi e^2 N/m}$, and N is the number density (concentration) of electrons in the equilibrium state. The behavior of the kernels of Eqs. (8) and (9) for different values of the chemical potential is described in Fig. 1. By the definition of the number density, we have

$$N = \int f_0(P, \alpha)\, d\Omega = \frac{2p_T^3}{(2\pi\hbar)^3} \int \frac{d^3P}{1 + e^{P^2 - \alpha}} = \frac{8\pi p_T^3}{(2\pi\hbar)^3} s_2(\alpha),$$

where

$$s_2(\alpha) = \int_0^{\infty} \frac{P^2\, dP}{1 + e^{P^2 - \alpha}} = \int_0^{\infty} P^2 f_0(P, \alpha)\, dP.$$

Hence, the numerical density of the plasma particles and the heat wave number $k_T = mv_T/\hbar$ are related by $N = (s_2(\alpha)/\pi^2) k_T^3$. Moreover,

$$\varkappa^2(\alpha) = \frac{\omega_{\rm p}^2}{\nu^2} \frac{s_0(\alpha)}{s_2(\alpha)} = \frac{\Omega_{\rm p}^2}{r(\alpha)} = \frac{1}{\varepsilon^2 r(\alpha)},$$

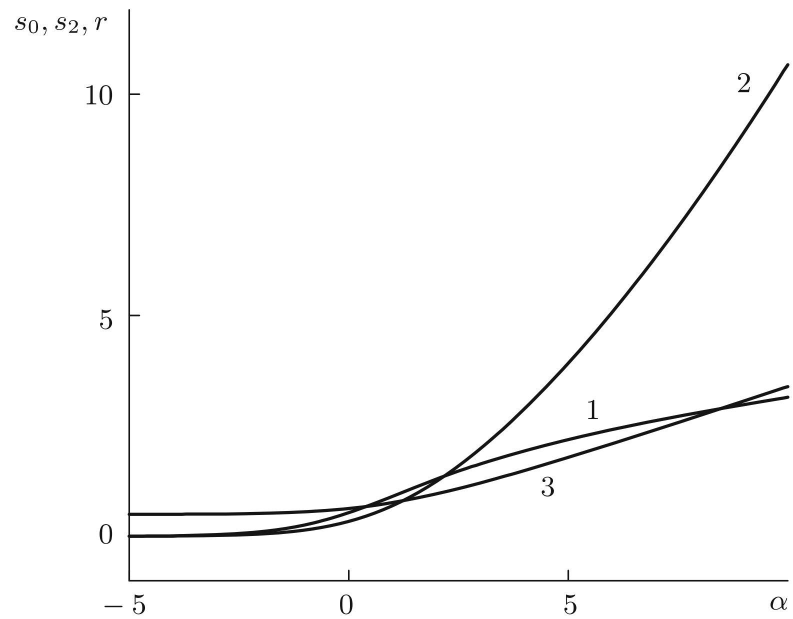

Fig. 2. Plots of the functions $s_0(\alpha)$ (curve 1), $s_2(\alpha)$ (curve 2), and $r(\alpha)$ (curve 3) as the chemical potential changes from $\alpha = -5$ to $\alpha = 10$.

where

$$r(\alpha) = \frac{s_2(\alpha)}{s_0(\alpha)}, \qquad \varepsilon = \frac{\nu}{\omega_\text{p}} = \frac{1}{\tau\omega_\text{p}} = \frac{1}{\Omega_\text{p}}, \qquad \Omega_\text{p} = \omega_\text{p}\tau.$$

Plots of the functions $s_0(\alpha)$, $s_2(\alpha)$, and $r(\alpha)$ are shown in Fig. 2.

With the expressions $\varkappa^2(\alpha)$, we can rewrite (9) as

$$\frac{de(x_1)}{dx_1} = \frac{1}{\varepsilon^2 r(\alpha)} \int_{-\infty}^{\infty} k(\mu', \alpha) H(x_1, \mu')\, d\mu'. \tag{10}$$

It is known that the frequency of plasma fluctuations is usually much higher than the frequency of collisions of electrons in metal [15]. The most typical values of ε are in the interval $10^{-4} \le \varepsilon \le 10^{-2}$.

In the case of the mirror reflection of electrons from the plasma boundary, we have the boundary conditions

$$f(\pm L, v_x, v_y, v_z, t) = f(\pm L, -v_x, v_y, v_z, t), \quad -\infty < v_x < +\infty,$$

for the electron distribution functions on the boundary of a layer of size $2L$. Hence, for the function $H(x_1, \mu)$, we obtain the mirror boundary conditions

$$H(l, \mu) = H(l, -\mu), \qquad H(-l, \mu) = H(-l, -\mu), \quad \mu > 0, \tag{11}$$

where $l = L/\lambda$ is the size of the layer in terms of the electron free path.

