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Jeans Instability of a Protoplanetary Gas Cloud with Radiation in Nonextensive Tsallis Kinetics

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Abstract—In the framework of Tsallis statistics, we study the effect of medium nonextensivity on the Jeans gravitational instability criterion for a self-gravitating protoplanetary cloud, the substance of which consists of a mixture of perfect gas and blackbody radiation. Generalized Jeans instability criteria have been found from the corresponding dispersion relations obtained both for a uniform cloud with radiation and for a rotating protoplanetary cloud. An integral generalized Chandrasekhar stability criterion for a gravitating spherical cloud has also been obtained. The presented results are analyzed for various values of deformation parameters q, dimensionality D of the velocity space and coefficient β, characterizing the fraction of radiation in the total pressure of the system. It is shown that radiation stabilizes the substance of nonextensive protoplanetary clouds, and for rotating clouds, the Jeans instability criterion is modified by the Coriolis force only in the transverse modes of perturbation wave propagation.

Keywords: Jeans and Chandrasekhar criteria, protoplanetary cloud, blackbody radiation, Tsallis nonextensive kinetics

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INTRODUCTION

As becomes clear now, Boltzmann–Gibbs statistical mechanics and standard thermodynamics are not completely universal theories, since they have limited domains of applicability. In particular, this is caused by the fact that Boltzmann–Gibbs statistics are based on the postulate of the complete mixing of the "phase points" flow in the phase space (the hypothesis of molecular chaos). This means that any allocated volume acquires such a well-developed chaotic structure with time that its points can be located in an arbitrary part of the phase space. Thus, in classical statistics, the phase space does not contain forbidden states and has the usual properties of continuity, smoothness, Euclideanity. In this case, the stochastic process has a Markov character, and a hypothesis of mixing, supplemented by the assumption of an infinite number of degrees of freedom, leads, ultimately, to the canonical (exponential) probability distribution of the Boltzmann–Gibbs states, which leads to the additivity of extensive thermodynamic variables, such as internal energy, entropy, etc., and in the case of kinetic theory, to the Maxwell velocity distribution. At the same time, numerous examples of anomalous systems with long-range force interaction, the fractal nature of the phase space and significant correlations between their individual parts are known in physics and other natural sciences where the methods of statistical mechanics are used. The complex spatiotemporal structure of such systems violates the additivity principle for important thermodynamic quantities such as entropy or internal energy.

In particular, a description of evolution of such systems characterized by an arbitrary phase space is possible in the framework of the so-called nonextensive statistical mechanics by Tsallis[1] (see Tsallis, 1988, 1999, 2009; Curado and Tsallis, 1991; Tsallis et al., 1998; Kolesnichenko, 2019), which is being intensively developed recently, mainly by foreign authors. An important advantage of nonextensive statistics over classical Boltzmann–Gibbs statistics is the asymptotic power law of the probability distribution.

Currently, theories of various nonextensive systems are developing rapidly, and new ideas emerge that enable a deeper understanding of their nature, capabilities and limitations. Each theory has a wide range of important applications related to the physics of statistical systems whose probabilistic properties are described by non-Gibbs (non-Gaussian) power-law distributions. In particular, nonextensive statistical

[1] Numerous journal articles, collections and monographs are devoted to surveys of studies within the framework of nonextensive Tsallis statistics. In addition, there is a constantly updated full bibliography (Nonextensive statistical mechanics and thermodynamics: Bibliography/ http://tsallis.cat.cbpf.br/biblio.htm), which today includes more than 5600 references.

mechanics is successfully applied to space systems with long-range interaction forces, which are the reason for their anomalous nature (statistical and thermodynamic nonextensivity).

In (Kolesnichenko and Chetverushkin, 2013), within the framework of the formalism of deformed Tsallis statistics, based on the modified kinetic equation (with the Bhatnagar-Gross-Krook collision integral), a generalized hydrodynamics (the so-called Navier-Stokes q-hydrodynamics) wased, which is suitable for modeling such systems. It is on the basis of this hydrodynamics we consider in this article the problem of Jeans gravitational instability for an extended gas–dust cloud that occupies the entire space of the present solar system, taking into account the blackbody radiation and a magnetic field influence on the critical wavelength of a perturbation leading to instability.

Gravitational instability is a fundamental process of fragmentation of gravitating cosmic matter. It causes the formation of stable astrophysical objects, such as stars, nebulae, protoplanetary dust condensations, accretion disks, etc. (see, Jeans, 1902, 2009; Chandrasekhar, 1981; Safronov, 1969; Gor'kavyi and Fridman, 1994; Fridman and Khoperskov, 2011). It is well known that self-gravitating cosmic matter becomes gravitationally unstable if arbitrarily small density perturbations that arise in it grow unlimitedly with time due to gravity, and the equilibrium is violated if the corresponding wavelengths exceed a certain value. Recently, a large number of publications were devoted to the problem of gravitational instability, among which are the following works: (Chandrasekhar and Fermi, 1953; Bonnor, 1957; Hunter, 1972; Goldreich and Lynden-Bell, 1965; Low and Lynden-Bell, 1976; Shakura and Sunyaev, 1976; Camenzind et al., 1986; Cadez, 1990; Pandey and Avinash, 1994; Owenet al., 1997; Tsiklauri, 1998; Maceet al., 1998; Limaet al., 2002; Radwan, 2004; Trigger et al., 2004; Sakagami and Taruya, 2004; Shukla and Stenflo, 2006; Tsintsadze et al., 2008; Masood et al., 2008; Cadez, 2010; Dhiman and Dadwal, 2012; Fridman and Polyachenko, 1984, 2012; Kaothekar and Chhajlani, 2013; Joshi and Pensia, 2017; Pensia et al., 2018; Kumar et al., 2018; Kolesnichenko, 2015, 2016, 2018, 2019; Kolesnichenko and Marov, 2014, 2016). In all these papers, various aspects of the Jeans instability of self-gravitating cosmic matter are considered both in the framework of the classical Navier–Stokes equations and MHD equations, and on the basis of the collisionless Boltzmann equation in the presence of gravitational fields and the Poisson equation.

It is known that in the case of normal stars, a large role in their hydrostatic equilibrium is played by radiation pressure. With allowance for this, in the framework of Tsallis' nonextensive kinetics, in the present article we consider the effect of radiation on the gravitational instability of the solar protoplanetary cloud (more precisely, its equatorial part, in which almost all radiation is long-wave, since it has already passed through multiple absorption and reemission by particles of matter). A local thermodynamic equilibrium is possible in this case, at which the particle temperature nearly coincides with the temperature of the black body. In this work, we find a functional dependence of the critical value of the perturbing wavelength on the entropy strain index q and the velocity-space dimensionality D. These free parameters should be determined in each case empirically from statistical or experimental data. We also examine the effect of rotation on the gravitational instability of a nonextensive protoplanetary cloud. According to the author, the results obtained here will help to better understand some astrophysical problems related, in particular, to modeling the processes of star and exoplanet formation from stellar nebulae.

EQUATIONS OF q-HYDRODYNAMICS IN NONEXTENSIVE KINETICS

Next, we consider a gaseous dynamic nonextensive system with a normalized distribution of particles $f(r,c,t)$ in the geometric space r and in the velocity space c with dimensionality D. The generalization of statistical mechanics (in the case of the Curado-Tsallis statistics) proposed by Tsallis is best described by the following two axioms (Curado and Tsallis, 1991; Kolesnichenko, 2018):

Axiom 1. The entropy functional associated with the normalized distribution of the probability function $f(z,t)$ is given by

$$S_q[f] = \frac{k}{q-1}\int dz\left\{f(z) - [f(z)]^q\right\}, \tag{1}$$

where q is the deformation parameter related to the fractal dimensionality, while for nonextensive systems, this parameter is the measure of their nonadditivity (Tsallis, 2009); $z = (r,c)$ is the phase space volume element; $dz \equiv drd^Dc$, where D is the dimensionality of the velocity space; k is the Boltzmann constant.

