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Final Touchdown Phases and a Guidance and Control Methodology for a Soft Moon Landing

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Abstract—This article continues our study of spacecraft guidance and control for a soft Moon landing (see our article “Main braking phase for a soft Moon landing as a form of trajectory correction”). Rationale is given for the objectives of the subsequent (final touchdown) phases. Analytical relations for the main parameters are obtained, and the impact of various disturbing factors is estimated. A methodology is proposed for calculating the main parameters for the whole braking sequence from the sighting altitude of the main braking phase termination to braking engine thrust and its throttle range.

Keywords: Moon landing, braking engine, lunar surface relief, guidance and control, thrust control, program control

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1. MAIN OBJECTIVES AND PARAMETERS OF THE FREE-FALL PHASE

It was shown in our previous article (Likhachev et al., 2012) that the core of the guidance and control methodology at the main braking phase is to predict the final approach orbit parameters from gyro inertial data with certain errors of execution. The true values of motion parameters at the subsequent flight phases are refined using Doppler instruments, in this case a four-beam radar. The frequency shift of the surface reflected radar signal is measured for each of the four narrow beams to determine the corresponding projection of the spacecraft’s velocity relative to the surface. However, due to the limited capacity of the transceivers, the Doppler instruments can operate in a limited range of beam deflection angles from the normal to the underlying surface. This angle depends on the flight altitude and radar reflective characteristics of the surface. For an altitude of about 3000 m, the limiting deflection angle for the spacecraft’s longitudinal axis from the gravitational vertical is estimated at about 25°. Therefore, the first action of the control system upon the completion of main braking is to reorient the spacecraft’s longitudinal axis toward the gravitational vertical. Due to the measuring circuit design, the data from the Doppler instruments may be delayed by 3 s from the time the instruments enter the operability range. Moreover, in order to reduce the horizontal velocity remaining after main braking, at the subsequent phase of second (final) braking, the thrust vector should be deflected from the gravitational vertical by an angle of up to 20°. Therefore, the maximum angle of reorientation of the longitudinal axis may be 85°. Provided that the maximum angular velocity is 5°/s, taking into account the time of raising and damping, the minimum time of the free-fall phase should not be less than $t_{ff} = 18$ s. During this time, the vertical velocity of descent increases by 29.2 m/s. Due to parameter dispersion at the end of the main braking phase, the residual velocity of motion may be $3\sigma_{vr} = 15$ m/s; therefore, the minimum vertical velocity of descent at the time of restarting the braking engine should not be less than 44.2 m/s.

2. OBJECTIVES AND PARAMETERS OF THE SECOND (FINAL) BRAKING PHASE

The main objectives of second (final) braking are:

—Reducing the spacecraft’s flight altitude until its landing supports touch the Moon’s surface.

—Reducing the spacecraft’s vertical velocity of descent to 2.5–3 m/s.

—Reducing the spacecraft’s horizontal velocity to 1 m/s.

—Correcting the angle between the spacecraft’s longitudinal axis and the gravitational vertical until it is 5°–7°.

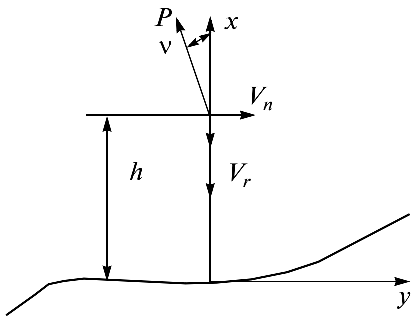

Fig. 1. Spacecraft motion diagram at the second (final) braking phase.

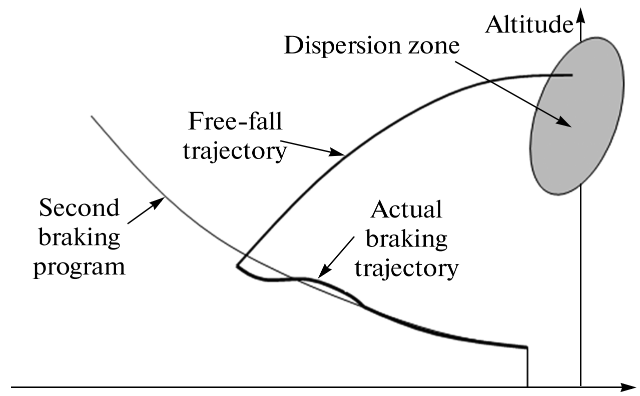

Fig. 2. Trajectory of the final touchdown phases.

It is assumed that second (final) braking is performed in the designated landing zone above a relatively even area with an average slant angle of 12° or less. Moreover, since the spacecraft flies only a short distance along the surface, its motion during this phase can be considered as motion in a plane–parallel gravitational field with a constant free-fall acceleration of $g_M = 1.623$ m/s^2.

The spacecraft's motion profile is shown in Fig. 1. Here, h is the spacecraft's altitude above the Moon's surface; V_n is the horizontal component of the spacecraft's velocity; V_r is the vertical velocity component; P is the braking engine thrust; ν is the angle of deflection of the thrust vector from the gravitational vertical; and g_M is the free-fall acceleration on the Moon's surface.

