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PARTIALLY COMMUTATIVE METABELIAN PRO-*P*-GROUPS

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Abstract: We prove two theorems about a partially commutative metabelian pro-p-group. The first theorem concerns the structure of annihilators for the commutators of nonadjacent vertices of the defining graph, and the second discusses the centralizers of the vertices of the defining graph.

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1. Introduction

This article deals with the centralizers of elements and the annihilators of commutators in partially commutative metabelian pro-p-groups. We obtain results that are rather similar to those in [1] for partially commutative metabelian abstract groups. Let us agree that, since here we consider pro-p-groups, when we say a subgroup, homomorphism, generating set, etc., we will mean a closed subgroup, a continuous homomorphism, a generating set, etc. in the topological sense.

Given an undirected finite graph Γ without multiple edges, let us define the partially commutative metabelian pro-p-group F_Γ. Let $X = \{x_1, \dots, x_n\}$ be the vertex set of Γ. Denote by F the free metabelian pro-p-group with basis X. Let $E = E(\Gamma)$ be the edge set of Γ. The group F_Γ is obtained from F by imposing the additional relations: $[x_i, x_j] = 1$ if the vertices x_i and x_j are adjacent in Γ, i.e. $(x_i, x_j) \in E$; so that $F_\Gamma = F/N_\Gamma$, where N_Γ is the normal subgroup in F generated by the commutators $[x_i, x_j]$ for which $(x_i, x_j) \in E$. We have the short exact sequence

$$1 \longrightarrow N_\Gamma \longrightarrow F \longrightarrow F_\Gamma \longrightarrow 1.$$

The graph Γ is called the *defining graph* of F_Γ.

Clearly, the quotient group of F_Γ by the commutant F'_Γ is a free abelian pro-p-group A with the basis $\{a_1, \dots, a_n\}$ that is the image of X. This group is isomorphic to the direct sum of n copies of the additive group of the ring of p-adic integers $\mathbb{Z}_p$. The action of F_Γ on F'_Γ by conjugations

$$x \to x^g = g^{-1}xg$$

defines the structure of a right module over the completed group algebra $\mathbb{Z}_p[[A]]$ on F'_Γ which is identified with the power series algebra $\mathbb{Z}_p[[y_1, \dots, y_n]]$ provided that $y_i = a_i - 1$. In the same manner, F' is a module over the same algebra $\mathbb{Z}_p[[y_1, \dots, y_n]]$. Therefore, each $f \in F$ is representable as

$$f = x_1^{l_1} \cdots x_n^{l_n} \prod_{1 \le i < j \le n} [x_i, x_j]^{\alpha_{ij}},$$

where $l_i \in \mathbb{Z}_p$ and $\alpha_{ij} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$.

Given a graph Γ and two vertices x_i and x_j of Γ, define the ideal $\mathscr{A}^\Gamma_{i,j}$ of $\mathbb{Z}_p[[y_1, \dots, y_n]]$. If x_i and x_j lie in the different connected components of Γ, then we put $\mathscr{A}^\Gamma_{i,j} = 0$. If x_i and x_j lie in the same connected component, then we consider each path without returns $(x_i, x_{i_1}, \dots, x_{i_r}, x_j)$ between these vertices and associate with the path $y_{i_1} \cdots y_{i_r}$ when the length of the path is greater than 1, whereas equals 1 when the length of the path is 1. By definition, $\mathscr{A}^\Gamma_{i,j}$ is generated by all these elements. In particular, if x_i and x_j are joined by an edge then $\mathscr{A}^\Gamma_{i,j}$ contains 1 and so coincides with $\mathbb{Z}_p[[y_1, \dots, y_n]]$.

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Let us formulate the main results of the article:

Theorem 1. *Let $x_1, \dots, x_n$, $n \geq 2$, be the vertices of the defining graph Γ of a partially commutative metabelian pro-p-group F_Γ. Then, for $i \neq j$, the annihilator of the commutator $[x_i, x_j]$ in $\mathbb{Z}_p[[y_1, \dots, y_n]]$ coincides with $\mathscr{A}^\Gamma_{i,j}$.*

Theorem 2. *Let $x_1, \dots, x_n$, $n \geq 2$, be the vertices of the defining graph Γ of a partially commutative metabelian pro-p-group F_Γ and let $x_2, \dots, x_m$ be the vertices adjacent to x_1. An element $g \in F_\Gamma$ lies in the centralizer of x_1 if and only if g can be written as*

$$g = x_1^{l_1} \cdots x_m^{l_m} \prod_{2 \leq i < j \leq m} [x_i, x_j]^{\gamma_{i,j}},$$

where $l_i \in \mathbb{Z}_p$ and $\gamma_{i,j} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$.