Axiom 2. The experimentally measured value of any macroscopic quantity $\langle A\rangle_q$ (thermodynamic characteristic of the q-system) is defined by the relation

$$\langle A\rangle_q \equiv \int\int dzA(r,t)[f(z)]^q, \tag{2}$$

where $A(r,t)$ is the corresponding microscopic value.

It is important to emphasize that entropy $S_q(A \cup B)$ of two independent systems is not an additive thermodynamic variable for $q \neq 1$, since

$$S_q(A \cup B) = S_q(A) + S_q(B) + k^{-1}(1-q)S_q(A)S_q(B).$$

Despite this fact, it was shown in the literature that there is a significant number of usual statistical and thermodynamic properties that are q-invariant and valid for any q. In particular, they include the property of convexity of the Tsallis entropy, the structure of equilibrium canonical ensembles, nonadditive thermodynamics, the structure of the Legendre transform, and much more (see Bibliography/http://tsallis.cat.cbpf.br/biblio.htm).

Basic Definitions and Equations

Tsallis entropy entails not only a generalization of statistical physics and thermodynamics, but also a generalization of physical kinetics and hydrodynamics (Kolesnichenko and Chetverushkin, 2013; Oliveira and Galvao, 2018). The simplest macroscopic quantity is the q-density of the number of particles, which is determined by the relation

$$n_q(r,t) \equiv \int [f(z)]^q d^D c. \tag{3}$$

Then, the mass q-density is $\rho_q(r,t) \equiv m n_q(r,t)$. Since a particle moving with velocity c has the momentum mc, then the expression

$$u_q(r,t) \equiv \frac{1}{\rho_q(r,t)} \int mc[f(z)]^q d^D c \tag{4}$$

determines the hydrodynamic velocity of the volume element. The quantity

$$\varepsilon_q(r,t) = \frac{1}{\rho_q(r,t)} \int \frac{m}{2} |c - u_q|^2 [f(z)]^q d^D c \tag{5}$$

is the specific internal energy (per unit mass) of the nonextensive system. The flows

$$\mathbf{P}_q(r,t) = m\int (c - u_q)(c - u_q)[f(z)]^q d^D c, \tag{6}$$

$$\mathbf{J}_q(r,t) \equiv \frac{m}{2} \int |c - u_q|^2 (c - u_q)[f(z)]^q d^D c \tag{7}$$

correspond to the tensors of pressure and heat flux. Hydrostatic q-pressure is defined as

$$p_q(r,t) = \frac{1}{3}\mathbf{P} : \mathbf{J} = \frac{1}{3} m \int |c - u_q|^2 [f(z)]^q d^D c, \tag{8}$$

where $\mathbf{I}$ is the unit second-order tensor. In particular, if shear stresses are equal to zero, and normal stresses are equal to each other, then $\mathbf{P}_q = p_q \mathbf{I}$.

In (Boghosian, 1999; Kolesnichenko and Chetverushkin, 2013), in the framework of Tsallis' nonextensive statistical mechanics and the moment method, hydrodynamic and quasi-hydrodynamic equations were obtained on the basis of the modified Boltzmann[2] kinetic equation taking into account self-gravitation with the Bhatnager–Gross–Krook collision integral:

$$\left(\frac{\partial}{\partial t} + c\mathrm{grad} + \mathbf{F}_q \mathrm{grad}_c\right)[f(r,c,t)]^q = -\frac{[f(r,c,t)]^q - \left[f^{(0)}(r,c,t)\right]^q}{\tau}. \tag{9}$$

Here $\mathrm{grad}_c \equiv \mathbf{i}_x \partial/\partial c_x + \mathbf{i}_y \partial/\partial c_y + \mathbf{i}_z \partial/\partial c_z$; $\mathbf{F}_q(r,t) = \mathbf{f}/m - \mathrm{grad}\Psi_q(r,t)$ is the velocity-independent external force (gravitation force) per unit of mass; $\mathbf{f}$ is the nongravitational force (for example, electromagnetic Lorentz force); $\Psi_q(\mathbf{r},t) \equiv -G\int \frac{m}{|\mathbf{r}-\mathbf{r}'|}[f(z',t)]^q dz'$ is the gravitation potential satisfying Poisson's equation $\Delta\Psi_q(r) = 4\pi G \int m f^q d^D c$; G is the gravitational constant; τ is the positive parameter, which is interpreted as the characteristic relaxation time of an arbitrary distribution function $f(r,c)$ to the local-Maxwell distribution $f^{(0)}(r,c)$ (in order of magnitude, τ coincides with the mean free path time of particles in the system). In the case $q > 1$, the equilibrium distribution $f^{(0)}(r,c)$, is determined by the following formula (see, for example, Kolesnichenko, 2019)

$$f^{(0)}(r,c) = \left\{c_{q,D}\frac{\rho_q}{m}\left(\frac{m}{2\pi kT}\right)^{D/2}\right\}^{1/q} \times \left\{1 - (1-q)\frac{m(c-u_q)^2}{2kT}\right\}^{1/(1-q)}, \tag{10}$$

where $c_{q,D} = \dfrac{(1-q)^{D/2}\Gamma\left(\frac{q}{1-q}\right)}{\Gamma\left(\frac{q}{1-q} - \frac{D}{2}\right)}$; $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$ is the gamma function.

When using the method of moments, the following hydrodynamic equations were obtained, which are a generalization of the usual Navier–Stokes hydrodynamic equations for nonextensive systems:

$$\frac{\partial \rho_q}{\partial t} + \mathrm{div}(\rho_q \mathbf{u}_q) = 0, \tag{11}$$

$$\frac{\partial(\rho_q u_q)}{\partial t} + \mathrm{div}(\mathbf{P}_q + \rho_q u_q u_q) = n_q f - \rho_q \mathrm{grad}\psi_q, \tag{12}$$

$$\frac{\partial(\rho_q \varepsilon_q)}{\partial t} + \mathrm{div}\{\mathbf{J}_q + \rho_q \varepsilon_q u_q\} + \mathbf{P}_q : \mathrm{Grad} u_q = 0. \tag{13}$$

In general, Eqs. (11)–(13) are not closed, since there is no necessary connection (defining relations) between the flow quantities $\mathbf{P}_q(r,t)$ and $\mathbf{J}_q(r,t)$, and

[2] In the cited paper, the kinetic theory was based on the Bhatnagar-Gross-Krook collision integral (BGK), which was generalized for an arbitrary value of parameter q.

the flow scalar characteristics $\rho_q(r,t)$, $u_q(r,t)$ and $T(r,t)$. This relationship can be found by solving model kinetic equation (9) with the help of the Chapman-Enskog method using the general asymptotic expansion of the distribution function with respect to the Knudsen number. When applying this method, the required relations were found closing the system (11)–(13) (Kolesnichenko and Chetverushkin, 2013). In particular, for the case of a zero-order approximation, when we have the distribution $f(r,t) \equiv f^{(0)}(r,t)$ (i.e., a generalized locally Maxwell distribution (10)), it was shown that the stress tensor $\mathbf{P}_q(r,t)$ reduces to the spherical tensor $\mathbf{P}_q^{(0)} \equiv p_q\mathbf{I}$, and the heat flux $\mathbf{J}_q = 0$. In this case, the internal energy $\varepsilon_q(r,t)$ and the hydrostatic pressure $p_q(r,t)$ are defined by the expressions

$$\varepsilon_q(r,t) = \frac{DkT}{2m}\left[1+(1-q)\,D/2\right]^{-1}, \tag{14}$$

$$p_q(r,t) = \frac{\rho_q kT}{m\left[1+(1-q)\,D/2\right]} = \frac{2}{D}\rho_q\varepsilon_q. \tag{15}$$

Since the concept of temperature in the q-statistics is rather arbitrary (it depends on the definition of temperature from the point of view of Lagrange multipliers), then we will interpret the value $T_{\rm eff}(r,t) \equiv T/\left[1+(q-1)\,D/2\right]$ as the generalized temperature of a complex nonadditive system. Naturally, this temperature is fundamentally different from the absolute thermodynamic temperature T, which characterizes the intensity of randomization of particles of the system. We note that if we write expression (14) for internal energy through the effective temperature $T_{\rm eff}$, then for ε_q we obtain the relation $\varepsilon_q(r,t) = DkT_{\rm eff}/2m$, which coincides for $q \to 1$ and $D = 3$ with the definition of internal energy in the Boltzmann-Gibbs statistics corresponding to the equal distribution of an ideal gas energy over the degrees of freedom. If the usual concept of temperature $T_{\rm eff}$ is kept, then inequality $\varepsilon_q > 0$ imposes a strict limit on the value of the entropy deformation parameter q: in this case, the entropy index must satisfy inequality $1 < q < 1 + 2/D$.