For the electric field, the boundary condition has the form

$$e(l) = 1, \qquad e(-l) = 1. \tag{12}$$

The boundary value problem for plasma fluctuations in a conductive medium layer is thus completely formulated and consists in finding a solution of Eqs. (8) and (10) such that boundary conditions (11) and (12) are satisfied.

3. Eigensolutions of the continuous spectrum

First, we seek a general solution of system of equations (8), (10). Separation of variables by the general Fourier method leads to the substitution

$$\begin{aligned} H_\eta(x_1,\mu) &= e^{-w_0x_1/\eta}\Phi_1(\eta,\mu)+e^{w_0x_1/\eta}\Phi_2(\eta,\mu),\\ e_\eta(x_1) &= \left[e^{-w_0x_1/\eta}+e^{w_0x_1/\eta}\right]E(\eta), \end{aligned} \tag{13}$$

where η is the spectral (or separation) parameter, which is usually complex. Substituting (13) in (8) and (10), we obtain the characteristic system of equations

$$(\eta-\mu)\Phi_1(\eta,\mu)=\eta\mu\frac{E(\eta)}{w_0}+\frac{\eta}{w_0}\int_{-\infty}^{\infty}k(\mu',\alpha)\Phi_1(\eta,\mu')\,d\mu', \tag{14}$$

$$(\eta+\mu)\Phi_2(\eta,\mu)=\eta\mu\frac{E(\eta)}{w_0}+\frac{\eta}{w_0}\int_{-\infty}^{\infty}k(\mu',\alpha)\Phi_2(\eta,\mu')\,d\mu', \tag{15}$$

$$-\frac{w_0}{\eta}E(\eta)=\frac{1}{\varepsilon^2r(\alpha)}\int_{-\infty}^{\infty}k(\mu',\alpha)\Phi_1(\eta,\mu')\,d\mu', \tag{16}$$

$$\frac{w_0}{\eta}E(\eta)=\frac{1}{\varepsilon^2r(\alpha)}\int_{-\infty}^{\infty}k(\mu',\alpha)\Phi_2(\eta,\mu')\,d\mu'. \tag{17}$$

Using (16) and (17), we transform (14) and (15) and obtain the system of equations

$$\begin{aligned} (\eta-\mu)\Phi_1(\eta,\mu) &= \frac{E(\eta)}{w_0}(\mu\eta-\eta_1^2),\\ (\eta+\mu)\Phi_2(\eta,\mu) &= \frac{E(\eta)}{w_0}(\mu\eta+\eta_1^2), \end{aligned} \tag{18}$$

where

$$\eta_1^2=w_0\varepsilon^2r(\alpha)=\frac{\nu^2}{\omega_{\text{p}}^2}\left(1-i\frac{\omega}{\nu}\right)r(\alpha)=\varepsilon(\varepsilon-i\Omega)r(\alpha).$$

For $\eta\in(-\infty,+\infty)$, we seek a solution of characteristic equations (18) in the space of generalized functions [16]:

$$\Phi_1(\eta,\mu)=\frac{E(\eta)}{w_0}(\mu\eta-\eta_1^2)\mathrm{P}\frac{1}{\eta-\mu}+g_1(\eta)\delta(\eta-\mu), \tag{19}$$

$$\Phi_2(\eta,\mu)=\frac{E(\eta)}{w_0}(\mu\eta+\eta_1^2)\mathrm{P}\frac{1}{\eta+\mu}+g_2(\eta)\delta(\eta+\mu), \tag{20}$$

where $\delta(x)$ is the Dirac delta function, the symbol $\mathrm{P}x^{-1}$ denotes the principal value of the integral of x^{-1}, the functions $g_1(\eta)$ and $g_2(\eta)$ play the role of arbitrary integration "constants," $\eta,\mu\in(-\infty,+\infty)$, and the set of values η filling the real line is called the continuous spectrum of the characteristic equation.

Solutions (19) and (20) of (15) are called the eigenfunctions of the characteristic equation.

To find the functions $g_1(\eta)$ and $g_2(\eta)$, we respectively substitute (19) and (20) in (16) and (17). As a result, we obtain

$$g_1(\eta)=-g_2(\eta),\qquad g_2(\eta)=\eta_1^2E(\eta)\frac{\Lambda(\eta)}{\eta k(\eta,\alpha)},$$

where we introduce the dispersion function

$$\Lambda(z)=\Lambda(z,\Omega,\varepsilon)=1+\frac{z}{w_0\eta_1^2}\int_{-\infty}^{\infty}\frac{\eta_1^2-\mu' z}{\mu'-z}k(\mu',\alpha)\,d\mu'. \tag{21}$$

Using the found functions $g_1(\eta)$ and $g_2(\eta)$, we represent eigenfunctions (19) and (20) of characteristic system of equations (18) as

$$\Phi_1(\eta,\mu)=\frac{E(\eta)}{w_0}F_1(\eta,\mu),\qquad \Phi_2(\eta,\mu)=\frac{E(\eta)}{w_0}F_2(\eta,\mu),$$

where

$$F_1(\eta,\mu)=P\frac{\mu\eta-\eta_1^2}{\eta-\mu}-\frac{w_0\eta_1^2\Lambda(\eta)}{\eta k(\eta,\alpha)}\delta(\eta-\mu),$$

$$F_2(\eta,\mu)=P\frac{\mu\eta+\eta_1^2}{\eta+\mu}+\frac{w_0\eta_1^2\Lambda(\eta)}{\eta k(\eta,\alpha)}\delta(\eta+\mu).$$

Family (13) of the eigensolutions of Eqs. (8) and (10), as previously mentioned, corresponds to the continuous spectrum. This family of the continuous spectrum is often called the Van Kampen mode.

