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A Nonequilibrium Figure of Saturn's Satellite Iapetus and the Origin of the Equatorial Ridge on Its Surface

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Abstract—The structure, dynamical equilibrium, and evolution of Saturn's moon Iapetus are studied. It has been shown that, in the current epoch, the oblateness of the satellite $\varepsilon_2 \approx 0.046$ does not correspond to its angular velocity of rotation, which causes the secular spherization behavior of the ice shell of Iapetus. To study this evolution, we apply a spheroidal model, containing a rock core and an ice shell with an external surface ε_2, to Iapetus. The model is based on the equilibrium finite-difference equation of the Clairaut theory, while the model parameters are taken from observations. The mean radius of the rock core and the oblateness of its level surface, $\varepsilon_1 \approx 0.028$, were determined. It was found that Iapetus is covered with a thick ice shell, which is 56.6% of the mean radius of the figure. We analyze a role of the core in the evolution of the shape of a gravitating figure. It was determined that the rock core plays a key part in the settling of the ice masses of the equatorial bulge, which finally results in the formation of a large circular equatorial ridge on the surface of the satellite. From the known mean altitude of this ice ridge, it was found that, in the epoch of its formation, the rotation period of Iapetus was 166 times shorter than that at present, as little as $T \approx 11^{\mathrm{h}}27^{\mathrm{m}}$. This is consistent with the fact that a driving force of the evolution of the satellite in our model was its substantial despinning. The model also predicts that the ice ridge should be formed more intensively in the leading (dark and, consequently, warmer) hemisphere of the satellite, where the ice is softer. This inference agrees with the observations: in the leading hemisphere of Iapetus, the ridge is actually high and continuous everywhere, while it degenerates into individual ice peaks in the opposite colder hemisphere.

Keywords: rock–ice satellites of the planets, Saturn's system, Iapetus, equilibrium figures, evolution models
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INTRODUCTION

Iapetus is the third largest moon of Saturn. It moves around Saturn under the 1 : 1 spin-orbital resonance along a far orbit with a semimajor axis of 3560820 km and a period T = 79.3215 d. Iapetus is axially symmetric in shape; its semiaxes are (Roatsch et al., 2009; Thomas, 2010):

$$a_1 = a_2 = 746 \text{ km}, \quad a_3 = 712 \text{ km}, \qquad R = (a_1 a_2 a_3)^{1/3} = 734.49 \text{ km}, \tag{1}$$

where R is the mean radius of the satellite. Its shape exhibits the moderate oblateness $\varepsilon_2 \approx 0.046$ (and the corresponding eccentricity e_2); the angular velocity of the axial rotation is

$$\Omega = 9.168012729 \times 10^{-7} \text{ s}^{-1}. \tag{2}$$

The satellite's mass is

$$M = 1.81 \times 10^{21} \text{ kg}. \tag{3}$$

An important property of Iapetus is its low mean density

$$\rho = 1.090 \text{ g/cm}^3. \tag{4}$$

This means that the portion of water ice in Iapetus is rather high.

In 2004, it was confirmed by the *Cassini* mission that the leading hemisphere of Iapetus is actually very dark, while the opposite side of this moon is very bright and its albedo is approximately 10 times higher than that of the dark hemisphere (Castillo-Rogez et al., 2007). Due to the light absorption, the difference in brightness of the hemispheres results in the temperature of the dark and bright sides of Iapetus being 129 and only 113 K, respectively (Spencer and Denk, 2010).

A unique feature of the landscape of Iapetus is its equatorial mountain ring (Porco et al., 2005). In the literature, it is often called the Iapetus wall or the Iapetus equatorial ridge. In the dark hemisphere this ridge is more than 1600 km long; its mean altitude is 13 km, and the width is 20 km. Some mountains reach 20 km in altitude. In the cold bright hemisphere, the ridge is interrupted, though high ice mountains are also observed there.