In the first-order approximation, the following governing equations are valid for the heat flux $\mathbf{J}_q(r,t)$ and viscous stress tensor $T_q(r,t) \equiv \mathbf{P}_q - p_q\mathbf{I}$:

$$\mathbf{J}_q(r,t) = -\lambda_q \,\mathrm{grad}\, T, \tag{16}$$

$$T(r,t) = \mu_q\left(\mathrm{grad}u + (\mathrm{grad}u)^T - \frac{2}{3}I\mathrm{div}u\right), \tag{17}$$

where $\lambda_q(r,t) = \tau\dfrac{kp_q}{m}\dfrac{1+D/2}{1+(1-q)(1+D/2)}$,

$\mu_q(r,t) = \tau p_q = \tau\dfrac{\rho_q kT}{m[1+(1-q)\,D/2]}$ are the heat conductivity and the shear viscosity coefficients, respectively.

EQUATIONS OF q-HYDRODYNAMICS FOR A PROTOPLANETARY CLOUD WITH EQUILIBRIUM RADIATION

Radiation pressure, as a factor of hydrostatic equilibrium, plays a large role in the evolution of many astrophysical objects. For the first time, an analysis of the instability in accretion disks with respect to axisymmetric perturbations was carried out by (Shakura and Sunyaev, 1976), taking into account the radiation pressure. In subsequent works, general polytropic models were treated (Camenzind et al., 1986), nonaxisymmetric disturbances were taken into account (McKee, 1990), sound and epicyclic vibrations (Khoperskov and Khrapov, 1995; Fridman and Khoperskov, 2011), etc.

Next, we will use the q-hydrodynamic system of Eqs. (11)–(15) to simulate the instability of a near-solar protoplanetary cloud (thick disk), the substance of which consists of a mixture of perfect gas and blackbody isotropic radiation with temperature T, propagating in all directions. We assume that the protoplanetary cloud is optically thick and the radiation field distribution is close to equilibrium. We also emphasize that the cloud has mainly axial symmetry, which is a consequence of its rotation around the central star. Further, we will also assume that the cloud is self-gravitating, for which the vertical structure (structure along the axis of rotation) is determined by the balance of pressure forces and the gravity of the disk itself.

Neglecting hydrodynamic dissipative processes and heating of cosmic matter due to dissipation, ionization and excitation processes, the initial system of q-equations, consisting of an analog of the Euler equations and the Poisson equation, has the form[3] (see, for example, Kolesnichenko, 2019):

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\mathbf{u}) = 0, \tag{18}$$

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\mathrm{grad}P - \mathrm{grad}\psi, \tag{19}$$

$$\Delta\psi = 4\pi G\rho, \tag{20}$$

$$\frac{d\varepsilon}{dt} = -\frac{P}{\rho}\mathrm{div}\mathbf{u} + \frac{dQ}{dt}. \tag{21}$$

where $d\mathbf{A}/dt \equiv \partial\mathbf{A}/dt + (\mathbf{u}\cdot\mathrm{grad})\mathbf{A}$ is the full time derivative of the structural quantity $\mathbf{A}(r,t)$.

Here

$$P(r,t) = p_q + p_{\rm rad} \equiv p_q + aT^4/3, \tag{22}$$

[3] In what follows, index "q" for some hydrodynamic and thermodynamic variables will be omitted.

$$\varepsilon(r,t) = \varepsilon_q + \varepsilon_{\rm rad} \equiv \varepsilon_q + aT^4/\rho \tag{23}$$

are, correspondingly, the total pressure and internal energy (per unit of mass) of a mixture of ideal q-gas and blackbody radiation; $\rho dQ/dt = -{\rm div}\mathbf{J}_Q$; $\mathbf{J}_Q$ is the resultant heat flux vector that accounts for, in principle, all thermodynamically reversible processes, which can carry away heat from the element of the system during its motion; $\varepsilon_{\rm rad}(r,t) = aT^4/\rho$ is the blackbody radiation energy per unit mass; $\varepsilon_q(r,t) = c_{vq}T(r,t) = \frac{D}{2+(1-q)D}\frac{kT(r,t)}{m}$ is the internal energy (per unit of mass of the gas component of a protoplanetary disk); $p_{\rm rad}(r,t) \equiv aT^4/3$ is the radiation pressure; $p_q(r,t) = \frac{2}{2+(1-q)D}\frac{k}{m}T\rho = \frac{2}{D}\rho\varepsilon_q$ is the gas pressure in the nonextensive disk system (analog of the equation of state in the kinetic theory of ideal gases); $T(r,t)$ is the absolute temperature; a is the Stefan-Boltzmann constant of radiation; $\psi(r,t) = -G\int_V \frac{\rho(r',t)}{|r-r'|}dr'$ is the gravitational potential, being a solution of Poisson's equation (8) (the integral here is taken over the entire volume V, occupied by the protoplanetary cloud); $c_{vq} = \frac{D}{2+(1-q)D}\frac{k}{m}$ is the specific isochoric heat capacity of the gas component of the mixture. We also define the adiabatic index of the gas substance of the disk as the ratio $\gamma_q \equiv \gamma_{\rm gas} = c_{pq}/c_{vq}$. Then $\gamma_q \equiv \gamma_{\rm gas} = 2 - q + 2/D$, $\gamma_1 = (2+D)/D$.

It is convenient to rewrite equation (21) for the total internal energy using continuity equation (18) in the usual form of the first law of thermodynamics

$$\frac{dQ}{dt} = \frac{d\varepsilon}{dt} + P\frac{dv}{dt}, \tag{21.1}$$

or in the form of the Gibbs relationship

$$TdS/dt \equiv dQ/dt = d\varepsilon/dt + Pdv/dt, \tag{24}$$

which expresses the rate of change of entropy $S(r,t)$ (per unit mass) of the disk substance and radiation when an element of the medium is moving along its trajectory. Here $v(r,t) = 1/\rho$ is the specific volume.

Isentropic Changes in the Medium Containing q-Gas and Radiation

Next, we will consider such motions of cosmic matter (in an ideal gas state) and blackbody radiation for which the entropy of each particle of the medium, in the first approximation, remains constant throughout the entire path of the particle, i.e., $dS/dt \equiv \partial S/\partial t + \mathbf{u}\cdot{\rm grad}S = 0$. Such reversible and adiabatic movements are isentropic. For them, Eq. (21) for the energy reduces to the form

$$\rho d\varepsilon/dt + P{\rm div}\mathbf{u} = 0, \tag{25}$$

which means that the rate of change in the total internal energy of a moving element of the medium is equal to the work of compression of this element performed by the surrounding medium.

However, for astrophysical purposes it is often convenient to use other forms of Eq. (25) (which were first obtained by Eddington (Eddington, 1988) and Chandrasekhar (Chandrasekhar, 1950)). These forms are valid when the pressure $P(r,t)$ and the internal energy $\varepsilon(r,t)$ can be calculated from the corresponding equations of state as functions of specific volume $v(r,t)$ and temperature $T(r,t)$ (or entropy $S(r,t)$) depending on the process under study. For a "slow" process characterized by a time that is much longer than the heat transfer time, any disturbances in the temperature profile will have time to relax. Therefore, this process can be considered as isothermal, in which $P = P(v,T_0) = P(v)$. The "fast" process (compared to the heat transfer process) can be considered adiabatic due to the lack of time for heat exchange between two neighboring areas: $S = S_0 = {\rm const}$ and $P = P(v,S_0) = P(v)$.