We note that $F_2(\eta,-\mu)=-F_1(\eta,\mu)$ and $F_2(-\eta,\mu)=F_1(\eta,\mu)$.

The dispersion function $\Lambda(z)$ can be represented as

$$\Lambda(z)=1-\frac{1}{w_0}+\frac{\eta_1^2-z^2}{w_0\eta_1^2}\lambda_0(z,\alpha), \tag{22}$$

where

$$\lambda_0(z,\alpha)=1+z\int_{-\infty}^{\infty}\frac{k(\mu,\alpha)\,d\mu}{\mu-z}.$$

By the Sokhotsky formulas, we can calculate the boundary values of the dispersion function from above and from below on the real line:

$$\Lambda^{\pm}(\mu)=\Lambda(\mu)\pm i\pi\frac{\mu k(\mu,\alpha)}{w_0\eta_1^2}(\eta_1^2-\mu^2),$$

where

$$\Lambda(\mu)=1+\frac{z}{w_0\eta_1^2}\int_{-\infty}^{\infty}\frac{\eta_1^2-\mu'\mu}{\mu'-\mu}k(\mu',\alpha)\,d\mu',$$

and the integral in this expression is singular and is understood as a Cauchy principal value.

4. Eigensolutions of the adjoint and discrete spectra

We find the roots of the dispersion equation,

$$\frac{\Lambda(z)}{z}=0. \tag{23}$$

For this, we expand the function $\lambda_0(z,\alpha)$ in a Laurent series in a neighborhood of infinity:

$$\lambda_0(z,\alpha)=-\frac{k_2(\alpha)}{z^2}-\frac{k_4(\alpha)}{z^4}+\dots,\quad z\to\infty.$$

Here,

$$k_{2n}(\alpha)=\int_{-\infty}^{\infty}k(\mu',\alpha)\mu'^{2n}\,d\mu'=\frac{1}{2s_0(\alpha)}\int_{-\infty}^{\infty}f_0(\mu',\alpha)\mu'^{2n}\,d\mu'=$$

$$=\frac{s_{2n}(\alpha)}{s_0(\alpha)},\quad n=0,1,2,\ldots,\qquad k_0(\alpha)\equiv 1,$$

where

$$s_{2n}(\alpha)=\int_{-\infty}^{\infty}f_0(\mu',\alpha)\mu'^{2n}\,d\mu'.$$

We substitute the obtained decomposition in (22) and obtain the expansion for the dispersion function in a Laurent series in a neighborhood of infinity:

$$\Lambda(z)=\Lambda_\infty+\frac{\Lambda_{-2}}{z^2}+\frac{\Lambda_{-4}}{z^4}+\ldots,\tag{24}$$

where

$$\Lambda_\infty=1-\frac{1}{w_0}+\frac{k_2(\alpha)}{w_0\eta_1^2},\qquad \Lambda_{-2}=\frac{k_4(\alpha)-\eta_1^2k_2(\alpha)}{w_0\eta_1^2},$$

$$\Lambda_{-4}=\frac{k_6(\alpha)-\eta_1^2k_4(\alpha)}{w_0\eta_1^2},\qquad \ldots.$$

It is easy to see that the value of the dispersion function at infinity is independent of the chemical potential and is equal to

$$\Lambda_\infty=\Lambda(\infty)=1-\frac{1}{w_0}+\frac{k_2(\alpha)}{w_0\eta_1^2}=-\frac{\omega^2-\omega_{\rm p}^2+i\nu\omega}{(\nu-i\omega)^2}.$$

Hence, we obtain $\Lambda_\infty\neq 0$ for any $\nu\neq 0$, i.e., in any collision plasma. By (23) and expansion (24), the point $z_j=\infty$ is a root of the dispersion equation. This point belongs to the spectrum adjoint to the continuous spectrum $(-\infty,+\infty)$. The point $z_j=\infty$ corresponds to the solution of the initial system of equations (8), (10)

$$H_\infty(x,\mu)=\frac{E_\infty}{z_0}\mu,\qquad e_\infty(x)=E_\infty,\tag{25}$$

where E_∞ is an arbitrary constant.

Solution (25) is independent of the chemical potential. It is natural to call it a Drude mode. It describes the volume conductance of the metal plasma considered by Drude (see, e.g., [17]).

