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Dynamics of Asteroid 3200 Phaethon Under Overlap of Different Resonances

T. Yu. Galushina[a, *] and G. E. Sambarov[a]
[a] *Tomsk State University, Tomsk, 634050 Russia*
**e-mail: volna@sibmail.com*

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Abstract—The paper is focused on studying the motion of asteroid 3200 Phaethon which approached the Earth in December 2017. We consider the dynamics of asteroid 3200 Phaethon, reveal its encounters with planets, mean motion and secular resonances, and estimate the predictability time and the causes of chaoticity. A peculiar feature in the dynamics of the object is that it passes through the unstable orbital resonance 3/7 with Venus and exhibits a gamut of apsidal–nodal resonances with Mercury, Venus, Earth, Mars, and Jupiter, as well as a large number of close encounters with terrestrial planets. These properties result in a chaotic character of the motion beyond a time interval between the years 1780 and 2350.

Keywords: 3200 Phaethon, asteroid, secular resonances, mean motion resonances, orbital evolution, OMEGNO
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INTRODUCTION

Asteroid 3200 Phaethon is a small asteroid of the Apollo group with a very small (0.14 AU) perihelion distance. This asteroid is interesting because of its unusual, extremely elongated, orbit, due to which its trajectory crosses, in projection to the ecliptic plane, the orbits of all terrestrial planets, from Mercury to Mars (Fig. 1). Since the asteroid moves close to the Sun, it was named after a hero of the Greek myth about Phaethon, who was a son of Helios, the god of the Sun.

3200 Phaethon was the first asteroid to be discovered using images obtained onboard a spacecraft. On October 11, 1983, Simon F. Green and John K. Davies found it in the images received from the Infrared Astronomical Satellite (IRAS). The discovery was formally announced on October 14 in the International Astronomical Union Circular (IAUC) along with the optical confirmation by Charles T. Kowal, who reported the object to be an asteroid in appearance. Its provisional designation was 1983 TB; and later, in 1985, it received the numerical designation and the full name 3200 Phaethon.

Asteroid 3200 Phaethon has attracted the attention of the scientific community for several reasons. In the main, it is worth mentioning the fact that this asteroid is a parent body of the Geminid meteor shower; however, its origin is not quite clear (Whipple, 1983; Fox et al., 1984; Gustafson, 1989; Williams and Wu 1993; Ryabova, 2007). Nevertheless, it has never exhibited any sign of the continuing loss of mass or any other form of cometary activity, which would indicate the shower replenishment. Moreover, the Japanese Space Exploration Agency considers 3200 Phaethon as a potential target for a space mission (Krüger et al., 2017). At present, the study of its dynamics has become especially urgent due to the recent encounter with the Earth in December 2017 (Ye, 2017; Jewitt, 2017), which may help to specify the orbit and physical parameters of the object with a higher precision; (Hanuš et al., 2016).

The ensemble of small bodies, including asteroid 3200 Phaethon, probably covers smaller objects, which have yet to be discovered; sometimes, it is called the Phaethon–Geminids complex (Ohtsuka et al., 2006). Probably, it is a product of disintegration of the precursor object; however, how these bodies disintegrated, whether from a catastrophic event or a continuous process, is unknown (Kasuga, 2009). The age of the Geminids is not determined; however, the shower is apparently young. Numerical models, considering the influence of different perturbing factors and planetary gravitational perturbations, yield an age in a range of 600 to 2000 yr (Gustafson, 1989; Williams and Wu, 1993; Ryabova, 2007).

In this paper, we intend to consider the dynamics of asteroid 3200 Phaethon, find the encounters with planets and the mean motion and secular resonances, estimate the predictability time, and reveal the causes of the chaoticity.

RESEARCH METHODS: A SHORT DESCRIPTION

To study the influence of mean motion and secular resonances on the motion of asteroid 3200 Phaethon,