How the ridge appeared on Iapetus is a real enigma. To explain its origin, different hypotheses were proposed. Kerr (2006) made an assumption that the origin of the ridge is connected with the despinning of the satellite. Denk et al. (2005) supposed that the ridge appeared due to volcanic activity. According to one more version (Ip, 2006), the mountain ring was formed during the fall (more exactly, the soft landing) of fragments of an ice ring rotating about Iapetus. Czechowski and Leliwa-Kopystynski (2008) and Roberts and Nimmo (2009) assert that the ridge appeared as a result of convection of the material in the interior of Iapetus. However, none of these hypotheses was thoroughly developed and compared to the observations in detail.

In this study, we are focused on the structure of Iapetus and its evolution in connection with the fact that its figure deviates from the equilibrium one. In the next section, we formulate the problem and derive the equilibrium finite-difference equation from the Clairaut theory for the model parameters. Further, we construct the "rock core + ice shell" model that is applied to Iapetus. This method has allowed us to calculate the size of the rock core and the thickness of the ice shell and to ascertain that the present oblateness of the satellite's surface does not correspond to the value of the angular velocity of its spin rotation. Because of this, the relaxation of Iapetus to the equilibrium figure of smaller oblateness was considered. At the qualitative level, it was shown how the equatorial mountain ring is formed in the process of spherization of the ice shell of the satellite to the equilibrium state. A key role of the rock core in the formation of this ridge was demonstrated. The developed mechanism of relaxation also allowed us to explain why the ridge is more massive in the dark (leading) hemisphere of Iapetus and becomes hardly noticeable on the equator in the opposite (trailing) hemisphere.

PROBLEM FORMULATION

We suppose that Iapetus' figure consists of two subsystems: an internal homogeneous spheroidal rock core covered with a homogenous spheroidal shell composed of ice. The density of the core is ρ_1, and its surface is described by

$$\frac{r^2}{a_1'^2}+\frac{x_3^2}{a_3'^2}=1,\ \ a_1' > a_3',\ \ r^2 = x_1^2 + x_2^2, \tag{5}$$

while the external surface of the shell is described by

$$\frac{r^2}{a_1^2}+\frac{x_3^2}{a_3^2}=1 \tag{6}$$

with semiaxes $a_1 > a_3$, and its density is ρ_2.

Let us consider the equilibrium equation for the liquid mass, rotating about axis Ox_3 with the angular velocity of solid-body rotation Ω

$$\operatorname{grad} p = \rho \operatorname{grad} \Phi. \tag{7}$$

Here, $p(x)$ is the pressure in the liquid and $\Phi(x)$ is the total potential, which is a sum of the gravitational $\varphi(x)$ and centrifugal potentials

$$\Phi(x) = \varphi(x) + \frac{1}{2}\Omega^2\left(x_1^2 + x_2^2\right). \tag{8}$$

According to Eq. (7), to provide for the equilibrium of the rotating system, the surfaces of constant pressure $p(x)$ = const and constant density $\rho(x)$ = const should coincide with the reference surfaces $\Phi(x)$ = const.

It is also known from the theory (see, e.g., Kondratyev (1989)) that the spheroidal reference surfaces for nonhomogeneous equilibrium figures may exist only under the weak-oblateness approximation. It is precisely the approximation under which Clairaut derived the differential equation connecting the density distribution $\rho(r)$ with the oblateness profile of the isosurfaces $\varepsilon(r)$

$$rD\varepsilon'' + (6D + 2rD')\varepsilon' + 2D'\varepsilon = 0. \tag{9}$$

Here, r is the mean radius of the intermediate spheroid, and $D(r) = \frac{1}{r^3}\int_0^r \rho(r)\,dr^3$ is the mean density of its interior. It is important that, for the two-layer model of the satellite, which is a system, consisting of a core and a shell with the densities ρ_1 and ρ_2 and the mean radii r_1 and r_2, respectively, the oblateness values for the core ε_1 and the shell ε_2 will be connected by the Clairaut finite-difference equation (9) (see, e.g., Kondratyev (1989; 2003))

