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Rotational Dynamics and Evolution of Planetary Satellites in the Solar and Exoplanetary Systems

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Abstract—In this review we consider the main rotation regimes that are inherent for planetary satellites of the Solar System, satellites of trans-Neptunian objects, and potential satellites of extrasolar planets. Both the findings of classical theoretical studies and the recent conclusions on the observed rotational dynamics of satellites and their long-term dynamical tidal evolution are described. We concentrate on a regime of a satellite's rotation that is synchronous with its orbital motion and is observed for all major planetary satellites (radius of the figure larger than ~500 km). We also consider irregularly shaped minor satellites (with a figure radius less than ~300 km), rotating regularly and much faster than in the case of synchronous rotation. The regime of chaotic rotation (tumbling) observed for the seventh satellite of Saturn, Hyperion, is analyzed at length. We also discuss the possibility of chaotic rotation of other minor satellites. Results and research prospects for the rotational dynamics of exomoons are presented.

Keywords: Solar System, exoplanetary systems, planetary satellites, celestial mechanics, rotational dynamics, resonances, dynamical chaos, tidal interaction

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INTRODUCTION

Each of the planets in the Solar System, except Mercury and Venus, has natural satellites. The total number of the currently known moons exceeds two hundred (see the JPL NASA site http://ssd.jpl.nasa.gov/). About 90% of the satellites are bodies of irregular shape with sizes (radii) ranging from one to three hundred kilometers; they are termed as minor satellites. For most minor satellites, the figure parameters, rotation states, and physical properties are unknown. The rotation parameters have been reliably determined only for ~25% of all known satellites, including all major satellites (larger than ~500 km in radius) (Archinal et al., 2018).

At present, the possibility of the existence of moons (Kipping et al., 2012, 2014; Heller, 2014, 2018; Heller et al., 2014; Sucerquia et al., 2019) and even submoons (see, Kollmeier and Raymond, 2019; Rosario-Franco et al., 2020) around exoplanets, i.e., planets beyond the Solar System, is under active investigation. Exomoons are actively searched for by analyzing the data of transit observations of exoplanets, particularly, within the "Hunt for Exomoons with Kepler" (HEK) project (Kipping et al., 2012, 2014). The studies of the rotational dynamics and evolution of known planetary satellites in the Solar System, as well as potentially existing exomoons, are in the focus of the present review. Here we also treat the papers dealing with the studies of the rotational dynamics of Pluto's moons (Showalter and Hamilton, 2015; Correia et al., 2015; Weaver et al., 2016) and the moons of the other large trans-Neptunian objects (Brown et al., 2006; Brown and Butler, 2018; Kiss et al., 2017; Sheppard et al., 2018; Parker et al., 2016).

In the review, we describe the findings of classical theoretical studies of the rotational dynamics observed for satellites and the long-term dynamical tidal evolution (Darwin, 1879, 1880; Kaula, 1964; MacDonald, 1964; Goldreich, 1966; Goldreich and Peale, 1966; Peale, 1977, 1999; Ferraz-Mello et al., 2008). We also consider recent results of the theory of tidal evolution (Efroimsky and Williams, 2009; Makarov and Efroimsky, 2013; Makarov, 2015). During the long-term evolution, a satellite passes through various spin-orbit resonance states and can be captured into one of them. The most probable final regime is the rotation in synchrony with the orbital motion; in particular, all major satellites of the planets reside in this regime. The typology of synchronous rotation of irregularly shaped satellites is one of the subjects of this review. An irregular shape should be also characteristic of submoons that potentially exist in the other planetary systems.

We also consider the dynamics of minor satellites rotating much faster than in the case of synchronous

Table 1. The number of known satellites of the planets of the Solar System

Planet	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Total
Number of satellites	1	2	79	82	27	14	205

rotation: to date, this type of rotation has been determined from observations of three dozen minor satellites. No doubt, this rotation type is most widespread among the irregular moons of planets. In addition to the regular rotation regime, a chaotic one may also occur. In a theoretical study, Wisdom et al. (1984) showed that a satellite of a strongly nonspherical shape being on an elliptical orbit may rotate chaotically, in an unpredictable manner. They found that, due to a pronouncedly asymmetric shape and a significant orbital eccentricity, the most probable candidate for chaotic rotation is the seventh moon of Saturn—Hyperion (S7). In this review, special attention is paid to the analysis of the results on the rotational dynamics of Hyperion. We consider the findings of the studies focused on simulations of the rotational dynamics and analysis of the light curves observed for Hyperion and other minor moons, starting from the papers by Klavetter (1989a, 1989b). These observational studies arrived at the conclusion that, at present, Hyperion rotates chaotically. As was determined by direct numerical modeling, the Lyapunov time (the predictable dynamics timescale) of Hyperion's rotation is around one month (Melnikov, 2002). According to the recent theoretical results, dynamical chaos is also probable for the rotational dynamics of Pluto's moons (Showalter and Hamilton, 2015; Correia et al., 2015).

In the review, substantial attention is paid to current numerical and experimental studies and theoretical models of the tidal evolution of the rotation states of satellites. Due to the tidal interaction between a planet and its satellite, the rotation state of a satellite evolves (for details on the tidal interactions, see, e.g., the book by Murray and Dermott, 2000); and, in the course of tidal evolution, the proper rotation of a satellite, which was inherent to it at the final formation stage or at the moment of being captured by a planet, becomes slower.

The review systematizes the results of research on the possibility of the existence of strange attractors in the phase space of the rotational motion of satellites that experience long-term tidal evolution. The probabilities of capturing satellites into various spin-orbit resonance states during the long-term tidal evolution are considered.

Various nonstandard, infrequently occurring, spin-orbital resonance states of satellites are discussed.

SATELLITES OF PLANETS: A GENERAL OVERVIEW

Satellites of planets form the next-largest population of minor bodies after asteroids, objects of the Kuiper belt, and cometary nuclei observed in the Solar System. In this section, we present the data on the statistics of known planetary satellites in the Solar System and their main physical and orbital parameters, mostly as contained in the database of the NASA JPL website (http://ssd.jpl.nasa.gov/). For a large portion of the satellites, the data on the physical parameters, which had been available to 2011, were described by Emelyanov and Uralskaya (2011) (see also http://lnfm1.sai.msu.ru/neb/rw/natsat/index.htm). The information on the nomenclature of planetary satellites and their physical and orbital parameters can be also found in the book by Emelyanov (2019).

The distribution of satellites by planetary affiliation is presented in Table 1. Note that dozens of satellites orbit the giant planets. For Pluto, which was regarded as a planet till recently, five moons are currently known (see, e.g., Showalter and Hamilton, 2015). The other large trans-Neptunian objects also possess satellites and even satellite systems (Brown et al., 2006; Brown and Butler, 2018; Kiss et al., 2017; Sheppard et al., 2018; Parker et al., 2016).

The size of a satellite is one of its key physical characteristics. The histogram (the differential distribution) of the mean radii R of satellites' figures shows (see Fig. 1) that $R < 300$ km for 90% of the known satellites of planets. In the following, we will call them minor satellites, while major satellites will be those with $R > 500$ km. To date, no satellites with R ranging from 300 to 500 km have been found around the planets of the Solar System. The data about the sizes and probable values of the inertial parameters of known satellites were statistically analyzed at length by Kouprianov and Shevchenko (2006).

As is known, the orbital motion of a satellite around a planet may occur only within the Hill sphere, the radius of which is defined as $r_H = a_p\,(m_p/3M_S)^{1/3}$, where a_p is the semimajor axis of the planetary orbit, m_p is the mass of the planet, and M_S is the mass of the host star (the Sun).

The planetary satellites are divided into two large groups: regular moons and irregular ones (for details, see Sheppard and Jewitt, 2003; Sheppard, 2006; Jewitt and Haghighipour, 2007; Nicholson et al., 2008). Regular satellites are deep inside the Hill sphere (the semimajor axis of a satellite's orbit is $a \le 0.05 r_H$), their

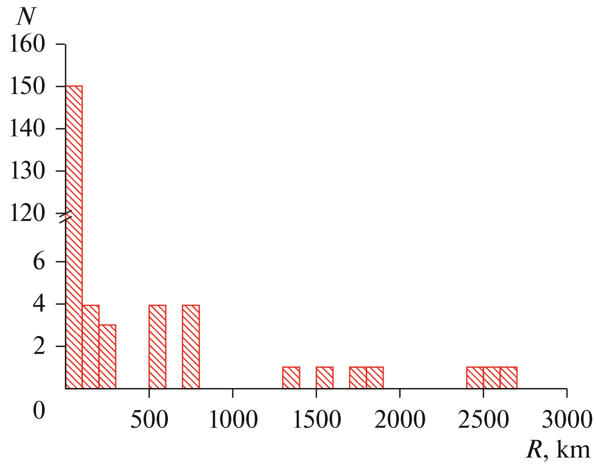

Fig. 1. The differential distribution (histogram) for the mean radii of figures of the known planetary satellites in the Solar System.

orbits are direct (prograde), and the orbital eccentricities and inclinations are low, $e \approx 0$ and $i \approx 0$. The classification of planetary satellites into prograde and retrograde ones is usually made on the basis of the value of the orbital inclination of a satellite relative to the equatorial plane of the planet; and it is specified that, if the orbital inclination is zero, the directions of the rotation of a planet and the orbital motion of a satellite coincide, whereas they are opposite if the inclination is 180° (see, e.g., Sheppard, 2006; Jewitt and Haghighipour, 2007). For prograde and retrograde orbits, $i \in [0°, 90°)$ and $i \in (90°, 180°]$, respectively.

The orbits of irregular satellites are mainly at larger distances from the planet ($0.05r_H < a \leq 0.65r_H$) and can be both prograde and retrograde. The values of e and i for these satellites are usually high: according to Sheppard (2006) (see Figs. 1, 2 in his study), for the majority of the known irregular satellites, $e \in [0.1, 0.6]$ and $i \in [25°, 60°]$ or $i \in [130°, 180°]$. This result is confirmed by the histograms of the e and i values (Fig. 2) constructed for all hitherto known planetary satellites. The irregular satellites are distributed by affiliation to the planets in the following way: Jupiter, Saturn, Uranus, and Neptune have 71, 38, 9, and 6 moons of this kind, respectively (Denk and Mottola, 2019). Thus, 124 irregular satellites are known, and, therefore, about 60% of all known planetary satellites are irregular.

