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Stefan-Maxwell Relations and Heat Flux with Anisotropic Transport Coefficients for Ionized Gases in a Magnetic Field with Application to the Problem of Ambipolar Diffusion

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Abstract—The defining relations for the thermodynamic diffusion and heat fluxes in a multicomponent, partially ionized gas mixture in an external electromagnetic field have been obtained by the methods of the kinetic theory. Generalized Stefan–Maxwell relations and algebraic equations for anisotropic transport coefficients (the multicomponent diffusion, thermal diffusion, electric and thermoelectric conductivity coefficients as well as the thermal diffusion ratios) associated with diffusion-thermal processes have been derived. The defining second-order equations are derived by the Chapman–Enskog procedure using Sonine polynomial expansions. The modified Stefan–Maxwell relations are used for the description of ambipolar diffusion in the Earth's ionospheric plasma (in the F region) composed of electrons, ions of many species, and neutral particles in a strong electromagnetic field.

Keywords: kinetic theory, Stefan–Maxwell relations, anisotropic kinetic coefficients, ambipolar diffusion

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INTRODUCTION

The defining transport relations ("fluxes via forces"), i.e., the expressions for the diffusion fluxes of various mixture components, the heat flux, and the viscous stress tensor linearly dependent on the gradients of hydrodynamic variables, are needed to close the hydrodynamic equations for multicomponent mixtures of gases and plasma (the differential mass, momentum, and energy conservation equations). These transport relations are currently derived by two fundamentally different methods. The first method is based on the classical thermodynamics of irreversible processes—in the form in which it was developed by Onsager, Prigogine, and other authors (see De Groot and Mazur, 1964; Gyarmati, 1974). Despite the great generality of the thermodynamic approach, which can be applied for a broad class of various media, unfortunately, it does not give a way of calculating the phenomenological coefficients (the diffusion, thermal diffusion, thermal conductivity, electric conductivity, and some other coefficients appearing in the case of a multicomponent neutral medium or a plasma with a magnetic field) figuring in the linear defining relations (see, e.g., Kolesnichenko, 2017a). The second method, a statistical approach, is based on solving the system of Boltzmann kinetic equations for a multicomponent mixture of neutral and charged particles. It allows not only the defining relations linking the thermodynamic fluxes with the particle number density and temperature gradients but also the computational formulas for kinetic coefficients to be obtained (Chapman and Cowling, 1960; Hirschfelder et al., 1961; Ferziger and Kaper, 1976; Marov and Kolesnichenko, 1987).

In classical notation the defining relations[1] obtained by both nonequilibrium thermodynamics (see, e.g., Kolesnichenko and Tirskii, 1976; Kolesnichenko and Marov, 2009) and gas-kinetic (Hirschfelder et al., 1961; Ferziger and Kaper, 1976) methods for the diffusion velocities $\mathbf{w}_\alpha$ $(\alpha = 1, 2, \ldots, N)$ and the heat flux $\mathbf{q}$ are

$$\mathbf{w}_\alpha = -\sum_{\beta=1}^{N} D_{\alpha\beta}\mathbf{d}_\beta - D_{T\alpha}\nabla \ln T, \quad (\alpha = 1, 2, \ldots, N), \tag{1}$$

$$\mathbf{q} = -\lambda' \nabla T - p\sum_{\alpha=1}^{N} D_{T\alpha}\mathbf{d}_\alpha + \sum_{\alpha=1}^{N} h_\alpha n_\alpha \mathbf{w}_\alpha, \tag{2}$$

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[1] Below our analysis will concern only the diffusion-thermal part of the transport processes in a multicomponent, partially ionized gas medium.

where $\mathbf{d}_\alpha = \nabla x_\alpha + (x_\alpha - c_\alpha)\nabla \ln p - \frac{\rho c_\alpha}{p}\left(\mathbf{F}_\alpha - \sum_{\beta=1}^{N} c_\beta \mathbf{F}_\beta\right)$ are the thermodynamic diffusion force vectors, $\sum_{\alpha=1}^{N}\mathbf{d}_\alpha = 0$; $c_\alpha = \rho_\alpha/\rho$, $\mathbf{v}_\alpha$, $\mathbf{w}_\alpha = (\mathbf{v}_\alpha - \mathbf{v})$, m_α, $\rho_\alpha = m_\alpha n_\alpha$, n_α, $x_\alpha = n_\alpha/n$ are, respectively, the mass concentration, mean mass velocity, diffusion velocity, molecular mass, mass density, number density, and mole fraction of component α $(\alpha = 1, 2, \ldots, N)$ in the mixture; h_α is the partial enthalpy per particle of component α (for an ideal gas $h_\alpha = \frac{5}{2}kT$); $n = \sum_\alpha n_\alpha$, $\rho = \sum_\alpha \rho_\alpha$, T, $p = kTn$, $\mathbf{v} = \rho^{-1}\sum_\alpha \rho_\alpha \mathbf{v}_\alpha$ are, respectively, the total number density, mass density, temperature, pressure, and hydrodynamic velocity of the gas mixture; k is the Boltzmann constant; $\mathbf{F}_\alpha$ is the non-electromagnetic mass force acting on component α; $D_{\alpha\beta}$, $D_{T\alpha}$, λ' are, respectively, the symmetric multicomponent diffusion[2] coefficients($D_{\beta\alpha} = D_{\alpha\beta}$, $D_{\alpha\alpha} > 0$), thermal diffusion coefficients, and partial thermal conductivity coefficient of the multicomponent mixture. The quantity λ' in (2) is interpreted as the thermal conductivity coefficient of the mixture (in the absence of all diffusive forces $\mathbf{d}_\alpha$). Since the phenomenological diffusion and thermal diffusion coefficients are linearly dependent in this case:

$$\sum_{\alpha=1}^{N} c_\alpha D_{\alpha\beta}(\mathbf{x}, t) = 0, \quad (\beta = 1, 2, \ldots, N), \tag{3a}$$

$$\sum_{\alpha=1}^{N} c_\alpha D_{T\alpha}(\mathbf{x}, t) = 0, \tag{3b}$$

for an N-component mixture there are only $N(N-1)/2$ independent diffusion coefficients and $(N-1)$ independent thermal diffusion coefficients. Relations (1) for the diffusion velocities $\mathbf{w}_\alpha$ $(\alpha = 1, 2, \ldots, N)$ are very difficult to use in the general multicomponent case, because in the literature, with rare exceptions, there is no practical information on the multicomponent diffusion coefficients $D_{\alpha\beta}$ (since $D_{\alpha\beta}$ depend on the mole fractions, they are usually not tabulated), while the existing experimental data refer mainly to the diffusion coefficients in binary gas mixtures $D_{\alpha\beta}$.

The kinetic theory of gas mixtures gives the following expressions for the kinetic transport coefficients in any approximation ξ when seeking them in the form of series in Sonine polynomials (Ferziger and Kaper, 1976):

$$\begin{cases} [D_{\alpha\beta}]_\xi = \dfrac{1}{2n} d_{\alpha,0}^{\beta}(\xi), \\ [D_{T\alpha}]_\xi = -\dfrac{1}{2n} a_{\alpha,0}(\xi) = -\dfrac{5}{4n}\displaystyle\sum_{\beta=1}^{N} n_\beta d_{\beta,1}^{\alpha}(\xi), \\ [\lambda']_\xi = \dfrac{5}{4} k \displaystyle\sum_{\alpha=1}^{N} x_\alpha a_{\alpha,1}(\xi). \end{cases} \tag{4}$$

In this case, the order of the approximation ξ with which the coefficients $a_{\alpha,p}(\xi)$ and $d_{\beta,p}^{\alpha}(\xi)$, are determined corresponds to the number of first terms retained in the expansion of the trial functions $\mathbf{a}_\alpha(\mathbf{c}_\alpha)$ and $\mathbf{d}_\alpha^\beta(\mathbf{c}_\alpha)$, characterizing the first-order perturbations $\phi_\alpha^{(1)} = -n^{-1}\mathbf{a}_\alpha \cdot \nabla \ln T - n^{-1}\sum_\beta \mathbf{d}_\alpha^\beta \cdot \mathbf{d}_\beta + \ldots$ of the distribution functions for individual components $f_\alpha = f_\alpha^{(0)}(1 + \phi_\alpha^{(1)})$ in series (in orthogonal Sonine polynomials $S_v^{(p)}(x)$in the Chapman–Cowling method). Here,

$$\mathbf{a}_\alpha = -\mathbf{V}_\alpha \sqrt{\frac{m_\alpha}{2kT}} \sum_{p=0}^{\xi} a_{\alpha,p}(\xi) S_{3/2}^{(0)}(\mathbf{V}_\alpha^2),$$

$$\mathbf{d}_\alpha^\beta = \mathbf{V}_\alpha \sqrt{\frac{m_\alpha}{2kT}} \sum_{p=0}^{\xi-1} d_{\alpha,p}^{\beta}(\xi) S_{3/2}^{p}(\mathbf{V}_\alpha^2);$$

$\mathbf{c}_\alpha$ is the velocity of a molecule of species α relative to the rest frame; $\mathbf{V}_\alpha = (\mathbf{c}_\alpha - \mathbf{v})\sqrt{m_\alpha/2kT}$ is the dimensionless thermal velocity; and $f_\alpha^{(0)}$ is the Maxwellian distribution function. The coefficients (4) are ultimately given as the ratios of determinants of order $N\xi + 1$ to determinants of order $N\xi$. The elements of these determinants are expressed via the so-called partial integral brackets [ ]' and [ ]", which, in turn, are expressed in a well-known way via the collision integrals (Ω, integrals) dependent on the interaction potentials between various pairs of particles, on their mass and temperature (Hirschfelder et al., 1961; Ferziger and Kaper, 1976).

It should be noted that the often arising need for taking into account the far approximations ($\xi = 3$ or more) when calculating the kinetic coefficients in an investigation of flows of partially ionized gas mixtures due to the slow convergence of the orthogonal expansions of the perturbed electron distribution function (when the electron-neutral particle cross sections increase rapidly with electron velocity) leads to the inversion of high-order matrices in the case of a multicomponent mixture and, thus, to a huge amount of calculations (see Devoto, 1966, 1967, 1968; Tirskii,

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[2] In this paper we use the symmetric multicomponent diffusion coefficients in the Ferziger–Kaper representation, which, in contrast to the analogous coefficients in the Hirschfelder et al. representation, are consistent with the Onsager thermodynamic principle of reciprocity (Curtiss, 1968).

1974). The expression for the true thermal conductivity coefficient λ turns out to be even more cumbersome. To calculate it, the vectors $\mathbf{d}_\alpha$ should be eliminated from (2) using the diffusion relations (1). A formal solution of the latter for $\mathbf{d}_\alpha$ gives

$$\mathbf{d}_\beta = \sum_{\alpha=1}^{N} E_{\beta\alpha}\mathbf{w}_\alpha - \nabla \ln T \sum_{\alpha=1}^{N} E_{\beta\alpha} D_{T\alpha}, \tag{5}$$

where $E_{\beta\alpha}$ are the elements of the matrix inverse to the matrix with elements $D_{\beta\alpha}$. Substituting (5) into (2), we will find

$$\mathbf{q} = -\lambda\nabla T + \frac{5}{2}kT\sum_{\alpha=1}^{N} n_\alpha \mathbf{w}_\alpha + p\sum_{\alpha,\beta=1}^{N} D_{T\alpha}E_{\beta\alpha}\mathbf{w}_\alpha, \tag{6}$$

where the true thermal conductivity coefficient is given by the expression

$$\lambda = \lambda' - kn\sum_{\alpha,\beta=1}^{N} D_{T\beta}E_{\alpha\beta}D_{T\alpha}. \tag{7}$$

The double sums in Eqs. (6) and (7) are also very complex expressions in the case of a multicomponent mixture of partially ionized gases, because the coefficients $E_{\beta\alpha}$, are the elements of the matrix inverse to the matrix $D_{\beta\alpha}$, where, in turn, the elements are the ratios of determinants of order $N\xi + 1$ and $N\xi$. Furthermore, the system of equations obtained after the substitution of $\mathbf{w}_\alpha$ from (1) into the system of mass conservation equations for the individual mixture components turns out to be unsolved for the higher derivatives. A numerical realization of such systems is known to involve great difficulties (see, e.g., Tirskii, 1974).