2. Proof of Theorem 1

2.1. We will use the representation of a p-group F which is based on the Magnus embedding for pro-p-groups; for this embedding the reader is referred to [2, 3].

Let T be a right topological free $\mathbb{Z}_p[[y_1, \dots, y_n]]$-module with basis $\{t_1, \dots, t_n\}$. Consider the matrix pro-p-group $M = \begin{pmatrix} A & 0 \\ T & 1 \end{pmatrix}$. Then the mapping

$$x_i \longmapsto \begin{pmatrix} a_i & 0 \\ t_i & 1 \end{pmatrix}, \quad i = 1, \dots, n,$$

extends to an embedding of the pro-p-group F into M. A matrix

$$\begin{pmatrix} a & 0 \\ \sum\limits_{i=1}^{n} t_i \alpha_i & 1 \end{pmatrix} \in M$$

belongs to the image of F if and only if

$$\sum_{i=1}^{n} \alpha_i y_i = a - 1. \tag{1}$$

Lemma 1. *Suppose that the vertices x_1 and x_{m+1} lie in the different connected components Γ_1 and Γ_2 of the defining graph Γ of F_Γ and α is a nonzero element in $\mathbb{Z}_p[[y_1, \dots, y_n]]$. Then $[x_1, x_{m+1}]^\alpha \neq 1$ in F_Γ.*

Proof. We may assume without loss of generality that Γ has two connected components. Let $x_1, \dots, x_m$ be the vertices of Γ_1 and let $x_{m+1}, \dots, x_n$ be the vertices of Γ_2.

Suppose that $[x_1, x_{m+1}]^\alpha = 1$ in F_Γ. Then the element $[x_1, x_{m+1}]^\alpha$ in the free metabelian pro-p-group F belongs to the normal subgroup N_Γ.

Consider the Magnus embedding of F into the matrix group M. The image of N_Γ is the topological $\mathbb{Z}_p[[y_1, \dots, y_n]]$-submodule generated by

$$t_j y_i - t_i y_j, \quad t_q y_p - t_p y_q, \quad 1 \leq i, j \leq m, \ m+1 \leq p, q \leq n.$$

Hence,

$$(t_1 y_{m+1} - t_{m+1} y_1)\alpha = \sum_{i,j} (t_j y_i - t_i y_j)\alpha_{ij} + \sum_{p,q} (t_p y_q - t_q y_p)\beta_{pq}$$

for some $\alpha_{ij}, \beta_{pq} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$. Therefore, $t_1 y_{m+1}\alpha = \sum (t_j y_i - t_i y_j)\alpha_{ij}$. Obviously, $\sum (t_j y_i - t_i y_j)\alpha_{ij}$ lies in F' but (1) implies $t_1 y_{m+1}\alpha \notin F'$ if $\alpha \neq 0$. This contradiction proves the lemma.

Lemma 2. *Suppose that $(x_i, x_j) \notin E$. Then $[x_i, x_j] \neq 1$ in F_Γ.*

PROOF. Consider the homomorphism φ of F onto the free metabelian pro-p-group with basis $\{x_i, x_j\}$ defined as

$$x_i \mapsto x_i, \quad x_j \mapsto x_j, \quad x_q \mapsto 1, \quad i \neq q, j \neq q.$$

Clearly, φ goes through F_Γ. Since $[x_i, x_j] \neq 1$ in $\varphi(F)$, we have $[x_i, x_j] \neq 1$ in F_Γ. The lemma is proved.