ROTATION STATES OF PLANETARY SATELLITES

One of the most important characteristics of a satellite is its rotation state. To date, the rotation states have been determined for fourscore planetary satellites in the Solar System (Archinal et al., 2018; Denk and Mottola, 2019; NASA JPL; Emelyanov, 2019). The rotation parameters were determined for all major satellites (Archinal et al., 2018; Emelyanov, 2019). In the mentioned studies, approximate expressions for the rotation elements of satellites are provided in the International Celestial Reference Frame (ICRF) (see Ma et al., 1998). For more than 70% of minor satellites, the rotation states are not known; moreover, only the sizes of satellites, which were estimated from their observed visual magnitudes under specific assumptions, are available; see the discussion in a paper by Emelyanov and Uralskaya (2011) and the book by Emelyanov (2019).

The information on the rotational dynamics and physical properties of satellites is obtained from the analysis and theoretical modeling of the observed light curves as well as from the analysis of high-resolution images of satellites acquired by spacecraft in interplanetary missions. The latter technique yields more accurate values for the figure parameters of a satellite and

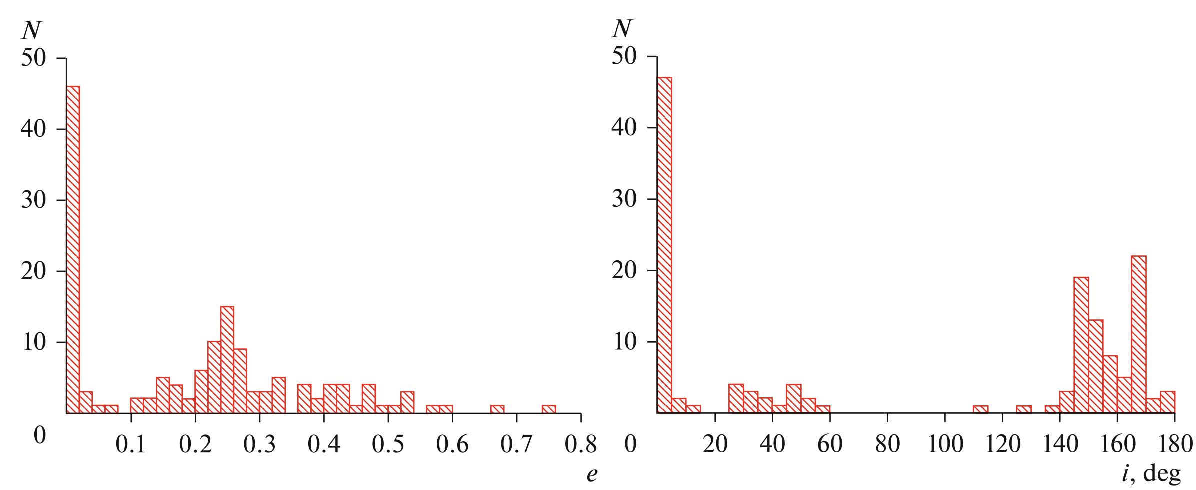

Fig. 2. The differential distributions (histograms) for the eccentricities and inclinations of orbits of the known planetary satellites in the Solar System.

reflectance characteristics of its surface and makes it possible to determine its spatial orientation and, sometimes, the angular velocity of rotation. However, this technique can be used only for a limited number of satellites, since the space missions are rarely accomplished. Among recent missions, the *Voyager 1* and *2*, *Galileo*, *Cassini–Huygens*, and *New Horizons* missions are worth mentioning as the most successful ones. Various advanced methods to study the orbital dynamics of planetary satellites, discover them, and determine their physical properties are discussed in papers by Emelyanov (2018, 2019).

By constructing theoretical light curves and comparing them to the observed ones, one may study the rotational dynamics of planetary satellites and their physical properties. An advantage of this approach is that the analyzed base of the initial observational data is large and extended in time: the number of points in an observational set may be very large and extend over decades. At the same time, if it is required, the temporal resolution of an observational set may be rather high.

The information obtained by modeling the light curves of planetary satellites makes it possible to plan space missions to them in detail; for example, the periodicity required for taking satellite images from the spacecraft and to select surface regions, images of which should be taken with a high spatial resolution, can be determined beforehand. If the dynamical parameters of a satellite were preliminarily determined by modeling its light curve, the spacecraft trajectory at the phase of approaching and orbiting the satellite can be calculated more accurately.

Using the ray tracing method (see, e.g., http://www.povray.org) and some simplifying assumptions (about the shape of an object, the reflectance of its surface, etc.) the model light curve can be built; and, by fitting the observed light curve with the model, the parameters of the object can be retrieved. With this method, Lacerda and Jewitt (2007) obtained the data (including the density estimates) for several Kuiper-belt contact-binary objects and one binary asteroid. The methods for modeling the light curves and retrieving the rotation parameters of asteroids from comparison of their theoretical and observed light curves are discussed by Masiero et al. (2009). We note that these modeling methods may also be successfully used to obtain the data about the rotational dynamics and physical parameters of planetary satellites.

Among all of the theoretically possible and observed rotational regimes of planetary satellites, three main ones may be distinguished: (i) rotation in synchrony with the orbital motion (the 1 : 1 spin-orbit resonance), (ii) regular rotation that is faster than synchronous, and (iii) chaotic rotation ("tumbling"). The satellites rotating rapidly or chaotically still compose a small fraction among the satellites with known rotation state. However, it is evident that the predominance of synchronously rotating satellites is caused by a selection effect, since the synchronous regime is typical for major planetary satellites, for which the rotation regime can be determined from observations in the first place. In Table 2, we list the satellites for which the rotation regime has been determined to date. Hereafter, in brackets after the satellite's name, the letter and the number refer to the first letter of the name of the planet the satellite belongs to and its sequence number, respectively; for example, M1 is for Phobos, the first moon of Mars.

We note that for all of the satellites presented in Table 2 as synchronously rotating, the rotation ephemerides are available (Archinal et al., 2018). For some of the minor satellites listed in Table 2, the presumed regime of synchronous rotation should be verified additionally (Archinal et al., 2018). For example, a relatively low resolution of the images acquired from the *Galileo* spacecraft did not allow the rotation regime of Adrastea (J15) to be determined; however, Thomas et al. (1998) suppose that Adrastea was captured into spin-orbit synchronous resonance, because, according to the estimates based on the theory by Peale (1977, 1999), a satellite, which initially rotates rapidly, is slowed down by tidal effects to the chaotic rotation regime in only a few thousand years.

In the next sections, we will consider all three regimes of rotation presented in Table 2 and will focus on the dynamics of those satellites, for which several modes of synchronous rotation may exist and the fast rotation regime has been determined. We will consider at length the studies of the chaotic dynamics of Hyperion (S7). However, first, we will summarize the main conclusions of the present-day theory for the long-term tidal evolution in the rotation of planetary satellites, since these data are needed to understand current developments in the statistics of the rotation states of satellites in the Solar System.

TIDAL ROTATIONAL EVOLUTION OF A SATELLITE

The present-day theory of the tidal evolution of planets and satellites stems from the studies by Darwin (1879, 1880), who formulated and argued for the hypothesis that the observed orientation of the Moon, which always presents the same face to the Earth, is caused by the dissipation of energy during the long-term evolution of the translational–rotational motion of a viscoelastic solid. The advanced theory of the spin-orbit tidal interaction between a satellite and a planet was developed in detail by Kaula (1964) and MacDonald (1964).

Let us consider briefly the findings of the theory of tidal interaction between a satellite and a planet. Due to the tidal interaction, the satellite's body is deformed. So-called tidal bulges are formed. The symmetry axis of the tidal bulges deviates from the planet-to-satellite direction by an angle that mainly depends

Table 2. Rotation regimes of planetary satellites in the Solar System as follows from the data of Archinal et al. (2018), Denk and Mottola (2019), and NASA JPL

Rotation	Synchronous	Rapid (the rotation velocity is greater than the synchronous one)	Chaotic
Satellite's name (planet, number)	Phobos (M1), Deimos (M2)	Himalia (J6)	Hyperion (S7)
	Moon (E1)		
	Io (J1), Europa (J2), Ganymede (J3), Callisto (J4), Amalthea (J5), Thebe (J14), Adrastea (J15), Metis (J16)	Phoebe (S9), Ymir (S19), Paaliaq (S20), Tarvos (S21), Ijiraq (S22), Suttungr (S23), Kiviuq (S24), Mundilfari (S25), Albiorix (S26), Skathi (S27), Erriapus (S28), Siarnaq (S29), Thrymr (S30), Narvi (S31), Bebhionn (S37), Bergelmir (S38), Bestla (S39), Fornjot (S42), Hati (S43), Hyrrokkin (S44), Kari (S45), Loge (S46), Skoll (S47), Greip (S51), Tarqeq (S52)	
	Mimas (S1), Enceladus (S2), Tethys (S3), Dione (S4), Rhea (S5), Titan (S6), Iapetus (S8), Janus (S10), Epimetheus (S11), Helene (S12), Telesto (S13), Calypso (S14), Atlas (S15), Prometheus (S16), Pandora (S17), Pan (S18)		
	Ariel (U1), Umbriel (U2), Titania (U3), Oberon (U4), Miranda (U5), Cordelia (U6), Ophelia (U7), Bianca (U8), Cressida (U9), Desdemona (U10), Juliet (U11), Portia (U12), Rosalind (U13), Belinda (U14), Puck (U15)		
	Triton (N1), Naiad (N3), Thalassa (N4), Despina (N5), Galatea (N6), Larissa (N7), Proteus (N8)	Caliban (U16), Sycorax (U17), Prospero (U18), Setebos (U19), Ferdinand (U24)	
		Nereid (N2)	
Total	49	32	1

on the difference between the angular velocity of the satellite's rotation with respect to its mass center and the angular velocity of the satellite's orbital motion. The tidal interaction between a satellite and a planet results in changing the angular velocity of rotation of the planet; for example, the rotation of the Earth is slowed down by tidal interaction between the Moon and the Earth. Due to gravitational attraction of the satellite's tidal bulges by the planet, the rotation velocity of the satellite either decreases, if it is larger than the orbital one, or increases, if it is smaller than the orbital one; whereas the angle between the proper rotation axis of the satellite and the normal to its orbital plane decreases. As a result, if the orbit of a satellite is fixed, the final stage of its tidal rotational evolution is the rotation around the axis perpendicular to the orbital plane, the rotation, which is synchronous with its orbital motion. If the nodal precession is accounted for, then, typically, at the final stage of the tidal evolution, the rotation axis of a satellite will be in one of the so-called Cassini states characterized by a small value of the oblique angle, i.e., the angle between the orbital plane normal and the rotation axis (Colombo, 1966; Peale, 1969, 1977, 1999). The capture of satellites into synchronous rotation and Cassini states is discussed at length by Gladman et al. (1996).