For these reasons, to describe the diffusion-thermal processes in a multicomponent mixture, it is convenient to have the defining relations (1) in the form of the so-called Stefan–Maxwell relations (i.e., in the form of relations solved for the thermodynamic diffusion forces $\mathbf{d}_\alpha$ via the fluxes $\mathbf{J}_\alpha = n_\alpha\mathbf{w}_\alpha$—“forces via fluxes”, but obtained by some special method – without inverting the matrices $D_{\beta\alpha}$), with the binary diffusion coefficients $\mathbf{D}_{\alpha\beta}$. entering into them instead of the multicomponent diffusion coefficients $D_{\beta\alpha}$. Such an inverse transformation has the following form:

$$\begin{gathered}\mathbf{d}_\beta = \sum_{\substack{\alpha=1\\ \alpha\neq\beta}}^{N} \frac{(n_\beta\mathbf{J}_\alpha - n_\alpha\mathbf{J}_\beta)}{n^2\mathbf{D}_{\alpha\beta}} \\ + \nabla\ln T\sum_{\substack{\alpha=1\\ \alpha\neq\beta}}^{N}\frac{n_\beta n_\alpha}{n^2\mathrm{D}_{\alpha\beta}}\left(D_{T\alpha} - D_{T\beta}\right), \quad (\beta = 1, 2, \ldots, N).\end{gathered} \tag{8}$$

An advantage of relations (8) is that the coefficients $\mathbf{D}_{\alpha\beta}$ in them at a low density of the gas mixture are almost independent of the concentrations of individual components (see, Chapman and Cowling, 1960).

A historical overview. Historically, relations (8) (without thermal diffusion, $D_{T\alpha} = 0$) were derived phenomenologically by Stefan (1871) and Maxwell (1868) under the assumption that the force acting on particles of species α from particles of species β is proportional to the difference of their diffusion velocities, while the total force of resistance to the motion of particles of species α in the mixture is equal to the sum of the independent forces of resistance from all the remaining particles (of other species). Generalized Stefan–Maxwell relations (including the thermal diffusion and the influence of external mass forces) were first obtained in the monograph by Hirschfelder et al. (1961) by the methods of the kinetic theory of monoatomic gases in the first approximation of the Chapman–Enskog theory for the multicomponent diffusion coefficients $[D_{\alpha\beta}]_1$ and in the second approximation for the thermal diffusion coefficients $[D_{T\alpha}]_2$. Muckenfuss and Curtiss (1958) and Muckenfuss (1973) derived these relations in the full second approximation of the kinetic coefficients. Subsequently, unsuccessful attempts to obtain relations (8) from the gas-kinetic theory in any approximation of the transport coefficients were made in a number of works. Therefore, Truesdell (1984) suggested that relations (8) are not universal but are an approximate result of the kinetic theory. Nevertheless, based on the methodology of solving the system of Boltzmann equations for partially ionized gases developed in the monograph by Ferziger and Kaper (1976), this author managed to derive the Stefan–Maxwell relation in higher-order approximations of the diffusion coefficients (Kolesnichenko, 1979, 1981, 1982). This result was also obtained by Kolesnikov and Tirskii (1982) based on the methodology developed in the monograph by Hirschfelder et al. (1961) and the paper by Zhadov and Tirskii (2003)—when using the symmetric multicomponent diffusion coefficients.

In addition, the Stefan–Maxwell relations were derived by one of us by nonequilibrium thermodynamics methods (Dissertation, 1973), which actually proves that they are absolutely universal, i.e., their application in modeling any mutually diffusing continuous media is justified. Kolesnichenko and Tirskii (1976) applied the developed thermodynamic approach to the description of nonideal (moderately dense) mixtures of gases and liquids.

It is important to note that, by analogy with the kinetic theory of transport processes in gases presented in the monograph by Ferziger and Kaper (1976), and also in the thermodynamic approach, it is convenient (see Kolesnichenko, 1994) to introduce new transport coefficients for the description of thermal diffusion effects – the so-called thermal diffusion ratios $k_{T\alpha}$:

$$\sum_{\beta=1}^{N} D_{\alpha\beta} k_{T\beta} = D_{T\alpha}, \quad (\alpha = 1, 2, \ldots, N), \tag{9a}$$

$$\sum_{\alpha=1}^{N} k_{T\alpha}(\mathbf{x}, t) = 0. \tag{9b}$$

In this case, Eq. (1) for the diffusion velocity $\mathbf{w}_\alpha$ can be written as a generalized Fick law:

$$\mathbf{w}_\alpha = -\sum_{\beta=1}^{N} D_{\alpha\beta}\left(\mathbf{d}_\beta + k_{T\beta}\nabla \ln T\right), \quad (\alpha = 1, 2, \ldots, N), \tag{10}$$

while Eq. (2) for the heat flux takes the form

$$\mathbf{q} = -\lambda \nabla T + p\sum_{\beta=1}^{\mathrm{N}}\left(k_{T\beta} + \frac{5}{2}\frac{n_\beta}{n}\right)\mathbf{w}_\beta. \tag{11}$$

Furthermore, the thermodynamic approach (Kolesnichenko, 1994; 2017a) makes it possible to obtain the following important relations that are identical to the analogous relations of the kinetic theory (Curtiss, 1968; Ferziger and Kaper, 1976):

—the Stefan–Maxwell relations in a form convenient for a practical solution of hydrodynamic problems,

$$\mathbf{d}_\beta = \sum_{\substack{\alpha=1 \\ \alpha\neq\beta}}^{N} \frac{x_\beta x_\alpha}{\mathcal{D}_{\alpha\beta}}\left(\mathbf{w}_\alpha - \mathbf{w}_\beta\right) - k_{T\beta}\nabla \ln T, \quad (\beta = 1, 2, \ldots, N); \tag{12}$$

—the expressions for the thermal diffusion ratios

$$k_{T\beta} = \sum_{\beta\neq\alpha}^{N} \frac{n_\alpha n_\beta}{n^2 \mathcal{D}_{\alpha\beta}}(D_{T\beta} - D_{T\alpha}), \quad (\beta = 1, 2, \ldots, N-1), \tag{13}$$

which relate $k_{T\beta}$ to the coefficients $D_{\gamma\beta}$ and $\mathcal{D}_{\alpha\beta}$;

—the expression for the true thermal conductivity coefficient of a multicomponent mixture

$$\lambda \equiv \lambda' - nk\sum_{\beta=1}^{N} k_{T\beta} D_{T\beta}; \tag{14}$$

—the system of equations to find the multicomponent diffusion coefficients of a mixture $D_{\gamma\beta}$ via the binary (tabulated) diffusion coefficients $\mathcal{D}_{\alpha\beta}$.

$$\sum_{\alpha\neq\beta=1}^{N}\left(\frac{n_\alpha n_\beta}{\mathcal{D}_{\alpha\beta}} + \sum_{\alpha\neq\delta=1}^{N} \frac{m_\beta}{m_\alpha}\frac{n_\alpha n_\delta}{\mathcal{D}_{\alpha\delta}}\right) D_{\gamma\beta} = n^2(c_\alpha - \delta_{\gamma\alpha}), \quad (\alpha, \gamma = 1, 2, \ldots, N). \tag{15}$$

Note that Eqs. (15) in the case of a binary mixture make it possible to obtain the following expressions for the coefficients $D_{\alpha\beta}$:

$$D_{11} = \frac{n^2 n_2 m_2^2}{n_1 \rho^2}\mathcal{D}_{12}, \quad D_{22} = \frac{n^2 n_1 m_1^2}{n_2 \rho^2}\mathcal{D}_{12}, \quad D_{12} = D_{21} = -\frac{n^2 m_1 m_2}{\rho^2}\mathcal{D}_{12}. \tag{16}$$

For a three-component mixture from (15) we can find

$$\begin{cases} D_{11} = \dfrac{n^2}{\rho^2 n_1}\dfrac{n_1 n_3 m_3^2 \mathcal{D}_{23}\mathcal{D}_{31} + n_1 n_2 m_2^2 \mathcal{D}_{12}\mathcal{D}_{23} + (\rho_2 + \rho_3)^2 \mathcal{D}_{31}\mathcal{D}_{12}}{n_1\mathcal{D}_{23} + n_2\mathcal{D}_{31} + n_3\mathcal{D}_{12}}, \\ D_{12} = \dfrac{n^2}{\rho_1^2}\dfrac{n_3 m_3^2 \mathcal{D}_{23}\mathcal{D}_{31} - m_2(\rho_1 + \rho_2)\mathcal{D}_{23}\mathcal{D}_{12} - m_1(\rho_2 + \rho_3)\mathcal{D}_{31}\mathcal{D}_{12}}{n_1\mathcal{D}_{23} + n_2\mathcal{D}_{31} + n_3\mathcal{D}_{12}}. \end{cases} \tag{17}$$

The remaining coefficients $D_{\alpha\beta}$ are derived from (17) by an appropriate permutation of indices.

We emphasize once again that, in contrast to the gas-kinetic approach, the thermodynamic derivation of all the above relations is not associated with the postulation of a specific form of intermolecular interaction. That is why they are universal, i.e., suitable in modeling diffusion-thermal processes for a broad class of continuous media (polyatomic mixtures of gases, dense gases, liquid solutions, etc.).

In addition to the above results, generalized Stefan–Maxwell relations for a heterogeneous medium and an expression for the total heat flux associated with them were derived thermodynamically (Kolesnichenko and Maksimov, 1997, 2001). Based on them, the filtering motions of a multiphase mixture in a porous solid (with capillary properties), when the mismatch between the pressures in the phases is taken into account, and a generalized Darcy filtering law extending the classical filtering theory to multiphase media was obtained. These relations were also used in constructing turbulent cosmic media that underlie the statements and numerical simulations of the problems of the formation, spatial structure, and evolution of various astrophysical objects (in particular, gas-dust protoplanetary disks (Marov and Kolesnichenko, 2013; Kolesnichenko, 2017b)).

Recently, based on the formalism of extended nonequilibrium thermodynamics (Jou et al., 1993), Zhdanov and Tirskii (2007) obtained a new (relaxation) modification of the Stefan–Maxwell relations that, given the contribution from the time and space derivatives of the thermodynamic fluxes to these relations, removes the paradox of the classical approach related to an infinite propagation speed of small component concentration and temperature perturbations (the result of using the "ordinary" linear dependences of nonequilibrium thermodynamics for the corresponding fluxes).

Finally, in a large series of works (Kolesnichenko, 1980, 1995, 1998, 2002, 2003, 2014a, 2014b, 2015, 2017a; Kolesnichenko and Marov, 1999, 2009; Marov and Kolesnichenko, 2001, 2013) devoted to studying the evolution of turbulent gas mixtures (partially ionized plasma) and turbulent gas-dust media for various natural and cosmic objects, the Onsager formalism of nonequilibrium thermodynamics made it possible to obtain various modifications of the Stefan–Maxwell relations for the turbulent diffusion fluxes and turbulent heat fluxes, which describe most completely the turbulent heat and mass transport processes at the first level of closing the averaged hydrodynamic-type equations.

At the same time, it should be noted that until recently a kinetic analysis of the diffusion-thermal processes in a partially ionized medium based on the generalized Stefan–Maxwell relations and the corresponding expression for the heat flux has been performed in the literature without allowance for the anisotropy of transport coefficients arising in the presence of an external magnetic field. However, these effects often become significant when adequately modeling the magnetized plasma of the ionosphere and especially the magnetosphere of the planet. In contrast to neutral gases that are characterized by three spatial parameters, namely the range of intermolecular forces, the mean free path, and the scale length for changes in macroscopic characteristics, at least three more spatial parameters—the Debye length and two cyclotron radii (one for electrons and the other for ions), play a significant role in plasma physics. For this reason, one of the difficult problems of the kinetic plasma theory is to derive the computational formulas for anisotropic transport coefficients. In this paper, based on the rigorous kinetic approach to modeling partially or fully ionized gas mixtures (composed of electrons, ions, and neutral atoms (molecules) of various types and satisfying the quasi-neutrality condition) developed in the monograph by Ferziger and Kaper (1976), we derive the generalized Stefan–Maxwell relations for the diffusion fluxes in both longitudinal and perpendicular and transverse directions to the magnetic field $\mathbf{B}$. We also derive the algebraic equations associated with diffusion-thermal processes to determine various anisotropic transport coefficients in a plasma under the influence of an external electromagnetic field: the multicomponent diffusion, thermal conductivity, electric conductivity coefficients etc.

As an example, the new form of the Stefan–Maxwell relations with anisotropic transport coefficients obtained here is used for the description of ambipolar diffusion in the Earth's ionospheric plasma (in the F region). It generalizes the well-known information on the ambipolar diffusion of a ternary mixture (electrons, ions, neutral particles) to a multicomponent plasma composed of electrons, ions of many species, and various neutral particles in a strong magnetic field.