2.2. Turn to proving Theorem 1. Let $A_{i,j}$ denote the annihilator $[x_i, x_j]$ in $\mathbb{Z}_p[[y_1, \dots, y_n]]$. We must prove that $A_{i,j}$ coincides with $\mathscr{A}^\Gamma_{i,j}$. The case when x_i and x_j are joined by an edge is obvious. If x_i and x_j lie in the different connected components of the defining graph; then, by Lemma 1, $A_{i,j} = 0$. By definition, in this case $\mathscr{A}^\Gamma_{i,j} = 0$ too.

Recall that, in every metabelian group G, we have the Jacobi identity

$$[[x, y], z][[y, z], x][[z, x], y] = 1,$$

which we conveniently rewrite as

$$[x, y]^{\bar{z}-1}[y, z]^{\bar{x}-1}[z, x]^{\bar{y}-1} = 1,$$

where the bar stands for the natural homomorphism $G \longrightarrow G/G'$. If $(x_i, x_l) \in E$ then we obtain

$$[x_i, x_j]^{a_l-1} = [x_l, x_j]^{a_i-1}. \tag{2}$$

Suppose that x_i and x_j lie in one connected component of Γ and are nonadjacent. Then $\mathscr{A}^\Gamma_{i,j}$ is nonzero. Show that its every generating element indicated above annihilates $[x_i, x_j]$. Suppose for notational simplicity that $i = 1, j = m$, and the path under consideration has the form $(x_1, x_2, \dots, x_m)$. We must prove that $[x_1, x_m]^{y_2 \cdots y_{m-1}} = 1$ in F_Γ. Suppose by induction that $[x_1, x_{m-1}]^{y_2 \cdots y_{m-2}} = 1$. From (2) we have $[x_1, x_{m-1}]^{y_m} = [x_1, x_m]^{y_{m-1}}$. Act on both sides of the last equality by $y_2 \cdots y_{m-2}$ and obtain $[x_1, x_m]^{y_2 \cdots y_{m-1}} = 1$. Hence, $\mathscr{A}^\Gamma_{i,j} \subseteq A_{i,j}$. Inducting on the number of the vertices n, prove the reverse inclusion.

If $n = 2$ then the graph contains just two vertices and they are either joined by an edge or lie in different connected components. For these cases, it is known that $A_{1,2} = \mathscr{A}^\Gamma_{1,2}$.

Suppose that, for the defining graphs with the number of vertices $< n$, all inclusions $A_{i,j} \subseteq \mathscr{A}^\Gamma_{i,j}$ hold. Prove that this inclusion holds for our graph Γ with n vertices. Assume for definiteness that $i = 1$ and $j = m$, our vertices are nonadjacent but lie in the same connected component; let $\{x_1, x_2, \dots, x_m\}$ be a path between x_1 and x_m. Note that $m \geq 3$.

Given $1 < s < m$, denote by Γ_s the graph obtained from Γ by removing $x_s \in X$ and also all edges incident to x_s. We have the homomorphism of pro-p-groups $\varphi_s : F_\Gamma \longrightarrow F_{\Gamma_s}$ defined as $x_s \rightarrow 1$ and $x_i \rightarrow x_i$, $i \neq s$. Note that φ_s induces the homomorphism of the abelianizations

$$A \longrightarrow \langle a_1, \dots, a_{s-1}, a_{s+1}, \dots, a_n \rangle$$

and the homomorphism (retraction) of the algebras

$$\psi_s : \mathbb{Z}_p[[y_1, \dots, y_n]] \longrightarrow \mathbb{Z}_p[[y_1, \dots, y_{s-1}, y_{s+1}, \dots, y_n]];$$

the kernel of this homomorphism is the ideal generated by y_s. It is easily checked that the ideal $\mathscr{A}^{\Gamma_s}_{1,m}$ of $\mathbb{Z}_p[[y_1, \dots, y_{s-1}, y_{s+1}, \dots, y_n]]$ is a subset of $\mathscr{A}^\Gamma_{1,m}$. Note that if $\alpha \in \mathbb{Z}_p[[y_1, \dots, y_n]]$ then $\alpha - \psi_s(\alpha) \in \ker \psi_s$.