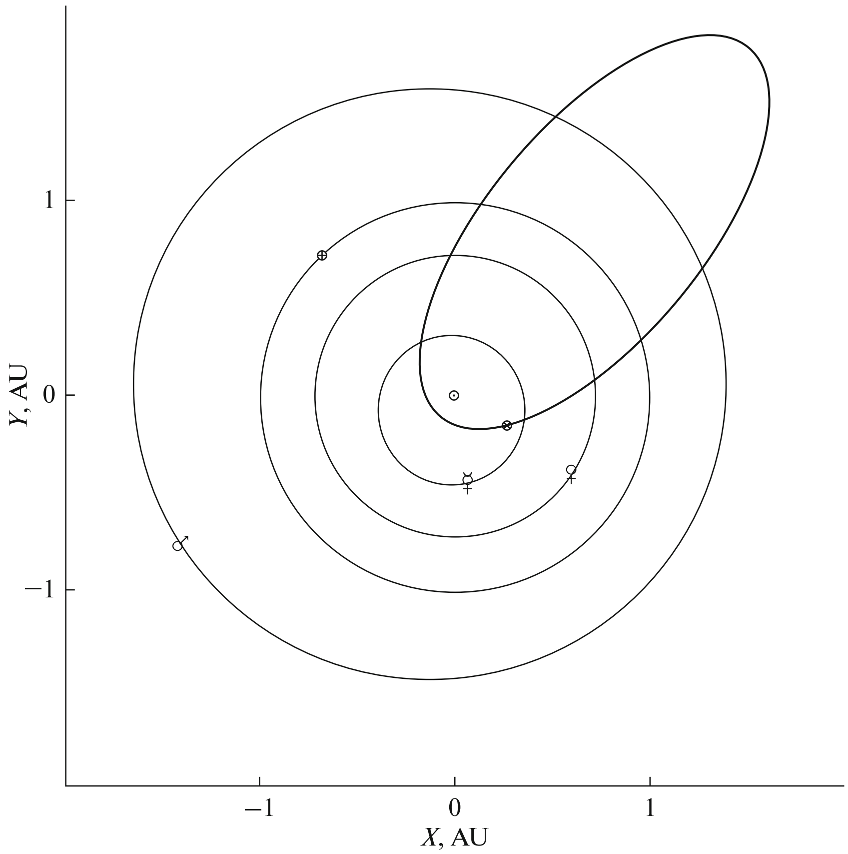

Fig. 1. Projection of the orbits of asteroid 3200 Phaethon and major planets (Mercury, Venus, the Earth, and Mars) onto the ecliptic plane in the fixed heliocentric coordinate system.

we carried out a numerical experiment containing the following procedures:

—The influence of different perturbing factors on the object was analyzed (Galushina et al., 2015).

—The probabilistic evolution of the orbit was numerically simulated with the IDA software package (Investigating Dynamics of Asteroids) (Bykova et al., 2012; Razdymakhina, 2011).

—Close encounters of the asteroid with the planets were found.

—The dynamical evolution of the objects was analyzed with the Orthogonal Mean Exponential Growth factor of Nearby Orbits (OMEGNO) method (Shefer and Koksin, 2013).

—The mean motion and apsidal–nodal resonances were revealed with the numerical analytical method.

The technique for studying the influence of different perturbing factors on the asteroid is presented by Skripnichenko and Galushina (2013) and Skripnichenko et al. (2014). In addition, the analysis of the influence of different perturbing factors on the motion of asteroid 3200 Phaethon is described in detail by Galushina et al. (2015).

An apsidal–nodal resonance is generally understood as the commensurability arising between the precession rates of the orbits of the asteroid and the planet ($\dot{\omega}$, $\dot{\omega}'$, $\dot{\Omega}$, $\dot{\Omega}'$). The technique for revealing these resonances was considered in depth by Galushina and Sambarov (2017). The theory of secular resonances, which we use here, is presented in the papers by Bordovitsyna et al. (2012) and Bordovitsyna and Tomilova (2016). Its essence is briefly as follows.

Let us write the argument of the perturbing function for the doubly averaged restricted three-body problem as

$$\psi = (l - 2p')\,\omega' - (l - 2p)\,\omega - m(\Omega - \Omega'), \qquad (1)$$

where $\omega = \omega_0 + \dot{\omega}(t - t_0)$, and $\Omega = \Omega_0 + \dot{\Omega}(t - t_0)$ are the argument and the longitude of the ascending node of the asteroid, respectively, $\omega' = \omega_0' + \dot{\omega}'(t - t_0)$, and $\Omega' = \Omega_0' + \dot{\Omega}'(t - t_0)$ are the argument and the longitude of the ascending node of the third body, respectively, l, p', l, p, and m are integer numbers, and t and t_0 are the current and initial moments of time, respectively.