By definition, a discrete spectrum of the characteristic system of equations is a set of finite complex roots of the dispersion equation that do not belong to the real line (the section of the dispersion function).

By expansion (24), in a neighborhood of infinity, there are two zeros $\pm\eta_0$ of the dispersion function $\Lambda(z,\alpha)$:

$$\pm\,\eta_0\approx\sqrt{\frac{\eta_1^2k_2(\alpha)-k_4(\alpha)}{\eta_1^2(w_0-1)+k_2(\alpha)}}=\sqrt{\frac{\varepsilon(\varepsilon-i\Omega)s_2^2(\alpha)-s_0(\alpha)s_4(\alpha)}{s_0(\alpha)s_2(\alpha)(1-\Omega^2+i\varepsilon\Omega)}}.\tag{26}$$

By the symmetry of the dispersion function, its zeros differ only in sign. For clarity, we understand the zero η_0 to be a value of the radical in (26) such that $\mathrm{Re}(w_0/\eta_0)>0$. The zeros $\pm\eta_0$ that form the discrete spectrum of the characteristic system correspond to the solution of the initial equations

$$\begin{aligned}&H_{\pm\eta_0}(x_1,\mu)=e^{-w_0x_1/\eta_0}\Phi_1(\eta_0,\mu)+e^{w_0x_1/\eta_0}\Phi_2(\eta_0,\mu),\\&e_{\pm\eta_0}(x_1)=\left[e^{-w_0x_1/\eta_0}+e^{w_0x_1/\eta_0}\right]E_0,\end{aligned}\tag{27}$$

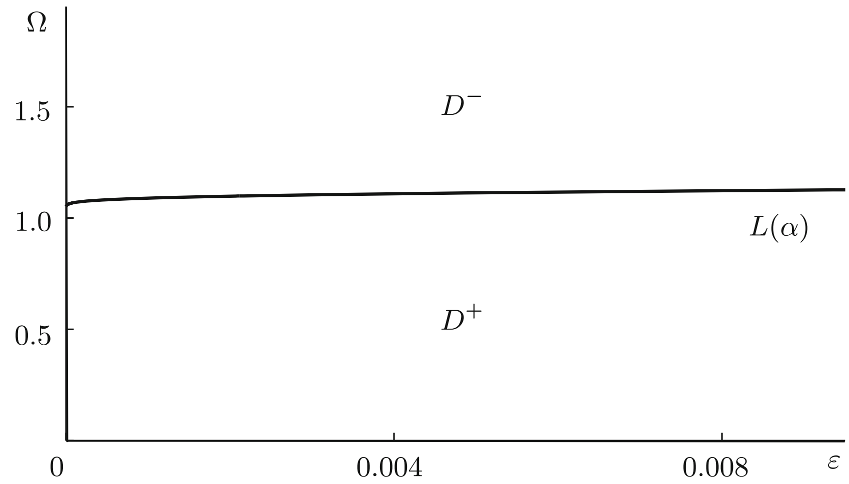

Fig. 3. The case $\alpha = 0$: the curve $L(\alpha)$ separates the regions of index one $D^+(\alpha)$ and index zero $D^-(\alpha)$. If $(\Omega, \varepsilon) \in D^+(\alpha)$, then $N = 2$, and if $(\Omega, \varepsilon) \in D^-(\alpha)$, then $N = 0$.

where E_0 is an arbitrary constant and

$$\Phi_1(\eta_0, \mu) = \frac{E_0}{w_0} \frac{\eta_0\mu - \eta_1^2}{\eta_0 - \mu}, \qquad \Phi_2(\eta_0, \mu) = \frac{E_0}{w_0} \frac{\eta_0\mu + \eta_1^2}{\eta_0 + \mu}.$$

Solution (27) is naturally called a Debye mode (it is a plasma mode). In the low-frequency case, it describes the famous Debye screening [17].

From (26), we can see that near the plasma resonance (for $\Omega \approx 1$), i.e., for $\omega \approx \omega_{\mathrm{p}}$, the absolute value of the zero η_0 is not bounded from above if $\varepsilon \ll 1$.

The set of physically meaningful parameters (Ω, ε) fills the quarter-plane $\{\Omega \geq 0,\ \varepsilon \geq 0\}$. The case $\Omega = 0$ (or $\omega = 0$) corresponds to a stationary external electric field, and the case $\varepsilon = 0$ (or $\nu = 0$) corresponds to the case of a collisionless plasma.

The question arises whether there are more complex zeros of the dispersion function except $\pm\eta_0$. As was done in [18], we can show that the number of zeros of the dispersion function is equal to twice the index of the function $G(\tau) = \Lambda^+(\tau)/\Lambda^-(\tau)$ on the real positive half-axis, $N = 2\varkappa(G)$, $\varkappa(\alpha) = \operatorname{ind}_{[0,+\infty]} G(\tau)$. As in [18], we can show that from the equations

$$\operatorname{Re} G(\mu, \Omega, \varepsilon, \alpha) = 0, \qquad \operatorname{Im} G(\mu, \Omega, \varepsilon, \alpha) = 0, \quad 0 \leq \mu \leq +\infty,$$

we can find a curve $L(\alpha)$ separating the regions $D^+(\alpha)$ and $D^-(\alpha)$ (see Fig. 3). If $(\Omega, \varepsilon) \in D^+(\alpha)$, then the number of zeros of the dispersion function is two (and these are the zeros $\pm\eta_0$), and if $(\Omega, \varepsilon) \in D^+(\alpha)$, then the dispersion function does not have zeros.