Since fuel consumption during second braking is 5% or less of its initial mass, the spacecraft motion during this phase can be considered as motion of a body with constant mass. Therefore, the acceleration produced by the braking engine at constant thrust can be considered a constant value of $W = P/m_{sc}$, where m_{sc} is the spacecraft's mass at the time of termination of the main braking phase.

Under these assumptions, the spacecraft motion equations are easily integrated, and we can write the relations for the free-fall phase:

$$h_{ff} = h_0 + V_{r0}t - 0.5g_M t^2$$

and

$$V_r = V_{r0} - g_M t,$$

and for the second braking phase:

$$h = h_{start} + V_{rstart}(t - t_{start}) + 0.5(W - g_M)(t - t_{start})^2,$$

$$V_r = V_{rstart} + (W - g_M)(t - t_{start}).$$

Excluding time from these relations, we get the relationship between the altitude and vertical velocity for each phase

$$h_0 - h = 0.5(V_{r0}^2 - V^2)/(W - g_M),$$

where $W = 0$ at the free-fall phase and $W = P/m_{sc}$ at the second (final) braking phase; h_0 and V_{r0} are the initial altitude and vertical velocity at the free-fall phase; $h = h_{start}$ and $V_{r0} = V_{rstart}$ are altitude and vertical velocity at the beginning of second braking (restarting the braking engine); and h and V are the current altitude and vertical velocity for a given phase.

We denote the excess acceleration (i.e., exceeding the free-fall acceleration on the Moon) as $W_{pr} = W - g_M$ and assume it is constant. Then, for altitude and vertical velocity to be simultaneously reduced to zero, the relationship $h = 0.5V^2/W_{pr}$ must be true both at the restart point and at every point during second braking. This relationship can be used as a spacecraft motion program during the second (final) braking phase. The program parameter is the spacecraft vertical velocity as a function of the measured altitude $V_{pr} = -\sqrt{2W_{pr}h_{meas}}$, and the deviation from the program is the difference between the measured vertical velocity (V_{meas}) and its program value $\Delta V = V_{meas} - V_{pr}$. If ΔV is positive, the braking engine thrust should be reduced, and if it is negative, the thrust should be increased. Figure 2 shows, in the altitude–vertical velocity phase plane, the free-fall and second braking trajectories for this guidance and control program.

The question arises at which time the system should issue a command to restart the engine? At the free-fall phase, the measured vertical velocity is greater than the program value. As is evident from Fig. 2, the transition from free-fall to second braking should be performed at the time when the measured and program velocities coincide. This is true of an instant transition of the engine to the nominal thrust regime. However, the time needed for the engine to develop the nominal thrust (t_{dev}) is significantly nonzero. Therefore, the restart command should be given beforehand, namely, before the deviation from the program reaches the value $\Delta V = (W_{pr} + g_M)t_{dev}$. A delay of t_{dev} against the nominal time will cause the motion parameters to deviate from the program, which, in its turn, will require a change in the level of thrust.

It should also be noted that the characteristic velocity at the second (final) braking phase (if the spacecraft follows the program curve) is determined by the velocity at the time of restarting the engine and by the program acceleration: $V_{char} = V_{start}(W_{pr} + g)/W_{pr}$. Here the ratio $(W_{pr} + g)/W_{pr}$ is the gravity loss factor of the braking maneuver. At $W_{pr} = 2g_M$ this factor is 2, and at $W_{pr} = 5$ m/s^2 it is 1.32. Naturally, efforts should be made to increase the program acceleration so as to reduce the characteristic velocity costs of the second braking phase.

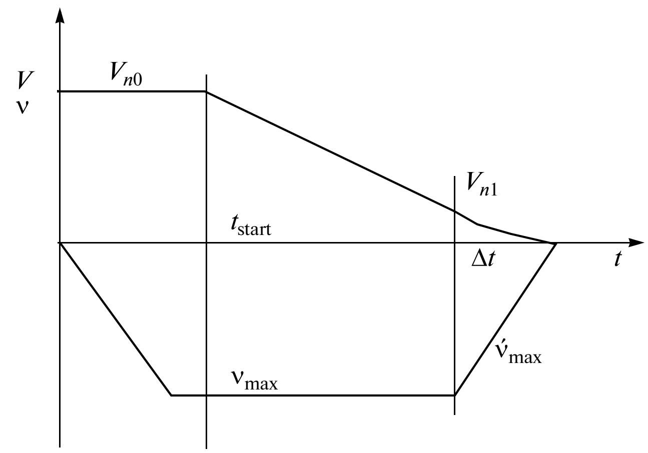

Fig. 3. Change in the horizontal velocity and deflection angle of the thrust vector from the gravitational vertical.