Then, the condition for the resonance occurrence can be written as

$$\dot{\psi} = 0.$$

The secular accelerations of the asteroid motion are determined by the influence of the third body and cal-

Table 1. Types of the apsidal–nodal resonance relations of low orders

No.	Resonance relation type	No.	Resonance relation type	No.	Resonance relation type
1	$(\dot{\Omega} - \dot{\Omega}'_j) + \dot{\omega} - \dot{\omega}'_j$	8	$(\dot{\Omega} - \dot{\Omega}'_j) - 2\dot{\omega} - 2\dot{\omega}'_j$	15	$(\dot{\Omega} - \dot{\Omega}'_j) - 2\dot{\omega}'_j$
2	$(\dot{\Omega} - \dot{\Omega}'_j) - \dot{\omega} + \dot{\omega}'_j$	9	$(\dot{\Omega} - \dot{\Omega}'_j) + 2\dot{\omega}$	16	$(\dot{\Omega} - \dot{\Omega}'_j) + 2\dot{\omega}'_j$
3	$(\dot{\Omega} - \dot{\Omega}'_j) + 2\dot{\omega} - 2\dot{\omega}'_j$	10	$(\dot{\Omega} - \dot{\Omega}'_j) - 2\dot{\omega}$	17	$(\dot{\Omega} - \dot{\Omega}'_j)$
4	$(\dot{\Omega} - \dot{\Omega}'_j) - 2\dot{\omega} + 2\dot{\omega}'_j$	11	$(\dot{\Omega} - \dot{\Omega}'_j) + \dot{\omega}$	18	$\dot{\omega} - \dot{\omega}'_j$
5	$(\dot{\Omega} - \dot{\Omega}'_j) + \dot{\omega} + \dot{\omega}'_j$	12	$(\dot{\Omega} - \dot{\Omega}'_j) - \dot{\omega}$	19	$\dot{\omega} + \dot{\omega}'_j$
6	$(\dot{\Omega} - \dot{\Omega}'_j) - \dot{\omega} - \dot{\omega}'_j$	13	$(\dot{\Omega} - \dot{\Omega}'_j) + \dot{\omega}'_j$	20	$\dot{\omega}$
7	$(\dot{\Omega} - \dot{\Omega}'_j) + 2\dot{\omega} + 2\dot{\omega}'_j$	14	$(\dot{\Omega} - \dot{\Omega}'_j) - \dot{\omega}'_j$		

culated by numerical integration with the formulas of the Lagrange polynomial derivative.

A complete set of the apsidal–nodal resonance relations to the second order is presented in Table 1. The table contains the resonance relations of the nodal and apsidal secular frequencies of the asteroid and perturbing bodies (Rosengren et al., 2015), as well as the geometric resonance of the proper frequencies of the asteroid $\dot{\omega} \approx 0$, which is a special case of the apsidal–nodal resonances—the Lidov–Kozai resonance (Lidov, 1961; Kozai, 1962). Relations 1–16 describe the mixed apsidal–nodal secular resonance, while the pure nodal resonance is described by relation 17, and the pure apsidal one, by relations 18 and 19. Relation 20 is the Lidov–Kozai resonance (Shevchenko, 2017), which is a geometric resonance in nature, because it is dependent only on the relative positions of the objects and not connected with the frequencies of the motion of perturbing bodies.

As resonance characteristics for mean motion resonances, we used the critical (resonance) argument β, which determines the longitude of the asteroid–planet conjunction (Murray and Dermott, 1999),

$$\beta = k_1\lambda_1 - k_2\lambda_2 - (k_1 - k_2)\omega_1, \qquad (2)$$

and its derivative with time α (the so-called resonant band (Grebenikov and Ryabov, 1978))

$$a \approx k_1 n_1 - k_2 n_2, \qquad (3)$$

where n_1 and n_2 are the mean motions, λ_1 and λ_2 are the mean longitudes of the asteroid and the planet, respectively, ω_1 is the argument of the asteroid pericenter, Ω_1 is the longitude of the ascending node of the asteroid, and k_1 and k_2 are positive integer numbers. If $\alpha = 0$, the asteroid is in the exact resonance with the planet, which is caused by the comparability k_2/k_1 of their mean motions. The asteroid moves in the resonance vicinity, if α and β oscillate near the exact comparability value, so that $|\beta - \beta_{mean}| \le 180°$ and $|\alpha| \le \alpha_{max}$ (β_{mean} is the center of the critical argument librations). The α_{max} value characterizes the boundaries of the resonance motion, and it is determined from the largest amplitude of oscillations of the critical argument β.