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{5}\frac{5\rho_1 - 3(\rho_1-\rho_2)\left[1-\left(\frac{r_1}{r_2}\right)^5\right]}{\rho_2 + (\rho_1-\rho_2)\left(\frac{r_1}{r_2}\right)^3}. \tag{10}$$

As is seen from this equation, the inequality for the oblateness values $\varepsilon_1 < \varepsilon_2$ will be satisfied, if $\rho_1 > \rho_2$. Moreover, Eq. (10) will also make it possible to determine the oblateness of the spheroidal reference surface of the rock core itself (see below).

CALCULATIONS FOR THE MODEL OF THE SATELLITE IAPETUS

Iapetus' Figure Is Not in Equilibrium

First of all, we know the mean radius of the external surface of the shell r_2 = 734.49 km (see Eq. (1)) and its oblateness $\varepsilon_2 \approx 0.0046$. It is important that we also know from observations the mean density of the body $\overline{\rho} = 1.090\ \mathrm{g/cm^3}$.

Then, from the equation

$$\overline{\rho} = \rho_2 + \alpha^3(\rho_1 - \rho_2), \quad \alpha = \frac{r_1}{r_2} \tag{11}$$

we derive the ratio of the radii of the core and the shell $\frac{r_1}{r_2} = \left(\frac{\overline{\rho} - \rho_2}{\rho_1 - \rho_2}\right)^{\frac{1}{3}}$. From the usual values of the densities of rock and water ice, $\rho_1 \approx 3$ g/cm^3 and $\rho_2 \approx 0.92$ g/cm^3, respectively, we obtain $\frac{r_1}{r_2} = 0.434$. Consequently, the mean radius of the rock core inside Iapetus is $r_1 \approx$ 318.75 km. Thus, the satellite Iapetus has a thick ice shell; its thickness is $\tau \approx 415.74$ km, which is larger than the core radius and amounts to 56.6% of the total radius of the figure. It is interesting that the mass of Iapetus is distributed almost equally between the rock core and the ice shell

$$\frac{M_{\mathrm{core}}}{M_{\mathrm{shell}}} = \frac{\rho_1}{\rho_2}\frac{\alpha^3}{1-\alpha^3} \approx 0.47.$$

By substituting the known values into the right-hand side of Eq. (10), we find the ratio $\frac{\varepsilon_2}{\varepsilon_1} \approx 1.625$, which yields the equilibrium oblateness of the surface of the rock core of the satellite $\varepsilon_1 \approx 0.0028$. This result is consistent with the equilibrium figure theory: the denser core of the satellite turns out to be actually rounder than the external surface of the ice shell of Iapetus, the oblateness of which is $\varepsilon_2 \approx 0.0046$.

With the use of one more formula of the theory

$$\varepsilon_2 = \frac{m}{2} + \frac{3}{5}\frac{\rho_1\varepsilon_1\alpha^5 + \rho_2\varepsilon_2}{\rho_2 + \rho_1\alpha^3}, \tag{12}$$

we obtain the rotation parameter of the model m:

$$m = \frac{\Omega^2 r_2^3}{MG} \approx 0.0466. \tag{13}$$

Since $\frac{\Omega^2}{2\pi G\overline{\rho}} = \frac{2}{3}m$, formula (13) yields the normalized squared angular velocity of the equilibrium model of Iapetus

$$\frac{\Omega_{\mathrm{eq}}^2}{2\pi G\overline{\rho}} \approx 0.0311. \tag{14}$$

On the other hand, the present value of the squared angular velocity of Iapetus is only

$$\frac{\Omega_{\mathrm{obs}}^2}{2\pi G\overline{\rho}} \approx 0.0016, \tag{15}$$

which leads us to the conclusion that Iapetus rotates very slowly and it is in the nonequilibrium state even now. This fact should be taken into account in the study of the evolution of Iapetus.