From Eq. (25) for the energy of a quasi-static process it follows

$$\begin{aligned}&\left(\frac{\partial\varepsilon}{\partial T}\right)_v dT + \left(\frac{\partial\varepsilon}{\partial v}\right)_T dv + Pdv\\ &= \frac{v}{T}\left(12p_{\rm rad} + \frac{c_{vq}}{c_{pq}-c_{vq}}p_q\right)dT\\ &\quad + (4p_{\rm rad} + p_q)dv.\end{aligned} \tag{26}$$

Therefore, for isentropic changes, we have

$$\begin{aligned}&\left(12p_{\rm rad} + \frac{1}{\gamma_q - 1}p_q\right)d\ln T\\ &+ (4p_{\rm rad} + p_q)d\ln v = 0.\end{aligned} \tag{27}$$

We now introduce the adiabatic exponents of the mixture of matter and radiation Γ_1, Γ_2, and Γ_3 by the relations

$$\frac{d}{dt}\ln P = \Gamma_1\frac{d}{dt}\ln\rho, \tag{28}$$

$$\frac{d}{dt}\ln T = (\Gamma_3 - 1)\frac{d}{dt}\ln\rho = \frac{\Gamma_2 - 1}{\Gamma_2}\frac{d}{dt}\ln P, \tag{29}$$

which can be used instead of energy Eq. (25). Taking into account equation of state (15) of an ideal gas, we can write

$$\begin{aligned}dP &= d(p_{\rm rad} + p_q)\\ &= (4p_{\rm rad} + p_q)d\ln T - p_q d\ln v.\end{aligned} \tag{30}$$

Therefore, Eq. (28) is nothing but

$$(4p_{\rm rad} + p_q)d\ln T + [\Gamma_1(p_{\rm rad} + p_q) - p_q]d\ln v = 0. \tag{31}$$

From (27) and (31) it follows that

$$\frac{12p_{\rm rad} + (\gamma_q - 1)^{-1}p_q}{4p_{\rm rad} + p_q} = \frac{4p_{\rm rad} + p_q}{\Gamma_1(p_{\rm rad} + p_q) - p_q}. \tag{32}$$

We now introduce parameter $\beta = p_q/P$, characterizing the gas fraction in the mixture[4]. Using this parameter, relation (32) can be rewritten in the form:

$$\Gamma_1 = \beta + \frac{(4 - 3\beta)^2(\gamma_q - 1)}{\beta + 12(\gamma_q - 1)(1 - \beta)}, \quad (\gamma_q - 1 = 1 - q + 2/D). \tag{33}$$

It can be easily shown that the following relations take place

$$\Gamma_2 = \frac{(4 - 3\beta)\Gamma_1}{\beta + 3(1 - \beta)\Gamma_1} = 1 + \frac{(4 - 3\beta)(\gamma_q - 1)}{3(\gamma_q - 1)(1 - \beta)(4 + \beta)}, \tag{33.1}$$

$$\Gamma_3 = 1 + \frac{\Gamma_1 - \beta}{4 - 3\beta} = 1 + \frac{\Gamma_1(\Gamma_2 - 1)}{\Gamma_2} = 1 + \frac{(4 - 3\beta)(\gamma_q - 1)}{\beta + 12(\gamma_q - 1)(1 - \beta)}. \tag{33.2}$$

If $p_{\rm rad} \ll p_q$, then all generalized exponents of adiabat Γ for "q-gas + radiation" system coincide with adiabatic exponent $(\gamma_q = 2/D + 2 - q)$, of a pure q-gas, while in the case of only the blackbody radiation, $p_q \ll p_{\rm rad}$, these exponents are equal to $4/3$. Thus, for a mixture of perfect q-gas and radiation, the generalized adiabatic exponents take intermediate values between $4/3$ and γ_q.

JEANS' GRAVITATIONAL INSTABILITY IN NONEXTENSIVE KINETIC THEORY

Let us now consider the simplest problem of instability in an infinite resting spherically homogeneous medium. We recall that when considering gravitational instability, Jeans considered a homogeneous state of a self-gravitating gas medium at rest, which is not entirely correct, since such a state is not an equilibrium state. Nevertheless, his derivation of the criterion of instability can be considered as a first approximation, which in the simplest cases gives the correct order of the lower critical wavelength of the perturbation leading to instability (see, for example, Safronov, 1969; Fridman and Khoperskov, 2011).

In the case of purely radial spherically symmetric motion, provided that the unperturbed state is equilibrium ($u = u_0 + u'$, $u_0 = 0$) and Poisson's equation (20) can be applied only to density perturbations (the condition $\psi_0 \cong 0$ is sometimes called "Jeans' fraud" (see Jeans, 1902, 2009)), the linearized main differential equations (18)–(21) have the form:

$$\frac{\partial\rho'}{\partial t} + \frac{\partial\rho_0 u}{\partial r} = 0, \tag{34}$$

$$\frac{\partial u}{\partial t} = \frac{1}{\rho_0}\frac{\partial P'}{\partial r} - \frac{\rho'}{\rho_0^2}\frac{\partial P_0}{\partial r} - \frac{\partial\psi'}{\partial r}, \tag{35}$$

$$d(P'/P_0)/dt = \Gamma_{1,0}d(\rho'/\rho_0)/dt, \tag{36}$$

$$\frac{\partial^2\psi'}{\partial r^2} = 4\pi G\rho'. \tag{37}$$

Hereinafter, index "0" refers to the unperturbed quantities.

Equation (36) is trivially integrable. Choosing the integration constant so that $P' = 0$ at $\rho' = 0$, we obtain

$$P'/P_0) = \Gamma_{1,0}\rho'/\rho_0. \tag{38}$$

We suppose now that the characteristic length associated with spatial changes in quantities P_0 and ρ_0 is large compared to other characteristic lengths of the problem (this is the so-called short-wave acoustics approximation), i.e., one can neglect derivatives $\partial P_0/\partial r$ and $\partial\rho_0/\partial r$. Under these additional simplifying assumptions, the equation of continuity, momentum, and energy can easily be combined into one equation for an adiabatic sound wave[5] (see, for example, Landau and Lifshitz, 1976)

$$\frac{\partial^2\rho'}{\partial t^2} + v_{S,q}^2\frac{\partial^2\rho'}{\partial r^2} - 4\pi G\rho_0\rho' = 0. \tag{39}$$

Here, the perturbed derivative of pressure $\partial P'/\partial r$ is expressed, according to (38), through the perturbed derivative of density $\partial\rho'/\partial r$ in the form $\partial P'/\partial r = (\Gamma_{1,0}P_0/\rho_0)\partial\rho'/\partial r = v_{S,q}^2\partial\rho'/\partial r$, where

$$v_{S,q} \equiv \sqrt{\Gamma_{1,0}\frac{P_0}{\rho_0}} = \left\{\frac{p_{q0}}{\rho_0}\left[1 + (\Gamma_{3,0} - 1)\frac{4 - 3\beta_0}{\beta_0}\right]\right\}^{\frac{1}{2}} \tag{40}$$

$$= \left\{\frac{1}{(\gamma_q - 1)D/2}\frac{kT_0}{m}\left[1 + \frac{(4 - 3\beta_0)^2(\gamma_q - 1)}{\beta_0^2 + 12\beta_0(\gamma_q - 1)(1 - \beta_0)}\right]\right\}^{\frac{1}{2}}$$

[4] Eddington first pointed out the special importance of quantity $(1 - \beta)$ for the theory of stellar structure. In a famous passage from his book "The Internal Structure of the Stars", Eddington associated this quantity with the "happening of the stars."