The non-synchronous rotation of Mercury, which was revealed from observations in 1965, directed the attention of researchers to development of an elaborated theory of the spin-orbit tidal evolution of celestial bodies that would make it possible to explain the observed rotation regime of Mercury. Later, the theory of the tidal rotational evolution of celestial bodies was also successfully used to describe the tidal rotational evolution of planetary satellites. According to theoretical studies performed both in the framework of the classical theory (Kaula, 1964; MacDonald, 1964; Peale and Gold, 1965; Goldreich, 1966; Goldreich and Peale, 1966; Peale, 1977, 1999; Ferraz-Mello et al., 2008) and with its state-of-art modifications (Efroimsky and Williams, 2009; Makarov and Efroimsky, 2013; Makarov, 2015), during the long-term tidal evolution, a satellite passes through various spin-orbit resonance states until it is captured into one of them. The ability of a satellite for residing in one of the resonance states is determined by the stability of the latter with respect to tilting the rotation axis. It is supposed that, at the final stage of the rotational evolution, the rotation axis of the satellite is orthogonal to the orbital plane. The stability relative to the rotation axis tilt means that small deviations of the rotation axis from the normal do not lead to substantial changes in the orientation of the satellite's figure and its rotation velocity. Because of this, to understand the character of the long-term dynamical evolution of planetary satellites, it is of key importance to analyze the stability of motion of satellites being in various spin-orbit resonance states and, above all, in synchronous resonance. The theory of spin-orbit evolution of strongly asymmetric and binary objects, including minor satellites of a strongly nonspherical shape, contact-binary minor bodies, and binary trans-Neptunian objects (TNOs), was developed in papers by Batygin and Morbidelli (2015) and Seligman and Batygin (2021).

It is also important to estimate the time required for the tidal slowing of the rotation to the synchronous state. From these estimates, one may ascertain whether the rotation of a satellite may become synchronous for the time passed from the moment of its formation. The satellites known as being captured into synchronous resonance, have the physical parameters that allow them to reach this state in a relatively short time (less or much less than the age of the Solar System (Peale, 1977, 1999)). According to the conclusions made by Peale (1977) from estimates of the duration of slowing down the rotation due to the tidal effects, most irregular satellites (recall that most minor moons are of this type) are still in the rotation states close to the initial ones.

In a paper by Aleshkina (2009), the spin-orbit tidal evolution of a number of major satellites ($R > 500$ km) with known parameters was considered. The numerical simulations showed that, during the tidal rotational evolution, major satellites rapidly pass through various spin-orbit resonances and are then captured into synchronous resonance. For all of the considered satellites, the time required for the tidal slowing of the initially rapid rotation to the synchronous state was estimated by theoretical and numerical methods; it was found that, for all satellites except Iapetus (S8), the duration of the tidal slowing is significantly shorter than the age of the Solar System. Simulations made by Castillo-Rogez et al. (2007, 2011) in the framework of an advanced model of tidal interaction for the long-term rotational evolution of Iapetus allowed its currently observed rotational dynamics to be theoretically grounded (also see Efroimsky and Williams (2009)).

Irregular satellites are usually rather small (less than ~10 km) and, therefore, nonspherical in shape (see the discussion in a paper by Kouprianov and Shevchenko (2006)); in addition, the eccentricities of their orbits are large ($e > 0.1$). Because of this, according to the theoretical results by Wisdom et al. (1984) and Wisdom (1987), in the phase space of the rotational motion, spin-orbit resonances may overlap each other. In accordance with the Chirikov overlap criterion for nonlinear resonances, dynamical chaos appears when the distance between the centers of neighboring resonances in the impulse variable is smaller than the sum of their halfwidths (Chirikov, 1979; Lichtenberg and Lieberman, 1982; Murray and Dermott, 2000; Morbidelli, 2002). Thus, under resonance overlapping, a zone of dynamical chaos appears in the phase space; and a satellite that reached this zone during its rotational evolution may turn out to be in the chaotic rotation regime. The chaotic rotational dynamics properties will be discussed in one of the following sections.

SYNCHRONOUS ROTATION

As already mentioned above, the most probable final rotation regime of a satellite is that in synchrony with the orbital motion. All major satellites of the planets, as well as a fraction of minor satellites, are in the synchronous rotation regime (see Table 2). For the satellites that have already finished the tidal rotational evolution, this observational fact is expected, since the 1 : 1 synchronous resonance with the orbital motion is the most probable final regime of the long-term tidal rotational evolution. Under synchronous resonance, the angular velocity of a satellite's rotation with respect to its mass center coincides with the angular velocity of its orbital motion; and, on average, the satellite's figure always faces the planet by the same side, whereas the rotation axis is perpendicular to the orbital plane.

During the translational–rotational motion of a satellite, the orientation of its figure relative to the direction toward the planet experiences oscillations. Beletskii (1959, 1965) derived an equation for the planar oscillations of the orientation of a satellite in an elliptical orbit. For the case of the planar rotation of a satellite (i.e., rotation in the orbital plane), its dynamics are determined by the eccentricity e and the parameter $\omega_0 = \sqrt{3(B-A)/C}$ that characterizes the asymmetry of the satellite's figure, where $A < B < C$ are the main central moments of inertia of the satellite. Theoretical studies of the periodic solutions of the Beletskii equation (Torzhevskii, 1964; Zlatoustov et al., 1964; Sarychev et al., 1977; Petrov et al., 1983; Bruno, 2002) showed in the following that, under the same values of the parameters, the equation may have several stable solutions (with various initial conditions) corresponding to synchronous rotation of a satellite, i.e., there exist several synchronous resonance modes. Among them, two main ones can be distinguished: if $e = 0$, the first mode exists for all possible values of the parameter $0 \le \omega_0 \le \sqrt{3}$ whereas the second one takes place only for satellites with a significantly asymmetrical figure ($\omega_0 \ge 1$). Melnikov (2001) thoroughly considered a possible mode of a satellite's rotation in one more synchronous resonance mode—the bifurcation mode—that exists in the parametric resonance zone ($\omega_0 \approx 1/2$). When a satellite rotates in the

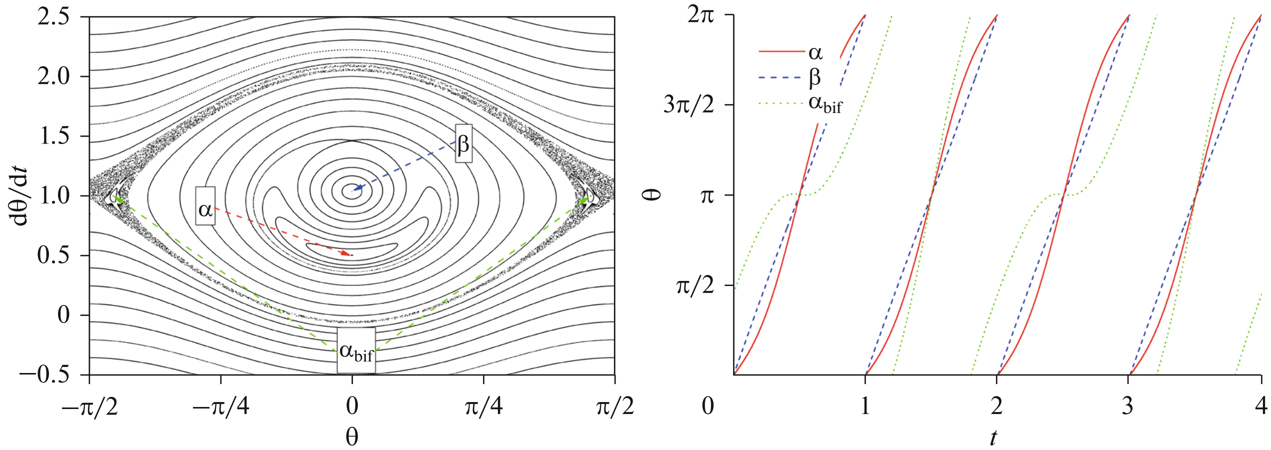

Fig. 3. Left: phase-space section of the planar rotational motion of a satellite defined at the orbital pericenter (therefore, the shown values of variables correspond to the pericenter passage time moments) for $e = 0.002$ and $\omega_0 = 1.058$ (Prometheus). The centers of the synchronous α- and β-resonances and the α_{bif} mode are indicated. Right: time dependence of the orientation of a satellite during its rotation in synchronous α-resonance (red curve), synchronous β-resonance (blue curve), and α_{bif} mode (green curve). The quantity θ is the angle between the line of apsides and the largest axis of a satellite's figure. The time t is expressed in orbital periods. (Adapted from a paper by Melnikov and Shevchenko (2007).)

bifurcation mode, a curve describing the variation of the satellite's orientation with time demonstrates a long oscillation with a period equal to two orbital periods of the satellite. This is seen in Fig. 3, where we present an example of the phase-space section with an indication of the centers of three modes (α, β, and α_{bif}) of synchronous resonance and the time dependence of the orientation of the satellite's figure during its rotation in the modes.