DEFINING EQUATIONS AND ANISOTROPIC MULTICOMPONENT DIFFUSION COEFFICIENTS

Under the main assumption that the gradients of hydrodynamic quantities and the external forces cause a small deviation of the distribution functions from the equilibrium Maxwellian distribution $f^{(0)}$, the modified Chapman–Enskog method of solving the system of Boltzmann equations for an ionized quasi-neutral N-component mixture of unexcited moderate-density monoatomic gases in the isothermal case leads to the following macroscopic first-order relations for the diffusion velocities $\mathbf{w}_\alpha$ $(\alpha = 1, 2, \ldots, N)$ of various components and the total heat flux $\mathbf{q}$ (Ferziger and Kaper, 1976):

$$\mathbf{w}_\beta = -\sum_s \sum_{\alpha=1}^{N} D^s_{\beta\alpha} \mathbf{d}_{\alpha s} - \sum_s D^s_{T\beta} \nabla_s \ln T, \tag{18}$$

$$\mathbf{q} = -\sum_s \lambda'_s \nabla_s T - p \sum_s \sum_{\alpha=1}^{N} D^s_{T\alpha} \mathbf{d}_{\alpha s} + \frac{5}{2} kT \sum_{\alpha=1}^{N} n_\alpha \mathbf{w}_\alpha, \tag{19}$$

where

$$\mathbf{d}_{\alpha s} = \nabla_s x_\alpha + (x_\alpha - c_\alpha) \nabla_s \ln p - \frac{\rho c_\alpha}{p} \left(\mathbf{F}_{\alpha s} - \sum_{\beta=1}^{N} c_\beta \mathbf{F}_{\beta s} \right) - \frac{\rho c_\alpha}{p} \left(\frac{e_\alpha}{m_\alpha} - \sum_{\beta=1}^{N} \frac{n_\beta e_\beta}{\rho} \right) \mathbf{E}'_s \tag{20}$$

is the thermodynamic diffusion force of the α-component, $\sum_{\alpha=1}^{N} \mathbf{d}_{\alpha s} = 0$; e_α is the electric charge of a particle of species α; $\mathbf{E}' = \mathbf{E} + c^{-1} \mathbf{v} \times \mathbf{B}$ is the electric field in the moving plasma, $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic fields; c is the speed of light in a vacuum. The index s means that the vector component parallel $(s = \|)$, perpendicular $(s = \perp)$ or transverse $(s = \wedge)$ to the magnetic field $\mathbf{B}$ is taken:

$$\mathbf{d}_{\alpha\|} = \mathbf{b}(\mathbf{b} \cdot \mathbf{d}_\alpha),\ \mathbf{d}_{\alpha\perp} = \mathbf{d}_\alpha - \mathbf{d}_{\alpha\|},\ \mathbf{d}_{\alpha\wedge} = \mathbf{b} \times \mathbf{d}_\alpha \quad (\mathbf{b} = \mathbf{B}/|\mathbf{B}|),$$

$$\nabla_\| T = \mathbf{b}(\mathbf{b} \cdot \nabla T),\ \nabla_\perp T = \nabla T - \nabla_\| T,\ \nabla_\wedge T = \mathbf{b} \times \nabla T.$$

The following should be noted regarding relations (18) and (19). Except for the term proportional to the electric field, all terms have the same form as that for the diffusion of ordinary (neutral) gases. However, it is important to keep in mind that particle fluxes perpendicular to both **B** and the directions of ordinary diffusion and thermal diffusion exist in the case of an ionized gas in the presence of a magnetic field. These transverse particle fluxes are characterized by the transverse diffusion, $D^{\wedge}_{\beta\alpha}$, and thermal diffusion, $D^{\wedge}_{T\beta}$., coefficients. The symmetric[3] multicomponent diffusion coefficients $D^{s}_{\beta\alpha}$ ($D^{s}_{\beta\alpha} = D^{s}_{\alpha\beta}$; $\sum_{\alpha} c_{\alpha} D^{s}_{\beta\alpha} = 0$; $D^{s}_{\alpha\beta} > 0$), the thermal diffusion coefficients $D^{s}_{T\beta}$ ($\sum_{\alpha} c_{\alpha} D^{s}_{T\alpha} = 0$), and the partial thermal conductivity coefficient λ'_{s} (in the absence of all diffusion forces) appearing in relations (19) and (20) are expressed via the coefficients $d^{\alpha(\xi)}_{\beta,p}$, $a^{(\xi)}_{\beta,p}$ of the expansion of the perturbation functionse $\phi^{(1)}_{\beta}$ of the equilibrium Maxwell distributions $f^{(0)}_{\alpha}$ in series in orthogonal Sonine polynomials as follows (see Ferziger and Kaper, 1976; Kolesnichenko, 1982):

$$\begin{aligned} &[D^{\parallel}_{\alpha\beta}]_{\xi} = \frac{1}{2n} d^{\alpha(1)}_{\beta,0}(\xi) = \frac{1}{2n} d^{\beta(1)}_{\alpha,0}(\xi), \\ &[D^{\perp}_{\alpha\beta}]_{\xi} + i[D^{\wedge}_{\alpha\beta}]_{\xi} = \frac{1}{2n} d^{\alpha(2)}_{\beta,0}(\xi) = \frac{1}{2n} d^{\beta(2)}_{\alpha,0}(\xi), \end{aligned} \tag{21a}$$

$$\begin{cases} [D^{\parallel}_{T\alpha}]_{\xi} = -\dfrac{1}{2n} a^{(1)}_{\alpha,0}(\xi) = -\dfrac{5}{4n} \displaystyle\sum_{\beta=1}^{N} x_{\beta} d^{\alpha(1)}_{\beta,1}(\xi), \\ [D^{\perp}_{T\alpha}]_{\xi} + i[D^{\wedge}_{T\alpha}]_{\xi} = -\dfrac{1}{2n} a^{(2)}_{\alpha,0}(\xi) = -\dfrac{5}{4n} \displaystyle\sum_{\beta=1}^{N} x_{\beta} d^{\alpha(2)}_{\beta,1}(\xi), \end{cases} \tag{21b}$$

$$\begin{aligned} &[\lambda'_{\parallel}]_{\xi} = \frac{5}{4} k \sum_{\beta=1}^{N} x_{\beta} a^{(1)}_{\beta,1}(\xi), \\ &[\lambda'_{\perp}]_{\xi} + i[\lambda'_{\wedge}\ ]_{\xi} = \frac{5}{4} k \sum_{\beta=1}^{N} x_{\beta} a^{(2)}_{\beta,1}(\xi). \end{aligned} \tag{21c}$$

The order of the approximation $\xi = 1, 2, \ldots,$with which the transport coefficients are determined corresponds to the number of first terms retained in the expansion of the coefficients of the perturbed distribution functions for the components in series in orthogonal polynomials. Thus, $D^{s}_{\alpha\beta}$ and $D^{s}_{T\alpha}$ are expressed only via the zeroth coefficients, while λ'_{s} are expressed only via the first expansion coefficient irrespective of the number of approximations. However, these coefficients will depend on the number of retained expansion terms, because they must be found from the system of linear algebraic equations

$$\begin{aligned} &\sum_{\beta=1}^{N} \sum_{p=0}^{\xi-1} \Lambda^{mp}_{\gamma\beta} d^{\alpha(s)}_{\beta,p}(\xi) + i \frac{4|\mathbf{B}|}{25ckp} \frac{4}{3\sqrt{\pi}} \frac{\Gamma(m+\frac{5}{2})}{\Gamma(m+1)} \\ &\quad \times \delta_{2s} \sum_{\beta=1}^{N} (\delta_{\gamma\beta} - c_{\gamma}\delta_{m0}) x_{\beta} e_{\beta} d^{\alpha(s)}_{\beta,m}(\xi) \\ &= \frac{8}{25k} (\delta_{\gamma\alpha} - c_{\gamma})\delta_{m0}, \quad (s = 1 \to \parallel,\ 2 \to \perp, \wedge; \\ &\quad m = 0, 1, \ldots, \xi - 1;\ \gamma, \alpha = 1, 2, \ldots, N), \end{aligned} \tag{22}$$

$$\begin{aligned} &\sum_{\beta=1}^{N} \sum_{p=0}^{\xi} \Lambda^{mp}_{\alpha\beta} a^{(s)}_{\beta,p}(\xi) + i \frac{4|\mathbf{B}|}{25ckp} \frac{4}{3\sqrt{\pi}} \frac{\Gamma(m+\frac{5}{2})}{\Gamma(m+1)} \\ &\quad \times \delta_{2s} \sum_{\beta=1}^{N} (\delta_{\gamma\beta} - c_{\gamma}\delta_{m0}) x_{\beta} e_{\beta} a^{(s)}_{\beta,m}(\xi) \\ &= \frac{4}{5k} x_{\alpha}\delta_{m1}, \quad (s = 1 \to \parallel,\ \ 2 \to \perp, \wedge; \\ &\quad m = 0, 1, \ldots, \xi - 1;\ \ \alpha = 1, 2, \ldots, N). \end{aligned} \tag{23}$$

Here, $\delta_{\gamma k}$ is the Kronecker symbol; $\Gamma(m)$ is the Gamma function; $\Lambda^{mp}_{\alpha\beta}$ are the coefficients (linear combinations of reduced Ω^{*}—integrals for simple types of ions) expressed via the partial integral brackets of Sonine polynomials of orders m and p as (see Hirschfelder et al., 1961)

$$\begin{aligned} \Lambda^{mp}_{\alpha\beta} = \Lambda^{pm}_{\beta\alpha} &= \frac{8\sqrt{m_{\alpha} m_{\beta}}}{75kT} \\ \times \Bigg\{ \delta_{\alpha\beta} \sum_{\gamma=1}^{N} x_{\alpha} x_{\gamma} &\left[\mathbf{V}_{\alpha} S^{(m)}_{3/2}(\mathrm{V}^{2}_{\alpha}), \mathbf{V}_{\alpha} S^{(p)}_{3/2}(\mathrm{V}^{2}_{\alpha}) \right]'_{\alpha\beta} \\ + x_{\alpha} x_{\beta} &\left[\mathbf{V}_{\alpha} S^{(m)}_{3/2}(\mathrm{V}^{2}_{\alpha}), \mathbf{V}_{\alpha} S^{(p)}_{3/2}(\mathrm{V}^{2}_{\alpha}) \right]''_{\alpha\beta} \Bigg\} \end{aligned}$$

and possessing (according to the momentum conservation law) the properties

$$\sum_{\alpha=1}^{N} \Lambda^{0p}_{\alpha\beta} = 0; \quad \sum_{\beta=1}^{N} \Lambda^{m0}_{\alpha\beta} = 0, \quad (\alpha = 1, 2, \ldots, N). \tag{24}$$

At $m = 0$ systems (22) and (23) are not linearly independent, which is easy to make sure by summing them over the index γ and comparing the result with identity (24). Therefore, two more independent relations (associated with the condition for solvability of the equations for the perturbations $\phi^{(1)}_{\alpha}$) between the coefficients $a^{(\xi)}_{\beta,0}$ and $d^{\alpha(\xi)}_{\beta,0}$; should be added to them:

$$\begin{aligned} &\sum_{\beta=1}^{N} c_{\beta} a^{(s)}_{\beta,0}(\xi) = 0, \quad (s = 1, 2); \\ &\sum_{\beta=1}^{N} c_{\beta} d^{\alpha(s)}_{\beta,0}(\xi) = 0, \quad (s = 1, 2;\ \ \alpha = 1, 2, \ldots, N). \end{aligned} \tag{25}$$

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[3] The symmetric multicomponent diffusion coefficients in complete agreement with the Onsager reciprocity relations in nonequilibrium thermodynamics (De Groot and Mazur, 1964; Ferziger and Kaper, 1976) are used in relations (18) and (19).

Much work was done in the literature to calculate the Ω^*—integrals as functions of the temperature for several specific intermolecular interaction potentials (see, e.g., Hirschfelder et al., 1961). The quantities $\Lambda_{\alpha\beta}^{mp}$ for $m, p = 0,1$, appearing in the expressions for the diffusion coefficients in the first and second approximations as well as for the thermal diffusion and thermal conductivity coefficients in the first approximation[4] written via the binary diffusion coefficients in the first approximation (Ferziger and Kaper, 1976)

$$[\mathbf{D}_{\alpha\beta}]_1 = \frac{3}{16n}\sqrt{\frac{2\pi kT}{m_{\alpha\beta}}}\frac{1}{\pi\sigma_{\alpha\beta}^2\Omega_{\alpha,\beta}^{(1,1)*}},$$

have the following form:

$$\Lambda_{\alpha\alpha}^{00} = \frac{4}{25kn}\sum_{\alpha\neq\gamma=1}^{N}\frac{x_\alpha x_\gamma}{[\mathbf{D}_{\alpha\gamma}]_1}, \quad \Lambda_{\alpha\beta}^{00} = \Lambda_{\beta\alpha}^{00} = -\frac{4}{25kn}\frac{x_\alpha x_\beta}{[\mathbf{D}_{\alpha\beta}]_1}$$
$$(\alpha \neq \beta),$$

$$\Lambda_{\alpha\alpha}^{01} = \Lambda_{\alpha\alpha}^{10} = -\frac{2}{25kn}\sum_{\alpha\neq\gamma=1}^{N}\frac{x_\alpha x_\gamma}{[\mathbf{D}_{\alpha\gamma}]_1}\frac{m_\gamma}{m_\alpha + m_\gamma}(6C_{\alpha\gamma}^* - 5),$$

$$\Lambda_{\alpha\beta}^{01} = \Lambda_{\beta\alpha}^{10} = \frac{2}{25kn}\frac{x_\alpha x_\beta}{[\mathbf{D}_{\alpha\beta}]_1}\frac{m_\beta}{m_\alpha + m_\beta}(6C_{\alpha\beta}^* - 5)$$
$$(\alpha \neq \beta),$$

