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CHARACTERIZATION OF SOME FINITE GROUPS BY ORDER AND LENGTH OF ONE CONJUGACY CLASS

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Abstract: We study the possibility of characterizing $S \in \{{}^2D_n(2), {}^2D_{n+1}(2)\}$ by simple conditions when $2^n+1>5$ is a prime. Furthermore, we will show that Thompson's conjecture is valid under some weak condition for these groups.

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1. Introduction

The *prime graph* of a finite group G, denoted by $\Gamma(G)$, is the graph whose vertices are the prime divisors of $|G|$ and where a prime p is defined to be *adjacent* to a prime q $(\neq p)$ if and only if G contains an element of order pq.

We denote by $\pi(G)$ the set of prime divisors of $|G|$. Let $t(G)$ be the number of connected components of $\Gamma(G)$ and let $\pi_1, \pi_2, \dots, \pi_{t(G)}$ be the connected components of $\Gamma(G)$. If $2 \in \pi(G)$, then we always suppose that $2 \in \pi_1$.

We can express $|G|$ as a product of integers $m_1, m_2, \dots, m_{t(G)}$, where $\pi(m_i) = \pi_i$ for each i. The numbers m_i are called the *order components* of G. In particular, if m_i is odd, then we call m_i an *odd component* of G. Write $OC(G)$ for the set $\{m_1, m_2, \dots, m_{t(G)}\}$ of order components of G and $T(G)$ for the set of connected components of $\Gamma(G)$. According to the classification theorem of finite simple groups and [1–3], we can list the order components of finite simple groups with disconnected prime graphs as in Tables 1–4 in [4].

Put $N(G) = \{n : G \text{ has a conjugacy class of size } n\}$. By Thompson's conjecture if L is a finite nonabelian simple group, G is a finite group with a trivial center, and $N(G) = N(L)$, then $L \cong G$.

Chen et al. in [5] showed that the projective special linear groups $A_1(p)$ are recognizable by order and length of one conjugacy class, where p is a prime. As a consequence, they showed that Thompson's conjecture is valid for $A_1(p)$.

Similar characterizations were found in [6, 7] for sporadic simple groups, simple K_3-groups (a finite simple group is called a *simple K_n-group* if its order is divisible by exactly n distinct primes), and alternating groups of degree p, where p is a prime.

Our result states that if $p = 2^n+1 > 5$ is a prime, then the groups $S \in \{{}^2D_n(2), {}^2D_{n+1}(2)\}$ are determined up to isomorphism by order and length of one conjugacy class.

If n is an integer, then denote the r-part of n by $n_r = r^a$ or $r^a \parallel n$; namely, $r^a \mid n$ but $r^{a+1} \nmid n$. If q is a prime, then we denote by $S_q(G)$, a Sylow q-subgroup of G and by $\mathrm{Syl}_q(G)$ the set of Sylow q-subgroups of G. The other notation and terminology in this paper are standard, and the reader is referred to [8] if need be.

2. Preliminary Results

Definition 2.1. Let a and n be integers greater than 1. Then a Zsigmondy prime of a^n-1 is a prime l such that $l \mid (a^n-1)$ but $l \nmid (a^i-1)$ for $1 \le i < n$.

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Lemma 2.1 [9]. *If a and n are integers greater than 1, then there exists a Zsigmondy prime of a^n-1, unless $(a, n) = (2, 6)$ or $n = 2$ and $a = 2^s - 1$ for some natural s.*

Remark 2.1. If l is a Zsigmondy prime of a^n-1, then Fermat's Little Theorem shows that $n \mid (l-1)$. Put $Z_n(a) = \{l : l \text{ is a Zsigmondy prime of } a^n - 1\}$. If $r \in Z_n(a)$ and $r \mid a^m - 1$, then $n \mid m$.