Formation of the Mountain Equatorial Ring on Iapetus

A unique feature of Iapetus' landscape—the large equatorial mountain ring (Fig. 1)—has been already mentioned in the Introduction. In the dark hemisphere, the mean altitude *h* and the width of the ridge are 13 and 20 km, respectively. Its arc extends over 1600 km, which is more than 70% of half the length of the satellite's equator. It is also worth noting that, in the cold bright hemisphere of the satellite, the ridge is broken: there are individual high ice mountains there.

In our opinion, it is the abovementioned deviation of Iapetus' figure from equilibrium (at present, the satellite rotates too slowly) that activates the formation of the ridge on the surface. This statement is drawn on the above-substantiated conclusion that the deviation of Iapetus' figure from equilibrium will inevitably result in the rounding of the satellite's shape. We will stress straight away that, due to a higher plasticity of ice than that of rock, the evolution of the shape of the external ice shell will evidently proceed faster than the spherization of the rock core (Fig. 2). Because of this, a subject of our further discussion is the evolution of the ice shell of the satellite.

Spherization, i.e., decreasing the oblateness of the ice shell of Iapetus under relaxation, leads to the continuous and, above all, differential-in-latitude lowering of the ice masses into the prior surface. In this process, on the equator itself, the mechanical resistance of the core makes the movement of the incompressible ice masses toward the satellite's center impossible; however, off the equator, the resistance of the core weakens: the normal to the core surface does not coincide any more with the direction of the columns' generatrices, which will lead to the bending of the ice columns. Thus, in the evolution of the shell's shape near the equatorial bulge, a strongly pronounced latitudinal variation in the subsidence degree of ice manifests itself: the out-of-equator masses of ice move under the action of gravity and deform the satellite's surface, while the ice masses on the equator are retarded by the pressure of the core and remain almost at the same distance from it. In this process, the neighboring masses of ice subside relative to the equatorial ring on the surface of Iapetus (Fig. 2). It is the subsiding of the ice masses neighboring the equator that resulted in the formation of the equatorial ridge on Iapetus.

Let us also pay attention to a subtle effect of the temperature difference between the leading and trailing hemispheres of the satellite. As the observations show (Spencer and Denk, 2010), due to light absorption, the temperature in the dark hemisphere of the satellite is somewhat higher (by 16 K) than that in the opposite (bright) side of Iapetus. Consequently, the ice in the dark hemisphere will be softer than that in

Fig. 1. An image of the equatorial ridge on Iapetus taken from the distance of 5620 km by the *Cassini* spacecraft on September 10, 2007 (https://saturn.jpl.nasa.gov/raw_images/154732/). Source: NASA/JPL-Caltech/Space Science Institute.

the bright hemisphere. Note that the blackening of the leading hemisphere of Iapetus exhibits a mosaic character and does not entirely cover the hemisphere. In the map of Iapetus' surface built from the *Cassini* observations (Roatsch et al., 2009), it is seen that the low-albedo blackened area in the leading hemisphere is elliptic in shape.

It should be expected that, since the darker hemisphere of the satellite is warmer, the ice will be moving more actively and the ridge will be formed more intensively there than in the colder bright hemisphere. This inference agrees with the observations. In point of fact, the equatorial ridge in the leading dark hemisphere is more massive and continuous everywhere. However, in the cold bright hemisphere, the ridge is broken up into individual ice peaks.

The sizes of the dirty dark spot along the equator in the leading hemisphere of the satellite are somewhat smaller than half the equator. This explains the fact that the length of the ridge on Iapetus is approximately 70% of half the equator of this satellite.