We note that a method for investigating the boundary regime was developed for $(\Omega, \varepsilon) \in L(\alpha)$ in [19].

The curve $L(\alpha)$ is defined by the parametric equations

$$L(\alpha)\colon \quad \Omega = \sqrt{L_1(\mu, \alpha)}, \qquad \varepsilon = \sqrt{L_2(\mu, \alpha)}, \quad 0 \leq \mu \leq +\infty,$$

where

$$L_1(\mu, \alpha) = \frac{s_0(\alpha)}{s_2(\alpha)} \frac{\mu^2[\lambda_0(\mu, \alpha)(1 + \lambda_0(\mu, \alpha)) + s^2(\mu, \alpha)]^2}{[-\lambda_0(\mu, \alpha)][(1 + \lambda_0(\mu, \alpha))^2 + s^2(\mu, \alpha)]},$$

$$L_2(\mu, \alpha) = \frac{s_0(\alpha)}{s_2(\alpha)} \frac{\mu^2 s^2(\mu, \alpha)}{[-\lambda_0(\mu, \alpha)][(1 + \lambda_0(\mu, \alpha))^2 + s^2(\mu, \alpha)]}.$$

5. Mirror reflection of electrons from the plasma boundary

We consider a boundary value problem consisting of Eqs. (5) and (6), condition (1), (2) for the mirror reflection of electrons from the plasma boundary, and conditions (3) and (4) on the electric field. We show that this problem has a solution that can be represented as an expansion in the eigenfunctions of the characteristic system:

$$H(x_1,\mu)=\frac{E_\infty}{w_0}\mu+\frac{E_0}{w_0}\left[\frac{\eta_0\mu-\eta_1^2}{\eta_0-\mu}e^{-w_0x_1/\eta_0}+\frac{\eta_0\mu+\eta_1^2}{\eta_0+\mu}e^{w_0x_1/\eta_0}\right]+ \\ +\int_{-\infty}^{\infty}\left[e^{-w_0x_1/\eta}F_1(\eta,\mu)-e^{w_0x_1/\eta}F_1(\eta,-\mu)\right]\frac{E(\eta)}{w_0}\,d\eta, \tag{28}$$

$$e(x_1)=E_\infty+E_0\left[e^{-w_0x_1/\eta_0}+e^{w_0x_1/\eta_0}\right]+ \\ +\int_{-\infty}^{\infty}\left[e^{-w_0x_1/\eta}+e^{w_0x_1/\eta}\right]E(\eta)\,d\eta, \tag{29}$$

where E_0 and E_∞ are unknown coefficients corresponding to the discrete spectrum (E_0 is the Debye amplitude and E_∞ is the Drude amplitude) and $E(\eta)$ is an unknown function called the coefficient of the continuous spectrum. For $(\Omega,\varepsilon)\in D^-(\alpha)$, we must take $E_0=0$ in expansions (28) and (29).

We transform expansion (28). For this, we use the antisymmetry of $F_1(\eta,\mu)$ with respect to the set of variables, $F_1(-\eta,-\mu)=-F_1(\eta,\mu)$, and the symmetry of $E(\eta)$. We then have

$$\int_{-\infty}^{\infty}e^{w_0x_1/\eta}F_1(\eta,-\mu)E(\eta)\,d\eta=\int_{-\infty}^{\infty}e^{-w_0x_1/\eta}F_1(-\eta,-\mu)E(-\eta)\,d\eta= \\ =-\int_{-\infty}^{\infty}e^{-w_0x_1/\eta}F_1(\eta,\mu)E(\eta)\,d\eta.$$

Using this relation, we can represent expansions (28) and (29) as

$$H(x_1,\mu)=\frac{E_\infty}{w_0}\mu+\frac{E_0}{w_0}\left[\frac{\eta_0\mu-\eta_1^2}{\eta_0-\mu}e^{-w_0x_1/\eta_0}+\frac{\eta_0\mu+\eta_1^2}{\eta_0+\mu}e^{w_0x_1/\eta_0}\right]+ \\ +\frac{2}{w_0}\int_{-\infty}^{\infty}e^{-w_0x_1/\eta}F_1(\eta,\mu)E(\eta)\,d\eta,$$

$$e(x_1)=E_\infty+2E_0\cosh\left(\frac{w_0x_1}{\eta_0}\right)+2\int_{-\infty}^{\infty}\cosh\left(\frac{w_0x_1}{\eta}\right)E(\eta)\,d\eta.$$