Lemma 2.2 [10, Remark 1]. *The equation $p^m - q^n = 1$, where p and q are primes and $m, n > 1$ has only one solution, namely $3^2 - 2^3 = 1$.*

Lemma 2.3 [11]. *Let q be a prime power which is not of the form $3^r 2^s \pm 1$, where $r = 0, 1$ and $s \geq 1$. Let $M = C_n(q)$, where $n = 2^m (m \geq 2)$ and $OC_2 = (q^n + 1)/(2, q + 1)$. If $x \in \pi_1(M)$, $x^\alpha \mid |M|$, and $x^\alpha - 1 \equiv 0 \bmod OC_2$, then $x^\alpha = q^{2kn}$, where $1 \leq k \leq n/2$.*

Corollary 2.1. *Let $S \in \{^2D_n(2), {}^2D_{n+1}(2)\}$, and $p = 2^n + 1 > 5$. If $x \in \pi(S) - \{p\}$ and $x^\alpha - 1 \equiv 0 \bmod p$, then either $x^\alpha \nmid |S|$ or $x = 2$.*

Proof. First let $S \neq^2 D_{n+1}(2)$ or $x \notin Z_{2(n+1)}(2)$. Then since $|S|_x \mid |C_n(2)|_x$ and $OC_2(C_n(2)) = p$, Lemma 2.3 completes the proof. Let $S =^2 D_{n+1}(2)$ and $x \in Z_{2(n+1)}(2)$. Then $|S|_x \leq (2^{n+1} + 1)/3 < p$ and hence $x^\alpha > |S|_x$, as claimed.

Lemma 2.4 [12, Corollary 3.8]. *Let $G =^2 D_n(q)$, where q is prime power. If $d = \gcd(q^n + 1, 4)$, then $\frac{|^2D_n(q)|d}{(q^n+1)}$, $\frac{|^2D_n(q)|d}{(q-1)(q^{n-1}+1)} \in N(G)$. Furthermore, $\frac{|^2D_n(q)|d}{(q^n+1)}$ and $\frac{|^2D_n(q)|d}{(q-1)(q^{n-1}+1)}$ are maximal in $N(G)$ by divisibility.*

Lemma 2.5 [4, Tables 1–4]. *Let $S \in \{^2D_n(2), {}^2D_{n+1}(2)\}$. If $S =^2 D_n(2)$, then $OC_1(S) = 2^{n(n-1)} \prod_{i=1}^{i=n-1}(2^{2i} - 1)$ and $OC_2(S) = 2^n + 1$, while if $S =^2 D_{n+1}(2)$, then $OC_1(S) = 2^{n(n+1)}(2^{n+1} + 1)(2^{n+1} - 1) \prod_{i=1}^{i=n-1}(2^{2i} - 1)$ and $OC_2(S) = 2^n + 1$.*

3. The Main Results

By Lemma 2.4, S has length $\frac{|S|}{2^n+1}$ of one conjugacy class. Note that since $2^n + 1$ is prime, we deduce that n is a power of 2.

Theorem 3.1. *Let G be a group. Then $G \cong S$ if and only if $|G| = |S|$ and G has length $\frac{|S|}{2^n+1}$ of a conjugacy class, where $S \in \{^2D_n(2), {}^2D_{n+1}(2)\}$ and $2^n + 1 = p > 5$ is a prime.*

Proof. The necessity of the theorem can be checked easily. We only need to prove sufficiency.

By hypothesis, there exists an element x of order p in G such that $C_G(x) = \langle x \rangle$ and $C_G(x)$ is a Sylow p-subgroup of G. By Sylow's Theorem, we have $C_G(y) = \langle y \rangle$ for every element y in G of order p. So, $\{p\}$ is a prime graph component of G and $t(G) \geq 2$. In addition, p is the maximal prime divisor of $|G|$ and an odd order component of G.

We will prove the main theorem in the following steps:

Step 1. G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a nonabelian simple group, and H is a nilpotent group.