2.1. Horizontal Velocity Control

One of the factors affecting the choice of the thrust control range is horizontal velocity control. The main task of the control loop is to zero the horizontal velocity by the time the spacecraft touches the surface. In fact, horizontal velocity damping begins when the engine develops the nominal thrust. As a result of the deviation of the thrust vector from the gravitational vertical, the acceleration has a component in the horizontal plane. Given perfect tracking of the braking programs, the vertical component of the total acceleration from the braking engine is $W_{ver} = W_{pr} + g_M$, the horizontal component is $W_{hor} = (W_{pr} + g_M)\tan\nu$, and the total acceleration produced by the braking engine is $W = (W_{pr} + g_M)/\cos\nu$.

Figure 3 shows the time-optimal process of horizontal velocity damping during second braking. The process in Fig. 3 consists of a portion with a constant deflection angle of the thrust vector from the gravitational vertical and a portion with a constant angular velocity of rotation to bring the thrust vector back to the vertical.

It was noted above that the maximum deflection angle is $\nu_{max} = 20°$ and the maximum angular velocity is $\dot{\nu}_{max} = 5°$/s. The limiting value of the deflection angle is reached during the free-fall phase. We need to determine the duration of velocity damping at this value of the angle while the duration of the final portion of descent is 4 s. In this portion the horizontal acceleration component is described by the relationship

$$dV_n/dt = W_{hor} = (W_{pr} + g_M)\tan(\nu_{max} - \dot{\nu}_{max}t).$$

After integrating this equation over a 4-s period with a zero final velocity, we have

$$V_{n1} = -(W_{pr} + g_M)\ln(\cos\nu_{max})/\dot{\nu}_{max},$$

where V_{n1} is the desired horizontal velocity at the time when the deflection is below the limit.

If horizontal velocity is controlled using the proportional control law $\nu_{pr} = k_{vn}V_{n\,meas}$, then at $k_{vn} = \nu_{max}/V_{n1}$ we can have a process similar to the one depicted in Fig. 3. We can also estimate the total duration of horizontal velocity damping. The final portion of the process lasts 4 s during which the horizontal velocity decreases by V_{nl} and the residual horizontal velocity $\Delta V_n = V_{n0} - V_{n1}$ decreases at the maximum value of the deflection angle during the time $\Delta t = (V_{n0} - V_{nl})/(W_{pr} + g_M)/\tan\nu_{max}$. The distance covered by the spacecraft along the Moon's surface during the period from giving the restart command to zeroing the horizontal velocity is (in the nominal case) is $L = V_{n0}t_{start} + 0.5(V_{n0} - V_{nl})\Delta t + 4V_{nl}/3$.

Thus, counteracting the horizontal velocity:

—Leads to a 6% reduction in the available range of accelerations from the braking engine to control the vertical velocity.

—Leads to a possible change in the altitude difference for the second (final) braking phase by changing the surface slant angle $\Delta h = L\tan(\varphi_p)$;

—Requires that the time of second braking be no less than the time of horizontal velocity damping $t_{sb} \geq 4$ s $+ \Delta t$ and that the vertical velocity at the time of restarting the braking engine satisfy the constraint $V_{start} \geq (4\text{ s} + \Delta t)W_{pr}$.

2.2. Estimating the Thrust Control Range at the Second Braking Phase

When using a fixed program for vertical velocity control, one of the key issues is to choose program acceleration so that, given the presence of disturbing factors, the average acceleration will not exceed the

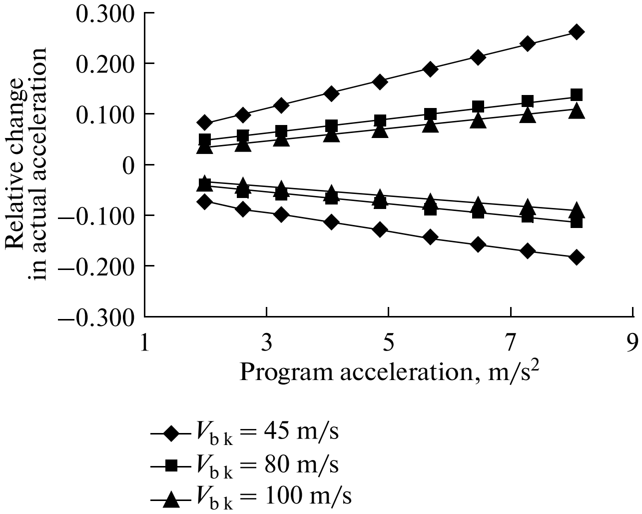

Fig. 4. Change in the actual excess acceleration relative to the program value, taking into account the delay ±0.8 s.

maximum net acceleration from the braking engine. In other words, the program acceleration must satisfy the condition

$$W_{pr} \le \alpha(\beta W_{max}\cos\nu - g_M).$$

Here the coefficient $\beta < 1$ must take into account the losses in acceleration due to the tolerance for the spacecraft's mass at the time of termination of main braking and the tolerance for the maximum thrust of the braking engine. The coefficient α must take into account the effects of the tolerances for parameters affecting spacecraft guidance, i.e., tolerance for the delay in developing the nominal thrust, effects of changes in surface relief height during second braking, effects of altitude and vertical velocity measurement errors from the data obtained with Doppler instruments, deviation of the thrust vector from the gravitational vertical to reduce horizontal velocity, and dynamic errors in angular motion control and thrust control. In what follows we assume that the control system works perfectly and dynamic errors can be ignored.