We start with the case $(\Omega,\varepsilon)\in D^+(\alpha)$. We calculate the value of the function $H(x_1,\pm\mu)$ on the upper boundary. We have

$$H(+l,\pm\mu)=\pm\frac{E_\infty}{w_0}\mu+\frac{E_0}{w_0}\left[\frac{\pm\eta_0\mu-\eta_1^2}{\eta_0\mp\mu}e^{-w_0l/\eta_0}+\frac{\pm\eta_0\mu+\eta_1^2}{\eta_0\pm\mu}e^{w_0l/\eta_0}\right]+ \\ +\frac{2}{w_0}\int_{-\infty}^{\infty}e^{-w_0l/\eta}F_1(\eta,\pm\mu)E(\eta)\,d\eta.$$

Substituting these equations in boundary condition (2), we obtain an integral Fredholm equation of the second kind:

$$\frac{E_\infty}{w_0}\mu+\frac{E_0\cosh(w_0l/\eta_0)}{w_0}\left[\frac{\mu\eta_0-\eta_1^2}{\eta_0-\mu}+\frac{\mu\eta_0+\eta_1^2}{\eta_0-\mu}\right]+ \\ +\frac{1}{w_0}\int_{-\infty}^{\infty}e^{-w_0l/\eta}(F_1(\eta,\mu)-F_1(\eta,-\mu))E(\eta)\,d\eta=0.$$

After simple transformations, we obtain the equation

$$\frac{E_\infty}{w_0}\mu+2\frac{E_0\cosh(w_0l/\eta_0)}{w_0}\mu\frac{\eta_0^2-\eta_1^2}{\eta_0^2-\mu^2}+$$
$$+\frac{2}{w_0}\int_{-\infty}^{\infty}F_1(\eta,\mu)E(\eta)\cosh\Big(\frac{w_0l}{\eta}\Big)\,d\eta=0,\quad -\infty<\mu<+\infty. \tag{30}$$

An explicit verification shows that the boundary conditions of the distribution function on the lower plate (boundary) of the layer lead to the same Eq. (30).

Substituting expansion (29) for the electric field in boundary condition (30), we obtain the equation

$$E_\infty+2E_0\cosh\Big(\frac{w_0l}{\eta_0}\Big)+2\int_{-\infty}^{\infty}E(\eta)\cosh\Big(\frac{w_0l}{\eta}\Big)\,d\eta=1. \tag{31}$$

We substitute the eigenfunctions $F_1(\eta,\mu)$ in (30) and obtain a singular integral equation with the Cauchy kernel [20] on the whole real axis $-\infty<\mu<+\infty$:

$$\frac{1}{2}E_\infty\mu+E_0\cosh\Big(\frac{w_0l}{\eta_0}\Big)\mu\frac{\eta_0^2-\eta_1^2}{\eta_0^2-\mu^2}+\int_{-\infty}^{\infty}\frac{\mu\eta-\eta_1^2}{\eta-\mu}E(\eta)\cosh\Big(\frac{w_0l}{\eta}\Big)\,d\eta-$$
$$-w_0\eta_1^2\frac{\Lambda(\mu)}{\mu k(\mu,\alpha)}E(\mu)\cosh\Big(\frac{w_0l}{\mu}\Big)=0. \tag{32}$$

We introduce an auxiliary function

$$M(z)=\int_{-\infty}^{\infty}\frac{z\eta-\eta_1^2}{\eta-z}E(\eta)\cosh\Big(\frac{w_0l}{\eta}\Big)\,d\eta. \tag{33}$$

The function $M(z)$ is analytic in the complex plane except the cut, the points of the whole real line $(-\infty,+\infty)$. The boundary values of the auxiliary function $M(z)$ from above and below on the real axis are related by the Sokhotsky formulas:

$$M^{\pm}(\mu)=\pm\pi i(\mu^2-\eta_1^2)E(\mu)\cosh\Big(\frac{w_0l}{\mu}\Big)+\int_{-\infty}^{\infty}\frac{\mu\eta-\eta_1^2}{\eta-\mu}E(\eta)\cosh\Big(\frac{w_0l}{\eta}\Big)\,d\eta,$$

where the singular integral

$$M(\mu)=\int_{-\infty}^{\infty}\frac{\mu\eta-\eta_1^2}{\eta-\mu}E(\eta)\cosh\Big(\frac{w_0l}{\eta}\Big)\,d\eta$$

is understood in the Cauchy principal value sense.