If $g \in G$ is an element of order p, then $C_G(g) = \langle g \rangle$. Set $H = O_{p'}(G)$ (the largest normal p'-subgroup of G). Then H is a nilpotent group since g acts fixed-point-freely on H. Let K be a normal subgroup of G such that K/H is a minimal normal subgroup of G/H. Then K/H is a direct product of copies of some simple group. Since $p \mid |K/H|$ and $p^2 \nmid |K/H|$, K/H is a simple group. Since $\langle g \rangle$ is a Sylow p-subgroup of K, $G = N_G(\langle g \rangle)K$ by the Frattini's argument and so $|G/K|$ divides $p - 1$.

If $|K/H| = p$, then by Lemma 2.1, there is a prime $r \in Z_{n-1}(2) \cap \pi(G)$ and so $|S|_r = |2^{n-1} - 1|_r \leq |G|_r$. Since $\pi(G) = \pi(K) \cup \pi(H) = \pi_1(G) \cup \pi_2(G)$; therefore, $r \in \pi(H)$. Since H is nilpotent, a Sylow r-subgroup is normal in G. It follows that the Sylow p-subgroup of G acts fixed-point-freely on the set of elements of order r and so $p \mid |S|_r - 1$. Thus $p \leq |S|_r \leq 2^{n-1} - 1 < p$; a contradiction. Hence, G has normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that K/H is a nonabelian simple group and p is an odd order component of K/H.

Step 2. $\pi(H) \subseteq \{2\}$.

Let $r \in \pi(H)$. Then $r \neq p$ and since H to be nilpotent, we deduce that $S_r(H) \trianglelefteq G$ and hence $S_p(G)$ acts fixed-point-freely on $S_r(H) - \{1\}$. Thus $p \mid |S_r(H)| - 1$. If $r \neq 2$, then $|S_r(H)| \mid |S|_r$ and so Corollary 2.1 leads us to get a contradiction. Thus $r = 2$, as claimed.

According to the classification theorem of finite simple groups and the results of Tables 1–4 in [4], K/H is an alternating group, a sporadic group, or a simple group of Lie type.

Step 3. K/H is not a sporadic simple group.

Suppose that K/H is a sporadic simple group. Since one of the odd order components of K/H is $p = 2^n + 1$, we get a contradiction by considering the odd order components of sporadic simple groups.

Step 4. K/H cannot be an alternating group Alt_m, where $m \geq 5$.

If $K/H \cong \mathrm{Alt}_m$ with $m \geq 5$, then since $p \in \pi(K/H)$, $m \geq 2^n + 1$. Thus there is a prime $u \in \pi(\mathrm{Alt}_m) \subseteq \pi(G)$ such that $\frac{p-1}{2} < u < p$. Since $|G| = |S|$, there exists $t \in \{2i, i : 1 < i < n-1\} \cup \{n\}$ such that $u \in Z_t(2)$. But $u > \frac{2^n-1+1}{2} = 2^{n-1}$ and so $u = 2^{n-1} + 1$ or $2^n - 1$. But n is a power of 2 and hence $3 \mid 2^{n-1} + 1$ and $2^n - 1$. Thus $3 \mid u$. This implies that $u = 3$ and so $n = 2$, which is a contradiction.

Step 5. $K/H \cong S$.

By Steps 3 and 4, and the classification theorem of finite simple groups, K/H is a simple group of Lie type such that $t(K/H) \geq 2$ and $p \in OC(K/H)$. Thus K/H is isomorphic to one of the groups of Lie type (in the following cases, r is an odd prime):

Case 1. Let $t(K/H) = 2$. Then $OC_2(K/H) = 2^n + 1$.