Suppose that

$$[x_1, x_m]^\alpha = 1, \quad \alpha \in \mathbb{Z}_p[[y_1, \dots, y_n]]. \tag{3}$$

Apply the homomorphism φ_2 to (3). In F_{Γ_2}, we obtain $[x_1, x_j]^{\psi_2(\alpha)} = 1$. By the induction assumption, $\psi_2(\alpha) \in \mathscr{A}^{\Gamma_2}_{1,m} \subseteq \mathscr{A}^\Gamma_{1,m}$. Since $\alpha - \psi_2(\alpha) \in \ker \psi_2$, this element is representable as $y_2\alpha_2$ for some $\alpha_2 \in \mathbb{Z}_p[[y_1, \dots, y_n]]$. Then $\alpha \equiv \alpha - \psi_2(\alpha) = y_2\alpha_2 \pmod{\mathscr{A}^\Gamma_{1,m}}$. Suppose that the comparison $\alpha \equiv y_2 \cdots y_l\alpha_l \pmod{\mathscr{A}^\Gamma_{1,m}}$ is proved for some $\alpha_l \in \mathbb{Z}_p[[y_1, \dots, y_n]]$, $2 < l < m - 1$.

Applying φ_{l+1} to the equality $[x_1, x_m]^{y_2 \cdots y_l \alpha_l} = 1$, we obtain

$$[x_1, x_m]^{y_2 \cdots y_l \psi_{l+1}(\alpha_l)} = 1$$

in $F_{\Gamma_{l+1}}$. By the induction assumption,

$$y_2 \cdots y_l \psi_{l+1}(\alpha_l) \in \mathscr{A}_{1,m}^{\Gamma_{l+1}} \subseteq \mathscr{A}_{1,m}^{\Gamma}.$$

Then the following equality and comparisons modulo $\mathscr{A}_{1,m}^{\Gamma}$ hold:

$$\alpha \equiv y_2 \cdots y_l \alpha_l - y_2 \cdots y_l \psi_{l+1}(\alpha_l) = y_2 \cdots y_l(\alpha_l - \psi_{l+1}(\alpha_l)) = y_2 \cdots y_l y_{l+1} \alpha_{l+1}$$

for some $\alpha_{l+1} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$. Therefore, there is $\alpha_{m-1} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$ for which

$$\alpha \equiv y_2 \cdots y_{m-1} \alpha_{m-1} \left(\operatorname{mod} \mathscr{A}_{1,m}^{\Gamma}\right).$$

Since $y_2 \cdots y_{m-1} \in \mathscr{A}_{1,m}^{\Gamma}$, we have $\alpha \in \mathscr{A}_{1,m}^{\Gamma}$. The theorem is proved.

3. Proof of Theorem 2

3.1. Observe that if Γ is a complete graph then F_Γ is a free abelian pro-p-group. If $E(\Gamma)$ is empty then $F_\Gamma = F$. Suppose that $\Gamma_1, \dots, \Gamma_m$ are the connected components of Γ, where each component Γ_i is a complete graph. In this case, F_Γ is the metabelian pro-p-product $B = A_1 \circ \dots \circ A_m$ of some free abelian pro-p-groups A_i generated by the vertices of the corresponding connected components.

The monograph [2] contains the construction (the Shmelkin embedding), using which we can obtain a representation of the pro-p-group B. Let us find this representation. The direct product $A_1 \times \dots \times A_m$ is the abelianization of B. Therefore, the former can be identified with A, so that the basis $\{a_1, \dots, a_n\}$ of the free abelian pro-p-group A is the union of the bases for $A_1, \dots, A_m$. Consider the free right $\mathbb{Z}_p[[y_1, \dots, y_n]]$-module with basis $\{t_1, \dots, t_m\}$ and the matrix pro-p-group $\begin{pmatrix} A & 0 \\ T & 1 \end{pmatrix}$. The embedding $B \longrightarrow \begin{pmatrix} A & 0 \\ T & 1 \end{pmatrix}$ is defined by the mapping $a_i \longmapsto \begin{pmatrix} a_i & 0 \\ t_j y_i & 1 \end{pmatrix}$; here $a_i \in A_j$. We focus the reader's attention on the fact that, in this mapping, t_j is the same for all elements a_i of the basis of A_j.

Lemma 3. *For the pro-p-group B defined above, its commutant B' is torsion-free as a $\mathbb{Z}_p[[y_1, \dots, y_n]]$-module; i.e., if*

$$1 \neq c \in B', \quad 0 \neq \gamma \in \mathbb{Z}_p[[y_1, \dots, y_n]]$$

then $c^\gamma \neq 1$.