The equations of motion and the equations of the OMEGNO parameters are simultaneously written in the inertial coordinate system and numerically integrated using the high-order Gauss–Everhart integrator (Avdyushev, 2010). In all of the cases, the coordinates of the major planets, Pluto, and the Moon were determined from the fundamental ephemerides DE431 (Planetary Development Ephemeris from the Jet Propulsion Laboratory) (Folkner et al., 2014). The relativistic effects are taken into account by adding the Schwarzschild terms to the equations of motion.

FITTING OF THE ORBIT AND CONSTRUCTION OF THE INITIAL CONFIDENCE REGION

In this paper, we consider the long-term orbital evolution of asteroid 3200 Phaethon. It is to be recalled that the influence of the perturbing factors on the asteroid was earlier estimated and the optimal force model was chosen (Galushina et al., 2015); the latter includes the influence of the major planets, the Moon, the oblateness the Sun, and the relativistic effects caused by the Sun. The other forces can be ignored without any loss of precision, since their effect is substantially smaller than the initial data uncertainties and not essential for the orbital evolution.

By July 2017, 3760 optical observations of the object were available in the database of the Minor Planet Center; however, to fit the orbit, we used 3697 observations selected after the rejection (Fig. 2). It is seen from Fig. 2 that the observations cover a large portion of the orbit except the perihelion vicinity that cannot be observed from the Earth. The results of the orbit fitting are presented in Table 2; the used designations are the following: N is the number of observations, Δt is the observational interval expressed in days, t_0 is the initial epoch, σ is the root-mean-square error of the presentation of observations expressed in

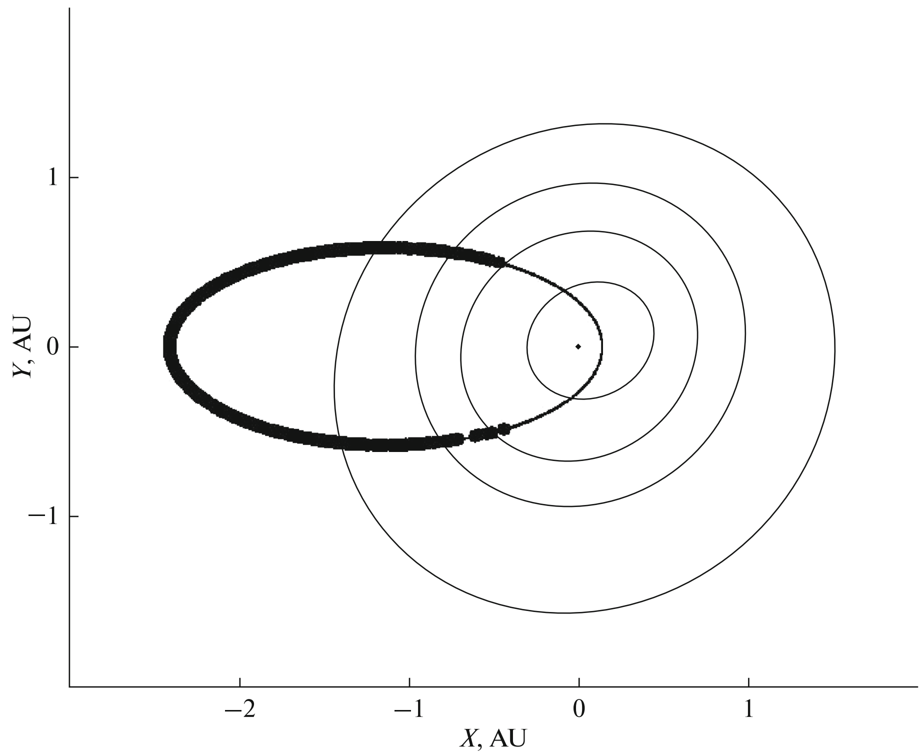

Fig. 2. Projection of the orbit of asteroid 3200 Phaethon onto the orbital plane of the asteroid in the fixed heliocentric coordinate system and the coverage by observations.

arcseconds, and $\Delta\mathbf{x}$ is the root-mean-square error of the coordinates. As the initial epoch, the arithmetical mean of the observational moments was chosen. The results confirm that the orbit of the asteroid is well determined. Moreover, Table 2 contains the orbital elements obtained after the fitting: the semimajor axis a, the eccentricity e, the inclination of the orbital plane to the ecliptic i, the longitude of the ascending node Ω, the pericenter argument ω, and the mean anomaly M.