$$\Lambda_{\alpha\alpha}^{11} = \frac{4}{25kn}\left[\frac{2x_\alpha^2 A_{\alpha\alpha}^*}{[\mathbf{D}_{\alpha\alpha}]_1}\right.$$
$$\left. + \sum_{\alpha\neq\gamma=1}^{N}\frac{x_\alpha x_\gamma\left(\frac{15}{2}m_\alpha^2 + \frac{25}{4}m_\gamma^2 - 3m_\gamma^2 B_{\alpha\gamma}^* + 4m_\alpha m_\gamma A_{\alpha\gamma}^*\right)}{[\mathbf{D}_{\alpha\gamma}]_1(m_\alpha + m_\gamma)^2}\right],$$

$$\Lambda_{\alpha\beta}^{11} = -\frac{4}{25kn}\frac{x_\alpha x_\beta}{[\mathbf{D}_{\alpha\beta}]_1}\frac{m_\alpha m_\beta A_{\alpha\gamma}^*}{(m_\alpha + m_\beta)^2}$$
$$\times\left[\frac{55}{4} - 3B_{\alpha\beta}^* - 4A_{\alpha\beta}^*\right] \quad (\alpha \neq \beta),$$

where $[\mathbf{D}_{\alpha\alpha}]_1$ is the self-diffusion coefficient; the combinations of reduced integrals $A_{\alpha\beta}^* = \Omega_{\alpha,\beta}^{(2,2)*}/\Omega_{\alpha,\beta}^{(1,1)*}$, $B_{\alpha\beta}^* = (5\Omega_{\alpha,\beta}^{(1,2)*} - 4\Omega_{\alpha,\beta}^{(1,3)*})/\Omega_{\alpha,\beta}^{(1,1)*}$, $C_{\alpha\beta}^* = \Omega_{\alpha,\beta}^{(1,2)*}/\Omega_{\alpha,\beta}^{(1,1)*}$ are close to unity (Chapman and Cowling, 1960; Ferziger and Kaper, 1976).

Thermal diffusion ratios. Thus, if the quantities $\Lambda_{\alpha\beta}^{mp}$ are assumed to be known, then the coefficients $D_{\alpha\beta}^s$, $D_{T\alpha}^s$, and λ_s' can be calculated with any accuracy via the solutions of system (22)–(24). It should be noted, however, that, as in the case of a neutral gas, the coefficients λ_s' are not the true thermal conductivity coefficients. To obtain the true thermal conductivity coefficients (calculated at $\mathbf{w}_\alpha = 0$), the vectors $\mathbf{d}_{\alpha s}$ should be eliminated from (19) using the diffusion relations (18). This can be done if, respectively, the parallel, perpendicular, and transverse thermal diffusion ratios $k_{T\alpha}^s(s = \|, \perp, \wedge)$ are introduced for each mixture component using the definitions

$$\sum_{\alpha=1}^{N} D_{\beta\alpha}^s k_{T\alpha}^s = D_{T\beta}^s, \quad (\beta = 1,2,\ldots,N);$$
$$\sum_{\alpha=1}^{N} k_{T\alpha}^s = 1 \quad (s = 1,2). \tag{26}$$

It should be noted that since the thermal diffusion ratios relate the diffusion coefficients describing the first-order transport effects to the thermal diffusion coefficients describing the second-order effects, the $\xi - \mathrm{e}$ approximation for the thermal diffusion ratio is defined as

$$\sum_{\alpha=1}^{N} d_{\alpha,0}^{\beta(s)}(\xi + 1)[k_{T\alpha}^s]_\xi = -\frac{5}{2}\sum_{\alpha=1}^{N} x_\alpha d_{\alpha,1}^{\beta(s)}(\xi + 1), \tag{27}$$
$$(\beta = 1,\ldots N; \quad s = \|, \perp, \wedge).$$

Given (26), the defining relations (18) and (19) take the form

$$\begin{cases}\mathbf{w}_\beta = \sum_s \mathbf{w}_{\beta s} = -\sum_s\left[\sum_{\alpha=1}^{N} D_{\beta\alpha}^s(\mathbf{d}_{\alpha s} + k_{T\alpha}^s\nabla_s \ln T)\right], & \text{(28a)}\\ \mathbf{q} = -\sum_s \lambda_s\nabla_s T + p\sum_s\left[\sum_{\alpha=1}^{N}\left(k_{T\alpha}^s + \frac{5}{2}x_\alpha\right)\mathbf{w}_{\alpha s}\right], & \text{(28b)}\end{cases}$$

where λ_s are the true thermal conductivity coefficients:

$$\lambda_s = \lambda_s' - nk\sum_{\alpha=1}^{N} k_{T\alpha}^s D_{T\alpha}^s, \quad (s = \|, \perp, \wedge). \tag{29}$$

Formula (29) is generally complex for the calculations of the thermal conductivity coefficients λ_s, because the corresponding procedure leads to a twofold inversion of the matrices. One inversion is related to finding the coefficients λ_s' and $D_{T\alpha}^s$ from systems (22), (23), and (25), and the other inversion is related to the resolution of system (27) (Devoto, 1966, 1967). The monograph by Ferziger and Kaper (1976) provides expressions for the coefficients λ_s and $k_{T\alpha}^s$, for mixtures of neutral gases, which allow them to be calculated in the first approximation directly via the coef-

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[4] Here, as in the monograph by Ferziger and Kaper (1976), by the first approximation to the coefficients $D_{T\alpha}$ and λ' we mean the first nonzero approximation.

ficients $\Lambda_{\alpha\beta}^{mp}$. This result was generalized by Kolesnichenko (1982) to the case of partially ionized mixtures of gases in a magnetic field:

$$\begin{cases} [\lambda_{\|}]_1 = \sum\limits_{\beta,\gamma=1}^{N} x_\beta (\Lambda^{11})^{-1}_{\beta\gamma} x_\gamma, \\ [\lambda_{\perp}]_1 = \sum\limits_{\beta,\gamma=1}^{N} x_\beta (L^{11})^{-1}_{\beta\gamma} x_\gamma, \\ [\lambda_{\wedge}]_1 = \dfrac{5ckp}{2|\mathbf{B}|} \sum\limits_{\beta=1}^{N} \dfrac{x_\beta}{e_\beta} \left[\sum\limits_{\gamma,\alpha=1}^{N} \dfrac{e_\beta}{e_\gamma} (L^{11})^{-1}_{\beta\alpha} \Lambda^{11}_{\alpha\gamma} - 1 \right] \end{cases} \tag{30}$$

$$\begin{gathered} [k_{T\alpha}^{\|}]_1 = \frac{5}{2} \sum_{\beta,\gamma=1}^{N} \Lambda^{10}_{\alpha\gamma} (\Lambda^{11})^{-1}_{\gamma\beta} x_\beta, \\ [k_{T\alpha}^{\perp,\wedge}]_1 = \frac{5}{2} \sum_{\beta,\gamma=1}^{N} \Lambda^{10}_{\alpha\gamma} (L^{11})^{-1}_{\gamma\beta} x_\beta, \quad (\alpha = 1, 2, \dots, N). \end{gathered} \tag{31}$$

This makes it possible to calculate the thermal conductivity coefficients λ_s and the thermal diffusion ratios $k_{T\alpha}^{s}$ in the first approximation directly via $\Lambda_{\alpha\beta}^{mp}$, i.e., without any precalculation of the coefficients λ', $D_{\beta\alpha}$, and $D_{T\alpha}$ and subsequent solution of the system of equations (26). Here, $(\Lambda^{11})^{-1}_{\gamma\delta}$ is the (γ, δ)th element of the matrix inverse to $\Lambda^{11}_{\gamma\delta}$; $\sum_\gamma (\Lambda^{11})^{-1}_{\beta\gamma} \Lambda^{11}_{\gamma\alpha} = \delta_{\beta\alpha}$, $(\alpha, \beta = 1, 2, \dots, N)$; $L^{11}_{\gamma\beta} = \Lambda^{11}_{\gamma\beta} + i\delta_{\gamma\beta}(2|\mathbf{B}|/5ckp) x_\beta e_\beta$.

Electric current density. Given (28a), the expression for the electric current density $\mathbf{J}$ (Ohm's law for a partially ionized gas) will be written as

$$\begin{aligned} \mathbf{J} &= \sum_s \mathbf{J}_s = \sum_s \sum_{\beta=1}^{N} n_\beta e_\beta \mathbf{w}_{\beta s} \\ &= -\sum_s \sum_{\alpha,\beta=1}^{N} n_\beta e_\beta D^{s}_{\alpha\beta} (\mathbf{d}_{\alpha s} + k_T^{s} \nabla_s \ln T) \\ &= \sum_s \left[\sum_{\alpha,\beta}^{N} \sigma^{s}_{\alpha\beta} \mathbf{d}'_{\beta s} - \sum_{\beta=1}^{N} \varphi^{s}_{\beta} \nabla_s T \right], \quad (s = \|, \perp, \wedge), \end{aligned} \tag{32}$$

where the quantity $\mathbf{d}'_\beta$ is related to the thermodynamic force $\mathbf{d}_\beta$ (3) by the formula

$$\begin{aligned} \mathbf{d}'_\beta &= \frac{p}{c_\beta} \left(\frac{\rho c_\beta}{m_\beta} - \sum_{\alpha=1}^{N} n_\alpha e_\alpha \right)^{-1} \mathbf{d}_\beta = -\mathbf{E}' \\ &+ \frac{p}{c_\beta} \left(\frac{\rho e_\beta}{m_\beta} - \sum_{\alpha=1}^{N} n_\alpha e_\alpha \right)^{-1} \\ &\times \left[\nabla x_\beta + (x_\beta - c_\beta) \nabla \ln p - \frac{\rho c_\beta}{p} \left(\mathbf{F}_\beta - \sum_{\alpha=1}^{N} c_\alpha \mathbf{F}_\alpha \right) \right], \end{aligned} \tag{33}$$

while the coefficients $\sigma^{s}_{\alpha\beta}$ and φ^{s}_{β} are expressed via the diffusion and thermal diffusion coefficients, respectively:

$$\sigma^{s}_{\alpha\beta} = \frac{e_\alpha n_\alpha}{p} D^{s}_{\alpha\beta} \left[e_\beta n_\beta - c_\beta \sum_{\gamma=1}^{N} e_\gamma n_\gamma \right], \tag{34a}$$

$$\varphi^{s}_{\beta} = \frac{e_\beta n_\beta}{T} \sum_{\alpha=1}^{N} k^{s}_{T\alpha} D^{s}_{\alpha\beta}, \quad (s = \|, \perp, \wedge). \tag{34b}$$

Here, $\sigma^{s}_{\alpha\beta}$, φ^{s}_{β} are the partial electric and thermoelectric conductivity coefficients, respectively. The elements of the complete electric conductivity, σ^{s}, and thermoelectric conductivity, φ^{s}, tensors are obtained by summing the partial coefficients over the indices.

Multicomponent diffusion coefficients. The system of equations (22) can be rewritten is a clearer form that allows us to determine the diffusion coefficients $D^{s}_{\alpha\beta}$, needed to calculate the diffusion velocities $\mathbf{w}_{\beta s}$. For simplicity, let $\xi = 2$; then,

$$\begin{gathered} \sum_{\beta=1}^{N} \left\{ d^{\alpha(2)}_{\beta,0} \left(\Lambda^{00}_{\gamma\beta} + i \frac{4|\mathbf{B}|}{25cpk} x_\beta e_\beta (\delta_{\gamma\beta} - c_\gamma) \right) + \Lambda^{01}_{\gamma\beta} d^{\alpha(2)}_{\beta,1} \right\} \\ = \frac{8}{25k} (\delta_{\gamma\alpha} - c_\gamma) \quad (m = 0) \quad \text{for all} \\ (\gamma, \alpha = 1, 2, \dots, N), \end{gathered} \tag{35}$$

$$\begin{gathered} \left\{ \sum_{\beta=1}^{N} \Lambda^{10}_{\gamma\beta} d^{\alpha(2)}_{\beta,0} + \left(\Lambda^{11}_{\gamma\beta} + i\delta_{\gamma\beta} \frac{2|\mathbf{B}|}{5ckp} x_\beta e_\beta \right) d^{\alpha(2)}_{\beta,1} \right\} = 0 \\ (m = 1) \quad \text{for all} \quad (\gamma, \alpha = 1, 2, \dots, N). \end{gathered} \tag{36}$$