Moreover, from the observational data on the altitude of the ridge, $h \approx 13$ km, some conclusions on the history of the evolution of Iapetus' figure can be made. If we assume that the volume remains constant during the changes in the satellite's figure, we may show that the oblateness of Iapetus at the beginning of the ridge formation ε' will be related to its current oblateness $\varepsilon_2 \approx 0.0046$ by the formula

$$\varepsilon_2' = 1 - (1 - \varepsilon_2)\left(1 - \frac{h}{R}\right)^3, \tag{16}$$

which yields $\varepsilon' \approx 0.0954$. Consequently, the axial rotation period of Iapetus in the epoch of the ridge formation was

$$T' = \frac{2\pi}{\sqrt{\frac{16}{15}\pi G\bar{\rho}\varepsilon'}} \approx 11^{\mathrm{h}}27^{\mathrm{m}}, \tag{17}$$

which is close to the rotation period of Saturn itself.

DISCUSSION

In recent years, there has been an upsurge in interest in the evolution of rock–ice celestial bodies of the Solar System (see, e.g., the review by Schubert et al.

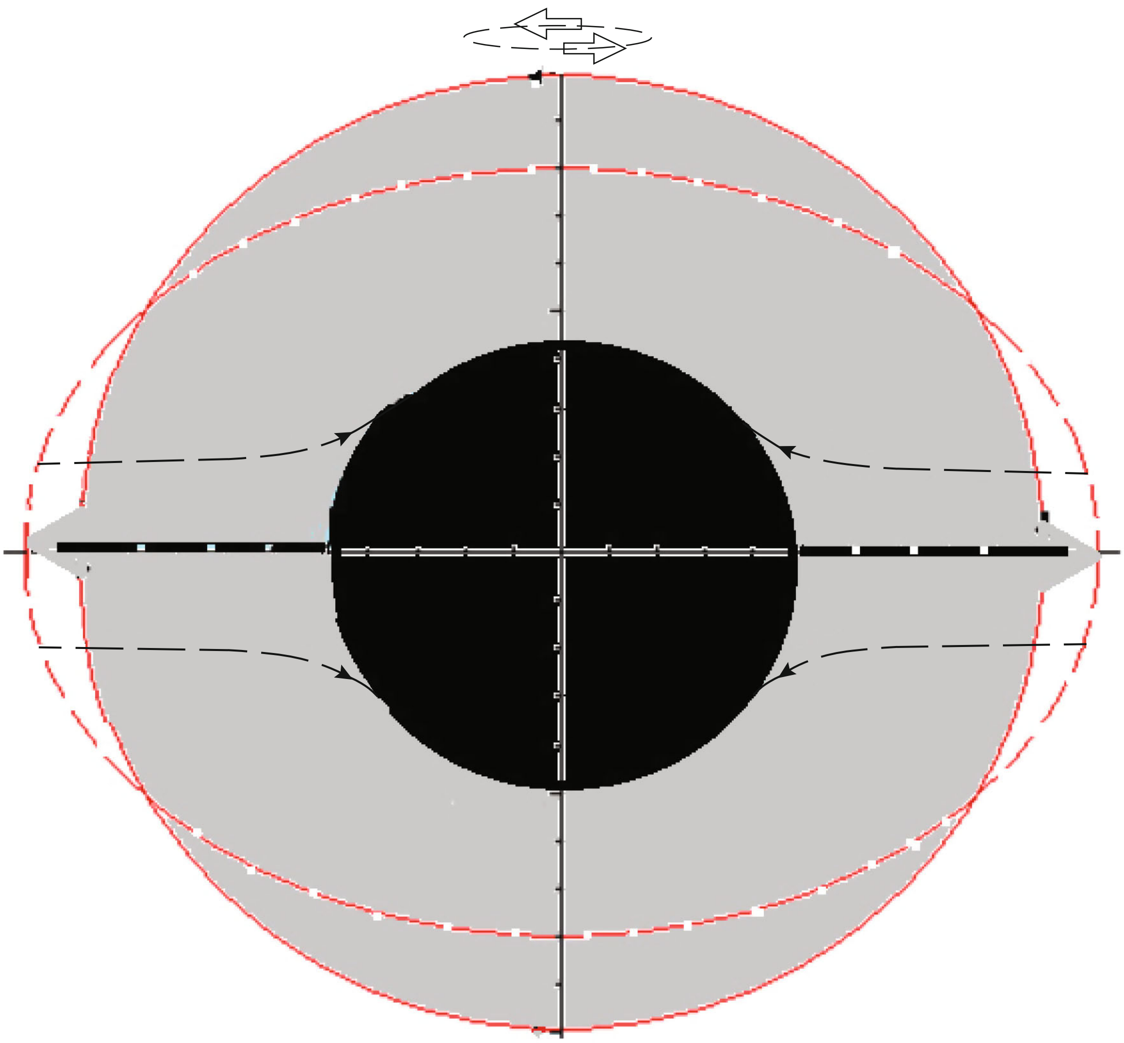

Fig. 2. A scheme of the initial (dashed ellipse) and present (solid circle) shapes of Iapetus. The ice shell and the rock core are shown gray and black, respectively. The deformation of some ice columns during the shell relaxation to a sphere is shown by black dashes. Under bending, the upper parts of the columns will sink (in the direction of arrows) relative to the column on the equator. A profile of the ridge formed on the surface is shown by triangles.

(2010) and the papers by Kondratyev (2016a; 2016b)). Iapetus, the moon of Saturn, is also among these bodies. To study the evolution of Iapetus, the spheroidal model of a rock core plus an ice shell has been proposed here. This model is based on the finite-difference equation that follows from the Clairaut theory. The model parameters were taken from observations. We showed that the oblateness of Iapetus $\varepsilon_2 \approx 0.046$ does not correspond to the current value of the angular velocity of its rotation, which results in the evolution and spherization of the figure of this satellite. Our method allowed us to determine the mean radius of the rock core $r_1 \approx 318.75$ km and the oblateness of its reference surface $\varepsilon_1 \approx 0.028$. It was shown that Iapetus has a thick shell of ice; its thickness is $\tau \approx 415.74$ km, which is 56.6% of the total mean radius of the figure. It is interesting to note that the mass of Iapetus is almost equally distributed between the core and the shell: $\frac{M_{\text{core}}}{M_{\text{shell}}} \approx 0.47.$