Table 2. Data on the observations and the results of the orbit fitting

Parameter	Value
N	3697
Δt, d	12164 (1983–2017)
t_0	15.01.2008
σ, arcsec	0.547
$\Delta\mathbf{x}$, AU	1.8×10^{-7}
a, AU	1.27120790
e	0.88999396
i, deg	22.18529429
Ω, deg	265.39339165
ω, deg	322.01903443
M, deg	0.83758333

The nonlinearity coefficient calculated for 3200 Phaethon for the moment t_0 does not exceed a critical value of 0.1 (Syusina et al., 2012; Avdyushev, 2015), which allows the linear methods to be used to construct the initial confidence regions. The initial confidence region was constructed as a six-dimensional ellipsoid according to the covariance matrix obtained in the fitting, and the nominal orbit was used as a center of the ellipsoid.

ANALYSIS OF THE PROBABILISTIC ORBITAL EVOLUTION

The probabilistic orbital evolution was analyzed by numerical integration of the differential equations of motion for the nominal orbit and 10 thousand clones, the positions and velocity components of which were distributed normally. The interval, which was defined by the integration accuracy, lasted from the year −1000 to the year 7000. Figure 3 presents the encounters of the asteroid with the major planets and the evolution of the resonance characteristics and the orbital elements.

Asteroid 3200 Phaethon belongs to the spherical subsystem of the Solar System: its inclination grows from 14° to 42° in the interval between the years 1 and 7000 (Fig. 3j). The shift from 0.126 to 0.228 in the perihelion distance (Fig. 3g) results in the successive approaches with Mercury, Venus, Earth, and Mars.