The Poincaré section method (the phase-space section method) is a widely known tool to study the properties of dynamical systems. In short, its essence is in choosing a surface in the phase space of a system and fixing the coordinates of the phase trajectory at the time moments it crosses this surface in the same direction. The algorithm for constructing the Poincaré sections is described, for example, in the book by Lichtenberg and Lieberman (1982).

The orientation of a satellite is defined by the angle θ, which is the angle between the line of apsides and the largest axis of the satellite's figure. The section presented in Fig. 3 was constructed in the following way: when the equations of the rotational motion of a satellite were numerically integrated, the values of the angle θ and the its rate of change with time, $d\theta/dt$, were fixed at the moments of passing the orbital pericenter. Then, in the plane (θ, $d\theta/dt$), the points with corresponding coordinates were marked.

Various examples of the phase-space sections built for various satellites can be found in the studies by Wisdom et al. (1984), Wisdom (1987), Klavetter (1989b), Dobrovolskis (1995), Black et al. (1995), Celletti et al. (2007), Melnikov and Shevchenko (2008), and Murray and Dermott (2000).

Further, we will consider three main synchronous resonance modes (including the bifurcation mode) in accordance with the papers by Melnikov and Shevchenko (2000, 2007) and Melnikov (2001). Wisdom et al. (1984) were the apparently first, who noted the existence of several synchronous resonance modes for planetary satellites. In papers by Melnikov and Shevchenko (2000, 2007), Melnikov (2001), and Kouprianov and Shevchenko (2006), the possibility that several regimes of the planar synchronous rotation exist in dynamics of known planetary satellites was addressed in detail. Real planetary moons, for which several synchronous resonance modes may exist, were revealed. For some of these moons, the stability of the planar synchronous rotation with respect to the tilt of the rotation axis was analyzed (Melnikov and Shevchenko, 1998, 2000, 2007, 2008; Kouprianov and Shevchenko, 2005; Pashkevich et al., 2021).

The synchronous rotation mode can be found from the analysis of observational data. For example, for Amalthea (J5), there are two modes (Melnikov and Shevchenko, 2000, 2007; Melnikov, 2001; Pashkevich et al., 2021). From the studies of the rotation stability of Amalthea, it was found that its rotation is unstable with respect to the tilt of the rotation axis in one of two modes of the planar synchronous rotation whereas the rotation in the second mode is stable (Melnikov and Shevchenko, 2000, 2007) (see Fig. 4). Amalthea cannot be captured into the nonstable synchronous resonance mode during its rotational evolution, since any small deviation of its rotation axis from the normal would result in leaving the planar synchronous rotation regime. A small libration amplitude (<5°) observed in the orientation of the largest axis of Amalthea's figure relative to the direction toward Jupiter, in

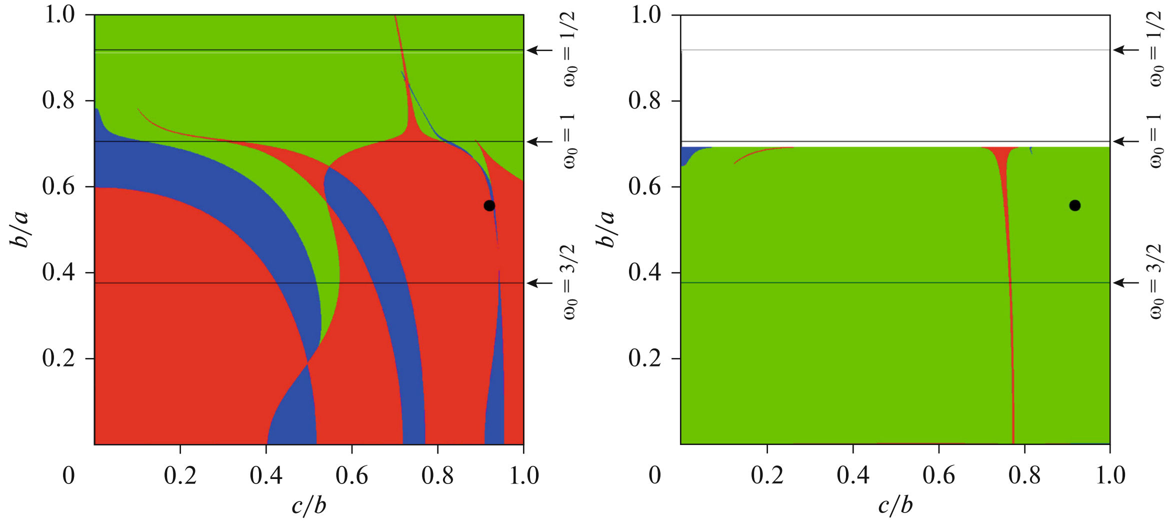

Fig. 4. The zones of stability (filled green) and instability (filled blue and red) with respect to tilting the rotation axis of a satellite for $e = 0.003$ (Amalthea): for the centers of the synchronous α- and β-resonances, the zones are shown in the left and right panels, respectively. The dot marks the position of Amalthea (J5). The horizontal dashed lines correspond to the values of ω_0 presented in the diagrams; $a > b > c$ are the axes of a three-axial ellipsoid approximating the satellite's figure. It is assumed that the density distribution inside the body is homogeneous (the density is constant and independent of the coordinates). The rotation of Amalthea is unstable in synchronous α- resonance and stable in synchronous β-resonance. (Adapted from a paper by Melnikov and Shevchenko (2007).)

the course of the satellite's orbital motion (Thomas et al., 1998), corresponds to the stable mode. If Amalthea were in the second synchronous mode, the libration amplitude could reach 30° (Pashkevich et al., 2021).

Thus, according to the theoretical results by Melnikov and Shevchenko (2000, 2007) and Melnikov (2001), the resonance rotational dynamics of minor moons of essentially nonspherical shape can be diverse: depending on the parameters of a satellite and the initial conditions, the satellite is captured into one of three regimes (modes) of synchronous rotation, if the rotation in this mode is stable; the modes significantly differ in terms of dynamics.

Comstock and Bills (2003) developed an analytical theory to estimate the magnitude of forced librations for various bodies of the Solar System. The amplitudes of forced librations for a number of satellites were numerically estimated. A theory of forced librations and a method to estimate their amplitudes for synchronous rotation of planets and satellites (including bodies of strongly asymmetric shape) were developed by Makarov et al. (2016). Noyelles (2008) and Noyelles et al. (2008) built an analytical model for the spatial synchronous rotation of a satellite and, on this basis, considered in detail physical librations in the synchronous rotation of four Galilean moons of Jupiter, Titan (S6), and Rhea (S5). For all of these satellites, the analytical and numerical estimates of the periods of harmonics (fundamental frequencies) in the spectrum of librations were obtained.

The amplitude of free librations of satellites is usually small, while the amplitude of forced librations can be rather substantial so that it can be determined from observations. The comparison of amplitudes of the observed librations with their theoretical magnitudes allows one to estimate the inertia characteristics of a satellite, i.e., to determine its moments of inertia and density. An analysis of this kind was performed for Janus (S10) and Epimetheus (S11) by Tiscareno et al. (2009), who calculated the theoretical libration amplitudes for these moons and determined the libration amplitudes from the analysis of the moons' images acquired by the *Cassini* spacecraft. For Epimetheus, the amplitude of the observed forced librations turned out to be substantial (~6°), which allowed the inertia moments of the satellite to be estimated with a higher accuracy.

Based on the results of Tiscareno et al. (2009), Noyelles (2010) developed a theory of the spatial rotation of Janus (S10) and Epimetheus (S11) and, on this basis, determined values for the libration periods of the satellites; moreover, the observed amplitude of librations of Janus and substantial uncertainties in the estimates of the libration amplitude of Epimetheus were theoretically explained.

Rambaux et al. (2012) considered at length the longitude librations in the rotational motion of Phobos (M1) and showed that, from the observed spectrum of librations, one may draw conclusions about the internal structure of this moon.

Noyelles (2009) considered the rotational dynamics of Callisto (J4) by numerical and analytical methods and developed a theory of its rotation in the ICRF coordinate system. Based on the comparison of the observational data of the *Galileo* and *Cassini* spacecraft and estimates inferred from the theory of rotation of planetary satellites, it was shown that the Cassini laws are obeyed for Callisto (Colombo, 1966; Peale, 1969, 1977, 1999; Gladman et al., 1996).

Noyelles et al. (2011) theoretically studied the rotational dynamics of Mimas (S1) and analyzed the interrelation between the satellite's internal structure, defined by various models, and the amplitude of the observed librations. In the paper by Tajeddine et al. (2014), based on examination of images obtained by the *Cassini–Huygens* spacecraft, physical librations in the rotation of Mimas were analyzed. It was found that most amplitudes of the spectrum of the observed librations agree well with their theoretical values. One of the amplitudes turned out to be twice as large as its theoretical value predicted by the hydrostatic equilibrium model; Tajeddine et al. (2014) supposed that the satellite is either not in the hydrostatic equilibrium state or, if Mimas is in hydrostatic equilibrium, there is a liquid ocean underneath a thick ice shell. Noyelles (2012, 2013, 2017, 2018) considered in detail theoretical models of rotation for satellites with complex internal structure.

Librations of a satellite influence its orbital elements; this results, particularly, in precession of the orbital pericenter (Borderies and Yoder, 1990). Consequently, from the analysis of astrometric observations of satellites' positions, one can estimate the magnitude of a satellite's librations and get information on its inertia parameters. Note that astrometric observations of a satellite can be rather numerous (tens of thousands), while high-resolution images taken by cameras onboard spacecraft, which are suitable for revealing librations, are available only for a small number of objects. This method was developed by Lainey et al. (2019), who estimated the libration amplitudes of some minor moons of Saturn based on the *Cassini* astrometric data and drew conclusions on their physical characteristics from comparison of these data to theoretical estimates.