Note that the present method fundamentally differs from the evolution mechanism that we developed earlier for the dwarf planet Haumea, which is also composed of a rock core and an ice shell (Kondratyev, 2016a). The cause is that the planet Haumea rotates very quickly, its shape is a distinctly expressed triaxial one, and it strongly differs from Iapetus in dynamics. Conversely, Iapetus is an oblate spheroid in shape; and its dynamical characteristics are completely different. For this very reason, the evolution of Iapetus is completely different from that of the trans-Neptunian object Haumea.

We have also ascertained that the deviation of Iapetus from dynamical equilibrium and the spherization of its shell lead to the settling of the ice masses of the equatorial bulge. Important corrections to the process of the mass subsidence are introduced by the internal rock core of Iapetus. The core most strongly hampers the ice mass subsiding on the satellite's equator itself; however, off the equator, the resistance of the core quickly decreases. Finally, the ice subsides nonuniformly, which results in the formation of a massive circular equatorial ridge on Iapetus. Our model also predicts that the ridge will be more actively formed exactly in the leading (dark and, consequently, wormer) hemisphere of the satellite, where the ice is softer. This conclusion agrees with the observations: in the colder opposite hemisphere of Iapetus, the ridge actually degrades into individual high peaks.

The strong cratering of the ridge observed in the images means that the ridge on Iapetus is a very ancient formation. Because of this, from the observational data on the altitude of the ridge $h \approx 13$ km, one may ascertain that the oblateness of Iapetus was rather noticeable in the early epoch, $\varepsilon' \approx 0.1$, while the period of its axial rotation was 166 times shorter than the present one, as little as $T \approx 11^{h}27^{m}$. This confirms our inference that an actual driving force of the evolution of the satellite was its substantial despinning.