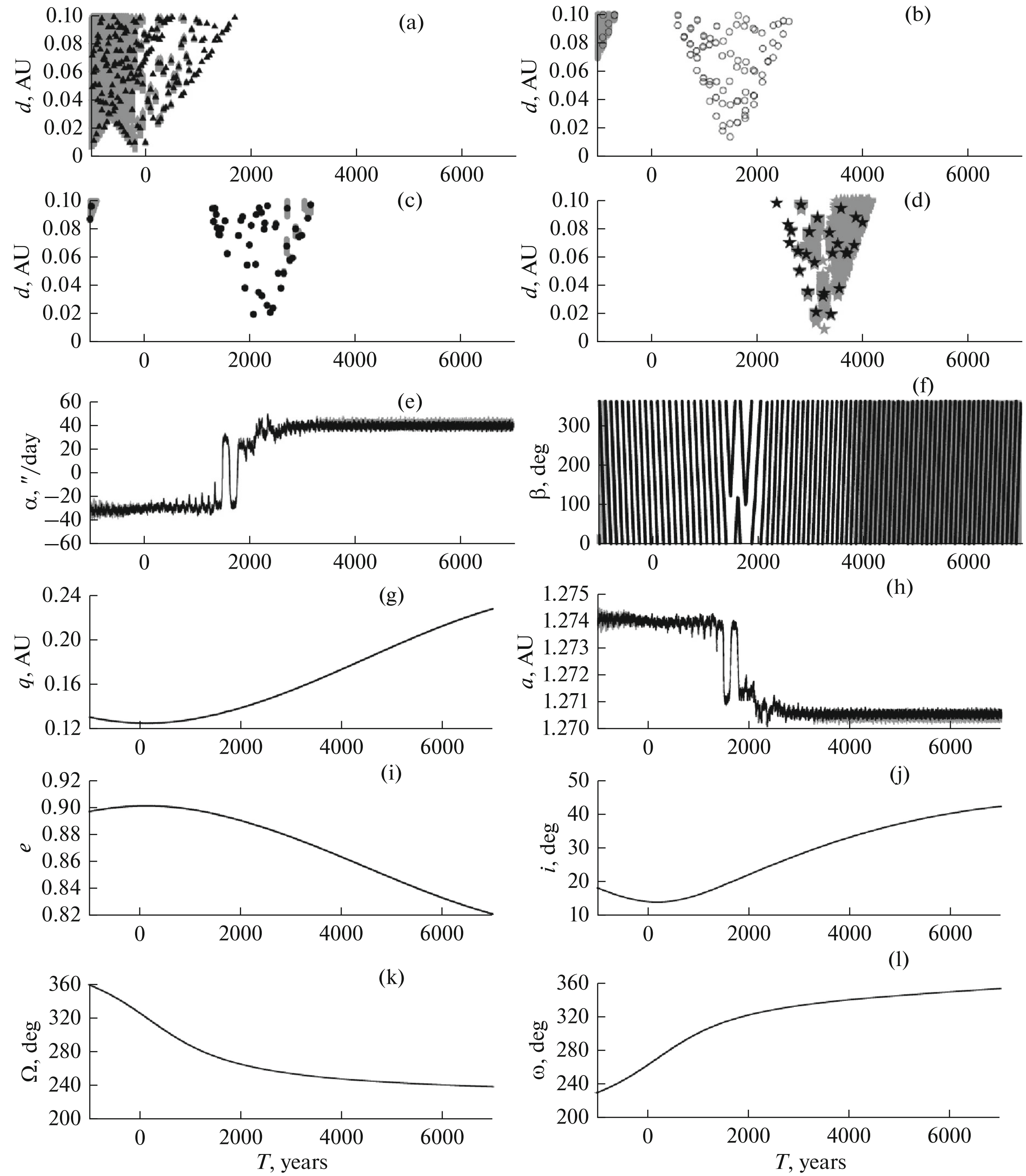

Fig. 3. Approaches of asteroid 3200 Phaethon with Mercury (a), Venus (b), the Earth (c), and Mars (d) (where d is the distance from the considered object to the center of the planet); the evolution of the resonant band α (e), the critical argument β (f), the perihelion distance q (g), the semimajor axis a (h), the eccentricity e (i), the inclination of the orbital plane to the ecliptic i (j), the longitude of the ascending node Ω (k), and the pericenter argument ω (l). The evolution for the clones and the nominal orbit is shown by a gray background and black marks, respectively.

When studying the probabilistic orbital evolution of the nominal orbit and clones, it was found that the configuration of the confidence region changes with time. After the years 1750 (backward) and 2350 (forward), the deviation of the clones from the nominal orbit becomes noticeable, which indicates unstable behavior.

Moreover, we also found several passages of the asteroid through the mean motion resonance 3/7 with Venus in the time interval between the years 1500 and 2000 (Figs. 3e and 3f). On this interval, the large-amplitude libration of the resonant band $\alpha = 7n_0 - 3n_2$ relative to 0 takes place, while the critical argument β