We note that oscillations of a satellite's figure relative to the direction toward the planet (described, in particular, by the Beletskii equation) may induce the development of sets of parallel grooves that are observed on the surfaces of some satellites (see, Veverka and Duxbury, 1977; Morrison et al., 2009; Thomas and Helfenstein, 2020).

Large icy satellites, residing in synchronous rotation, may exhibit regular changes in the rotation velocity, because their rotation is not solid-body (Van Hoolst et al., 2013; Coyette et al., 2018). For example, for Titan (S6), the varying duration of its day (its period of synchronous rotation) can be explained by the presence of a liquid underneath the satellite's surface (Lorenz et al., 2008). As Makarov (2015) showed, if a satellite is modeled by a semiliquid body, it can be captured into a stable pseudosynchronous rotation, the angular velocity of which is higher than the orbital velocity.

RAPID REGULAR ROTATION

One more type of rotation state, which is characteristic of minor satellites and is known from observations, is a state of rapid regular rotation (which is more rapid than the synchronous one). More than 30 minor satellites rotating in this way are currently known (see Table 2), and all of them are irregular. These satellites have not completed their tidal rotational evolution yet.

Satellites of Jupiter

From the analysis of the long-term ground-based photometric observations of selected moons of Jupiter, Degewij et al. (1980) found that the rotation period of Himalia (J6) is $P_{\rm rot} \sim 9.5$ h (whereas its orbital period $P_{\rm orb} \approx 250$ d). Later, from the analysis of the ground-based observations of Himalia, Pilcher et al. (2012) determined with high accuracy that $P_{\rm rot} = 7.7891 \pm 0.0005$ h. Thus, the rotation of Himalia is significantly more rapid that the synchronous one: $P_{\rm orb}/P_{\rm rot} \approx 630$.

In a paper by Emelyanov (2005), the mass of Himalia was estimated by assessing the gravitational influence of Himalia on other minor moons of Jupiter during close encounters. Usually, the masses of irregular satellites are estimated from their observed brightness by assuming the values of their surface albedo and density to be known (see the discussion in papers by Emelyanov et al. (2007), Emel'yanov and Uralskaya (2011), and Emelyanov (2019)).

From the results of an analysis of ground-based photometric observations, Luu (1991) estimated the rotation periods for satellites J9–J12 as being in the range 8–12 h. Thus, these satellites should most likely be in a state of rapid rotation (note that they are not included in Table 2).

Satellites of Saturn

By analyzing photometric observations of Phoebe (S9), Andersson (1972) found that the period of its proper rotation is within an interval of 9 to 13 h. Thus, the angular velocity of the rotation of Phoebe is substantially (by three orders of magnitude) greater than its orbital velocity.

The orbital period of Phoebe is $P_{\rm orb} \approx 550$ d, the semimajor axis of the orbit is about $220R_{\rm Saturn}$, where $R_{\rm Saturn} \approx 57600$ km is the mean radius of Saturn, and the orbital inclination is $i \approx 175°$. Thus, Phoebe is an irregular satellite that moves around Saturn along a rather distant orbit and rotates rapidly and non-synchronously ($P_{\rm orb}/P_{\rm rot} \approx 1400$).

Degewij et al. (1980) reported the rotation period of Phoebe as 11.25 or 21.1 h. From the analysis of the

Voyager 2 images of Phoebe, Thomas et al. (1983) specified the period of its proper rotation with a higher precision: $P_{rot} = 9.4 \pm 0.2$ h. Ground-based observations of Phoebe (Kruse et al., 1986) showed that the light curve of the satellite is close to a sinusoidal curve with a period of 9.282 ± 0.015 h. Later, from the analysis of ground-based observations of Phoebe, Bauer et al. (2004) determined the period of its proper rotation with a high accuracy: $P_{rot} = 9.2735 \pm 0.0006$ h.

A thorough modeling of the rotational dynamics of Phoebe made it possible to build an analytical model of its rotation and determine possible values of the precession and nutation of the axis of its proper rotation (Cottereau et al., 2010). On the basis of a modeled figure of Phoebe, which was built from the analysis of its images taken from the *Cassini* spacecraft (Gaskell, 2013), the ephemeris of the Phoebe rotation (Archinal et al., 2018), and the profiles of occultations of some stars by Phoebe, Gomes-Júnior et al. (2020) estimated its rotation period with an even higher accuracy: $P_{rot} = 9.27365 \pm 0.00002$ h.

By analyzing in detail the observational data acquired from the *Cassini* spacecraft, Denk and Mottola (2019) found that 25 irregular satellites of Saturn rotate with the periods ranging from 5 to 76 h; this is significantly less than their orbital periods. For 20 satellites (including Phoebe), the uncertainties in determining the rotational periods are rather small (<2%); for three satellites, the periods were estimated ambiguously; and for two more satellites, the estimates are tentative. Note that all of the satellites are rather small (for almost all of them, $R < 2$ km). The data obtained by Denk and Mottola (2019) allowed the known information about rotation states of planetary satellites to be enlarged by about one third in volume.

A Satellite of Neptune—Nereid (N2)

Among currently known fourteen moons of Neptune, the second one—Nereid—is of particular interest in what concerns the rotational dynamics. Nereid is an irregular satellite, and the diameter of its figure is approximately 350 km (Smith et al., 1989; Thomas et al., 1991). Its orbit is extremely elongated ($e \approx 0.75$). For some time, it was believed that Nereid may rotate chaotically (Dobrovolskis, 1995; Schaefer, B.E. and Schaefer, M.W., 2000). Grav et al. (2003) analyzed their ground-based observations and determined that the rotation period of Nereid is $P_{rot} = 11.52 \pm 0.14$ h (whereas $P_{orb} = 360$ d). Thus, its rotation velocity is ~750 times higher than the synchronous rotation velocity. The main problem in determining the rotation period of Nereid is that the amplitudes of changes in its light curve found by various observers (in different epochs) differ tenfold (Schaefer, B.E. and Schaefer, M.W., 2000; Grav et al., 2003; Schaefer et al., 2008). Schaefer et al. (2008) supposed that these amplitude differences may be explained by a very elongated figure of the moon, a substantial precession of its proper rotation axis, and strong variations in the albedo over the surface. A thorough numerical modeling of the rotational dynamics of Nereid was performed by Alexander et al. (2011), who, in particular, estimated the influence of the figure asymmetry on the shape of the light curve observed. Relations between the shape, the rotation, and the observed light curve of Nereid were analyzed in detail by Hesselbrock et al. (2013) by constructing the model light curves for various values of the parameters specifying the figure, orientation, and rotation characteristics of the satellite. Hesselbrock et al. (2013) fitted the light curves constructed in ground-based observations with the model curves and proposed that amplitude variations of the light curves of Nereid in different observational epochs (see Fig. 4 in a paper by Schaefer et al. (2008)) may actually be explained by its extremely elongated figure with additional accounting for probable changes in the albedo over the surface. From the analysis of observations carried out with two space telescopes (Spitzer Space Telescope and Herschel Space Observatory), Kiss et al. (2016) derived the rotation period of Nereid with a high precision ($P_{rot} = 11.594 \pm 0.017$ h) and defined its figure parameters more accurately. According to their results, the peculiarities observed in the light curve of Nereid are caused only by its elongated figure, which is approximated by a three-axial ellipsoid with the largest and smallest semiaxes related as 1.3 : 1; while the fact that Nereid has "a very rough, highly cratered surface" may also contribute.

Satellites of Uranus

Maris et al. (2001) observed Caliban (U16) and Sycorax (U17) from the Earth and found that the rotation periods of these moons are ~3 and ~4 h, respectively. Later, the photometric observations performed by Maris et al. (2007) confirmed the estimate for the rotation period of Sycorax (U17) and allowed the rotation periods of Prospero (U18) (~4.6 h) and Setebos (U19) (~4.4 h) to be determined. According to Maris et al. (2001, 2007), the orbital periods of Caliban (U16), Sycorax (U17), Prospero (U18), and Setebos (U19) exceed the periods of their proper rotation by 5200, 8600, 10300, and 12200 times, respectively. Based on the analysis of the observational data acquired with the *Kepler* space telescope, Farkas-Takács et al. (2017) specified more precisely the rotation periods of the above-mentioned moons of Uranus (their values turned out to be somewhat higher than those found by Maris et al. (2001, 2007)); moreover, they were the first to determine the rotation period of Ferdinand (U24) (~11.8 h).

Thus, rapid non-synchronous rotation has been revealed for five moons of Uranus so far. According to Sheppard et al. (2005), the largest of these irregular moons of Uranus—Sycorax (U17)—has the radius

$R \approx 75$ km, while the radius of the smallest one—Ferdinand (U24)—is $R \approx 10$ km.

It is evident that the progress in current techniques of observations, including, primarily, the use of space telescopes (Farkas-Takács et al., 2017; Kiss et al., 2016), makes it possible to study the rotational dynamics of rather small distant satellites of planets.

As regards the rotational dynamics of satellites beyond the orbit of Neptune, four moons of Pluto are observed as rotating rapidly (Weaver et al., 2016). Apparently, all known satellites of TNOs rotate rapidly (see a paper by Thirouin et al. (2014) for the discussion about the time required for the tidal effects to slow down satellites of large TNOs). The rotational dynamics of satellites of large TNOs is briefly overviewed below.

CHAOTIC ROTATION

A third type of rotation state, which was predicted theoretically and revealed from observations, is a state of chaotic rotation (tumbling). Chaotic behavior can be observed in the rotational dynamics of various celestial bodies (planets, planetary satellites, asteroids, and cometary nuclei) (Murray and Dermott, 2000; Morbidelli, 2002; Shevchenko, 2020). Dynamical chaos manifests itself in the exponential divergence of close trajectories in the phase space of a dynamical system (Lichtenberg and Lieberman, 1982); consequently, its dynamics are unpredictable on timescales larger than the so-called Lyapunov time of a system (Chirikov, 1979). To reveal dynamical chaos, various numerical tools are used; among them are calculations of the Lyapunov characteristic exponents, the Mean Exponential Growth factor of Nearby Orbits (the MEGNO parameter), and the frequency analysis (see reviews by Maffione et al. (2013), Morbidelli (2002), and Melnikov (2018)).