1.1. If $K/H \cong^2 D_{n'}(q)$, where $n' = 2^u \geq 4$, then $\frac{q^{n'}+1}{(2,q-1)} = 2^n+1$. If q is odd, then $q^{n'} = 2^{n+1}+1$, which is a contradiction with Lemma 2.2. Hence $q = 2^t$ and so $q^{n'} = 2^n$. But $p \in Z_{2n}(2)$ and $p \in Z_{2n't}(2)$. Thus Remark 2.1 forces $n't = n$. We claim that $t = 1$. If not, then $Z_{n-1}(2) \cap \pi(K/H) = \varnothing$. But Lemma 2.1 forces $Z_{n-1}(2) \neq \varnothing$ and hence since $|G| = |S|$, $\pi(G)$ contains a prime $r \in Z_{n-1}(2)$. Since $r \nmid |\mathrm{Out}(K/H)|$ and $G/K \lesssim \mathrm{Out}(K/H)$, we deduce that $r \mid |H|$. Step 2 shows that $r = 2$, which is a contradiction. Thus $t = 1$ and hence $K/H \cong^2 D_n(2)$.

If $K/H \cong B_{n'}(q)$, where $n' = 2^u \geq 4$, then arguing as above $n' = n$ and $q = 2$. Thus $K/H \cong C_n(2)$.

Also if $K/H \cong C_{n'}(q)$, where $n' = 2^u > 2$, then arguing as above $n' = n$ and $q = 2$. Thus $K/H \cong C_n(2)$.

1.2. If $K/H \cong C_r(3)$ or $B_r(3)$, then $\frac{3^r-1}{2} = 2^{n+1}$. Thus $2^n + 1 = 3^r - 3$, which is a contradiction. The same reasoning rules out the case when $K/H \cong D_r(3)$ or $D_{r+1}(3)$.

1.3. If $K/H \cong C_r(2)$, then $2^r - 1 = 2^n + 1$ and hence $2^r = 2^n + 2$, which is a contradiction. The same reasoning rules out the case when $K/H \cong D_r(2)$ or $D_{r+1}(2)$.

1.4. If $K/H \cong D_r(5)$, where $r \geq 5$, then $(5^r - 1)/4 = (2^n + 1)$. Thus $5^r - 5 = 2^{n+2}$, which is a contradiction.

1.5. If $K/H \cong^2 D_{n'}(3)$, where $9 \leq n' = 2^m + 1$ and n' is not prime, then $\frac{3^{n'}-1}{2} = 2^{n+1}$ and hence $3^{n'} - 1 = 2^{n+1} + 1$. Thus Lemma 2.2 forces $n + 1 = 3$, which is a contradiction.

1.6. If $K/H \cong^2 D_{n'}(2)$, where $n' = 2^m + 1 \geq 5$, then $2^{n'-1} + 1 = 2^n + 1$ and hence $n' - 1 = n$. Thus $K/H \cong^2 D_{n+1}(2)$, as claimed.

1.7. If $K/H \cong^2 D_r(3)$, where $5 \leq r \neq 2^m + 1$, then $\frac{3^r+1}{4} = 2^n + 1$ and hence $3^r = 2^{n+2} + 3$, which is a contradiction.

1.8. If $K/H \cong G_2(q)$, where $2 < q \equiv \varepsilon \bmod 3$ and $\varepsilon = \pm 1$, then $q^2 - \varepsilon q + 1 = 2^n + 1$. Thus $q(q - \varepsilon) = 2^n$, which is impossible. The same reasoning rules out the case when $K/H \cong F_4(q)$, where q is odd.

1.9. If $K/H \cong^2 F_4(2)'$, then since $|^2F_4(2)| = 2^{11} \times 3^3 \times 5^2 \times 13$, $2^n + 1 = 13$; a contradiction. Also we can rule out $K/H \cong {}^2A_3(2)$.

1.10. Let $K/H \cong A_{r-1}(q)$, where $(r, q) \neq (3, 2)$, $(3, 4)$. Since $\frac{q^r-1}{(r,q-1)(q-1)} = p$, $p \in Z_r(q)$ and so Remark 2.1 shows that $r \mid p - 1 = 2^n$. Thus $r = 2$, which is a contradiction. The same reasoning excludes the case when $K/H \cong^2 A_{r-1}(q)$.