Proof. It suffices to check the Shmelkin embedding for B, observe that B' is identified with some $\mathbb{Z}_p[[y_1, \dots, y_n]]$-submodule in T, and recall that T is torsion-free as a free module over an integral ring. The lemma is proved.

3.2. Suppose the fulfillment of the hypotheses of Theorem 2. Recall that $x_2, \dots, x_m$ are all vertices of Γ adjacent to x_1.

Lemma 4. *The projection to A of the centralizer of x_1 in F_Γ coincides with $\langle a_1, a_2, \dots, a_m\rangle$.*

Proof. By hypothesis, $x_1, x_2, \dots, x_m$ centralize x_1, and so the indicated projection contains $\langle a_1, a_2, \dots, a_m\rangle$. It suffices to prove that if some element $x_{m+1}^{l_{m+1}} \cdots x_n^{l_n} c$, where $l_i \in \mathbb{Z}_p$ and $c \in F'_\Gamma$, centralizes x_1, then all l_i's are zero. Suppose the contrary; for example, $l_n \neq 0$. Consider the homomorphism φ of F_Γ onto the free metabelian pro-p-group with basis $\{x_1, x_2\}$ defined as

$$x_1 \mapsto x_1,\ x_2 \mapsto 1, \dots,\ x_{n-1} \mapsto 1,\ x_n \mapsto x_2.$$

Applying φ to the equality

$$\left[x_{m+1}^{l_{m+1}} \cdots x_n^{l_n} c, x_1\right] = \left[x_{m+1}^{l_{m+1}} \cdots x_n^{l_n}, x_1\right] c^{y_1} = 1,$$

we get

$$\left[x_2^{l_n}, x_1\right] \varphi(c^{y_1}) = 1. \tag{4}$$

Under the Magnus embedding of the free metabelian pro-p-group with basis $\{x_1, x_2\}$, we obtain

$$x_2^{l_n} = \begin{pmatrix} a_2^{l_n} & 0 \\ t_2\alpha & 1 \end{pmatrix}, \quad c\varphi = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix},$$

where $\alpha \in \mathbb{Z}_p[[y_1, y_2]]$ and $t \in T$. Equality (4) is equivalent to $t_2\alpha y_1 + t_1\left(1 - a_2^{l_n}\right) + t y_1 = 0$. The last relation implies that $1 - a_2^{l_n}$ belongs to the ideal of the algebra $\mathbb{Z}_p[[y_1, y_2]]$ generated by y_1. But $a_2^{l_n} = (1+y_2)^{l_n} = 1 + l_n\, y_2 + \cdots$, and the inclusion $1 - a_2^{l_n} \in \mathbb{Z}_p[[y_1, y_2]] \cdot y_1$ implies $l_n = 0$; a contradiction. The lemma is proved.

Given a partially commutative metabelian pro-p-group F_Γ and a vertex x of Γ, denote by $C_\Gamma(x)$ the intersection of the centralizer of x in F_Γ with the commutant F_Γ'. We have

Lemma 5. *Suppose that the graph Δ is obtained from a graph Γ by removing the edges $(x_i, x_j) \in \Gamma$ for which $i \neq 1$ and $j \neq 1$. The homomorphism of graphs $\Delta \longrightarrow \Gamma$ acting identically on the set of vertices $V = V(\Gamma) = V(\Delta) = \{x_1, \dots, x_n\}$ induces a group homomorphism $F_\Delta \longrightarrow F_\Gamma$. The restriction of this homomorphism to $C_\Delta(x_1)$ is an epimorphism $C_\Delta(x_1) \longrightarrow C_\Gamma(x_1)$.*