By the Sokhotsky formulas, we have the equalities

$$M^+(\mu)-M^-(\mu)=2\pi i(\mu^2-\eta_1^2)E(\mu)\cosh\Big(\frac{w_0l}{\mu}\Big),\quad \mu\in(-\infty,+\infty), \tag{34}$$

$$M(\mu)=\frac{M^+(\mu)+M^-(\mu)}{2},\quad \mu\in(-\infty,+\infty).$$

By the Sokhotsky formulas for auxiliary function (33) and dispersion function (21), we transform singular equation (32) into a boundary value problem:

$$\frac{1}{2}E_\infty\mu+E_0\cosh\Big(\frac{w_0l}{\eta_0}\Big)\mu\frac{\eta_0^2-\eta_1^2}{\eta_0^2-\mu^2}+\frac{1}{2}(M^+(\mu)+M^-(\mu))+$$
$$+\frac{1}{2}\frac{\Lambda^+(\mu)+\Lambda^-(\mu)}{\Lambda^+(\mu)-\Lambda^-(\mu)}(M^+(\mu)-M^-(\mu))=0,\quad -\infty<\mu<+\infty.$$

From this equation, we obtain the boundary value problem of reconstructing the analytic function from its zero jump on the cut:

$$\Lambda^+(\mu)\left[M^+(\mu)+\frac{1}{2}E_\infty\mu+E_0\cosh\left(\frac{w_0 l}{\eta_0}\right)\mu\frac{\eta_0^2-\eta_1^2}{\eta_0^2-\mu^2}\right]-$$
$$-\Lambda^-(\mu)\left[M^-(\mu)+\frac{1}{2}E_\infty\mu+E_0\cosh\left(\frac{w_0 l}{\eta_0}\right)\mu\frac{\eta_0^2-\eta_1^2}{\eta_0^2-\mu^2}\right]=0,\quad \mu\in(-\infty,+\infty). \tag{35}$$

Problem (35) has the solution

$$M(z)=-\frac{1}{2}E_\infty z-E_0\cosh\left(\frac{w_0 l}{\eta_0}\right)z\frac{\eta_0^2-\eta_1^2}{\eta_0^2-z^2}+\frac{C_1 z}{\Lambda(z)}, \tag{36}$$

where C_1 is an arbitrary constant.

Eliminating the pole at infinity, we obtain $C_1 = E_\infty\Lambda_\infty/2$. A Debye amplitude can be found by eliminating the poles in solution (36) at the points $\pm\eta_0$. By the symmetry of the dispersion function, these poles can be eliminated by a single solution,

$$E_0=\frac{E_\infty\Lambda_\infty\eta_0}{(\eta_1^2-\eta_0^2)\Lambda'(\eta_0)\cosh(w_0 l/\eta_0)}.$$

The coefficient of the continuous spectrum can be found by substituting solution (36) in Sokhotsky formula (34),

$$E(\mu)=\frac{E_\infty\Lambda_\infty}{2w_0\eta_1^2}\frac{\mu^2\,k(\mu,\alpha)}{\cosh(w_0 l/\mu)\Lambda^+(\mu)\Lambda^-(\mu)}.$$

To find E_∞, we rewrite Eq. (31) using the symmetry of $E(\eta)$:

$$E_\infty+2E_0\cosh\left(\frac{w_0 l}{\eta_0}\right)+2\int_{-\infty}^{\infty}E(\eta)\cosh\left(\frac{w_0 l}{\eta}\right)d\eta=1$$

or, explicitly,

$$\frac{1}{\Lambda_\infty}+\frac{2\eta_0}{(\eta_1^2-\eta_0^2)\Lambda'(\eta_0)}+\frac{1}{2\pi i}\int_{-\infty}^{\infty}\left(\frac{1}{\Lambda^+(\eta)}-\frac{1}{\Lambda^-(\eta)}\right)\frac{\eta\,d\eta}{\eta^2-\eta_1^2}=\frac{1}{\Lambda_\infty E_\infty}. \tag{37}$$

The integral in (37),

$$J=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\left(\frac{1}{\Lambda^+(\eta)}-\frac{1}{\Lambda^-(\eta)}\right)\frac{\eta\,d\eta}{\eta^2-\eta_1^2},$$

can be calculated analytically.

The function

$$\varphi(z)=\frac{z}{\Lambda(z)(z^2-\eta_1^2)},$$

for which $\varphi(z)=O(z^{-1})$, $z\to\infty$, is analytic in the upper and lower complex half-planes (outside the cut, the real axis) except the points $\pm\eta_1$ and $\pm\eta_0$. Hence, this integral is equal to

$$J=\left[\operatorname*{Res}_{\eta_0}+\operatorname*{Res}_{-\eta_0}+\operatorname*{Res}_{\eta_1}+\operatorname*{Res}_{-\eta_1}+\operatorname*{Res}_{\infty}\right]\varphi(z).$$

Noting that

$$\operatorname*{Res}_{\pm\eta_1} \varphi(z) = \frac{1}{2\Lambda_1}, \qquad \Lambda_1 = \Lambda(\eta_1) = 1 - \frac{1}{1 - i\Omega}, \qquad \operatorname*{Res}_{\pm\eta_0} \varphi(z) = \frac{\eta_0}{\Lambda'(\eta_0)(\eta_0^2 - \eta_1^2)},$$

we obtain

$$J = \frac{2\eta_0}{\Lambda'(\eta_0)(\eta_0^2 - \eta_1^2)} + \frac{1}{\Lambda_1} - \frac{1}{\Lambda_\infty}.$$