1.11. Let $K/H \cong A_r(q)$, where $(q-1) \mid (r+1)$. Since $\frac{q^r-1}{(r,q-1)} = p$, $p \in Z_r(q)$ and hence Remark 2.1 shows that $r \mid p-1 = 2^n$. Thus $r = 2$, which is a contradiction. The same reasoning rules out the case when $(q+1) \mid (r+1)$, $(r,q) \neq (3,3)$, $(5,2)$ and $K/H \cong^2 A_r(q)$.

1.12. If $K/H \cong E_6(q)$, where $q = u^\alpha$, then $\frac{q^6+q^3+1}{(3,q-1)} = p$. Thus $p \in Z_6(q)$ and hence Remark 2.1 shows that $6 \mid p-1 = 2^n$, which is a contradiction. The same reasoning rules out the case when $K/H \cong^2 E_6(q)$, where $q > 2$.

Case 2: Let $t(K/H) = 3$. Then $p = 2^n + 1 \in \{OC_2(K/H), OC_3(K/H)\}$.

2.1. If $K/H \cong A_1(q)$, where $4 \mid q+1$, then $\frac{q-1}{2} = 2^n + 1$ or $q = 2^n + 1$. If $q = 2^n + 1$, then $q + 1 = 2^n + 2$ and hence $4 \nmid q+1$, which is a contradiction. If $\frac{q-1}{2} = p$, then $q \equiv -1 \bmod 4$. Let $q = u^\alpha$, where u is a prime. Thus $p \in Z_\alpha(u)$ and hence Remark 2.1 shows that $\alpha \mid p - 1 = 2^n$. So $\alpha = 2^t$ and hence $q = u^\alpha \equiv 1 \bmod 4$, which is a contradiction.

2.2. If $K/H \cong A_1(q)$, where $4 \mid q+1$, then $q = 2^n + 1$ or $\frac{q+1}{2} = p$.

• If $q = 2^n + 1$, then $q = p$ and so $|K/H| = p(p^2-1)/2 = 2^n p(2^{n-1}+1)$. Since $G/K \lesssim \mathrm{Out}(K/H) \cong Z_2$, we deduce that $Z_n(2) \subseteq \pi(H)$, which is a contradiction with Step 2.

• If $\frac{q+1}{2} = p$, then $q = 2^{n-1} + 1$. Thus $3 \mid q$ and so $3^\alpha = 2^{n+1} + 1$, which is a contradiction with Lemma 2.2.

2.3. If $K/H \cong A_1(q)$, where $q > 2$ and q is even, then $p \in \{q-1, q+1\}$. If $q - 1 = 2^n + 1$, then $q = 2(2^{n-1}+1)$, which is a contradiction. If $q+1 = 2^n+1$, then $q = 2^n$ and hence $|K/H| = 2^n(2^n-1)(2^n+1)$. But $G/K \lesssim \mathrm{Out}(K/H) \cong Z_n$, and so $Z_{n-1}(2) \subseteq \pi(H)$, which is a contradiction with Step 2.

2.4. If $K/H \cong^2 A_5(2)$ or $A_2(2)$, then $|K/H| = 2^{15} \times 3^6 \times 7 \times 11$ or $8 \times 3 \times 7$. Clearly, $2^n + 1 \neq 11$ and $2^n + 1 \neq 7$, which is a contradiction.

2.5. If $K/H \cong^2 D_r(3)$, where $r = 2^t+1 \geq 5$, then $\frac{3^r+1}{4} = 2^n+1$ or $\frac{3^r-1}{2} = 2^n+1$. If $\frac{3^r+1}{4} = 2^n+1$, then $3^r = 2^{n+2} + 3$, which is a contradiction. If $\frac{3^r-1}{2} = 2^n + 1$, then $2^{n+1} + 1 = 3^{r-1}$, which is a contradiction with Lemma 2.2.

2.6. If $K/H \cong G_2(q)$, where $q \equiv 0 \bmod 3$, then $q^2 - q + 1 = 2^n + 1$ or $q^2 + q + 1 = 2^n + 1$ and so $q(q \pm 1) = 2^n$, which is impossible.

Similarly we can rule out $K/H =^2 G_2(q)$.