An indicator of chaos, which was theoretically rigorously proved, is the Lyapunov characteristic exponents (LCEs). The LCEs indicate the rate of the exponential divergence of initially close trajectories in the phase space of a system (Oseledets, 1968; Lichtenberg and Lieberman, 1982; Murray and Dermott, 2000; Morbidelli, 2002; Shevchenko, 2020). For the Hamiltonian system, the LCE spectrum contains $2N$ exponents, where N is the number of degrees of freedom of the system. For regular dynamics all LCEs are equal to zero, while for chaotic motion at least the maximum LCE is larger than zero. In papers by Benettin et al. (1976, 1980) the effectiveness of LCEs as a tool to study dynamical systems was demonstrated, and basic algorithms for their calculations were proposed. The Householder QR-based (HQRB) method developed by von Bremen et al. (1997) is based on the QR-decomposition of the tangent map matrix by using the Householder transformation. Shevchenko and Kouprianov (2002) developed a software package based on the HQRB method and used it to study the rotational dynamics of planetary satellites. Analytical methods for estimating the maximum Lyapunov exponents are described in papers by Shevchenko (2002, 2020); these methods are mainly based on the theory of separatrix maps (Shevchenko, 1999, 2020).

In papers by Wisdom et al. (1984) and Wisdom (1987), it was theoretically shown that a nonspherical satellite in an elliptic orbit may rotate in a chaotic (unpredictable) manner. Wisdom et al. (1984) were the first who noticed that Hyperion (S7) is a candidate for being in chaotic rotation, since its figure is strongly asymmetric and its orbit eccentricity is high. Later, the modeling of the observed light curves and the rotational dynamics of Hyperion, which was performed by a number of researchers (Klavetter, 1989b; Black et al., 1995; Devyatkin et al., 2002; Melnikov, 2002; Harbison et al., 2011), confirmed the chaotic nature of Hyperion's rotation. The results of studies of the rotational dynamics of Hyperion are considered at length below.

The Chaotic Rotational Dynamics of Hyperion

The possibility of chaos in the rotational dynamics of planetary satellites was first ever noticed in a paper by Wisdom et al. (1984); and Hyperion, the seventh moon of Saturn, was pointed out as the most probable candidate for being in chaotic rotation.

Hyperion (S7) was discovered by W. Bond (1848), G. Bond, and, independently, W. Lassell (1848). Its orbit has a noticeable eccentricity ($e \approx 0.1$) and a small inclination to Saturn's equator ($i = 0.43°$). The orbital period of Hyperion is $P_{\rm orb} \approx 21.28$ d; and the semimajor axis of the orbit is $a \approx 25R_{\rm Saturn}$, where $R_{\rm Saturn} \approx 57600$ km is the mean radius of Saturn. As was revealed from the observational data from the *Voyager 2* spacecraft (Thomas and Veverka, 1985; Thomas et al., 1995) and the *Cassini* spacecraft (Thomas et al., 2007; Thomas, 2010), the figure of Hyperion is strongly elongated; the semiaxes of a three-axial ellipsoid approximating its figure are 180, 133, and 103 km. According to Thomas et al. (2007) and Rossignoli et al. (2019), there are numerous small craters and several large ones on the surface of Hyperion; and the largest crater is 150 km across, which is comparable to the radius of the satellite's figure. Hyperion is the largest among spherically asymmetric satellites in the Solar System.

The results of the theoretical analysis by Wisdom et al. (1984) suggest that it is the strongly asymmetric geometric figure of Hyperion coupled with the substantial eccentricity of its orbit that distinguishes it among the known planetary satellites as the most probable candidate for being in chaotic rotation. Moreover, a numerical analysis of the stability of the modeled rotation of Hyperion in synchronous resonance, which was performed by Wisdom et al. (1984), showed that its synchronous rotation is unstable with

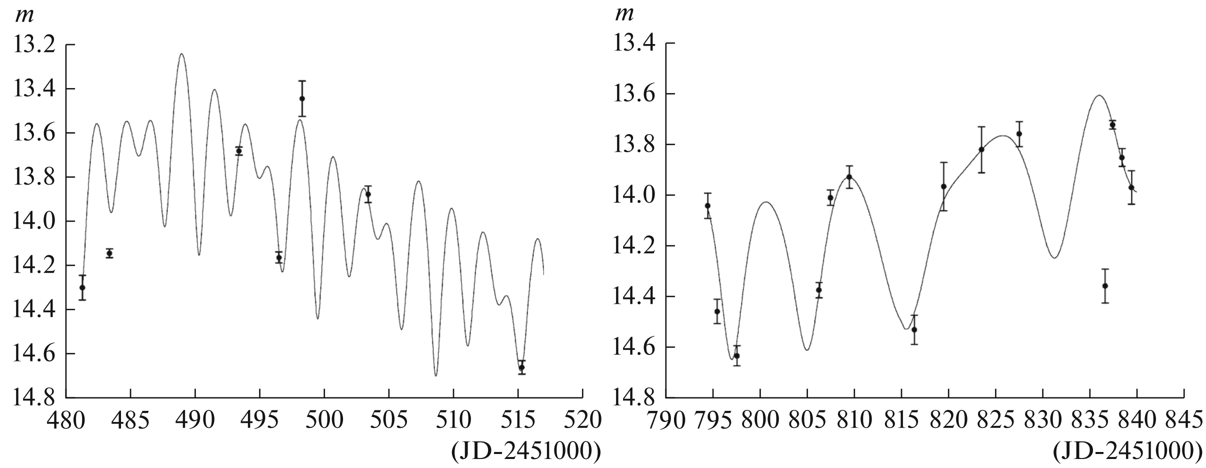

Fig. 5. The model light curves of Hyperion (S7) based on the Pulkovo observations carried out in September–December 1999 (left panel) and September–October 2000 (right panel). The dots with bars show the observed magnitudes of Hyperion. The time scale is given in Julian days. (Adapted from a paper by Devyatkin et al. (2002).)

respect to tilting the rotation axis. Even a small deviation of the rotation axis from the normal makes the satellite tumble chaotically.

In 1984, an analysis of the *Voyager 2* observational data drove Thomas et al. (1984) to the conclusion that the rotation period of Hyperion is approximately 13 d and the rotation axis is close to the orbital plane. This rotation state, which is unusual for a satellite, and the potentially chaotic regime of Hyperion's rotation, which was theoretically predicted by Wisdom et al. (1984), motivated researchers to study the dynamics and organize further observations of Hyperion. The rotational dynamics and the light curves of Hyperion were modeled by Klavetter (1989b) on the basis of observational data he obtained in 1987 (Klavetter, 1989a). Black et al. (1995) modeled the rotational dynamics of Hyperion on the basis of the *Voyager 2* observational data (Thomas et al., 1995). Pulkovo observations and the observational data of Klavetter (1989a) were used by Devyatkin et al. (2002) and Melnikov (2002) to model the light curves and the rotational dynamics of Hyperion. The *Voyager 2* and *Cassini* observations served as a basis for modeling the rotational dynamics of Hyperion by Harbison et al. (2011). The main purpose of all of the above-listed studies was to elucidate the character of Hyperion's rotation.

Klavetter (1989b), from modeling the light curves he observed, found that, most likely, Hyperion rotates chaotically. Black et al. (1995) and Harbison et al. (2011) determined that the rotation angular velocity of Hyperion exceeds by approximately four times the angular velocity of its orbital motion. If the rotation velocity is so high and far from synchronous resonance, the rotation may as well be regular. However, based on results of numerical simulations of the long-term rotational dynamics, Black et al. (1995) tended to conclude that the rotation of Hyperion is chaotic.

By modeling the observed light curves of Hyperion (both those obtained in Pulkovo and built by Klavetter), Devyatkin et al. (2002) estimated the parameters and initial conditions that defined the rotation state of Hyperion in the epochs of observations (in 1987 and 1999–2000). Examples of the model light curves are presented in Fig. 5. The data obtained from the comparison of the phase-space section and the initial conditions, resulting in the observed light curves, suggested that the rotation is chaotic. The LCEs calculated for the acceptable initial conditions of Hyperion's motion confirmed that it actually rotates chaotically (Melnikov, 2002), since the maximum LCE turned out to be larger than zero (see, e.g., the results of LCE calculations in Fig. 6). Thus, Melnikov (2002) rigorously ascertained the chaotic character of Hyperion's rotation.

The Lyapunov Time Values for the Chaotic Rotation of Hyperion

By calculating the LCEs for the chaotic rotation of a satellite having the parameters of Hyperion, Wisdom et al. (1984) estimated the Lyapunov time (which is defined to be equal to $1/L_1$, where L_1 is the maximum LCEs) as approximately equal to two orbital periods (~42 d). According to Melnikov (2002), the Lyapunov time for the rotational dynamics of Hyperion is in an interval from 38 to 51 d. The Lyapunov time estimated for Hyperion theoretically is ~30 d (Shevchenko, 2002). The Lyapunov time for the rotation of Hyperion as estimated numerically by Kouprianov and Shevchenko (2005) is 54 d.