Proof. Let c be an element in the free metabelian pro-p-group F whose image under the natural homomorphism $F \to F_\Gamma$ lies in $C_\Gamma(x_1)$. For its image, find a preimage in $C_\Delta(x_1)$. We may assume without loss of generality that

$$c = \prod [x_i, x_j]^{\alpha_{i,j}},$$

where $(x_i, x_j) \notin E$ and $\alpha_{i,j} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$. By hypothesis, $c^{y_1} = 1$ in F_Γ and

$$\prod [x_i, x_j]^{\alpha_{i,j} y_1} = \prod [x_s, x_t]^{\beta_{s,t}},$$

where $(x_s, x_t) \in E$ and $\beta_{s,t} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$. Each of the elements $\beta_{s,t}$ is representable as $\beta_{s,t} = y_1\beta'_{s,t} + \beta''_{s,t}$, where $\beta''_{s,t}$ does not depend on y_1. Consider the element

$$c_1 = \prod [x_i, x_j]^{\alpha_{i,j}} \prod [x_s, x_t]^{-\beta'_{s,t}}$$

in F. By hypothesis, $x_2, \dots, x_m$ are all vertices adjacent to x_1 in Γ. We have

$$c_1^{y_1} = \prod_{j=2}^{m} [x_1, x_j]^{\beta''_{1,j}} \prod [x_u, x_v]^{\beta''_{u,v}}, \tag{5}$$

with $(x_u, x_v) \in E$, $u \neq 1$, and $v \neq 1$. Apply to (5) the retraction $F \longrightarrow \langle x_2, \dots, x_n \rangle$, which consists in replacing x_1 with 1. We infer

$$\prod [x_u, x_v]^{\beta''_{u,v}} = 1;$$

hence,

$$c_1^{y_1} = \prod_{j=2}^{m} [x_1, x_j]^{\beta''_{1,j}}.$$

The above equality means that the image of the element c_1 of F in F_Δ lies in the centralizer of x_1. The images of c and c_1 in F_Γ coincide. Therefore, each element in $C_\Gamma(x_1)$ has a preimage in $C_\Delta(x_1)$. The lemma is proved.

Lemma 6. *Every element in F_Γ belonging to $C_\Gamma(x_1)$ is representable as*

$$c = \prod_{2 \le i < j \le m} [x_i, x_j]^{\gamma_{i,j}},$$

where $\gamma_{i,j} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$.

Proof. Let c be an element in F whose image in F_Γ lies in $C_\Gamma(x_1)$. Consider the graph Δ defined in Lemma 5. Add (if need be) some additional edges to this graph so that all vertices $x_1, \dots, x_m$ of the new graph become adjacent. Denote the so-obtained graph by Λ. By Lemma 5, we assume that c represents an element of $C_\Delta(x_1)$ in F_Δ. There is a canonical isomorphism of pro-p-groups $F_\Delta \longrightarrow F_\Lambda$. By construction, F_Λ is the metabelian pro-p-product of a free abelian pro-p-group of rank m and free abelian pro-p-groups of rank 1. Since $c^{y_1} = 1$ in F_Δ, we have $c^{y_1} = 1$ in F_Λ. Then, by Lemma 3, $c = 1$ in F_Λ. It follows that c lies in the normal subgroup of F generated by $[x_i, x_j]$, $2 \le i < j \le m$. The lemma is proved.

Now, we can easily prove Theorem 2. Suppose that $g \in F_\Gamma$ centralizes x_1. By Lemma 4, g is representable as $x_1^{l_1} \cdots x_m^{l_m} c$, where $l_i \in \mathbb{Z}_p$ and $c \in F'_\Gamma$. Clearly, $c \in C_\Gamma(x_1)$. By Lemma 6, we have the representation

$$c = \prod_{2 \le i < j \le m} [x_i, x_j]^{\gamma_{i,j}},$$

where $\gamma_{i,j} \in \mathbb{Z}_p[[y_1, \dots, y_n]]$. Theorem 2 is proved.

References

1. Gupta Ch. K. and Timoshenko E. I., “Partially commutative metabelian groups: centralizers and elementary equivalence,” Algebra and Logic, vol. 48, no. 3, 173–192 (2009).
2. Wilson J. S., *Profinite Groups*, Clarendon Press, Oxford (1988).
3. Remeslennikov V. N., “Embedding theorems for profinite groups,” Math. USSR-Izv., vol. 14, no. 2, 367–382 (1980).
4. Romanovskii N. S., “On Shmelkin embeddings for abstract and profinite groups,” Algebra and Logic, vol. 38, no. 5, 326–334 (1999).

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