The tolerance for braking engine thrust is, as a rule, about 5–6% of its maximum thrust.

The mass tolerance at the beginning of second braking depends on the entire flight history, i.e., on whether all the planned corrections were performed en route from the Earth to the Moon and in the artificial lunar satellite orbit, on the fuel consumption for angular motion control, and on the change in fuel consumption due to the tolerance for specific thrust during braking in going from the Earth–Moon route to an orbit around the Moon and during the main braking phase. According to preliminary estimates, the mass tolerance can be 3 to 5%. Therefore, the coefficient β can take values in the range 0.92–0.94.

Provided that thrust control is perfect, the system tracks the program dependence $2h_{meas}\hat{W}_{pr} = V^2_{meas}$, or $2(1 \pm \delta_h)h\hat{W}_{pr} = (1 \pm \delta_v)^2V^2$. Here, δ_h and δ_v are the relative measurement errors for altitude and vertical velocity. For Doppler instrument measurements, each of these errors can be no more than 1%; the true excess acceleration can be estimated by the formula $\hat{W}_{pr} = (1 \pm \delta_h \pm 2\delta_v)W_{pr}$. We use the notation $\delta_i = \hat{W}_{pri}/W_{pr} - 1$ to denote the relative change in the actual excess acceleration due to the effect of the ith factor.

Given the nominal engine thrust development time, the vertical velocity and altitude at the time of transition to the nominal regime will be consistent with the program trajectory. The delay tolerance ($\Delta\tau > 0$) leads to altitude and vertical velocity "drawdowns" (relative to the program trajectory) of, respectively,

$$\Delta h = -V_{start}\Delta\tau$$

and

$$\Delta V = -g_M\Delta\tau.$$

To counteract the effects of these disturbances, by the time of termination of second braking, the average value of the actual excess acceleration must be

$$\hat{W}_{pr} = 0.5(V_{start} + g\Delta\tau)^2/(h_{start} - V_{start}\Delta\tau).$$

The ratio between the actual excess and program accelerations for vertical velocities of 45, 80, and 100 m/s and a delay tolerance of ±0.8 s for the braking engine to develop the nominal thrust is presented in Fig. 4.

The larger the program acceleration and the lower the vertical velocity at the engine restart point, the greater the difference between the actual excess acceleration and the program acceleration; in some cases this difference can be as large as 50%. This difference is almost linearly dependent on the program acceleration. Provided that the program acceleration is 6 m/s² and the tolerance $\Delta\tau$ is +0.8 s, the excess acceleration is greater than the program acceleration by 35%.

If the engine develops the nominal thrust more rapidly (i.e., the delay tolerance $\Delta\tau$ is negative), the altitude will be higher than the program curve. Therefore, to successfully complete second braking, the system needs to reduce the thrust. The required decrease in excess acceleration for program acceleration of 6 m/s² and tolerance $\Delta\tau = -0.8$ s is –22%.

We now consider the surface slant at the landing site. We assume that the slant angle is in the range ±12°. The change in altitude due to the surface slant was considered at the free-fall phase in the measurements made with Doppler instruments until the braking engine restart command was issued; after the restart, the change in relief height should be compensated by controlling the braking engine thrust. If we assume that the horizontal velocity component is 15 m/s or less, then the change in altitude during second braking must not exceed 20 m. Using the above procedure for determining the actual acceleration, we obtain the relative change in the actual excess acceleration to compensate for the slant angle of ±12° (Fig. 5).

The pattern of changing the actual acceleration is the same as in taking into account the engine delay. Given an acceleration of 6 m/s², the actual excess acceleration increases by 5.5% for a positive slant and decreases by about 2% for a negative slant.

Taking into account the random nature of all the above tolerances and surface slant, the coefficients α and β are as follows:

$$\alpha = 1 \pm \sqrt{(2\delta_v)^2 + (\delta_h)^2 + (\delta_\tau)^2 + (\delta_\varphi)^2}$$

and

$$\beta = 1 - \sqrt{(\delta_m)^2 + (\delta_p)^2},$$

where δ_τ is the relative change in the actual excess acceleration relative to the program acceleration due to the delay tolerance for the engine to develop the nominal thrust; δ_φ is the relative change in the actual acceleration to compensate for the change in the surface relief height; δ_m is the tolerance for the spacecraft's mass; and δ_p is the tolerance for the maximum thrust of the braking engine.

According to Figs. 4 and 5, the coefficient α is largely dependent on the program acceleration as such, the vertical velocity at the time of restarting the braking engine, and the said tolerances.

The dependence of the maximum required acceleration on the program acceleration is

$$W_{\max} = (W_{\mathrm{pr}}\alpha + g_{\mathrm{M}})/(\beta\cos\nu_{\max}),$$

where all the perturbing factors lead to $\delta_i > 0$ and $\alpha > 1$. Correspondingly, the dependence for the minimum required acceleration is calculated by the formula:

$$W_{\min} = W_{\mathrm{pr}}\alpha + g_{\mathrm{M}},$$

where all the perturbing factors lead to the value $\delta_i < 0$ and $\alpha < 1$.