Estimates of the Lyapunov time for the observed rotational dynamics of Hyperion, which were found

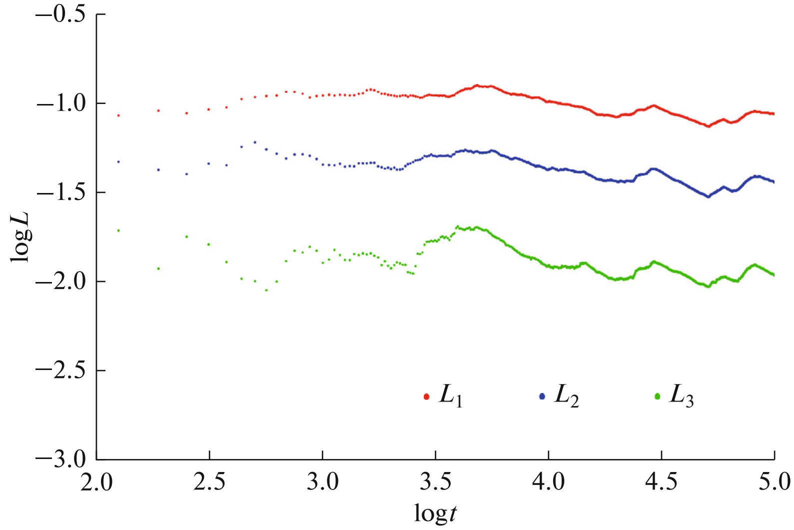

Fig. 6. The dependence of the LCEs current values ($L_1 > L_2 > L_3$) on the time t, for which they are calculated, for the allowed initial conditions) of the rotational motion of Hyperion (S7). The time unit is $1/(2\pi)$ of the orbital period. (Adapted from a paper by Melnikov (2002).)

by Harbison et al. (2011), also point to the chaotic character of its rotation. Specifically, 61.4 ± 3.6 d was derived as an average estimate.

Thus, any information on the initial conditions, specifying the rotational motion of Hyperion, is lost in the satellite's dynamics in one–two months, i.e., during one and a half to three orbital periods. Therefore, useful information about the rotational dynamics of this satellite may be retrieved from its light curve, if the segments of the latter are modeled on time intervals smaller than that mentioned.

Chaos in the rotation of Hyperion can be found by analyzing the data acquired from both ground-based and space-borne observations. From further modeling and examination of the observed light curves and analysis of the data obtained from spacecraft, it will be possible to estimate more accurately the chaotic rotation parameters of Hyperion and its Lyapunov time.

By application of methods of nonlinear analysis of time series to the observational data obtained by Klavetter (1989b) and Devyatkin et al. (2002), Tarnopolski (2015) specified requirements for a temporal series of ground-based photometric observations, which would allow the chaotic behavior of Hyperion's dynamics to be revealed by calculating the maximum LCE directly from the observational series. According to Tarnopolski (2015), photometric observations of Hyperion should last for not less than a year and be performed with several telescopes in order to ensure the continuity of the observational series without time gaps. To date, Hyperion is the only satellite in the Solar System, for which the chaotic rotational dynamics has been rigorously confirmed.

As follows from the observational data statistics, the smaller the satellite is, the more asymmetric shape it may have (see, e.g., Kouprianov and Shevchenko, 2006; Melnikov and Shevchenko, 2007, 2010). The observed dependence of the parameter ω_0, characterizing the asymmetry of a satellite's figure, on the mean radius of a satellite is presented in Fig. 7. The dependence suggests that the chaotic rotation regime is more probable for the smaller satellites of planets. Let us discuss this issue in more detail.

Chaotic Rotation of Other Minor Satellites

Is it possible that other minor satellites of planets in the Solar System, except Hyperion, currently reside in observable regimes of chaotic rotation? Wisdom (1987) showed that, for all satellites that are irregularly-shaped (essentially nonspherical) and have nonzero orbital eccentricities, chaotic rotation is the case for all of them before they are captured into synchronous resonance in the course of tidal evolution. Melnikov and Shevchenko (2010) considered the problem on typical present-day rotational regimes of planetary satellites and showed that more than 60% of the known minor satellites, the rotation regime of which has not been determined yet, rotate either regularly (more rapidly than in the synchronous regime) or chaotically. Chaotic rotation should be observed for those minor satellites whose tidal evolution has already been completed; in other words, in the phase space of the rotational motion, the satellite has approached the zone corresponding to synchronous rotation, but the synchronous resonance is unstable or does not exist at all. The conclusion that a substantial fraction of known minor satellites currently rotate rapidly was made by Melnikov and Shevchenko (2010) from the

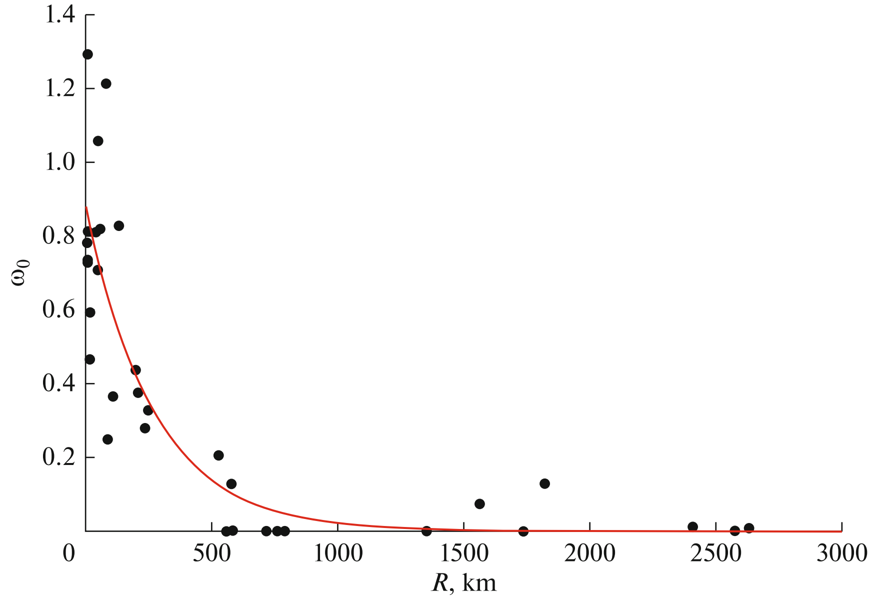

Fig. 7. The diagram for the mean radius of a satellite versus the parameter ω_0. The dots indicate the positions of satellites with known parameters. The solid curve represents an exponential approximation. (Adapted from a paper by Melnikov and Shevchenko (2007).)

viewpoint of the dynamical stability of synchronous rotation for these satellites.

Kouprianov and Shevchenko (2005) and Melnikov and Shevchenko (2008) showed that two moons of Saturn—Prometheus (S16) and Pandora (S17)—may rotate chaotically, since their planar synchronous rotation is most likely unstable with respect to tilting the rotation axis. The Lyapunov time of the potentially chaotic rotation of these satellites is rather small, less than a day (Kouprianov and Shevchenko, 2005). Model numerical experiments (Melnikov and Shevchenko, 2008; Melnikov, 2020) demonstrated that, in the chaotic rotation regime of some minor satellites, e.g., Prometheus (S16) and Pandora (S17), the largest axis of the satellite's figure tends to be oriented toward the planet; therefore, the rotation of the satellite may resemble the synchronous one. For the first time, this effect was noticed by Wisdom (1987) in numerical experiments that modeled the chaotic spatial rotation of Phobos (M1). As Melnikov (2020) showed, the effect of preferred orientation of a satellite's figure toward the planet should be noticeable in the chaotic dynamics of satellites with small orbital eccentricities ($e < 0.005$); this may make it difficult to identify the chaotic rotational regime for these satellites, if the observational series is not sufficiently long.

Quillen et al. (2020) studied the rotational dynamics of Phobos and Deimos assuming that the satellites were initially in the chaotic tumbling regime. The numerical experiments confirmed the hypotheses of Wisdom (1987) that these satellites can reside in the chaotic rotation regime for thousands of orbital periods, even if the initial eccentricities of their orbits were small. After being captured into synchronous rotation due to the tidal effects, viscoelastic satellites may rotate for a long time around the axis that does not coincide with the axis of its largest moment of inertia; this is accompanied by the enhanced dissipation of energy in the satellite's body. During the chaotic tumbling, the scattering of energy is several orders of magnitude larger than that during regular synchronous rotation (Quillen et al., 2020). According to Wisdom (1987), the sets of parallel grooves observed on the surface of Phobos could appear just in the epoch of its chaotic rotation in the past.

Strange Attractors in the Chaotic Rotational Dynamics of Satellites

In papers by Celletti and MacKay (2007), Celletti et al. (2007) and Celletti and Chierchia (2008), it was shown that, at certain values of parameters in the problem on the rotational motion of satellite, if one accounts for dissipative forces, periodic and quasi-periodic attractors exist in the phase space of motion (about the attractor types, see a book by Lichtenberg and Lieberman (1982)). According to Celletti and Chierchia (2008), if the eccentricity is small, a large fraction of the phase space is occupied by a periodic attractor corresponding to the 1 : 1 synchronous resonance. When the eccentricities are large, periodic attractors, corresponding to resonances of higher orders, and quasi-periodic attractors dominate.

Khan et al. (1998) studied the dynamics of the planar rotational motion of a satellite with taking into account tidal perturbations and showed that, on the

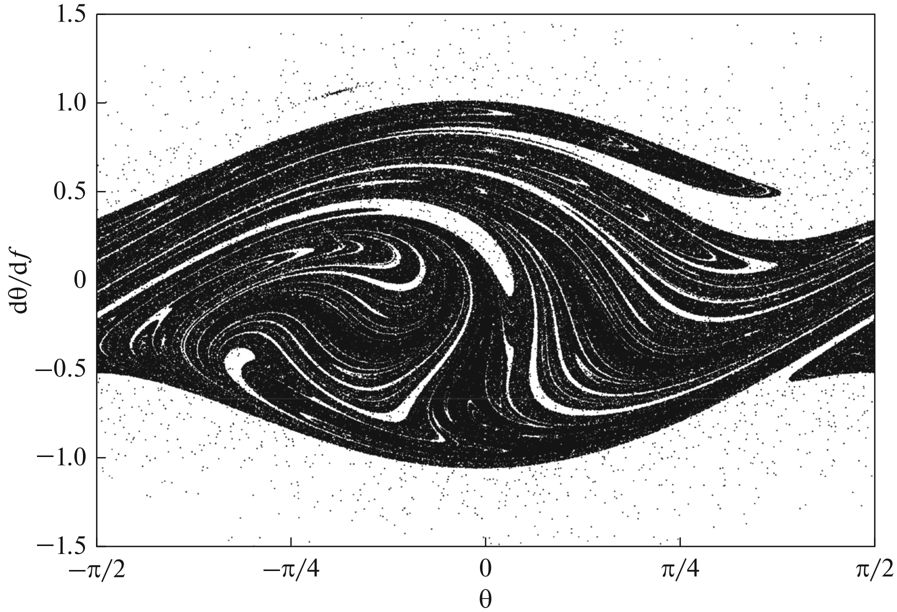

Fig. 8. An example of a strange attractor in the phase-space section of the planar rotational motion of a satellite. The designations θ and *f* are for the angle between the line of apsides and the largest axis of a satellite's figure and the true anomaly, respectively. (Adapted from a paper by Melnikov (2014).)

phase-space sections, given certain values of the parameters of the problem (the eccentricity, the parameter describing the asymmetry of a satellite's figure, and the parameter defining the magnitude of the tidal dissipation), a structure that is characteristic of a strange attractor appears (for details of the theory of strange attractors, see a book by Lichtenberg and Lieberman (1982)). The appearance of a strange attractor in this problem was pointed out by Beletskii (2007), who arrived at this conclusion by analyzing the phase-space sections.