Substituting this equality in (37), we obtain

$$E_\infty = \frac{\Lambda_1}{\Lambda_\infty}, \qquad C_1 = \frac{1}{2}\Lambda_1. \tag{38}$$

Expansions (28) and (29) have thus been found. We found the coefficients of expansions (28) and (29). The coefficient E_∞ of the adjoint spectrum can be found by formula (38), the coefficient E_0 of the discrete spectrum, and the coefficient $E(\eta)$ of the continuous spectrum can be found from the formulas

$$\begin{aligned} E_0 &= \frac{\Lambda_1 \eta_0}{(\eta_1^2 - \eta_0^2)\Lambda'(\eta_0)\cosh(w_0 l/\eta_0)}, \\ E(\eta) &= \frac{\Lambda_1 \eta_0}{4\pi i(\eta^2 - \eta_1^2)\cosh(w_0 l/\eta)} \left(\frac{1}{\Lambda^+(\eta)} - \frac{1}{\Lambda^-(\eta)}\right). \end{aligned} \tag{39}$$

The structure of the electric field generally has the form

$$\begin{aligned} e(x_1) = \frac{\Lambda_1}{\Lambda_\infty} &+ \frac{2\Lambda_1 \eta_0}{\Lambda'(\eta_0)(\eta_1^2 - \eta_0^2)} \frac{\cosh(w_0 x_1/\eta_0)}{\cosh(w_0 l/\eta_0)} + \\ &+ \frac{\Lambda_1}{w_0 \eta_1^2} \int_{-\infty}^{\infty} \frac{\eta^2 k(\eta, \alpha)}{\Lambda^+(\eta)\Lambda^-(\eta)} \frac{\cosh(w_0 x_1/\eta)}{\cosh(w_0 l/\eta)}\, d\eta. \end{aligned} \tag{40}$$

According to (28), we can construct the distribution function from formulas (38) and (39):

$$\begin{aligned} H(x_1, \mu) = \frac{\Lambda_1}{\Lambda_\infty} \frac{\mu}{w_0} &+ \frac{\Lambda_1 \eta_0}{w_0(\eta_1^2 - \eta_0^2)\Lambda'(\eta_0)\cosh(w_0 l/\eta_0)} \times \\ &\times \left[e^{-w_0 x_1/\eta_0} \frac{\mu\eta_0 - \eta_1^2}{\eta_0 - \mu} + e^{w_0 x_1/\eta_0} \frac{\mu\eta_0 + \eta_1^2}{\eta_0 + \mu}\right] + \frac{\Lambda_1}{w_0^2 \eta_1^2} \int_{-\infty}^{\infty} \frac{e^{-w_0 x_1/\eta} F_1(\eta, \mu)\eta^2 k(\eta, \alpha)}{\Lambda^+(\eta)\Lambda^-(\eta)\cosh(w_0 l/\eta)}\, d\eta. \end{aligned} \tag{41}$$

We stress that formulas (40) and (41) hold for $(\Omega, \varepsilon) \in D^+(\alpha)$. In the case $(\Omega, \varepsilon) \in D^-(\alpha)$, the zero η_0 of the dispersion function does not exist. Hence, we can assume that $\eta_0 = 0$ in this case. The second summand in (40) and (41) then disappears, and these formulas simplify.

6. Conclusion

We have analytically solved the classic problem of fluctuations of the electron plasma in a conductive layer with an arbitrary degree of degeneracy of the electron gas in an external varying electric field. We found the electron distribution function and the screened electric field inside the plasma in explicit form. The method that we used here can also be used to solve the boundary value problems for Vlasov–Poisson systems of equations (see, e.g., [21]).

We again stress that to solve a complicated problem like the plasma fluctuation problem analytically, we must first reduce it to a one-dimensional problem with a single velocity. For this, we direct the strength vector of the external electric field along the axis orthogonal to the surface of the layer with the plasma. In the considered Landau–Vlasov problem, this is the x axis, and the field has the form

$$\mathbf{E}_{\text{ext}} = E_0 e^{-i\omega t}(1, 0, 0). \tag{42}$$

It generates the self-consistent field $E(x)e^{-i\omega t}(1, 0, 0)$ inside the plasma. The electron dispersion function has the form $f = f(x, v_x, t)$. If the external field has three nonzero coordinates, i.e., $\mathbf{E}_{\text{ext}} = e^{-i\omega t}(E_1, E_2, E_3)$, then the distribution function becomes a function of the general form

$$f = f(x, y, z, v_x, v_y, v_z, t).$$

Hence, because our aim is an analytic solution, we take the external field in form (42).

We note another special feature of the problem. Nonhomogeneous boundary value problem (35) in the theory of functions of a complex variable does not require factoring the coefficient (or solving the corresponding homogeneous problem). In fact, because the boundary conditions are symmetric, problem (35) turns out to be defined on the same cut to which the dispersion function of the problem belongs.

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