The ratio between the maximum and minimum required acceleration characterizes the braking engine thrust throttle range necessary to counteract the perturbing factors

$$\varepsilon = W_{\max}/W_{\min}.$$

The dependence for the maximum and minimum required accelerations and the engine thrust throttle range for a delay tolerance of $\Delta\tau = \pm 0.8$ s is presented in Table 1. When building the dependence, we assumed that $\beta = 0.932$.

It is evident that the required throttle range at the second braking phase increases with increasing pro-

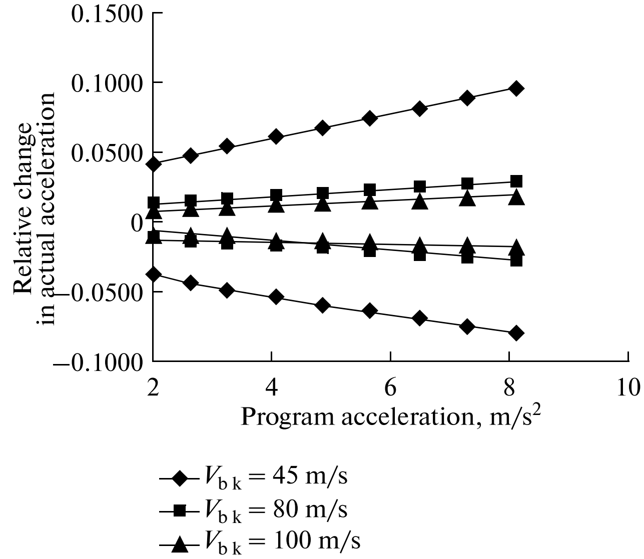

Fig. 5. Change in the actual acceleration to compensate for a slant angle of ±12°.

gram acceleration. For $W_{\mathrm{pr}} = 3.25$ m/s², the throttle range must be no less than 1.5, and for $W_{\mathrm{pr}} = 6.21$ m/s², it must be no less than 2.0. Note that the throttle range at the main braking phase must be 1.2.

3. DESCENT AT CONSTANT SPEED

The objective of descent at constant speed is to terminate all the transition processes and compensate for altitude dispersion from the previous phase. The nominal velocity of descent is selected so that the dynamic control errors and Doppler measurement inaccuracies would not result in exceeding the upper limit for vertical velocity at the time when the landing supports touch the Moon's surface. At this phase, spacecraft use, as a rule, special landing engines, which balance, on average, the spacecraft's lunar weight by controlling thrust in the range ±10% of the nominal value. If Doppler instruments do not work at small altitudes, the control is based only on the gyroscopic and accelerometric instruments of the motion control system.

4. METHODOLOGY FOR CALCULATING THE BRAKING PARAMETERS

Using the above materials together with (Likhachev et al., 2012) on the braking sequence, we can determine not only the main parameters for each braking phase but also some design parameters to optimize the total fuel consumption during braking. Below is the proposed methodology, which is essentially a sequence of computing operations (steps):

(1) The following parameters are determined: the possible spacecraft mass range m_{mb} at the beginning of the main braking phase, the possible nominal thrust range $P_1, P_2, ..., P_i$, and the specific thrust of the main braking engine. The complete combustion times Tp_i are calculated. Based on the design experience, each of the calculated Tp in the series is assigned a sighting altitude H_{sc} of the main braking phase termination.

(2) For the selected final approach orbit parameters and each Tp in the said range, a main-braking boundary problem is calculated at the zero value of the true anomaly of the final approach orbit exit point.

(3) A value is given for the true anomaly before the periapsis, which is set equal to the angular distance along the surface calculated from the boundary problem solution. The boundary problem is recalculated.

(4) The calculated results for the last boundary problem are used to determine:

—The energy costs to perform main braking V_{charmb}.

—The length of the main braking phase t_{mb}.

—The spacecraft's final mass m_{sc}.

—The nominal acceleration from the braking engine at the time of its restart $\hat{W}$.

—The program values for the initial deflection angle of the thrust vector from the gravitational vertical ν_{pr} and rotational velocity relative to the pitch angle $\dot{\nu}_{pr}$.

—The deflection angle of the thrust vector from the gravitational vertical at the time of termination of main braking ν_t.

(5) The end parameters of the main braking phase are statistically calculated to determine the standard deviation in altitude σ_h at the end of the phase.

(6) The standard deviations are determined (or adopted) for the vertical (σ_{vr}) and horizontal (σ_{vn}) velocity components at the time of termination of main braking.

(7) The maximum acceleration developed by the engine during second braking is calculated: $W_{max} = 1.1\hat{W}$.

(8) The minimum time of the free-fall phase t_{ffmin} is estimated from the values of the deflection angle of the thrust vector from the gravitational vertical at the time of termination of main braking, the maximum angular velocity of the spacecraft, and the maximum deflection angle of the thrust vector from the gravitational vertical during residual horizontal velocity damping at the second braking phase.