Considering the tidal interaction widens the set of possible regimes of the rotational motion of a satellite in the neighborhood of synchronous resonance, since, in a dissipative system, chaotic motion on a strange attractor may take place. A strange attractor appears in that part of the phase space where a chaotic layer near the spin-orbit resonance separatrices is located when the tidal interaction is absent. An example of the phase-space section of the planar rotational motion of a satellite with accounting for the tidal interaction with a planet is shown in Fig. 8. By assuming the orbital eccentricity of a satellite to be small, Khan et al. (1998) estimated analytically the parameter characterizing the tidal dissipation under which the strange attractor is formed.

In a paper by Melnikov (2014), the problem of the potential existence of a strange attractor in the rotational dynamics of planetary satellites is considered in detail. By calculating and analyzing the LCE values on a set of allowed values of the parameters in the problem, it was shown that a strange attractor may emerge during the tidal evolution of the rotational motion of some minor satellites, e.g., in the dynamics of Hyperion (S7). We recall that, as follows from the works by Klavetter (1989b), Black et al. (1995), Devyatkin et al. (2002), Melnikov (2002), and Harbison et al. (2011), in the present epoch, Hyperion rotates chaotically.

Rotation of Nearby Moons of Giant Planets within the Relativistic Approximation

Biscani and Carloni (2015) and Pashkevich et al. (2021) showed that, when studying the long-term evolution of the rotational motion of moons of giant planets, one should take into account relativistic effects. In rotation of celestial bodies, the most significant relativistic effects are those of geodesic precession and nutation, constituting together geodesic rotation. The effects of geodesic precession (De Sitter, 1916) and nutation (Fukushima, 1991) manifest themselves in systematic and periodic changes, respectively, of the direction of the rotation axis of a celestial body, due to parallel transfer of the angular momentum vector along the orbit in the curved spacetime. Since Jupiter is the second most massive body in the Solar System, it should be expected that it would induce relativistic perturbations in the dynamics of nearby objects. Theoretical estimates of the geodesic precession of two of Jupiter's moons—Io (J1) and Metis (J16)—were obtained within a simplified model of their rotation by Biscani and Carloni (2015). Based on available ephemerides (Archinal et al., 2018), Pashkevich and Vershkov (2019) and Pashkevich et al. (2021) thoroughly analyzed the rotational dynamics of the both satellites of Mars and the inner satellites of Jupiter (Amalthea (J5), Thebe (J14), Adrastea (J15), and Metis (J16)) within the relativistic approximation. It turned out that the geodesic precession magnitude for the inner satellites of Jupiter, the orbits of which are rather close to the planet, is comparable to the magnitude of their precession in the Newton approximation. Thus, the relativistic effects in the rotational dynamics of nearby satellites of giant planets should be taken into account in modeling of their long-term tidal evolution. The relativistic effects may play an important part in the rotational dynamics of hypothetical satellites of exoplanets (see the discussion below).

ROTATIONAL DYNAMICS OF SATELLITES OF LARGE TRANS-NEPTUNIAN OBJECTS

Many of the TNOs have satellites (Brown et al., 2006; Parker et al., 2016; Kiss et al., 2017; Sheppard et al., 2018), or they are binary objects with components of comparable masses (Thirouin et al., 2014;, 2016) or components in contact (Grishin et al., 2020). To date, six TNOs (including Pluto) that are larger than 1000 km across have been found; all of them possess satellites (Arakawa et al., 2019). According to the results of numerical simulations carried out by Arakawa et al. (2019), most satellites of large TNOs were formed due to collisions of large bodies at early stages of the formation of the Solar System.

The rotational dynamics of circumbinary satellites (those orbiting around gravitating binaries), for example, minor satellites in the Pluto—Charon system, can be chaotic. Showalter and Hamilton (2015) pointed out that two circumbinary satellites in the Pluto—Charon system—Nix (P2) and Hydra (P3)—may rotate chaotically. Correia et al. (2015) confirmed the potentially chaotic rotational dynamics of minor satellites in the Pluto—Charon system by numerical calculations.

However, somewhat later, based on a detailed analysis of the *New Horizons* data, Weaver et al. (2016) determined that the rotation of four minor moons of Pluto (including Nix and Hydra) is more rapid, by 6—88 times, than the synchronous one. The fact that the minor satellites of Pluto are rapidly rotating is confirmed by the results of numerical simulations performed jointly for their orbital and rotational dynamics (Quillen et al., 2017). Thus, in the course of the long-term tidal evolution of the rotational motion, the satellites of the Pluto—Charon system have not reached the zone of chaos yet.

Apparently, rapid non-synchronous rotation is also characteristic of the other known satellites of TNOs just because they have not yet completed their tidal evolution (Brown et al., 2006; Brown and Butler, 2018; Ćuk et al., 2013; Kiss et al., 2017; Sheppard et al., 2018; Parker et al., 2016).

ROTATIONAL DYNAMICS OF EXOMOONS

At present, studies of the possibility for moons to exist orbiting around exoplanets (Kipping et al., 2012, 2014, Heller, 2014, 2018; Heller et al., 2014; Sucerquia et al., 2019) and even for submoons to exist orbiting around exomoons (see, Kollmeier and Raymond, 2019; Rosario-Franco et al., 2020) are under active development. As is known, there no objects of the second type in the Solar System, though they may exist in other systems.

Exomoons are of great interest because, first of all, the problem of habitability of exoplanetary systems is highly topical. Indeed, the conditions for life to exist may be more suitable on satellites of exoplanets than on parent exoplanets, which are often gaseous giants, though located in a zone of potential habitability of a host star (Williams et al., 1997; Kaltenegger, 2010; Heller et al., 2014). Exomoons can be discovered in observations of transits, since a moon orbiting a planet introduces specific variations into a shape of the light curve of the transit. This concerns both the intervals between transits and the duration of transits, while the curve shape during the transit-induced obscuration of a host star is also influenced (see Kipping, 2011; Heller, 2014).

A real potential candidate for an exomoon was proposed by Teachey and Kipping (2018). By analyzing the Hubble Space Telescope observations and by using the "Transit-Timing Variation" (TTV) method, they found an evidence for the existence of a moon of the planet Kepler-1625b. Planet Kepler-1625b is similar to Jupiter in size, and the size of its moon is similar to that of Neptune (Heller, 2018). The discovery of the moon orbiting planet Kepler-1625b became a topic for discussion (Heller et al., 2019; Teachey et al., 2020), which revealed the complexity of the problem.

With the TTV method, Kipping (2020) found an evidence for the presence of exomoons in six planetary systems; Fox and Wiegert (2021) found eight more systems, the planets of which exhibit signs of the presence of moons.

The rotational dynamics of exomoons can be chaotic rather often, since the configurations of planetary systems (and, consequently, their satellite systems) widely vary. For example, there exist planets in systems of multiple stars; and the eccentricities and inclinations of orbits of exoplanets are often rather high. Note that chaos may take place in the rotational dynamics of coorbital satellites (among them are, for example, some satellites in the system of Saturn) in quasi-circular orbits (Correia and Robutel, 2013), as well as for coorbital exoplanets with significant orbital eccentricities (Leleu et al., 2016). In exoplanetary systems, the rotational dynamics of satellites can be more complicated due to perturbations from additional satellites (Tarnopolski, 2017).

In many cases, it could be essential to account for relativistic effects. Based on the results of studies of the rotational dynamics within the relativistic approximation for some satellites of Jupiter (Biscani and Carloni, 2015; Pashkevich et al., 2021), one may suggest that the relativistic effects in the rotational dynamics could be apparently important for many exomoons that will be discovered in future. Iorio (2021) showed that, for moons of exoplanets, due to the relativistic effects, the tilt of the proper rotation axis can vary in wide ranges (10°–100°) during time intervals less than a million years. This conclusion agrees with the results of Pashkevich et al. (2021) for planetary satellites in the Solar System.

In most studies of exomoons, only the aspects of their possible orbital dynamics and the problems of their formation are considered. However, even now, the problem of the rotational dynamics and rotational evolution of exomoons has gained in relevance primarily due to the issue of habitability of exosystems.

CONCLUSIONS

Here we reviewed the works focused on studying the rotational dynamics and rotational evolution of planetary satellites in the Solar System and potentially existing satellites of exoplanets. The data on the observed rotation states of known satellites were presented. The main findings of the theoretical studies concerning the long-term dynamical tidal evolution of the rotational motion of satellites were analyzed. We discussed the main rotational regimes of a satellite, which are theoretically possible and are observed—rotation in synchrony with the orbital motion, rotation that is more rapid than the synchronous, and chaotic rotation. The results of the analysis performed for the chaotic rotational dynamics of the seventh moon of Saturn—Hyperion—were considered at length. Recent activity in studying the rotational dynamics of satellites of TNOs (including the moons of Pluto) and potential exomoons was surveyed.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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