2.7. If $K/H \cong F_4(q)$, where q is even, then $q^4+1 = 2^n+1$ or $q^4-q^2+1 = 2^n+1$. If $q^4-q^2+1 = 2^n+1$, $q^2(q^2-1) = 2^n$, which is impossible. If $q^4 + 1 = 2^n + 1$, then $q^4 = 2^n$. So $(q^{12}-1) = (2^{3n}-1) \mid |K/H|$ and $Z_{3n}(2) \subseteq \pi(G) = \pi(S)$, which is a contradiction again.

2.8. If $K/H \cong^2 F_4(q)$, where $q = 2^{2t} + 1 > 2$, then $q^2 + \sqrt{2q^3} + q + \sqrt{2q} + 1 = 2^n + 1$ or $q^2 - \sqrt{2q^3} + q - \sqrt{2q} + 1 = 2^n + 1$. Thus $2^n + 1 = 2^{2(2t+1)} + \varepsilon 2^{3t+2} + 2^{2t+2} + \varepsilon 2^{t+1} + 1$, where $\varepsilon = \pm 1$ and hence $2^n = 2^{t+1}(2^{3t+1} + \varepsilon 2^{2t+1} + 2^t + \varepsilon)$, which is a contradiction.

2.9. If $K/H \cong E_7(2)$, then $2^n + 1 \in \{73, 127\}$, which is impossible.

2.10. If $K/H \cong E_7(3)$, then $2^n + 1 \in \{757, 1093\}$, which is impossible.

Case 3: Let $t(K/H) = \{4,5\}$. Then $p = 2^n+1 \in \{OC_2(K/H), OC_3(K/H), OC_4(K/H), OC_5(K/H)\}$.

3.1. If $K/H \cong A_2(4)$ or ${}^2E_6(2)$, then $2^n + 1 = 7$ or $2^n + 1 = 19$, which is impossible.

3.2. If $K/H \cong^2 B_2(q)$, where $q = 2^{2t}+1$ and $t \geq 1$, then $2^n+1 \in \{q-1, q \pm \sqrt{2q}+1\}$. If $q-1 = 2^n+1$, then $2^{2t} + 1 = 2^n + 2$ and if $q \pm \sqrt{2q} + 1 = 2^n + 1$, then $2^{t+1}(2^t \pm 1) = 2^n$, which is impossible.

3.3. If $K/H \cong E_8(q)$, then $2^n + 1 \in \{q^8 - q^7 + q^5 - q^4 + q^3 - q + 1, q^8 + q^7 - q^5 - q^4 - q^3 + q + 1, q^8 - q^6 + q^4 - q^2 + 1, q^8 - q^4 + 1\}$. Thus $q^t = 2^n$, where $t > 1$ is a natural such that $(t, q) = 1$, which is a contradiction.

The above cases show that $K/H \cong C_n(2)$, ${}^2D_n(2)$, or ${}^2D_{n+1}(2)$. Let $S = {}^2D_n(2)$. If $K/H \ncong S$, then $K/H \cong C_n(2)$. Thus $|G|_2 \mid 2^n|S|_2/n$. But $|K/H|_2 \geq 2^{n^2}$ and so $|G|_2 \geq 2^{n^2}$, which is a contradiction.

Let $S =^2 D_{n+1}(2)$. Then applying the previous argument shows that $Z_{n+1}(2) \subseteq \pi(K/H) \subseteq \pi(G) = \pi(S)$. Thus $K/H \cong S$.

Since $|G| = |S|$, $H = 1$ and $K = G \cong S$. Theorem 3.1 is proved.

Corollary 3.1. *Thompson's conjecture holds for the simple groups $S \in \{^2D_n(2), {}^2D_{n+1}(2)\}$, where $2^n + 1 > 5$ is a prime.*

PROOF. Let G be a group with a trivial central and $N(G) = N(S)$. Then it is proved in [13, Lemma 1.4] that $|G| = |S|$. Hence, the corollary follows from Theorem 3.1.

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