(9) Given that the vertical velocity at the time of restarting the braking engine is

$$V_{startmin} = t_{ffmin}g_M + 3\sigma_{vr},$$

the following values are calculated for the program acceleration range $W_{pr} = 3$ to 8 m/s^2: the velocity components and the coefficient α_+ for a positive delay time of thrust development and a positive surface slant

$$\alpha^+ = 1 + \sqrt{(2\delta_v)^2 + (\delta_h)^2 + (\delta_\tau)^2 + (\delta_\varphi)^2},$$

where $\delta_v = 0.01$; $\delta_h = 0.01$; $\delta_\tau = \hat{W}_{pr\tau}/W_{pr} - 1$; $\hat{W}_{pr\tau} = 0.5(V_{startmin} + g_M\Delta\tau)^2/(h_{startmin} - V_{startmin}\Delta\tau - h_{sc})$; $\Delta\tau > 0$, $h_{startmin} = 0.5(V_{startmin})^2/W_{pr} + h_{sc}$; $\delta_\varphi = \hat{W}_{pr\varphi}/W_{pr} - 1$, $\hat{W}_{pr\varphi} = 0.5(V_{startmin})^2/(h_{startmin} - \Delta h - h_{sc})$; $\Delta h = [0.5(3\sigma_{vn} - V_{n1})\Delta t + 4V_{n1}/3]\tan\varphi$; $\varphi > 0$ ($\varphi = 12$ deg $= \pi/15$); $V_{n1} = -(W_{pr} + g_M)\ln(\cos\nu_{max})/\dot{\nu}_{max}$; $\nu_{max} = 20$ deg $= \pi/9$; $\dot{\nu}_{max} = 5$ deg/s $= 5\pi/180$ 1/s; $\Delta t = (3\sigma_{vn} - V_{n1})(W_{pr} + g_M)/\tan\nu_{max}$; $t_{sbmin} = V_{startmin}/W_{pr}$.

If $t_{sbmin} < \Delta t + 4$ s, then reiterate step 9 with the value

$$V_{startmin} = (\Delta t + 4\text{ s})W_{pr}.$$

(10) For the adopted W_{pr}, the corresponding values are calculated for the maximum required acceleration from the braking engine

$$W_{max} = (W_{pr}\alpha^+ + g_M)/(\beta\cos\nu_{max}),$$

(11) For the same values of program acceleration, one calculates the minimum acceleration from the braking engine

$$W_{min} = \alpha^- W_{pr} + g_M,$$

where $\alpha^- = 1 - \sqrt{(2\delta_v)^2 + (\delta_h)^2 + (\delta_\tau)^2 + (\delta_\varphi)^2}$; $\delta_v = -0.01$; $\delta h = -0.01$; $\delta_\tau = \hat{W}_{pr\tau}/W_{pr} - 1$; $\hat{W}_{pr\tau} = 0.5(V_{startmin} + g_M\Delta\tau)^2/(h_{startmin} - V_{startmin}\Delta\tau - h_{sc})$; $\Delta\tau < 0$; $\delta_\varphi = \hat{W}_{pr\varphi}/W_{pr} - 1$, $\hat{W}_{pr\varphi} = 0.5(V_{startmin})^2/(h_{startmin} - \Delta h - h_{sc})$; $\Delta h = [0.5(3\sigma_{vn} - V_{n1})\Delta t + 4V_{n1}/3]\tan\varphi$; $\varphi = -12$ deg $= -\pi/15$).

(12) The required throttle range is determined for the braking engine thrust at the second braking phase

$$\varepsilon = W_{max}/W_{min}.$$

(13) The dependence W_{max} is plotted as a function of program acceleration:

$$W_{max} = f(W_{pr}).$$

(14) For the selected length of the complete combustion period Tp and maximum acceleration values (step 7), an allowable value of program acceleration W_{pr} is determined using the plot built at step (13).

(15) Values are calculated for the minimum altitude of the dispersion zone and the sighting altitude of main braking termination

$$H_{\min} = h_{\text{startmin}} + 0.5(V_{\text{startmin}})^2/g_{\text{M}};$$
$$\hat{H}_{\text{sc}} = H_{\min} + 3\sigma_{\text{h}}.$$

(16) The resulting sighting altitude $\hat{H}_{\text{sc}}$ is compared to the value set in the boundary problem (H_{sc}). If they differ by more than 20 m, than it is assumed that $H_{\text{sc}} = \hat{H}_{\text{sc}}$ and steps (3)–(16) are repeated. (If the values of H_{sc} and $\hat{H}_{\text{sc}}$ differ by no more than 200 m, it is not necessary to refine the statistical characteristics of parameter dispersion at the end of main braking (step 5).)

(17) The maximum altitude is estimated for the termination of the main braking phase: $H_{\text{sc}} = \hat{H}_{\text{sc}} + 3\sigma_{\text{h}}$.