[5] When studying perturbed states of self-gravitating cosmic matter, one often has to deal with some form of sound waves.

is the adiabatic (or Laplace) velocity of sound in the nonextensive radiation hydrodynamics. When writing (40), it is taken into account that

$$\frac{P_0}{\rho_0} = \frac{p_{q,0} + p_{\text{rad},0}}{\rho_0} = \frac{1}{\beta_0}\frac{p_{q,0}}{\rho_0} = \frac{1}{\beta_0}\frac{1}{(\gamma_q - 1)D/2}\frac{kT_0}{m} = \frac{1}{\beta_0}\frac{1}{1+(1-q)D/2}\frac{kT_0}{m}. \quad (41)$$

We note that in the particular case when $q = 1$ and $D = 3$, we have $\gamma_1 = 5/3$ (classic perfect monatomic gas). Whence it follows that

$$v_{S,1} \equiv \left\{\frac{k}{m}T_0\left[1 + \frac{2(4-3\beta_0)^2}{3\beta_0(8-7\beta_0)}\right]\right\}^{\frac{1}{2}}. \quad (40.1)$$

When radiation is also absent, then $(v_{S,1})_{\beta_0=1} \equiv v_{\text{gas},1} = \sqrt{\gamma_1 kT_0/m}$ is the adiabatic sound velocity in an ideal gas.

If $q \neq 1$ (perfect q-gas) and radiation is absent ($\beta_0 = 1$), then

$$(v_{S,q})_{\beta_0=1} = \left[\frac{kT_0}{m}\frac{2\gamma_q}{(\gamma_q-1)D}\right]^{\frac{1}{2}} = \left[\frac{kT_0}{m}\frac{2-q+2/D}{(1-q)D/2+1}\right]^{\frac{1}{2}}. \quad (40.2)$$

Equation (39) is a linear and homogeneous partial differential equation; therefore, the method of normal oscillations (mode method) is applicable to it. By solving Eq. (39) for the perturbed density in the form $\rho' \sim \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$, describing the waves with angular frequency ω and wave vector $\mathbf{k}$ in direction r[6], and wavelength $\lambda_r = 2\pi/k$, we obtain the dispersion equation for propagating sound wave

$$\omega^2 - k^2\frac{p_{q,0}}{\rho_0}\left\{1 + \frac{\Gamma_{1,0}-\beta_0}{4-3\beta_0}\left(1 + 4\frac{1-\beta_0}{\beta_0}\right)\right\} + 4\pi G\rho_0 = 0, \quad (42)$$

which with allowance for (40) and (41) take the “standard” form

$$\omega^2 = k^2 v_{S,q}^2 - 4\pi G\rho_0. \quad (42.1)$$

Here the adiabatic sound velocity $v_{S,0}$ is determined by formula (40).

For stable waves with frequencies ω we have $\omega^2 > 0$, whereas the instability corresponds to the condition $\omega^2 < 0$. These two types are discriminated by the case of neutral stability $\omega^2 = 0$, which corresponds to the modes with the critical wavelength of the perturbation

$$\lambda_{\text{cr}} = 2\pi/k_{\text{cr}}, \quad k_{\text{cr}}^2 = \omega_{\text{cr}}^2/v_{Sq}^2, \quad \omega_{\text{cr}}^2 = 4\pi G\rho_0. \quad (43)$$

From (42.1) it follows that the boundary value $k = k_{\text{cr}}$ separates the stable ($k > k_{\text{cr}}$) and unstable ($k < k_{\text{cr}}$)density oscillations. At small k (long waves) the oscillations will increase with time and the Jeans instability appears, whereas the density oscillations with small wavelength (large k and small wavelength) are oscillating and propagate in the form of sound waves.

Therefore, the critical wavelength of the perturbation

$$\lambda_{\text{cr}} = \frac{2\pi v_{Sq}}{\omega_{\text{cr}}} = \sqrt{\frac{\pi v_{Sq}^2}{G\rho_0}} \equiv \left\{\frac{2\pi kT_0}{mG\rho_0 D}\left[\frac{1}{\gamma_q-1} + \frac{(4-3\beta_0)^2}{\beta_0^2 + 12\beta_0(\gamma_q-1)(1-\beta_0)}\right]\right\}^{\frac{1}{2}}. \quad (44)$$

is the size of the smallest “droplets” of “fractal” gas matter with radiation that can be held together by its own gravitational attraction. Therefore, the Jeans instability criterion for a mixture of q-gas and blackbody radiation, modified in the framework of nonextensive kinetic theory, will look as follows: the length λ_r of unstable perturbation wave must satisfy inequality

$$\lambda_r \geq \lambda_{\text{cr}} = v_{Sq}\sqrt{\frac{\pi}{G\rho_0}} \equiv \left\{\frac{\pi kT_0}{mG\rho_0}\frac{2}{(\gamma_q-1)D} \times\left[1 + \frac{(4-3\beta_0)^2(\gamma_q-1)}{\beta_0^2 + 12\beta_0(\gamma_q-1)(1-\beta_0)}\right]\right\}^{\frac{1}{2}}. \quad (45)$$

In conventional literature, the length

$$\lambda_{\text{J}} = \sqrt{\frac{\pi v_{\text{gas}}^2}{G\rho_0}} = \left(\gamma_1\frac{\pi kT_0}{mG\rho_0}\right)^{\frac{1}{2}}, \quad (46)$$

corresponding to the compression region of a self-gravitating ideal gas is called the Jeans length. With allowance for (45), the Jeans instability criterion in nonextensive kinetics can be rewritten in the form:

$$\frac{\lambda_r}{\lambda_{\text{J}}} \geq \frac{v_{Sq}}{v_{\text{gas}}} = \left\{\frac{1}{\gamma_1}\frac{2}{(\gamma_q-1)D} \times\left[1 + \frac{(4-3\beta_0)^2(\gamma_q-1)}{\beta_0^2 + 12\beta_0(\gamma_q-1)(1-\beta_0)}\right]\right\}^{\frac{1}{2}} = \left\{\frac{1}{\gamma_1}\frac{2/D}{(1-q+2/D)} \times\left[1 + \frac{(4-3\beta_0)^2(1-q+2/D)}{\beta_0^2 + 12\beta_0(1-q+2/D)(1-\beta_0)}\right]\right\}^{\frac{1}{2}} \equiv \Xi_q. \quad (45.1)$$

[6] It should be noted that the linearized equation of momentum requires that the velocity u be parallel to the wave vector $\pm\mathbf{k}$ (see Landau, Lifshitz, 1976). Consequently, the velocities of the fluid particles associated with adiabatic sound waves are parallel to the direction of wave propagation.

Whence it follows:

(1) If $q = 1$ (in this case $\gamma_1 = 1 + 2/D$), then we have

$$\Xi_1 \equiv \left[\frac{1}{\gamma_1}\left(1 + \frac{(4-3\beta_0)^2\, 2/D}{\beta_0^2 + 24\beta_0(1-\beta_0)/D}\right)\right]^{\frac{1}{2}} > 1. \quad (47)$$

Therefore, the critical wavelength of perturbation λ_r in the considered case is higher than the Jeans wavelength λ_J, i.e., the cloud matter is stabilized due to radiation pressure, and the equality meets the condition of limiting stability.

(2) If $q \neq 1$, but radiation is absent, $\beta_0 = 1$, then we have

$$\Xi_q = \left[\frac{1}{\gamma_1}\left(2/D + \frac{2/D}{1-q+2/D}\right)\right]^{\frac{1}{2}}, \quad 0 < q < 1 + 2/D. \quad (48)$$

In this case, the criterion of gravitational instability depends on the numerical values of the entropy strain index q and dimensionality D of the velocity space. In this case, a situation is possible in which a gravitationally stable (based on the classical Boltzmann-Gibbs statistics) gas cloud will be unstable according to nonextensive statistics by Tsallis (see Kolesnichenko and Marov, 2014, 2016).

The critical mass associated with λ_{cr} (the mass contained inside a sphere of diameter λ_{cr}) is determined by the relation

$$M_{cr} = (\pi/6)\rho_0\lambda_{cr}^3 = M_J\Xi^3, \quad (49)$$

where $M_J \equiv (\pi/6)\rho_0\lambda_J^3 = (\pi/6)\rho_0\left(\gamma_1\pi k T_0/mG\rho_0\right)^{3/2}$ is the critical Jeans mass. Perturbations with a mass M_r exceeding the critical Jeans mass ($M_J(\Xi > 1)$ can grow, forming gravitationally limited structures, while perturbations with a mass M_r less than M_J do not grow and behave like acoustic waves. In this case, for self-gravitating nonextensive media with radiation, the critical values of the wavelength and mass clearly depend on the entropy index q, dimensionality D of the velocity space, and coefficient β, which, being free parameters, should be determined empirically from experimental data in each case. This allows one to more correctly simulate a real situation when studying the instability of self-gravitating space objects in the framework of nonextensive statistics.