Solving system (36) for $d^{\beta(2)}_{\alpha,1}$ and substituting the result into Eqs. (35), we will obtain the following system of algebraic equations that allow the anisotropic multicomponent diffusion coefficients $D^{s}_{\alpha\beta}$ to be found via the binary diffusion coefficients $D_{\alpha\beta}$ (Kolesnichenko, 1982):

$$\begin{gathered} \sum_{\alpha\neq\beta=1}^{N} \frac{x_\alpha x_\beta}{[D^{(1)}_{\alpha\beta}]_2} \left\{ [D^{\|}_{\gamma\alpha}]_2 - [D^{\|}_{\gamma\beta}]_2 \right\} = \delta_{\alpha\gamma} - c_\alpha, \\ (\alpha, \gamma = 1, 2, \dots, N), \end{gathered} \tag{37}$$

$$\begin{gathered} \sum_{\alpha\neq\beta=1}^{N} \frac{x_\alpha x_\beta}{[D^{(2)}_{\alpha\beta}]_2} \left\{ [D^{\perp}_{\gamma\alpha}]_2 - [D^{\perp}_{\gamma\beta}]_2 \right\} = \delta_{\alpha\gamma} - c_\alpha \\ + \frac{|\mathbf{B}|}{cp} \sum_{\alpha\neq\beta=1}^{N} (\delta_{\alpha\beta} - c_\alpha) n_\beta e_\beta [D^{\wedge}_{\gamma\beta}]_2, \quad (\alpha, \gamma = 1, 2, \dots, N), \end{gathered} \tag{38}$$

$$\sum_{\alpha\neq\beta=1}^{N}\frac{x_\alpha x_\beta}{[D^{(2)}_{\alpha\beta}]_2}\left\{[D^{\wedge}_{\gamma\alpha}]_2-[D^{\wedge}_{\gamma\beta}]_2\right\} = -\frac{|\mathbf{B}|}{cp}\sum_{\alpha\neq\beta=1}^{N}(\delta_{\alpha\beta}-c_\alpha)n_\beta e_\beta[D^{\perp}_{\gamma\beta}]_2, \quad (\alpha,\gamma=1,2,\ldots,N), \tag{39}$$

$$\begin{cases}\displaystyle\sum_{\beta=1}^{N}c_\beta[D^{\parallel}_{\alpha\beta}]_2=0, & \text{(40a)}\\ \displaystyle\sum_{\beta=1}^{N}c_\beta[D^{\perp}_{\gamma\beta}]_2=0, & \text{(40b)}\\ \displaystyle\sum_{\beta=1}^{N}c_\beta[D^{\wedge}_{\gamma\beta}]_2=0, & \text{(40c)}\end{cases}$$

where $[D^{(k)}_{\alpha\beta}]_\xi=[D_{\alpha\beta}]_1[f^{(k)}_{\alpha\beta}]_\xi$ are the binary diffusion coefficients in the ξ approximation ("parallel" at $k=1$ and "perpendicular" at $k=2$); $[f^{(k)}_{\alpha\beta}]_\xi$ are the correction factors taking into account the higher approximations in the coefficients $D^{(k)}_{\alpha\beta}$:

$$[f^{(k)}_{\alpha\beta}]_\xi=[1-\Delta^{(k)}_{\alpha\beta}(\xi)]^{-1},\quad \Delta^{(k)}_{\alpha\beta}(1)=0,$$
$$\Delta^{\parallel}_{\alpha\beta}(2)=\frac{1}{\Lambda^{00}_{\alpha\beta}}\sum_{\gamma,\delta=1}^{2}\Lambda^{01}_{\alpha\gamma}(\Lambda^{11})^{-1}_{\gamma\delta}\Lambda^{01}_{\delta\beta}, \tag{40}$$
$$\Delta^{\perp,\wedge}_{\alpha\beta}(2)=\frac{1}{\Lambda^{00}_{\alpha\beta}}\sum_{\gamma,\delta=1}^{2}\Lambda^{01}_{\alpha\gamma}(L^{11})^{-1}_{\gamma\delta}\Lambda^{01}_{\delta\beta}.$$

Transport coefficients $D^{\parallel}$. Systems (37) and (40a) can be combined into the following algebraic relations analogous to (15):

$$\sum_{\alpha\neq\beta=1}^{N}\left[\frac{x_\alpha x_\beta}{D^{(1)}_{\alpha\beta}}+\sum_{\alpha\neq\gamma=1}^{N}\frac{m_\beta}{m_\alpha}\frac{x_\gamma x_\beta}{D^{(1)}_{\alpha\gamma}}\right][D^{\parallel}_{\beta\delta}]_2=c_\alpha-\delta_{\alpha\delta}, \tag{41}$$
$$(\alpha,\delta=1,2,\ldots,N),$$

which link the multicomponent longitudinal diffusion coefficients $D^{\parallel}$ with the diffusion coefficients $\mathbf{D}^{(1)}$ of binary mixtures in the first approximation. Relations (42) are analogous to relations (15) for a mixture of neutral gases. Thus, in the case of a plasma, Eqs. (16) and (17) for a binary mixture and a three-component mixture remain valid for the longitudinal components of the diffusion transport tensors. For example, for a quasi-neutral mixture of singly ionized atoms with mass m_2 and electrons with mass m_1 it follows from (42) that

$$[D^{\parallel}_{11}]_2=p\tau^{(1)}_{12}/n_1m_1,\quad [D^{\parallel}_{12}]_2=-\varepsilon[D^{\parallel}_{11}]_2,$$
$$[D^{\parallel}_{22}]_2=\varepsilon^2[D^{\parallel}_{11}]_2, \tag{42}$$

where $\tau^{(1)}_{\alpha\beta}=n^2m_{\alpha\beta}\mathbf{D}^{(1)}_{\alpha\beta}/pn_\beta$ is the mean inverse frequency of collisions (along the magnetic field $\mathbf{B}$) between particles of species α and β ($\tau^{(1)}_{\alpha\beta}\neq\tau^{(1)}_{\beta\alpha}$); $m_{\alpha\beta}=m_\alpha m_\beta/(m_\alpha+m_\beta)$ is the reduced mass of a pair of particles; $\varepsilon=m_1/m_2\ll 1$.

Transport coefficients $D^{\perp}$ and $D^{\wedge}$. It is convenient to calculate the coefficients $D^{\perp}$ and $D^{\wedge}$ jointly. Introducing the complex quantities $\tilde{D}_{\alpha\beta}=D^{\perp}_{\alpha\beta}+iD^{\wedge}_{\alpha\beta}$, we will rewrite (38) and (39) as follows:

$$\sum_{\alpha\neq\beta=1}^{N}\frac{x_\alpha x_\beta}{[D^{(2)}_{\alpha\beta}]_2}\left\{[\tilde{D}_{\gamma\alpha}]_2-[\tilde{D}_{\gamma\beta}]_2\right\} + i\frac{|\mathbf{B}|}{cp}\sum_{\beta=1}^{N}(\delta_{\alpha\beta}-c_\alpha)n_\beta e_\beta[D^{\wedge}_{\gamma\beta}]_2=\delta_{\alpha\gamma}-c_\alpha, \tag{43}$$
$$(\alpha,\gamma=1,2,\ldots,N).$$

Hence, given (40b), (40c), we will obtain the relations linking the multicomponent diffusion coefficients $D^{\perp,\wedge}_{\alpha\beta}$ with the diffusion coefficients of binary mixtures in the second approximation:

$$\sum_{\alpha\neq\beta=1}^{N}\left[\frac{x_\alpha x_\beta}{\mathbf{B}^{(2)}_{\alpha\beta}}+\sum_{\alpha\neq\gamma=1}^{N}\frac{m_\beta}{m_\alpha}\frac{x_\gamma x_\beta}{\mathbf{B}^{(2)}_{\alpha\gamma}}\right][\tilde{D}_{\beta\delta}]_2=c_\alpha-\delta_{\alpha\delta}, \tag{44}$$
$$(\alpha,\delta=1,2,\ldots,N),$$

where

$$\mathbf{B}^{(2)}_{\beta\alpha}=\frac{kTx_\alpha}{m_{\beta\alpha}}\frac{\tau^{(2)}_{\beta\alpha}}{1+ic_\alpha\omega_{\beta\alpha}\tau^{(2)}_{\beta\alpha}}. \tag{45}$$

Here,

$$\omega_{\beta\alpha}=e_\beta|\mathbf{B}|/cm_{\beta\alpha},\quad \tau^{(2)}_{\beta\alpha}=\frac{m_{\beta\alpha}}{kTx_\alpha}\mathbf{D}^{(2)}_{\beta\alpha}, \tag{46}$$

are, respectively, the generalized gyrofrequency of the β component and the mean time between the collisions in the perpendicular direction (in the second approximation) of the α component with the β one ($\tau^{(2)}_{\beta\alpha}\neq\tau^{(2)}_{\alpha\beta}$).

For a quasi-neutral binary mixture ($e_1=-e_2=e$) it follows from (45) that

$$D^{\perp}_{11}=\frac{p}{n_1m_1}\frac{\tau^{(2)}_{12}}{1+[\omega_{12}\tau^{(2)}_{12}]^2},\quad D^{\perp}_{12}=-\varepsilon D^{\perp}_{11},$$
$$D^{\perp}_{22}=\varepsilon^2D^{\perp}_{11}, \tag{47}$$

$$D^{\wedge}_{11}=-\frac{p}{n_1m_1}\tau^{(2)}_{12}\frac{\omega_{12}\tau^{(2)}_{12}}{1+[\omega_{12}\tau^{(2)}_{12}]^2},\quad D^{\wedge}_{12}=-\varepsilon D^{\wedge}_{11},$$
$$D^{\wedge}_{22}=\varepsilon^2D^{\wedge}_{11}, \tag{48}$$

where $\omega_{12} = -\omega_{21}$. The correction $\Delta_{12}^{2(2)}$ to the coefficients $D_{12}^{(2)}$ can be easily inferred from (43) (Kolesnichenko, 1979).

Let us also write out the formulas for the coefficients $D_{\alpha\beta}^{\perp,\wedge}$ in the case of a quasi-neutral gas formed by electrons ($\alpha = 1$), neutral atoms ($\alpha = 3$), and ions (singly ionized atoms of the same species). Using then the notation for the degree of ionization $\alpha = n_1/(n_1 + n_3)$ and neglecting the terms of order $\zeta(= m_1/m_2)$, from (45) we will obtain

$$\begin{cases} D_{11}^{\perp} = \dfrac{kT}{m_1}\dfrac{1+\alpha}{\alpha}\tau_{21}^{(2)}\dfrac{(1-\alpha^2)(\tilde{\omega}_1^2 W_1 + W_2) + \left[\alpha(1-\alpha)\,\tau_{21}^{(2)}/\tau_{31}^{(2)} + \tilde{\omega}_1/\tilde{\omega}_3\right]W_2}{W_2^2 + \tilde{\omega}_1^2 W_1^2}, \\ D_{13}^{\perp} = -\dfrac{kT}{m_1}(1-\alpha^2)\tau_{21}^{(2)}\left(1 - \tau_{21}^{(2)}/\tau_{31}^{(2)}\right)\dfrac{W_2}{W_2^2 + \tilde{\omega}_1^2 W_1^2}, \\ D_{33}^{\perp} = \dfrac{kT}{m_1}\alpha(1+\alpha)\tau_{21}^{(2)}\dfrac{\left(1 + (1-\alpha)\,\tau_{21}^{(2)}/\alpha\tau_{31}^{(2)}\right)W_2 - \tilde{\omega}_1^2 W_1}{W_2^2 + \tilde{\omega}_1^2 W_1^2}, \\ D_{23}^{\perp} = -\dfrac{1-\alpha}{\alpha}D_{33}^{\perp}, \quad D_{12}^{\perp} = -\dfrac{1-\alpha}{\alpha}D_{13}^{\perp}, \quad D_{22}^{\perp} = \left(\dfrac{1-\alpha}{\alpha}\right)^2 D_{33}^{\perp}; \end{cases} \tag{49}$$

$$\begin{cases} D_{11}^{\wedge} = \dfrac{kT}{m_1}\dfrac{1+\alpha}{\alpha}\tau_{21}^{(2)}\dfrac{(1-\alpha^2)(W_1 - W_2)\tilde{\omega}_1 + \left[\alpha(1-\alpha)\,\tau_{21}^{(2)}/\tau_{31}^{(2)} + \tilde{\omega}_1/\tilde{\omega}_3\right]W_2\tilde{\omega}_1}{W_2^2 + \tilde{\omega}_1^2 W_1^2}, \\ D_{13}^{\wedge} = \dfrac{kT}{m_1}(1-\alpha^2)\tau_{21}^{(2)}\left(1 - \tau_{21}^{(2)}/\tau_{31}^{(2)}\right)\dfrac{\tilde{\omega}_1 W_2}{W_2^2 + \tilde{\omega}_1^2 W_1^2}, \\ D_{33}^{\wedge} = -\dfrac{kT}{m_1}\alpha(1+\alpha)\tau_{21}^{(2)}\tilde{\omega}_1\dfrac{\left(1 + (1-\alpha)\,\tau_{21}^{(2)}/\alpha\tau_{31}^{(2)}\right)W_1 + W_2}{W_2^2 + \tilde{\omega}_1^2 W_1^2}, \\ D_{23}^{\wedge} = -\dfrac{1-\alpha}{\alpha}D_{33}^{\wedge}, \quad D_{12}^{\wedge} = -\dfrac{1-\alpha}{\alpha}D_{13}^{\wedge}, \quad D_{22}^{\wedge} = \left(\dfrac{1-\alpha}{\alpha}\right)^2 D_{33}^{\wedge}. \end{cases} \tag{50}$$