(18) Values are determined for the maximum altitude and the vertical velocity at the time of restarting the braking engine to begin the second braking phase.

$$h_{\text{startmax}} = 0.5(V_{\text{startmax}})^2/W_{\text{pr}};$$
$$V_{\text{startmax}} = \sqrt{2g(H_{\max} - h_{\text{sc}})(1 + W_{\text{pr}}/g_{\text{M}})}$$

(19) A value is calculated for the engine acceleration resulting in the maximum characteristic velocity costs at the second braking phase, which correspond to the maximum velocity at the time of restarting the engine, the rapid development of the nominal thrust, and a downward surface slant during braking.

$$W_{\text{char}} = \alpha_{\text{char}} W_{\text{pr}} + g_{\text{M}},$$

where $\alpha_{\text{char}} = 1 - \sqrt{(2\delta_v)^2 + (\delta_h)^2 + (\delta_\tau)^2 + (\delta_\varphi)^2}$; $\delta_v = -0.01$; $\delta_h = -0.01$; $\delta_\tau = \hat{W}_{\text{pr}\tau}/W_{\text{pr}} - 1$; $\hat{W}_{\text{pr}\tau} = 0.5(V_{\text{startmax}} + g_{\text{M}}\Delta\tau)^2/(h_{\text{startmax}} - V_{\text{startmax}}\Delta\tau - h_{\text{sc}})$; $\Delta\tau < 0$; $\delta_\varphi = \hat{W}_{\text{pr}\varphi}/W_{\text{pr}} - 1$, $\hat{W}_{\text{pr}\varphi} = 0.5(V_{\text{startmax}})^2/(h_{\text{startmax}} - \Delta h - h_{\text{sc}})$; $\Delta h = [0.5(3\sigma_{\text{vr}} - V_{n1})\Delta t + 4V_{n1}/3]\tan\varphi$; $\varphi = -12\ \text{deg} = -\pi/15$.

(20) The characteristic velocity at the second (final) braking phase is calculated:

$$V_{\text{charsb}} = V_{\text{startmax}}(\alpha_{\text{char}} W_{\text{pr}} + g_{\text{M}})/(\alpha_{\text{char}} W_{\text{pr}}).$$

(21) The characteristic velocity costs at the phase of constant-speed descent are taken at $V_{\text{charcsd}} = 10g_{\text{M}}$.

(22) The characteristic velocity costs are calculated for the entire braking trajectory

$$V_{\text{char}\Sigma} = V_{\text{charmb}} + V_{\text{charsb}} + V_{\text{charcsd}}.$$

(23) The ratio is calculated between the spacecraft mass at the time of touching the Moon's surface and its mass before the main braking phase

$$\ln(\mu) = -V_{\text{char}\Sigma}/(P_{\text{sp}} g_{\text{M}}).$$

Tables 2 and 3 show the results obtained at the final iteration step of the proposed technique for two values of the delay tolerance for nominal thrust development

Table 1. Ratio between the maximum and minimum required accelerations

W_{pr}, m/s^2	$W_{\max}$, m/s^2	$W_{\min}$, m/s^2	ε
1.62	5.19	3.36	1.370
2.16	5.68	3.71	1.413
2.71	6.17	4.06	1.456
3.25	6.5	4.34	1.497
4.06	7.98	4.94	1.615
4.87	9.64	5.5	1.752
5.68	11.56	6.04	1.914
6.49	13.83	6.55	2.111
7.30	16.60	7.04	2.357
8.12	20.10	7.51	2.677

(±0.5 and ±0.8 s) as the main parameter affecting the characteristics of second braking. The calculations were performed for a final approach orbit with a periapsis of 18 km and apoapsis of 100 km. The vertical and horizontal velocity dispersion at the end of main braking were taken at $\sigma_{\text{vr}} = \sigma_{\text{vn}} = 5$ m/s.

The data presented in Tables 2 and 3 give, in the first approximation, the values of all the braking parameters, depending on the thrust-to-weight ratio and mass flow rate. Obviously, to make a soft landing with a maximum final mass of the spacecraft, efforts should be made to achieve the least length of the complete combustion period or, equivalently, to increase the thrust-to-weight ratio given the maximum specific impulse of the braking engine. Increasing the thrust-to-weight ratio for second braking results in a larger thrust throttle range, which greatly complicates the braking engine design and raises the financial expenses. If the throttle range is limited by 1.5, then a decrease in the spacecraft's final mass against the maximum limit is 0.35 to 0.68% of the initial mass, depending on the tolerance for the delay in developing the nominal thrust. In addition, the braking engine should have a complete combustion period of $Tp =$

Table 2. Parameters of the braking sequence with a random delay tolerance of ±0.8 s for the engine to develop the nominal thrust