Note that the further development of the approach proposed here can be related with account of the influence of medium rotation, magnetic field, viscosity, and other dissipative effects on the Jeans instability.

STABILITY INTEGRAL CONDITION FOR A SPHERICALLY SYMMETRIC DISTRIBUTION OF MATTER AND RADIATION IN TSALLIS' NONEXTENSIVE KINETICS

In this section, we will proceed from the hypothesis by Hoyle (1960) about the joint formation of a star (the Sun) and a protoplanetary cloud from the matter of a single rotating stellar nebula. There is an integral theorem for a spherical configuration of a nebula in gravitational equilibrium of matter (perfect gas) and blackbody radiation (see Chandrasekhar, 1950; Theorem 6 on p. 111). The theorem states that the pressure P_{ce} at the center of attraction of the gravitating cloud with a mass M, at which the density $\rho(r)$ at a point located a distance r from the center does not exceed the average density $\bar{\rho}(r)$ of the inner part with radius r, must satisfy the inequality

$$\frac{1}{2}G\left(\frac{4\pi}{3}\right)^{1/3}\bar{\rho}^{4/3}M^{2/3} \le P_{ce} \le \frac{1}{2}G\left(\frac{4\pi}{3}\right)^{1/3}\rho_{ce}^{4/3}M^{2/3}. \quad (50)$$

Here $\bar{\rho}$, ρ_{ce} are the corresponding average density of the cloud and its density in the center. This means that the pressure acting in the center of the cloud of mass M must be intermediate between the pressures at the centers of two configurations with a uniform density—one with a density equal to the mean density $\bar{\rho}$ of the cloud, and the other with a density equal to the density ρ_{ce} in its center. In the case where there are some areas in which opposite density gradients predominate, inequality (50) is violated, and this indicates the instability. Thus, we can assume that inequality (50) is equivalent to the integral stability condition for "maternal" star nebula.

We now obtain a generalization of this stability condition to the case of a nonextensive spherical gas mass with radiation. Using the definition of parameter β and the equation of state for the radiation pressure, as well as formula (15) for the q-gas pressure, we obtain

$$P = \frac{1}{\beta}\frac{1}{1+(1-q)D/2}\frac{k}{m}T\rho = \frac{1}{1-\beta}\frac{1}{3}aT^4. \quad (51)$$

Whence it follows that

$$T = \left[\frac{3(1-\beta)}{a\beta}p_q\right]^{1/4} = \left[\frac{3}{a}\frac{1}{1+(1-q)D/2}\frac{(1-\beta)}{\beta}\right]^{1/3}\rho^{1/3}. \quad (52)$$

Then

$$P = \left[\left(\frac{k}{m}\right)^4\frac{3(1-\beta)}{a\beta^4}\right]^{1/3}\left[\frac{1}{1+(1-q)D/2}\right]^{4/3}\rho^{4/3}. \quad (53)$$

Consequently, in the center of the gas sphere

$$P_{ce} = \left[\left(\frac{k}{m}\right)^4 \frac{3(1-\beta_{ce})}{a\beta_{ce}^4}\right]^{1/3} \times \left[\frac{1}{1+(1-q)\,D/2}\right]^{4/3} \rho_{ce}^{4/3}. \tag{54}$$

On the other hand, according to inequality (50), we have

$$P_{ce} \le \frac{1}{2} G \left(\frac{4\pi}{3}\right)^{1/3} M^{2/3} \rho_{ce}^{4/3}. \tag{55}$$

Comparing (55) and (50), we obtain:

$$\left[\left(\frac{k}{m}\right)^4 \frac{3(1-\beta_{ce})}{a\beta_{ce}^4}\right]^{1/3} \left[\frac{2}{2+(1-q)D}\right]^{4/3} \le \left(\frac{\pi}{6}\right)^{1/3} G M^{2/3}, \tag{56}$$

or

$$M \ge \left[\frac{2}{2+(1-q)D}\right]^2 \left(\frac{6}{\pi}\right)^{1/2} \left(\frac{k}{m}\right)^2 \times \left[\frac{3}{a}\frac{(1-\beta_{ce})}{\beta_{ce}^4}\right]^{1/2} G^{-3/2}. \tag{57}$$

In the previous inequalities, β_{ce} is the quantity β in the center of the gas sphere.

Substituting the numerical value of the Stefan-Boltzmann constant $a = 8\pi^5 k^4 / 15 h^3 c^3$ into (57) (here h is the Planck constant, c is the speed of light in vacuum), we have:

$$\mu^2 M \left[\frac{2+(1-q)D}{2}\right]^2 \times \left(\frac{\beta_{ce}^4}{1-\beta_{ce}}\right)^{1/2} \ge 0.1873 \left(\frac{hc}{G}\right)^{3/2} m_H^{-2} \cong 5.48 M_\odot. \tag{58}$$

Here we used the relations: $m = \mu m_H$, where μ is the average molecular weight, m_H is the mass of the hydrogen atom; $M_\odot$ is the solar mass; $(hc/G)^{3/2} m_H^{-2} \approx 29.2 M_\odot$. Then the right-hand side of the inequality

$$M \ge 5.48 M_\odot \mu^{-2} \left(\frac{1-\beta_{ce}}{\beta_{ce}^4}\right)^{1/2} \left[\frac{2}{2+(1-q)D}\right]^2, \quad (0 < q < 1 + 2/D) \tag{59}$$

gives (within the framework of nonextensive Tsallis kinetics) the lower stability limit of the gravitating cloud (spherical gas configuration) with mass M.

Note that if we introduce the Chandrasekhar parameter β_*, which is uniquely determined by the mass M of the gas configuration and the average molecular weight μ using the fourth-order equation[7] (see Chandrasekhar, 1950, 1985)

$$\mu^2 M \equiv \left(\frac{6}{\pi}\right)^{\frac{1}{2}} \left[\left(\frac{k}{m_H}\right)^4 \frac{3}{a} \frac{(1-\beta_*)}{\beta_*^4}\right]^{\frac{1}{2}} G^{-3/2} = 5.48 \left(\frac{(1-\beta_*)}{\beta_*^4}\right)^{\frac{1}{2}} M_\odot, \tag{60}$$

then inequality (59) can be written in the form

$$\frac{(1-\beta_*)}{\beta_*^4} \ge \frac{1-\beta_{ce}}{\beta_{ce}^4} \left[\frac{2}{2+(1-q)D}\right]^4, \quad (0 < q < 1 + D/2),$$

or, since the function $(1-\beta)\beta^{-4}$ is monotonically increasing with increasing $(1-\beta)$, as follows:

$$(1-\beta_{ce})\left[1+(1-q)\,D/2\right]^{-4} \le (1-\beta_*). \tag{61}$$

Thus, for the stability of nonextensive spherical gas cloud with radiation, the numerical values of parameters β_{ce}, D and q must satisfy inequality (61).

JEANS INSTABILITY OF A ROTATING PROTOPLANETARY CLOUD WITH RADIATION IN TSALLIS' KINETICS

Since rotation is a very common phenomenon in the Universe, the question arises: how does rotation affect the Jeans gravitational instability? In this regard, we consider in a simplified formulation the problem of the Coriolis force effect on the gravitational instability of a nonextensive gaseous medium of a protoplanetary radiating cloud.