In these formulas: $\tilde{\omega}_1 = \omega_{21}\tau_{21}^{(2)}$, $\tilde{\omega}_3 = \omega_{23}\tau_{23}^{(2)}$,

$$\begin{aligned} W_1 &= \frac{(1-\alpha)(1-2\alpha)}{\alpha}\frac{\tau_{21}^{(2)}}{\tau_{31}^{(2)}} - \frac{\tilde{\omega}_1}{\tilde{\omega}_3}, \\ W_2 &= (1-\alpha)^2\tilde{\omega}_1^2 + \frac{(1-\alpha)}{\alpha}\frac{\tau_{21}^{(2)}}{\tau_{31}^{(2)}}\left(1 + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}\right) + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}. \end{aligned} \tag{51}$$

Finally, here we will give the expressions for the partial anisotropic electric conductivity coefficients in the case of a three-component mixture of a partially ionized gas. According to (32), for the coefficients $\sigma_{\alpha\beta}^{s}$ we have

$$\sigma^{s} = \frac{ne^2}{kT}\left(\frac{\alpha}{1+\alpha}\right)^2\left[D_{11}^{s} - 2D_{21}^{s} + D_{22}^{s}\right], \quad (s = \|, \perp, \wedge), \tag{52}$$

whence, given (51) and (52), for the electric conductivity coefficients we obtain the following formulas for the longitudinal, transverse, and Hall conductivities:

$$\begin{aligned} \sigma^{\|} &= \frac{ne^2}{m_1}\frac{\alpha}{1+\alpha}\tau_{21}^{(2)} \\ &\times\left[\frac{(1-\alpha)}{\alpha}\frac{\tau_{21}^{(2)}}{\tau_{31}^{(2)}} + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}\right]\Big/\left[\frac{(1-\alpha)}{\alpha}\frac{\tau_{21}^{(2)}}{\tau_{31}^{(2)}}\left(1 + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}\right) + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}\right], \end{aligned} \tag{54a}$$

$$\begin{aligned} \sigma^{\perp} &= \frac{ne^2}{m_1}\frac{\alpha}{1+\alpha}\tau_{21}^{(2)}W_2 \\ &\times\left[\frac{(1-\alpha)}{\alpha}\frac{\tau_{21}^{(2)}}{\tau_{31}^{(2)}} + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}\right]\Big/\left(W_2^2 + \tilde{\omega}_1^2 W_1^2\right), \end{aligned} \tag{54b}$$

$$\begin{aligned} \sigma^{\wedge} &= -\frac{ne^2}{m_1}\frac{\alpha}{1+\alpha}\tau_{21}^{(2)} \\ &\times W_1\left[\frac{(1-\alpha)}{\alpha}\frac{\tau_{21}^{(2)}}{\tau_{31}^{(2)}} + \frac{\tilde{\omega}_1}{\tilde{\omega}_3}\right]\Big/\left(W_2^2 + \tilde{\omega}_1^2 W_1^2\right) \end{aligned} \tag{54c}$$

Collision Ω^*-integrals. Thus, using relations (42) and (45), the expressions for the multicomponent diffusion coefficients can be reduced to combinations of binary diffusion coefficients for various mixture components that, in turn, are combinations of collision Ω^*-integrals. The reduced collision integrals for the intermolecular interaction potentials $\varphi(r) = (\sigma_{\alpha\beta}/r)^{\nu}$

(ν is the repulsion index) are deduced strictly analytically and have the following form (Hirschfelder et al., 1961):

$$\Omega_{\alpha,\beta}^{(l,r)*} = \frac{4(l+1)(\nu/kT)^{2/\nu}\Gamma(r+2-2/\nu)}{(r+1)!\,[2l+1-(-1)^{l}]} \times \int_{0}^{\infty}[1-\cos^{l}\chi_{\alpha\beta}(z,\nu)z\,dz. \qquad (57)$$

Here, $z = b(m_{\alpha\beta}\,g^{2}/2\nu)^{1/\nu}\sigma_{\alpha\beta}^{-1}$; $\chi_{\alpha\beta}(z,\nu)$ is the deflection angle of the molecules in the center-of-gravity coordinate system; for a given value of the parameter ν the angle $\chi_{\alpha\beta}$ does not depend on the relative velocity of the molecules g and the impact parameter b separately, but is a function of the variable z. Below the integral over z in (57) will be denoted by the symbol $A_{l}(\nu)$. The difficulties of calculating (57) arise, because the Coulomb potential is a long-range one. Nevertheless, the $\Omega_{\alpha,\beta}^{(l,r)*}$-integrals can be calculated for point centers of repulsion (this requires setting $\sigma_{\alpha\beta} = e_{\alpha}e_{\beta}$ and $\nu = 1$). For an unscreened Coulomb potential with the cutoff radius[5] at $r = R_{D}$

$$\varphi(r) = \begin{cases} e_{\alpha}e_{\beta}/r, & r < R_{D} \\ 0, & r > R_{D} \end{cases}, \qquad (58)$$

(where $R_{D} = 1\Big/\sqrt{\sum_{\alpha}4\pi n_{\alpha}\,e_{\alpha}^{2}/kT}$ is the Debye screening length) the factor $A_{l}(\nu)$ can then be approximated by the following quantity (Ferziger and Kaper, 1976):

$$A_{1}(1) \approx \frac{1}{4}\ln(1+4z_{0}^{2}),$$
$$A_{2}(1) \approx \frac{1}{2}\left[\ln(1+4z_{0}^{2}) - \frac{4z_{0}^{2}}{4z_{0}^{2}+1}\right], \qquad (59)$$

where $z_{0} \equiv (2kT/e_{\alpha}\,e_{\beta})R_{D}$. In this case, the integral (57) is calculated trivially, and we have the following result: $\Omega_{\alpha,\beta}^{(l,r)*} = (\pi kT/2m_{\alpha\beta})^{1/2}A_{1}(1) \times (r-1)!(e_{\alpha}\,e_{\beta}/kT)^{2}$.

Given this expression, the binary diffusion coefficient (in the first approximation) and the mean inverse collision frequency are defined as follows:

$$[\mathbf{D}_{\alpha\beta}]_{1} = \frac{3}{2\sqrt{2\pi m_{\alpha\beta}}}\frac{(kT)^{5/2}}{ne_{\alpha}^{2}e_{\beta}^{2}\ln(1+4z_{0}^{2})},$$
$$\tau_{\alpha\beta}(1) = \frac{3}{2\sqrt{2\pi}}\frac{m_{\alpha\beta}^{1/2}(kT)^{3/2}}{n_{\beta}e_{\alpha}^{2}e_{\beta}^{2}\ln(1+4\Lambda_{\alpha\beta}^{2})}. \qquad (60)$$

"FORCES VIA FLUXES" RELATIONS. GENERALIZED STEFAN–MAXWELL RELATIONS

The system of equations (37)–(40), which relates the multicomponent diffusion coefficients to the diffusion coefficients of binary mixtures for various pairs of mixture components, is often difficult to use in solving specific problems. Furthermore, as has already been noted, substituting Eqs. (18) and (19) for the fluxes into the magnetohydrodynamic equations leads to a system of equations that are unsolved for the higher derivatives, which, in view of the absence of general methods, leads to some difficulties in numerically solving such equations. Therefore, it is convenient to have relations (18) solved for the thermodynamic diffusion forces $\mathbf{d}_{\alpha}$ via the fluxes $\mathbf{w}_{\alpha}$ and written in the form of Stefan–Maxwell relations that include the diffusion coefficients in binary mixtures of gases $D_{\alpha\beta}^{(k)}(\xi)$. instead of the multicomponent diffusion coefficients $D_{\alpha\beta}^{s}(\xi)$.

Let us multiply both sides of Eqs. (37)–(39) by $\mathbf{d}_{\alpha s} + k_{T\beta}^{s}\nabla_{s}\ln T$ $(s = \|, \perp, \wedge)$ and sum the result over the index β. Taking into account (28a), we will obtain the following generalized Stefan–Maxwell relations with anisotropic kinetic coefficients for multicomponent diffusion in a strong magnetic field:

$$\mathbf{d}_{\alpha\|} = \sum_{\alpha\neq\beta=1}^{N}\frac{x_{\alpha}x_{\beta}}{[D_{\alpha\beta}^{(1)}]_{\xi}}(\mathbf{w}_{\beta\|} - \mathbf{w}_{\alpha\|}) - [k_{T\alpha}^{\|}]_{\xi-1}\nabla_{\|}\ln T, \qquad (61a)$$

$$\mathbf{d}_{\alpha\perp} = \sum_{\alpha\neq\beta=1}^{N}\frac{x_{\alpha}x_{\beta}}{[D_{\alpha\beta}^{(2)}]_{\xi}}(\mathbf{w}_{\beta\perp} - \mathbf{w}_{\alpha\perp}) - [k_{T\alpha}^{\perp}]_{\xi-1}\nabla_{\|}\ln T - \frac{n_{\alpha}}{cp}\mathbf{B}\times\left(e_{\alpha}\mathbf{w}_{\alpha\wedge} - \frac{m_{\alpha}}{\rho}\mathbf{J}_{\wedge}\right), \qquad (61b)$$

$$0 = \sum_{\alpha\neq\beta=1}^{N}\frac{x_{\alpha}x_{\beta}}{[D_{\alpha\beta}^{(2)}]_{\xi}}(\mathbf{w}_{\beta\wedge} - \mathbf{w}_{\alpha\wedge}) - \frac{n_{\alpha}}{cp}\mathbf{B}\times\left(e_{\alpha}\mathbf{w}_{\alpha\perp} - \frac{m_{\alpha}}{\rho}\mathbf{J}_{\perp}\right), \quad (\alpha = 1, 2, \ldots, N). \qquad (61c)$$

Relations (61a) (along with (61b), (61c)) are not independent: their sum over α gives an identity. An advantage of these equations is that the coefficients $D_{\alpha\beta}^{(k)}$ in them are almost independent of the composition of gas mixtures at a low density (i.e., $\Delta_{\beta\alpha}^{N(k)} \cong \Delta_{\beta\alpha}^{2(k)}$ (Braginskii, 1963).

In the case of a weak magnetic field, where it can be assumed that $D_{\alpha\beta}^{(1)} \approx D_{\alpha\beta}^{(2)} = D_{\alpha\beta}$ and $k_{T\alpha}^{\|} \approx k_{T\alpha}^{\perp} = k_{T\alpha}$, summing relations (61) over $s(=\|, \perp, \wedge)$, leads to Stefan–Maxwell relations in the following standard form:

$$\mathbf{d}_{\alpha}^{*} = \sum_{\alpha\neq\beta=1}^{N}\frac{x_{\alpha}x_{\beta}}{D_{\alpha\beta}}(\mathbf{v}_{\beta} - \mathbf{v}_{\alpha}) - k_{T\alpha}\nabla\ln T, \qquad (62)$$

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[5] The cutoff means that when the impact parameter b exceeds the Debye length R_{D}, no collisions occur ($\chi = 0$), while the upper limit of integration in the integral (57) should be replaced by R_{D}. This replacement does not change noticeably the integral, except for the case where the impact parameter b is very close to R_{D}.

where $\mathbf{d}_\alpha^* = \nabla x_\alpha + (x_\alpha - c_\alpha)\nabla \ln p - \frac{n_\alpha}{p}\left(\mathbf{F}_\alpha^* - \sum_{\beta=1}^{N} c_\beta \mathbf{F}_\beta^*\right)$; $\mathbf{F}_\beta^* = \mathbf{F}_\beta - \frac{e_\beta}{m_\beta}\left(\mathbf{E} + c^{-1}\mathbf{v}_\beta \times \mathbf{B}\right)$; $\mathbf{v}_\alpha$ is the mean mass velocity for particles of species α.