Tp, s	1027	863	640	512	427
ν_{pr}, deg	–89.4	–92.60	–97.80	–102.19	–106.25
ν'_{pr}, deg/s	0.0969	0.1128	0.1526	0.2027	0.2637
V_{charmb}, m/s	1774.4	1757.6	1744.5	1743.5	1747.7
μ_{mb}	0.5682	0.5714	0.5738	0.5740	0.5732
φ_{mb}, deg	13.06	10.84	8.12	6.51	5.44
H_{sc}, m	3100	2650	2200	2000	1800
σ_H, m	700	582	456	390	332
$\hat{W}_{mb}$, m/s^2	5.39	6.44	8.55	10.58	12.83
W_{max}, m/s^2	5.93	7.08	9.41	11.75	14.11
α	0.807	0.771	0.699	0.647	0.610
β	0.932	0.932	0.932	0.932	0.932
W_{pr}, m/s^2	2.992	3.725	5.085	6.405	7.724
W_{min}, m/s^2	4.15	4.69	5.63	6.54	7.47
$h_{startmin}$, m	358	292	219	198	192
h_{min}, m	982	916	843	821	816
$\hat{H}_{sc}$, m	3092	2672	2220	1992	1812
h_{max}, m	5202	4428	3600	3161	2808
V_{max}, m/c	104.4	99.83	93.84	90.2	86.5
W_{char}, m/s^2	4.4	5.03	6.16	7.19	8.15
t_{ffmin}, s	18	18	18	18	18
t_{ffmax}, s	73.6	70.8	67.1	64.8	62.5
t_{sbmin}, s	11.4	9.6	8.3	7.5	7.0
t_{sbmax}, s	37.6	29.3	20.7	16.2	13.2
t_{csdmin}, s	5	5	5	5	5
t_{sbmax}, s	10	10	10	10	10
$t_{\Sigma min}$, s	476	399	303	248	212
$t_{\Sigma max}$, s	563	476	371	309	271
V_{charsb}, m/s	165.45	147.34	127.44	116.5	108
$V_{charcsd}$, m/s	16.23	16.23	16.23	16.23	16.23
$V_{char\Sigma}$, m/s	1956.4	1938.3	1918.4	1907.4	1898.9
μ_Σ	0.5362	0.5393	0.5428	0.5447	0.5461
ε no less than	1.43	1.51	1.67	1.80	1.89

Table 3. Parameters of the braking sequence with a random delay tolerance of ±0.5 s for the engine to develop the nominal thrust

Tp, s	1027	863	640	512	427
ν_{pr}, deg	–89.4	–92.60	–97.80	–102.19	–106.25
$\dot{\nu}_{pr}$, deg/s	0.0969	0.1128	0.1526	0.2027	0.2637
V_{charmb}, m/s	1774.4	1757.6	1744.5	1743.5	1747.7
μ_{mb}	0.5682	0.5714	0.5738	0.5740	0.5732
φ_{mb}, deg	13.06	10.84	8.12	6.51	5.44
H_{sc}, m	3100	2650	2200	2000	1800
σ_H, m	700	582	456	390	332
$\hat{W}_{mb}$, m/s^2	5.39	6.44	8.55	10.58	12.83
W_{max}, m/s^2	5.93	7.08	9.41	11.75	14.11
α	0.872	0.849	0.803	0.772	0.750
β	0.932	0.932	0.932	0.932	0.932
W_{pr}, m/s^2	3.16	3.98	5.53	7.06	8.58
W_{min}, m/s^2	4.43	5.09	6.29	7.46	8.64
$h_{startmin}$, m	340	274.7	203.1	181.2	174.6
h_{min}, m	964	899	827	805	798
$\hat{H}_{sc}$, m	3074	2655	2205	1975	1795
h_{max}, m	5184	4411	3582	3145	2790
V_{max}, m/c	105.25	100.6	94.56	90.82	86.96
W_{char}, m/s^2	4.63	5.36	6.73	8.02	9.25
t_{ffmin}, s	18	18	18	18	18
t_{ffmax}, s	74.1	71.2	67.5	65.2	62.8
t_{sbmin}, s	10.65	9.4	7.8	6.8	6.0
t_{sbmax}, s	35	26.9	18.5	14.2	11.4
t_{csdmin}, s	5	5	5	5	5
t_{sbmax}, s	10	10	10	10	10
$t_{\Sigma min}$, s	476	398	304	248	211
$t_{\Sigma max}$, s	561	474	369	308	266
V_{charsb}, m/s	162.11	144.25	124.6	113.85	105.48
$V_{charcsd}$, m/s	16.23	16.23	16.23	16.23	16.23
$V_{char\Sigma}$, m/s	1953.0	1935.18	1915.53	1904.78	1896.41
μ_Σ	0.5368	0.5394	0.5432	0.5451	0.5466
ε no less than	1.34	1.39	1.50	1.58	1.63

640 s for a delay tolerance of $\Delta\tau = \pm 0.5$ s and Tp = 865 s at $\Delta\tau = \pm 0.8$ s.

CONCLUSIONS

The proposed methodology, which is based on analysis of guidance and control methodologies for spacecraft making a soft landing in the near-polar regions of the Moon, allows one to select, in the first approximation, the main parameters of each braking phase and formulate the requirements for the braking thruster. The final requirements for the thruster should consider the mass of the braking engine and fuel tanks. In addition, it is necessary to analyze the feasibility and costs of developing an engine with a large throttle range.

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