For simplicity, we will assume that the self-gravitating cloud rotates uniformly around the i_z axis with constant angular velocity $\mathbf{\Omega} = (0, 0, \Omega)$, and in the direction of the i_x axis there is a q-gas flow with velocity $\mathbf{u}_0 = (U, 0, 0)$[8]. In this case, the following changes should be made in the original system of q-hydrodynamic equations (18)–(21): on the right-hand side of equation of motion (19), an additional term $2\mathbf{u} \times \mathbf{\Omega}$ appears, associated with the Coriolis force, and it is convenient to write equation of energy (21) in the form (29)

$$d \ln T / dt = (\Gamma_3 - 1) d \ln \rho / dt.$$

Then, the linearized equations (18)–(21), obtained under the condition that in the unperturbed state of the cloud there is a uniform gas flow $\mathbf{u}_0 \equiv \mathbf{i}_x U$ (where

[7] In particular, it follows from (60) that for a star with a mass equal to the solar mass and with an average molecular weight equal to unity, the radiation pressure in the center of the star cannot exceed three percent of the total pressure, i.e., $1 - \beta_* \cong 0.03$ (Chandrasekhar, 1985).

$U = \text{const}$), and Poisson's equation (20) is applicable only to density perturbations, have the form

$$\frac{\partial \rho'}{\partial t} + U\frac{\partial \rho'}{\partial x} + \rho_0 \mathrm{div}\mathbf{u}' = 0, \tag{62}$$

$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{u}_0 \cdot \mathrm{grad})\mathbf{u}' - 2\mathbf{u}' \times \mathbf{\Omega} + \frac{\beta_0 P_0}{\rho_0} \times \left\{ \frac{4-3\beta_0}{\beta_0} \mathrm{grad}\left(\frac{T'}{T_0}\right) + \mathrm{grad}\left(\frac{\rho'}{\rho_0}\right) \right\} + \mathrm{grad}\psi' = 0, \tag{63}$$

$$\Delta\psi' = 4\pi G\rho', \tag{64}$$

$$\frac{dT'}{dt} = (\Gamma_{3,0} - 1)\frac{T_0}{\rho_0}\frac{d\rho'}{dt}, \tag{65}$$

where

$$2\mathbf{u}' \times \mathbf{\Omega} = \{2v'\mathbf{\Omega}, -2\mathbf{u}'\mathbf{\Omega}, 0\}. \tag{66}$$

Here the quantities ρ_0, T_0, P_0, $\mathbf{u}_0$ and β_0 describe a certain stationary solution of system (18)–(21), while the values ρ', T', $\mathbf{u}'(= \mathbf{i}_x\mathbf{u}' + \mathbf{i}_y v' + \mathbf{i}_z w')$ and ψ' are small perturbations of hydrodynamic parameters weakly violating an unperturbed state. The system of equations (62)–(65) describes the development of small adiabatic perturbations in a fractal gas medium with radiation compared to the main solution in space and time. It is a system of linear and homogeneous partial differential equations. Therefore, the method of normal oscillations (mode method) is applicable to it. Let us represent all the perturbed hydrodynamic parameters ξ in the form

$$\xi \sim \exp[-i\omega t + i(k_x x + k_y y + k_z z)], \tag{67}$$

describing a set of perturbation waves with angular frequency ω and real wave number $\mathbf{k} = \{k_x, k_y, k_z\}$, the components of which are directed along the i_x, i_y and i_z axes. Substituting them into system (62)–(67) and using relation

$$\rho_0 T' = (\Gamma_{3,0} - 1)T_0\rho' \tag{68}$$

(consequence of Eq. (65)), we obtain as a result the system of linear algebraic equations with respect to small perturbations of hydrodynamic parameters:

$$(-\omega + k_x U)\rho' + \rho_0 \mathbf{k} \cdot \mathbf{u}' = 0, \tag{69}$$

$$i\mathbf{u}'(-\omega + k_x U) - 2\mathbf{u}'\times\mathbf{\Omega} + i\mathbf{k}\left\{ \frac{\rho'}{\rho_0}\frac{\beta_0 P_0}{\rho_0}\left[\frac{4-3\beta_0}{\beta_0}(\Gamma_{3,0}-1)+1\right] + \psi' \right\} = 0, \tag{70}$$

$$k^2\psi' + 4\pi G\rho' = 0. \tag{71}$$

The following algebraic relation follows from (69)–(71):

$$i(-\omega + k_x U)\mathbf{u}' - 2\mathbf{u}' \times \mathbf{\Omega} + i\frac{k^2 v_{S,q}^2}{\omega - k_x U}\mathbf{k}\frac{\mathbf{k}\cdot\mathbf{u}'}{k^2} - i\frac{4\pi G\rho_0}{\omega - k_x U}\mathbf{k}\frac{\mathbf{k}\cdot\mathbf{u}'}{k^2} = 0, \tag{72}$$

or

$$(\omega - k_x U)^2\mathbf{u}' - i2(\mathbf{u}' \times \mathbf{\Omega})(\omega - k_x U) - \mathbf{k}^2 v_{S,q}^2 \mathbf{u}' + 4\pi G\rho_0 \mathbf{u}' = 0. \tag{73}$$

Here $v_{S,q} = \left\{ \frac{p_{q0}}{\rho_0}\left[\frac{4-3\beta_0}{\beta_0}(\Gamma_{3,0}-1)+1\right]\right\}^{\frac{1}{2}}$ is the adiabatic velocity of sound in nonextensive gas medium with radiation. Note that, when writing (73), vector identity $\mathbf{u}' \equiv \mathbf{k}(\mathbf{k}\cdot\mathbf{u}')/k^2 + k^{-2}\mathbf{k}\times(\mathbf{u}'\times\mathbf{k})$ was used (Kochin, 1961), which for longitudinal sound waves in liquid (see footnote no. 6) takes the form: $\mathbf{u}' = \mathbf{k}(\mathbf{k}\cdot\mathbf{u}')/k^2$.

Let us analyze relationship (73).

(1) Assume that equilibrium self-gravitating gas cloud does not rotate ($U = 0$ and $\Omega = 0$), then (73) coincides with dispersion relation (42.2), from which it follows the above-considered Jeans instability criterion (46) for a stationary uniform cloud with radiation in the case of nonextensive kinetics.

(2) If $U = \text{const}$ and $\Omega = 0$, then from (73) we obtain the dispersion relation of the form:

$$(-\omega + k_x U)^2 = \mathbf{k}^2 v_{S,q}^2 - 4\pi G\rho_0. \tag{74}$$

From this relation it follows that a uniform flow of matter has a destabilizing effect on the stability of the gas cloud, contributing to an increase in the critical value of the Jeans wave number $\mathbf{k}_J$ (Radwan, 2004).

(3) If the velocity of the flow is $U = 0$ and $\Omega \neq 0$ then:

$$\mathbf{u}'(\omega^2 - \mathbf{k}^2 v_{S,q}^2 + 4\pi G\rho_0) = i2(\mathbf{u}' \times \mathbf{\Omega})\omega. \tag{75}$$

When we multiply this ratio scalarly by oscillating velocity $\mathbf{u}'$, we obtain the dispersion relation

$$\omega^2 - \mathbf{k}^2 v_{S,q}^2 + 4\pi G\rho_0 = 0, \tag{76}$$

[8] It is known that the problem of stability of a self-gravitating two-dimensional gas cloud cannot be described, in principle, in the framework of the two-dimensional approximation, since it is a priori very unstable (see, for example, Fridman and Khoperskov, 2011). However, when the angular velocity of rotation is sufficiently high, in the presence of a strong external gravitational field with cylindrical geometry and with a generatrix along the axis of rotation of the cloud, it is possible to ensure its stability. In this case, the structure of the protoplanetary cloud along the axis of rotation will be determined solely by its self-gravity. It is clear that this case is artificial, since such cylindrical fields, if they occur in real astrophysical systems, are without embedded disks. At the same time, the analysis of such a self-gravitating thick gas disk embedded in the cylinder is of certain theoretical interest, since only in this case one can allocate the effects arising under the action of pure gravity. It is precisely such models were studied in most classical works on astrophysical disks (see, for example, Goldreich and Lynden-Bell, 1965; Hunter, 1972; Toomre, 1964).

from which it follows that the Coriolis rotation force does not overcome the stabilizing radiation effect of a self-gravitating cloud, i.e., Jeans' instability criterion (46) considered above also holds for a rotating cloud with radiation.

(4) We now consider the case when a perturbation wave propagates in the xy plane perpendicular to the direction of the cloud rotation axis, $\mathbf{\Omega} = \mathbf{i}_z\mathbf{\Omega}$, i.e., when $\mathbf{u}' \cdot \mathbf{\Omega} = 0$.