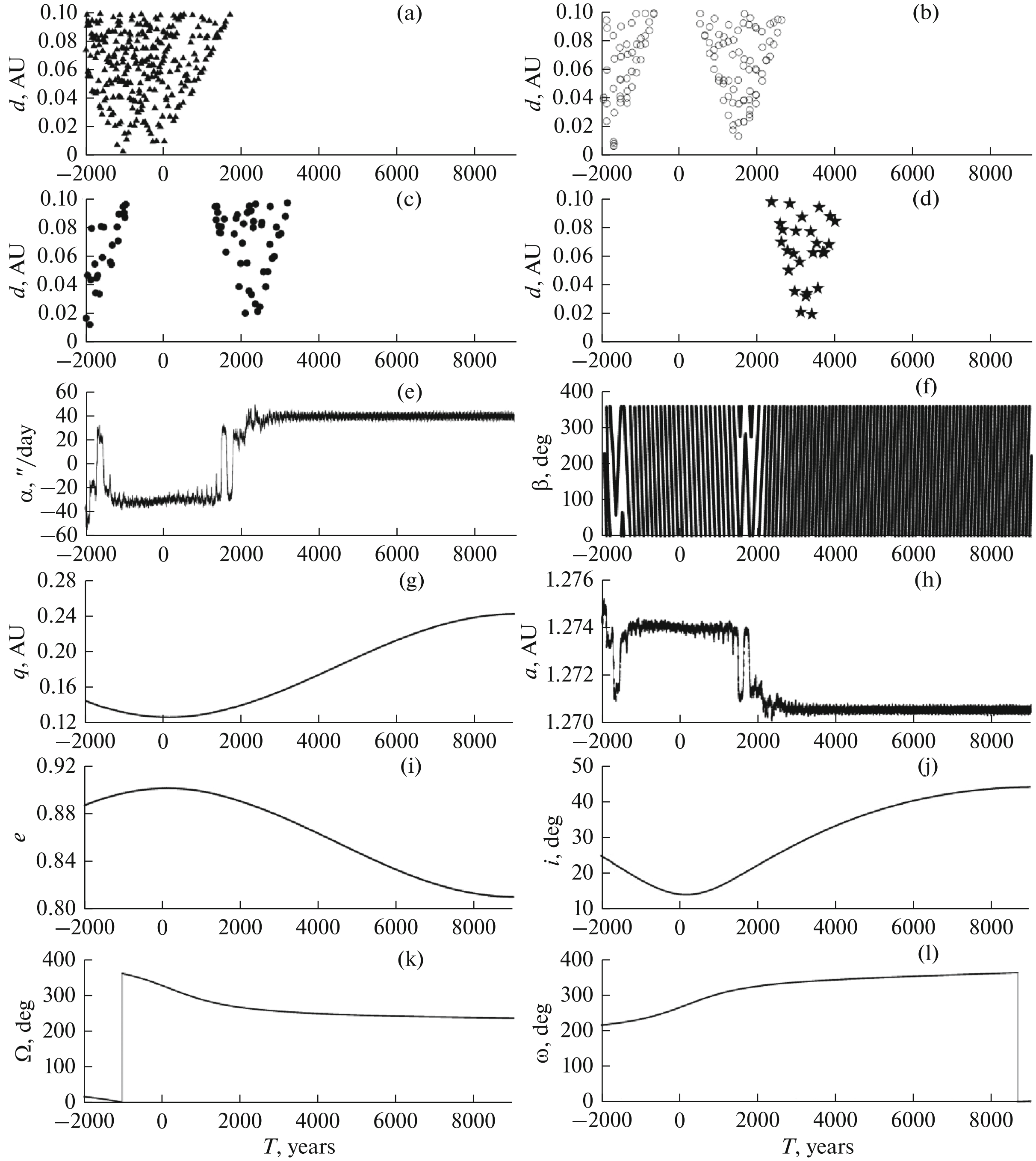

Fig. 4. Approaches of asteroid 3200 Phaethon with Mercury (a), Venus (b), the Earth (c), and Mars (d) (where d is the distance from the considered object to the center of the planet); the evolution of the resonant band α (e), the critical argument β (f), the perihelion distance q (g), the semimajor axis a (h), the eccentricity e (i) , the inclination of the orbital plane to the ecliptic i (j), the longitude of the ascending node Ω (k), and the pericenter argument ω (l). The interval for nominal orbit integration is enlarged: from the year −2000 to the year 9000.

transits from the circulation mode to the libration one. Beyond the 1500–2000 interval, the critical argument always circulates. One more amazing feature is that, before the year 1500, the libration center of the resonance characteristic α with a small amplitude of oscillations was shifted to the negative side (−35″ per day), while it moved to the positive side (40″ per day) after the year 2000. In other words, for the entire interval under consideration, Phaethon is moving in the resonance vicinity, but passes through the sharp resonance only a few times.

Naturally, the question arises whether there are more than those passages through the sharp reso-

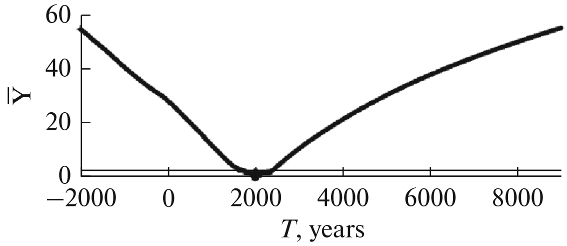

Fig. 5. The OMEGNO parameter evolution.

nance. Unfortunately, any attempt to widen the interval considered in the analysis of the probabilistic evolution led to an unacceptable drop in precision. However, we have succeeded in increasing the integration interval for the nominal orbit and examining the behavior of the resonance characteristics in more detail by this example.

ANALYSIS OF THE CHAOTIC AND REGULAR DYNAMICS UNDER OVERLAP OF DIFFERENT RESONANCES

Figure 4 presents the results of the analysis of the orbital evolution of the nominal orbit on the time interval from the year −2000 to the year 9000. Several passages trough the exact mean motion resonance with Venus were additionally found in the interval between the years −1400 and −2000 (Fig. 4e); in the same interval, the asteroid approaches Mercury, Venus, and the Earth. In the study of the future period, after the year 4000, no encounters or passages through the exact resonance were revealed. This fact allows us to suppose that the passage through the commensurability is stimulated by the encounters with Venus.