In sum, we note that the ridge-forming process is analyzed here at a qualitative level. Such a consideration is needed at the first stage of the study of a challenging problem, when one hypothesis should be chosen from many others and developed in its main aspects. Naturally, for the further analysis of this problem in more detail, one should use differential equations of motion of the inhomogeneous gravitating spheroidal figure taking account of the high viscosity of the medium. It will not be easy to take into account all of the features of this problem even in the framework of the mathematical scheme (primarily, this refers to the different temperatures of Iapetus' hemispheres known from observations). Nevertheless, the fundamental solution of this problem is of great importance for answering many questions about the origin and evolution of rock–ice bodies of the Solar System.

REFERENCES

Castillo-Rogez, J.C., Johnson, T.V., Lunine, J.I., and Thomas, P.C., Iapetus' geophysics: rotation rate, shape, and equatorial ridge, *Icarus*, 2007, vol. 190, pp. 179–202.

Czechowski, L. and Leliwa-Kopystynski, J., The Iapetus's ridge: possible explanations of its origin, *Adv. Space Res.*, 2008, vol. 42, pp. 61–69.

Denk, T., Neukum, G., Roatsch, Th., Burns, J.A., Helfenstein, P., and Porco, C.C., First imaging results from the Iapetus B/C flyby of the Cassini spacecraft, *Lunar Planet. Sci.*, 2005, vol. 36, p. 262.

Ip, W.H., On a ring origin of the equatorial ridge of Iapetus, *Geophys. Res. Lett.*, 2006, vol. 33, p. L16203.

Kerr, R.A., How Saturn's icy moons get a (geologic) life, *Science*, 2006, vol. 311, p. 29.

Kondratyev, B.P., *Dinamika ellipsoidal'nykh gavitiruyushchikh figur (Dynamics of Ellipsoidal Gravitating Objects)*, Moscow: Nauka, 1989.

Kondratyev, B.P., *Teoriya potentsiala i figury ravnovesiya (The Theory of Potential and Equilibrium Objects)*, Izhevsk: Regulyarnaya Khaoticheskaya Dinamika, 2003.

Kondratyev, B.P., The near-equilibrium figure of the dwarf planet Haumea and possible mechanism of origin of its satellites, *Astrophys. Space Sci.*, 2016a, vol. 361, no. 5, p. 9.

Kondratyev, B.P., Dynamics of Centaur Chariklo and evolution of its rings, *Astrophys. Space Sci.*, 2016b, vol. 361, no. 12, p. 389.

Porco, C.C., Baker, E., Barbara, J., Beurle, K., Brahic A., Burns, J.A., Charnoz, S., Cooper, N., Dawson, D.D., Del Genio, A.D., Denk, T., Dones, L., Dyudina, U., Evans, M.W., Giese, B., et al., Cassini imaging science: initial results on Phoebe and Iapetus, *Science*, 2005, vol. 307, pp. 1237–1242.

Roatsch, T., Jaumann, R., Stephan, K., and Thomas, P.C., Cartographic mapping of the icy satellites using ISS and VIMS Data, in *Saturn from Cassini-Huygens*, New York: Springer-Verlag, 2009, p. 763.

Roberts, J.H. and Nimmo, F., Tidal dissipation due to despinning and the equatorial ridge on Iapetus, *Lunar Planet. Sci.*, 2009, vol. 40, p. 1927.

Schubert, G., Hussmann, H., Lainey, V., Matson, D.L., McKinnon, W.B., Sohl, F., Sotin, C., Tobie, G., Turrini, D., and van Hoolst, T., Evolution of icy satellites, *Space Sci. Rev.*, 2010, vol. 153, pp. 447–487.

Spencer, J.R. and Denk, T., Formation of Iapetus' extreme albedo dichotomy by exogenically triggered thermal ice migration, *Science*, 2010, vol. 327, p. 432.

Thomas, P.S., Sizes, shapes, and derived properties of the Saturnian satellites after the Cassini nominal mission, *Icarus*, 2010, vol. 208, pp. 395–401.

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