Then the algebraic relation stems from (25):

$$\begin{gathered}|\mathbf{u}'|^2(\omega^2 - \mathbf{k}^2 v_{S,q}^2 + 4\pi G\rho_0)^2 \\ = -4\omega^2(\mathbf{u}' \times \mathbf{\Omega})\cdot(\mathbf{u}' \times \mathbf{\Omega}) = 4\omega^2 |\mathbf{u}'|^2 \mathbf{\Omega}^2,\end{gathered} \tag{77}$$

which is written with the use of the vector algebra formula $(\mathbf{a}\times\mathbf{b})\cdot(\mathbf{a}\times\mathbf{b}) = (\mathbf{a}\cdot\mathbf{b})^2 - \mathbf{a}^2\mathbf{b}^2$ (see Kochin, 1961) and the condition $\mathbf{u}' \cdot \mathbf{\Omega} = 0$.

From (77) it follows the dispersion relation

$$\begin{gathered}\omega^4 + 2\omega^2(-k^2 v_{S,q}^2 + 4\pi G\rho_0 - 2\Omega^2) \\ + (k^2 v_{S,q}^2 - 4\pi G\rho_0)^2 = 0.\end{gathered} \tag{78}$$

Let ω_1^2 and ω_2^2 be the roots of equation (78), then

$$\begin{gathered}\omega_1^2 + \omega_2^2 = -2(-\mathbf{k}^2 v_{S,q}^2 + 4\pi G\rho_0 - 2\Omega^2), \\ \omega_1^2\omega_2^2 = (\mathbf{k}^2 v_{S,q}^2 - 4\pi G\rho_0)^2.\end{gathered} \tag{79}$$

Whence it follows that the condition of instability of the cloud $\omega_{1,2}^2 < 0$ for a set of disturbance waves takes the form

$$v_{S,q}^2\mathbf{k}^2 < 4\pi G\rho_0 - 2\Omega^2. \tag{80}$$

In this case, the critical wavelength of perturbation, $\lambda_{\rm cr} = 2\pi/k_{cr}$ and the critical wave number $k_{\rm cr} = |\mathbf{k}|_{\rm cr}$, separating stable ($k_r > k_{\rm cr}$) and unstable ($k_r < k_{\rm cr}$) perturbation waves, are given by relations

$$\begin{aligned}|\mathbf{k}|_{\rm cr} &= \frac{1}{v_{S,q}}\left(4\pi G\rho_0 - 2\Omega^2\right)^{1/2} \\ &= 2\left(\frac{\pi G\rho_0}{v_{S,q}^2}\right)^{1/2}\left(1 - \frac{\Omega^2}{2\pi G\rho_0}\right)^{1/2},\end{aligned} \tag{81}$$

$$\lambda_{\rm cr} = \frac{2\pi}{|\mathbf{k}|_{\rm cr}} = \sqrt{\frac{\pi v_{S,q}^2}{G\rho_0}}\left(1 - \frac{\Omega^2}{2\pi G\rho_0}\right)^{-1/2}. \tag{82}$$

Thus, taking into account radiation for perturbation waves propagating in a direction perpendicular to the direction of the cloud rotation axis, we obtain the following representation for the Jeans instability criterion of a rotating gas cloud:

$$\begin{aligned}\lambda_r > \lambda_{\rm cr} &= v_{S,q}\sqrt{\frac{\pi}{G\rho_0}}\left(1 - \frac{\Omega^2}{2\pi G\rho_0}\right)^{-1/2} \\ &= v_{S,q}\sqrt{\frac{\pi}{G\rho_0}}\left(1 - \frac{\Omega^2}{2\pi G\rho_0}\right)^{-1/2},\end{aligned} \tag{83}$$

which, with allowance for formula (46) for the Jeans length can be written in the form

$$\begin{aligned}\frac{\lambda_r}{\lambda_J} &> \frac{v_{S,q}}{v_{\rm gas}}\left(1 - \frac{\Omega^2}{2\pi G\rho_0}\right)^{-1/2} = \left\{\frac{1}{\gamma_1}\frac{1}{(\gamma_q - 1)D/2}\right. \\ &= \left.\left[1 + \frac{(4 - 3\beta_0)^2(\gamma_q - 1)}{\beta_0^2 + 12\beta_0(\gamma_q - 1)(1 - \beta_0)}\right]\right\}^{1/2} \\ &\times \left(1 - \frac{\Omega^2}{2\pi G\rho_0}\right)^{-1/2}.\end{aligned} \tag{84}$$

It is important to bear in mind that criterion (84) makes sense only in the case when the condition $\Omega^2/2\pi G\rho_0 < 1$ is satisfied (the condition of the rotating cloud stability by Toomre (1964)).

CONCLUSIONS

It is known that dynamic chaos arises with the instability of nonequilibrium systems (in particular, various astrophysical objects), which makes it possible to form more complex ordered (generally fractal) structures. The occurrence of fractal formations is confirmed for many astrophysical systems, in particular, stars, interstellar molecular clouds, accretion protoplanetary disks, etc. When taking into account the effects of a strong gravitational field in models of fractal space structures, fundamental difficulties arise, since traditional gas-dynamic methods are often not applicable to them. Overcoming these difficulties requires a new approach to solving evolutionary problems in space gas dynamics. One of the possible approaches to studying the evolution of such anomalous systems can be based on the methods of nonextensive statistical Tsallis' mechanics. It is this mechanics designed to describe the evolution of fractal systems with strong gravitational interaction (see, for example, Kolesnichenko and Marov, 2013, 2014, 2019).

Bearing in mind the great cosmogonic significance of the problem of gravitational instability, in the present work, within the framework of the Tsallis kinetics, we studied the effect of nonextensivity of the medium on the Jeans gravitational instability criterion for a self-gravitating protoplanetary cloud, the substance of which consists of a mixture of perfect q-gas and blackbody radiation. Dispersion equations are obtained, on the basis of which an analysis of axisymmetric vibrations of protoplanetary self-gravitating clouds is carried out. For simple model systems, such as infinite spherically uniform medium at rest and rotating gas clouds, we obtained the modified Jeans' gravitational instability criteria with allowance for the radiation pressure. In addition, within the framework of nonextensive kinetics, a modified integral Chandrasekhar stability condition is obtained for the spherical mass of a mixture consisting of q-gas and radiation. For these self-gravitating objects, critical values of wavelengths and masses are found that explicitly depend on several free parameters, namely, on the strain index q of entropy, the dimensionality D of the velocity space and coefficient β, which characterizes the fraction of the substance in the total pressure of the mixture. This enables us to simulate real astrophysical objects and to find more justifiably appropriate criteria for their gravitational instability.

An approach in describing the evolution of relatively simple (model) astrophysical formations based on nonextensive kinetics can be extended to more realistic physical situations, in particular, taking into account the dynamics of perturbations in inhomogeneous and anisotropic fractal disk media, to the studies of gravitational perturbations of dissipative disks and natural oscillation frequencies of vertically inhomogeneous magnetic disks, etc. (Fridman and Khoperskov, 2011). Since the physical meaning and numerical values of the entropy deformation index q are very important in understanding the evolution of many anomalous astrophysical objects, the problem of determining q seems to be extremely urgent. Unfortunately, this problem is still open. At the same time, at present, there are serious achievements in modern helioseismology, which reliably explores the internal structure and dynamics of the Sun (see Gough, 2011). Millions of resonant vibrational modes were found and studied in the solar atmosphere. Their frequencies were measured with sufficiently high accuracy, which allows one to study the internal structure of the Sun at large depths (Gough and Hindman, 2010). These results make it possible to solve not only some well-known cosmological problems, but also raise a number of theoretical issues, the answers to which are necessary to understand how an ordinary star actually evolves. In particular, helioseismology enables us, generally speaking, to find experimental evidence for the presence of nonextensive effects in the depths of a star from the determined velocities of sound. Therefore, there is hope that in the very near future it will be possible to obtain astronomical data on the numerical values of parameter q other than unity.

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