On the other hand, using the complete equation of motion

$$\rho d\mathbf{v}/dt = -\nabla p + c^{-1}\mathbf{J}\times\mathbf{B} + \sum_{\beta=1}^{N}\rho_\beta \mathbf{F}_\beta$$

for the continuum modeling an ionized gas as a whole and the relation $p_\alpha = x_\alpha p$ for the partial pressure of the α component, relations (61) can be written as the equations of motion for the individual mixture components. In the case of a quasi-neutral plasma, they take the form

$$\rho_\alpha\left(\frac{d\mathbf{v}}{dt}\right)_{\parallel} = -\nabla_{\parallel} p_\alpha - p k_{T\alpha}^{\parallel}\nabla_{\parallel}\ln T + \sum_{\alpha\neq\beta=1}^{N}\frac{n_\beta m_{\beta\alpha}}{\tau_{\beta\alpha}^{(2)}}\left(\mathbf{v}_{\alpha\parallel} - \mathbf{v}_{\beta\parallel}\right) + \rho_\alpha \mathbf{F}_{\alpha\parallel} + n_\alpha e_\alpha \mathbf{E}_{\parallel}, \tag{63}$$

$$\rho_\alpha\left(\frac{d\mathbf{v}}{dt}\right)_{\perp} = -\nabla_{\perp} p_\alpha - p k_{T\alpha}^{\perp}\nabla_{\perp}\ln T + \sum_{\alpha\neq\beta=1}^{N}\frac{n_\beta m_{\beta\alpha}}{\tau_{\beta\alpha}^{(2)}}\left(\mathbf{v}_{\alpha\perp} - \mathbf{v}_{\beta\perp}\right) + \rho_\alpha \mathbf{F}_{\alpha\perp} + n_\alpha e_\alpha\left(\mathbf{E}_{\perp} + \frac{1}{c}\mathbf{v}_{\beta\perp}\times\mathbf{B}\right). \tag{64}$$

Here, $\mathbf{v}_{\alpha\parallel} = \mathbf{w}_{\alpha\parallel} + \mathbf{b}(\mathbf{b}\cdot\mathbf{v})$ and $\mathbf{v}_{\alpha\perp} = \mathbf{w}_{\alpha\perp} + \mathbf{v} - \mathbf{b}(\mathbf{b}\cdot\mathbf{v})$ are, respectively, the mean longitudinal and mean transverse velocities of a particle of species α.

Anisotropic electric conductivity coefficients for a multicomponent plasma in a strong magnetic field. When using relations (42) for the multicomponent diffusion coefficients $D_{\alpha\beta}^{\parallel}$ and formulas (34a) for the electric conductivity coefficients $\sigma_{\alpha\beta}^{\parallel} = \frac{e_\alpha e_\beta n_\alpha n_\beta}{p} D_{\alpha\beta}^{\parallel}$, it is easy to obtain the following algebraic equations relating the longitudinal electric conductivity coefficients $\sigma_{\alpha\beta}^{\parallel}$ for a quasi-neutral multicomponent plasma to the diffusion coefficients of binary mixtures:

$$\sum_{\alpha\neq\beta=1}^{N}\left[\frac{x_\alpha x_\beta}{[\mathbf{D}_{\alpha\beta}^{(1)}]_2} + \sum_{\alpha\neq\gamma=1}^{N}\frac{m_\beta}{m_\alpha}\frac{x_\gamma x_\beta}{[\mathbf{D}_{\alpha\gamma}^{(1)}]_2}\right]\frac{[\sigma_{\beta\delta}^{\parallel}]_2}{e_\beta x_\beta} = \frac{e_\delta n_\delta}{kT}(c_\alpha - \delta_{\alpha\delta}), \quad (\alpha, \delta = 1, 2, \ldots, N). \tag{65}$$

Using relations (45) linking the multicomponent diffusion coefficients $\tilde{D}_{\alpha\beta} = D_{\alpha\beta}^{\perp} + iD_{\alpha\beta}^{\wedge}$ with the diffusion coefficients of binary mixtures and the formulas $\tilde{\sigma}_{\alpha\beta} = \sigma_{\alpha\beta}^{\perp} + i\sigma_{\alpha\beta}^{\wedge}$, we can obtain algebraic equations convenient for determining the perpendicular and transverse electric conductivity coefficients for a multicomponent plasma:

$$\sum_{\alpha\neq\beta=1}^{N}\left[\frac{x_\alpha x_\beta}{\mathbf{B}_{\alpha\beta}^{(2)}} + \sum_{\alpha\neq\gamma=1}^{N}\frac{m_\beta}{m_\alpha}\frac{x_\gamma x_\beta}{\mathbf{B}_{\alpha\gamma}^{(2)}}\right]\frac{[\tilde{\sigma}_{\beta\delta}]_2}{e_\beta x_\beta} = \frac{e_\delta n_\delta}{kT}(c_\alpha - \delta_{\alpha\delta}), \quad (\alpha, \delta = 1, 2, \ldots, N), \tag{66}$$

where

$$\frac{1}{\mathbf{B}_{\beta\alpha}^{(2)}} \equiv \frac{1}{\mathbf{D}_{\beta\alpha}^{(2)}} + i\frac{e_\beta|\mathbf{B}|}{c}\frac{c_\alpha}{kTx_\alpha}. \tag{67}$$

Analogous relations can be derived for the partial thermoelectric conductivity coefficients φ_α^s $(s = \parallel, \perp, \wedge)$.

AMBIPOLAR IONOSPHERIC DIFFUSION IN THE PRESENCE OF AN EXTERNAL ELECTROMAGNETIC FIELD

An important factor in ionospheric aeronomy is the transport of charged particles by diffusion, which, along with the photochemical processes, controls the vertical distribution of ionization (Ratcliff, 1975). The role of transport processes is particularly great in the *F* region, where the mean free path and the lifetime of charged particles increase rapidly with height (Johnson, 1951; Gershman, 1974). A number of additional factors should be taken into account when describing the diffusion of charged particles in a partially ionized ionospheric plasma compared to the diffusion of minor components under neutral atmosphere conditions (Ferraro, 1945). First, these include the influence of the electric field of ambipolar diffusion that arises upon the diffusion separation of lighter and "faster" electrons (whose scale height is great) and considerably "slower" ions hindering the relative diffusion of these charged particles. Second, these include the electrostatic forces arising during the collisions of not only charged particles between themselves but also charged and neutral particles, when an electric dipole moment is induced in the latter. One peculiarity of the diffusion of charged particles is the anisotropy due to the action of a magnetic field (Spitzer, 1957; Goland, 1963; Rose and Clark, 1963).

In the case of a three-component plasma (e, i, m), its electron and ion components under the polarization electric field arising in the gravitational field are known (Dougherty, 1961; Rishbeth, 1964; Whitten and Popov, 1977) to diffuse jointly (ambipolar diffusion) with a common diffusion coefficient and a common hydrodynamic velocity. For an ionospheric plasma composed of electrons, ions of many species, and neutral particles the diffusion velocities of the individual charged components are unequal, especially in the presence of concentration gradients and a strong electromagnetic field, and must be determined by taking into account the condition for the longitudinal electric field be approximately equal to zero (Ferraro, 1964; Schunk and Walker, 1972, 1973).

As an example of the approach proposed here, consider the case where the interaction only between the charged (indices α and β) and neutral (index m) plasma components is taken into account and the collisions of electrons and ions (of all species) between themselves[6] are disregarded. In this case, we will assume the hydrodynamic velocity $\mathbf{v}$ to be coincident with the neutral wind velocity. We will use Eqs. (63) and (64) that (for a singly ionized quasi-neutral plasma and with the above constraints) can be rewritten as

$$p\sum_m \frac{x_\alpha x_m}{\mathbf{D}^{(1)}_{\alpha m}}\mathbf{w}_{\alpha\|} = n_\alpha e_\alpha \mathbf{E}_\| - \mathbf{G}_{\alpha\|}, \quad (\alpha = 1,2,\ldots,N), \tag{68}$$

$$\begin{aligned} p\sum_m \frac{x_\alpha x_m}{\mathbf{D}^{(1)}_{\alpha m}}(\mathbf{w}_{\alpha\perp} + \mathbf{w}_{\alpha\wedge}) &= n_\alpha e_\alpha \mathbf{E}'_\perp \\ &+ c^{-1}|\mathbf{B}|\, n_\alpha e_\alpha(\mathbf{w}_{\alpha\perp} + \mathbf{w}_{\alpha\wedge})\times\mathbf{b} - \mathbf{G}_{\alpha\perp}, \end{aligned} \tag{69}$$

where

$$\mathbf{G}_{\alpha s} \equiv \nabla_s p_\alpha - \rho_\alpha \mathbf{F}_{\alpha s} + \rho_\alpha(\mathrm{d}\mathbf{v}/\mathrm{d}t)_s + p k^s_{T\alpha}\nabla_s \ln T, \quad (s = \|, \perp, \wedge). \tag{70}$$

Here, for the purposes of simplification, we will restrict ourselves to the first approximation for the kinetic equations and will neglect the inertia of charged particles and assume that $\mathbf{F}_\alpha \equiv \mathbf{g}$.

Introducing the mobility coefficients for particles of species β

$$\begin{aligned} b_{\beta\|} &\equiv \frac{e_\beta}{kT}\left[\sum_m \frac{x_m}{\mathbf{D}_{\beta m}}\right]^{-1} \\ &= e_\beta\left[\sum_m m_{\beta m}\tau^{-1}_{\beta m}\right]^{-1} = \frac{c}{|\mathbf{B}|}\gamma_\beta^{-1}, \end{aligned} \tag{71}$$

$$\begin{aligned} b_{\beta\perp} &\equiv \frac{b_{\beta\|}}{1 + c^{-2}|\mathbf{B}|^2 b^2_{\beta\|}} = \frac{c}{|\mathbf{B}|}\frac{\gamma_\beta}{1+\gamma_\beta^2}, \\ b_{\beta\wedge} &\equiv \frac{c^{-1}|\mathbf{B}|\, b^2_{\beta\|}}{1 + c^{-2}|\mathbf{B}|^2 b^2_{\beta\|}} = \frac{c}{|\mathbf{B}|}\frac{1}{1+\gamma_\beta^2}, \end{aligned} \tag{72}$$

(where $\gamma_\beta = \sum_m (\omega_{\beta m}\tau_{\beta m})^{-1}$), we will rewrite (68) and (69) as

$$\mathbf{w}_{\beta\|} = b_{\beta\|}(\mathbf{E}_\| - \mathbf{G}_{\beta\|}/n_\beta e_\beta), \tag{73}$$

$$\begin{aligned} \mathbf{w}_{\beta\perp} + \mathbf{w}_{\beta\wedge} &= b_{\beta\perp}(\mathbf{E}'_\perp - \mathbf{G}_{\beta\perp}/n_\beta e_\beta) \\ &+ b_{\beta\wedge}(\mathbf{E}'_\perp - \mathbf{G}_{\beta\perp}/n_\beta e_\beta)\times\mathbf{b}; \end{aligned} \tag{74}$$

hence it follows that

$$\begin{aligned} (1+\gamma_\beta^2)\mathbf{w}_\beta &= \frac{c}{\gamma_\beta|\mathbf{B}|}\left(\mathbf{E}_\| - \frac{\mathbf{G}_{\beta\|}}{n_\beta e_\beta}\right) + \frac{c}{|\mathbf{B}|}\gamma_\beta \\ &\times\left(\mathbf{E}' - \frac{\mathbf{G}_{\beta\perp}}{n_\beta e_\beta}\right) - \frac{c}{|\mathbf{B}|}\mathbf{b}\times\left(\mathbf{E}' - \frac{\mathbf{G}_\beta}{n_\beta e_\beta}\right), \quad (\beta = 1,2,\ldots). \end{aligned} \tag{75}$$

In this case, according to (73) and (74), Ohm's law for the current density vector $\mathbf{J}$ takes the form

$$\begin{aligned} \mathbf{J} &= \sigma'_\|\mathbf{E}_\| + \sigma'_\perp\mathbf{E}'_\perp - \sigma'_\wedge\mathbf{b}\times\mathbf{E}'_\perp \\ &- \sum_\beta\left(b_\beta\mathbf{G}_{\beta\|} + b_{\beta\perp}\mathbf{G}_{\beta\perp} - b_{\beta\wedge}\mathbf{b}\times\mathbf{G}_{\beta\perp}\right), \end{aligned} \tag{76}$$

where

$$\begin{aligned} \sigma'_\| = \sum_\beta e_\beta n_\beta b_{\beta\|}, \quad \sigma'_\perp = \sum_\beta e_\beta n_\beta b_{\beta\perp}, \\ \sigma'_\wedge = \sum_\beta e_\beta n_\beta b_{\beta\wedge}. \end{aligned} \tag{77}$$

Any electric field produced by the tendency for the electrons to rise above the ions will be vertical, because the ionosphere is stratified horizontally. Let us now choose the origin of a Cartesian coordinate system at some point of the ionosphere in such a way that the $0x$ and $0y$ axes are directed to the magnetic south and east, respectively, and the $0z$ axis is directed upward. Let us introduce the following notation: I is the coordinate-independent (by our assumption) magnetic inclination (dip); $\mathbf{l}, \mathbf{m}$, and $\mathbf{n}$ are unit vectors in the $0x$, $0y$, and $0z$ directions, respectively. Then,

$$\begin{gathered} \mathbf{v} = v_x\mathbf{l} + v_y\mathbf{m}, \quad \mathbf{B} = -B\cos I\mathbf{l} - B\sin I\mathbf{n}, \\ \mathbf{E} = E_x\mathbf{l} + E_y\mathbf{m} + E_z\mathbf{n}; \\ \mathbf{E}_\| = (E_x\cos^2 I + E_z\cos I\sin I)\mathbf{l} \\ + (E_x\cos I\sin I + E_z\sin^2 I)\mathbf{n}, \\ \mathbf{v}_\perp = v_x\sin^2 I\mathbf{l} + v_y\mathbf{m} -_x \cos I\sin I\mathbf{n}, \\ \mathbf{b}\times\mathbf{E}'_\perp = c^{-1}|\mathbf{B}|\mathbf{v}_\perp + \mathbf{b}\times\mathbf{E}_\perp, \\ \mathbf{b}\times\mathbf{E} = E_y\sin I\mathbf{l} + \\ + (E_z\cos I - E_x\sin I)\mathbf{m} - E_y\cos I\mathbf{n}. \end{gathered}$$