As is known, the presence of overlapping resonances (Chirikov, 1977), some of which are unstable, may result in the unstable and chaotic character of the motion (Bordovitsyna and Tomilova, 2016). Consequently, the next step of our study was to analyze the behavior of the OMEGNO parameter (Fig. 5). The asteroid motion is quasiperiodic (Fig. 5), close to stable periodic, in the 1780−2350 interval. Thus, the predictability time of the motion is approximately 340 yr forward and 230 yr backward. It can be found in Fig. 3 that, between the years 1780 and 2350, the evolution of clones almost coincides with that of the nominal orbit. Beyond this interval, the differences become noticeable, which is clearly seen in the diagrams of the approaches (Figs. 3a−3d).

The transition from the resonant motion in the past to nonresonant promotes the chaotic behavior. However, the comparison of Figs. 4 and 5 does not help us to answer the question on the causes of a rapid transition to the chaotic state in the future. To reveal the causes of the considered phenomenon, let us turn to the apsidal−nodal resonances (Table 1). We analyzed the resonances with the major planets, from Mercury to Saturn. For all 20 resonances, the resonance argument makes libration motions on the considered time interval (sometimes, with the extreme amplitude); however, the behavior of the resonance relations differs slightly. All of the resonance relations oscillate around zero, though their amplitudes are different (see Fig. 6). Thus, all of the resonances remain stable and cannot induce chaotic behavior, which manifests itself, according to the results, in the encounters with the Earth and Mars.

CONCLUSIONS

In this paper, we considered the orbital evolution of asteroid 3200 Phaethon that experienced a close encounter with the Earth in December 2017. Moreover, the studied object approaches the other terrestrial planets on the time interval from the year −2000

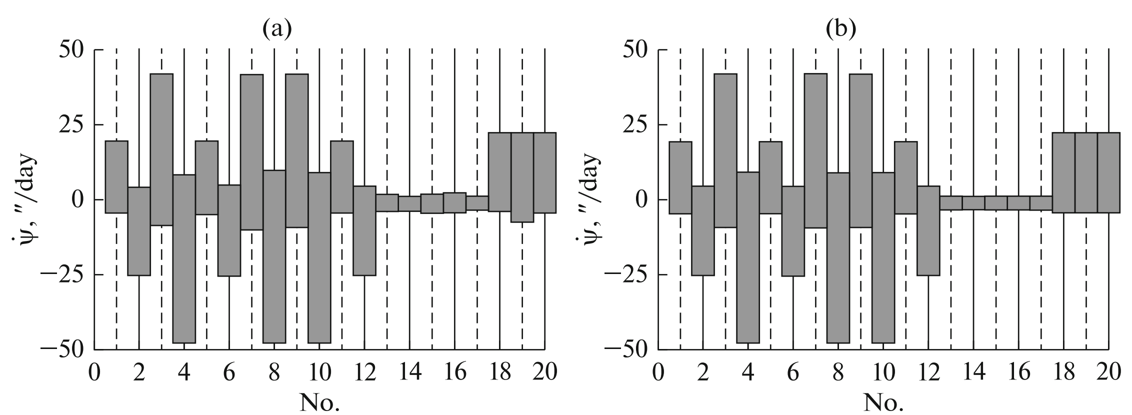

Fig. 6. The boundaries of changes in the resonance interrelations for the apsidal−nodal resonances with the Earth (a) and other planets (b) on the interval between the years −2000 and 9000.

to the year 9000. Due to the encounters with Venus, the asteroid passes through the mean motion resonance with this planet, though the object does not stay there long. The unstable geometric configuration of the 3/7 resonance with Venus (an unsuccessful attempt to capture the object into resonance), more precisely, the transition between the resonant and nonresonant states, may be a cause of such a strong manifestation of the chaotic character of the object's motion, as that shown in Fig. 5, where the OMEGNO parameter passes a value of 2 in the year 1750 and starts to rise steeply.

When examining the future motion of asteroid 3200 Phaethon, the analogous trend to the chaotic behavior is seen. The study has shown that, in this case, the chaoticity may be induced by close encounters with the Earth and Mars.

Finally, it is worth noting that, due to the presence of unstable resonances and a large number of encounters with the planets, the motion of asteroid 3200 Phaethon can be considered as regular only in the interval between the years 1780 and 2350, while the manifestations of chaotic behavior are observed beyond this range.

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