We will also restrict ourselves to the case of a stratified ionosphere. Then,

$$\mathbf{G}_{\alpha s} = \mathrm{G}_\alpha\,\mathbf{n}_s \cong (\partial p/\partial z + \rho_\alpha g)\mathbf{n}_s, \quad (s = \|, \perp, \wedge), \tag{78}$$

where $\mathbf{n}_\| = -\sin I\cos I\mathbf{l} + \sin^2 I\mathbf{n}$, $\mathbf{n}_\perp = -\sin I\cos I\mathbf{l} + \cos^2 I\mathbf{n}$, $\mathbf{n}_\wedge = \cos I\mathbf{m}$. In addition, it follows from the Maxwell equations for the electric field $\partial E_x/\partial z = 0$, $\partial E_y/\partial z = 0$, and $\partial E_z/\partial z = 4\pi e\left(n_e - \sum_\alpha Z_\alpha n_\alpha\right)$ that E_x and E_y do not depend on the z coordinate and are completely determined by the boundary conditions,

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[6] A generalization of the theory of ambipolar diffusion to the case where the collisions of electrons with ions are taken into account can be found, for example, in the monograph by Gershman (1974).

while the variable polarization field $E_z = E_z(z)$, resulting from a partial compensation of positive and negative charges should be excluded from consideration for a quasi-neutral plasma (here, $e_\alpha = Z_\alpha e$ is the charge of ions of species α).

Considering below the ionosphere as a thin shell with a current-nonconducting lower boundary, we may set $\mathrm{J}_z \cong 0$ (in view of the electric charge conservation). It then follows from (76) that

$$\mathrm{J}_z = (\sigma'_{\|} - \sigma'_{\perp})\left(E_x \cos I \sin I + E_z \sin^2 I\right) + \sigma'_{\perp} E'_z + \sigma'_{\wedge} E'_y \cos I - \frac{c}{|\mathbf{B}|} \sum_\beta \mathrm{G}_\beta \frac{\gamma_\beta^2 + \sin^2 I}{\gamma_\beta (1 + \gamma_\beta^2)} \approx 0. \tag{79}$$

Let us numerically estimate the conductivity coefficients σ'_s in the $F2$: region: $|\mathbf{B}| \approx 0.3$ G; $\tau_{\alpha m}^{-1} \approx 1\ \mathrm{s}^{-1}$; $\tau_{em}^{-1} \approx 35\ \mathrm{s}^{-1}$ ($T = 1480\,\mathrm{K}$); $m \approx 10 m_\mathrm{H}$, where m_H is the hydrogen atomic mass), $m_{em} \approx m_e = 9.1 \times 10^{-28}$ g, $\omega_e = e|\mathbf{B}|/m_e = 6.2 \times 10^6\ \mathrm{s}^{-1}$, $\omega_\alpha = e|\mathbf{B}|/m_\alpha = 1.5 \times 10^2\ \mathrm{s}^{-1}$, and the inequalities $\omega_{em}^{-1}\tau_{em}^{-1} \ll \omega_{\alpha m}^{-1}\tau_{\alpha m}^{-1}$. hold in a wide range of heights. Then, $\gamma_e \ll \gamma_\alpha \ll 1$, $b_{\beta\perp} \ll b_{\beta\wedge} \ll b_{\beta\|}$, $|b_e| \gg |b_\alpha|$, and

$$b_{\beta\wedge} \approx c/|\mathbf{B}|, \ \sigma'_{\|} \approx e b_e n_e, \ \sigma'_{\wedge} \approx 0. \tag{80}$$

If the small terms are neglected, then relation (79), given (80), turns into

$$0 = \sigma'_{\|}\left(E_x \cos I \sin I + E_z \sin^2 I\right) + \frac{c}{|\mathbf{B}|} E'_z \sum_\beta e_\beta n_\beta \gamma_\beta - \frac{c}{|\mathbf{B}|} \sum_\beta \mathrm{G}_\beta \frac{\gamma_\beta^2 + \sin^2 I}{\gamma_\beta}. \tag{81}$$

Let us now consider a region far from the magnetic equator (when $I > 3°$); in this case, the terms $\sim (\gamma_\beta/\sin I)^2$, small compared to unity, may also be neglected in Eq. (81); it then follows from (81) that

$$E_z = -E_x \cot I + \frac{c}{|\mathbf{B}|\sigma'_{\|}} \sum_\beta \gamma_\beta^{-1} G_\beta \simeq -E_x \cot I + \frac{1}{e n_e} G_e + \sum_\alpha \frac{n_\alpha}{e n_e} \frac{\gamma_e}{\gamma_\alpha} \left(\frac{G_e}{n_e} + \frac{G_\alpha}{n_\alpha}\right). \tag{82}$$

We will now use this relation to eliminate the vertical electric field component E_z from Eqs. (75) written under the same assumptions as (82). For the diffusion velocities of ions of species α and electrons we will then obtain the following expressions:

$$\begin{cases} w_{\alpha x} = -v_x + w_{\alpha z} \cot I - c|\mathbf{B}|^{-1} E_y \cos \cos ec I, \\ w_{\alpha y} = -v_y + c|\mathbf{B}|^{-1} E_x \cos ec I + \gamma_\alpha c |\mathbf{B}|^{-1} E_y + \gamma_\alpha v_x \sin I - c|\mathbf{B}|^{-1} E_{p\alpha} \cos I, \\ w_{\alpha z} = v_x \sin I \cos I + c|\mathbf{B}|^{-1} E_y \cos I - E_{p\alpha} \sin^2 I \Big/ \sum_m m_{\alpha m} \tau_{\alpha m}^{-1}; \end{cases} \tag{83}$$

$$\begin{cases} w_{ex} = -v_x + w_{ez} \cot I - c|\mathbf{B}|^{-1} E_y \cos ec I, \\ w_{ey} = -v_y + c|\mathbf{B}|^{-1} E_x \cos ec I + \gamma_e c |\mathbf{B}|^{-1} E_y + \gamma_e v_x \sin I - c|\mathbf{B}|^{-1} \sum_\alpha \frac{n_\alpha \gamma_e}{n_e \gamma_\alpha} E_{p\alpha} \cos I, \\ w_{ez} = v_x \sin I \cos I + c|\mathbf{B}|^{-1} E_y \cos I - \sin^2 I \sum_\alpha \left[\frac{n_\alpha E_{p\alpha}}{n_e \sum_m m_{\alpha m} \tau_{\alpha m}^{-1}}\right]. \end{cases} \tag{84}$$

The following notation is introduced here:

$$E_{p\alpha} \equiv \frac{G_e}{n_e} + \frac{G_\alpha}{n_\alpha} = kT\left[\frac{\partial \ln n_e}{\partial z} + \frac{\partial \ln n_\alpha}{\partial z} + \frac{1}{H_\alpha} + 2\frac{\partial \ln T}{\partial z}\right]; \tag{85}$$

$$H_\alpha = \frac{kT}{m_\alpha g}$$

is the scale height for ions of species α.

On the whole, the results obtained here are consistent with the data of classic works (Dougherty, 1961; Stubbe, 1970). The only difference is that when deriving the expressions for the ion velocities $\mathbf{w}_\alpha$ via the concentrations n_α and temperature T in a weakly ionized mixture, these authors used a simplified equation of motion for electrons, $-\nabla p_e / e n_e = \mathbf{E}' + c^{-1} \mathbf{w}_e \times \mathbf{B}$, without any quantitative estimation of some of the discarded terms. This allowed Eq. (83) to be obtained for the electrostatic polarization field E_z (but without the last term on the right-hand side). However, the expressions for the diffusion velocities $\mathbf{w}_{e,\alpha}$ derived in this approach do not give an accurate approximation when discarding the terms ~ γ_α^2.in them. At the same time, in this paper, refined expressions for $\mathbf{w}_{e,\alpha}$ were derived using relations (75) and (79).

If the production rate of particles of species α in the ionosphere through all aeronomic reactions is denoted by $\dot{n}_\alpha$ (see, e.g., Stubbe, 1970), then the following equation can serve as a good approximation of the continuity equation for the α ion component:

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial z}\left\{-D_\alpha\left[\frac{1}{2}\left(\frac{\partial \ln n_e}{\partial z} + \frac{\partial \ln n_\alpha}{\partial z} + \frac{1}{H_\alpha}\right) + \frac{\partial \ln T}{\partial z}\right] \times \sin^2 I + v_x \cos I \sin I + \frac{cE_y}{|\mathbf{B}|}\cos I\right\} = \dot{n}_\alpha, \tag{86}$$

where $D_\alpha = 2kT\left[\sum_m m_{\alpha m}\tau_{\alpha m}^{-1}\right]^{-1} = 2\left[\sum_m \frac{x_m}{D_{\alpha m}}\right]^{-1}$ is the effective ambipolar diffusion coefficient. This equation confirms the conclusions about the necessity of modifying the classical theory of ambipolar diffusion when a magnetic field is present.

CONCLUSIONS

The derivation of the closed systems of hydrodynamic equations for multicomponent, partially or fully ionized gas mixtures also necessarily includes the derivation of the heat and mass transport relations that link the thermodynamic mass and energy fluxes with the gradients of the main hydrodynamic variables and external forces (gravitational and electromagnetic) acting on the individual components of the medium. A general form of such defining relations derived on the basis of both thermodynamic and rigorous gas-kinetic approaches is presented in many known monographs (De Groot and Mazur, 1964; Chapman and Cowling, 1960; Hirschfelder et al., 1961; Ferziger and Kaper, 1976; Sedov, 1984; Frank-Kamenetskii, 1987). However, so far in the literature there is still no consensus on the efficiency and appropriateness of using a specific form of the transport relations in the case of multicomponent gas mixtures and plasma derived using different approaches. The classical transport relations derived in both nonequilibrium thermodynamics and gas-kinetic theory correspond to linear relations between the thermodynamic diffusion and heat fluxes on the one hand and the thermodynamic diffusion forces and the temperature gradient on the other (this method of description may be called "fluxes via forces"). In this case, the standard Chapman–Enskog method of solving the system of Boltzmann equations for N-component mixtures of monoatomic molecules leads to fairly complex formulas for calculating the transport coefficients (in particular, the multicomponent diffusion and thermal diffusion coefficients) in the form of the ratios of determinants of order $N\xi + 1$ to determinants of order $N\xi$ (where ξ is the order of the approximation with which these coefficients are determined). At the same time, another representation of the transport relations is also possible, where the interrelated thermodynamic fluxes and forces change places, so that we can write the equations for the diffusion fluxes in the form of the so-called Stefan –Maxwell relations ("forces via fluxes") and the expression for the heat flux in a form resolved for the temperature gradient. In this case, it is important to note that only the binary diffusion coefficients for multicomponent mixtures of gases and plasma (given the correction factors for the higher approximations) enter into this new form of the transport relations.

Until recently, a kinetic analysis of the diffusion-thermal processes in a partially ionized medium based on the generalized Stefan–Maxwell relations and the corresponding expression for the heat flux has been performed in the literature without allowance for the anisotropy of transport coefficients arising in the presence of an external magnetic field. In this paper, based on the rigorous kinetic approach to modeling partially ionized gases developed in the monograph by Ferziger and Kaper (1976), we have derived the generalized Stefan–Maxwell relations for the diffusion fluxes in the longitudinal, perpendicular, and transverse directions to the magnetic field and the system of algebraic equations associated with the diffusion-thermal processes in a plasma with an external magnetic field to determine various anisotropic transport coefficients. It is important to note that allowance for the anisotropy of transport coefficients caused by the action of a magnetic field still remains a key problem in multicomponent plasma physics (Spitzer, 1957; Goland, 1963; Rose and Clarke, 1963; Gershman, 1974). The system of anisotropic Stefan–Maxwell relations derived here, which is convenient for the solution of MHD equations, allows these equations to be written in a normal Cauchy form, i.e., in a form resolved for the first coordinate derivatives of the temperature, concentrations, and fluxes. The continuum-kinetic models and efficient numerical algorithms for solving heat and mass transport problems in a wide range of Knudsen numbers (see, e.g., Zhdanov and Tirskii 2007) are known to have been developed precisely for such cases.

The results obtained here are used to for the description of ambipolar diffusion in the ionospheric plasma of the planet (in the F region), which generalizes the known information on the ambipolar diffusion of a ternary mixture (composed of electrons, ions, and neutral particles) to a multicomponent plasma in a strong electromagnetic field.

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