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**THEORY  
OF METALS**

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## Spincaloritronic in Magnetic Nanostructures<sup>1</sup>

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Received March 26, 2018

**Abstract**—Using one of the methods of quantum nonequilibrium statistical physics we have investigated the spin transport transverse to the normal metal/ferromagnetic insulator interface in hybrid nanostructures. An approximation of the effective parameters, when each of the interacting subsystems (electron spin, magnon, and phonon) is characterized by its own effective temperature have been considered. We have constructed the macroscopic equations describing the spin-wave current caused by both the resonantly exciting spin subsystem of conduction electrons and an inhomogeneous temperature field in the ferromagnetic insulator. In addition, we have derived the generalized Bloch equations describing the spin-wave current propagation in the insulator and considering the resonant-diffusion nature of the propagation of magnons and their relaxation processes. We have shown that the spin-wave current excitation under combined resonance conditions bears a resonant nature. The formation of the two: injected and thermally excited, different in energies magnon subsystems and the influence of its interaction with phonon drag effect under spin Seebeck effect conditions in the magnetic insulator part of the metal/ferromagnetic insulator/metal structure is studied.

**Keywords:** spin current, magnon diffusion, thermomagnetic phenomena

**DOI:** 10.1134/S0031918X18130173

### INTRODUCTION

The term *spincaloritronics* was coined to refer to all transport phenomena that involve spin and heat in condensed-matter structures and devices. The term *spin caloritronics* was coined to refer to all transport phenomena that involve spin and heat. One of the central issues of spintronics is the generation and control of spin currents in solids. It has been found that thermal perturbations may also cause the spin effects to occur. The recently invigorated field of spin caloritronics focuses on the interaction of spins with heat currents, motivated by newly discovered physical effects and strategies to improve existing thermoelectric devices. The first effect that has opened a new direction in spintronics—spincaloritronics—the influence of thermal perturbations on spin effects, is the spin Seebeck effect (SSE) [1]. Afterwards, the SSE could be observed in various materials such as semiconductors and metallic ferromagnets. Besides, later the spin Nerst effect (or the thermal spin Hall effect), the spin Peltier effect, and others have been discovered.

Studying of the SSE in a non-conducting magnet in the system of a nonmagnetic conductor/magnetic insulator (N/F)  $\text{LaY}_2\text{Fe}_5\text{O}_{12}$  has shown that this effect cannot be described by standard approaches as regards a description of thermoelectric effects. As distinct from conducting crystals where the transfer of the spin

angular momentum is due to band charge carriers, the spin Seebeck effect can be realized in non-conducting magnetic materials through the excitation of a localized spin system. In this case, the excitations (magnons) underlying the spin-wave current causes the transfer of the angular momentum. Thus, conducting crystals and a nonconductive magnet differ from each other in type of the spin current, namely, spin-wave one. It is a new type of the spin current.

The spin effects in nonconductive magnetic materials under thermal perturbations were studied in several papers. In this case, the theoretical description boiled largely down to the consideration of the evolution of the localized moments subsystem. Given thermal fluctuations derived from the fluctuation-dissipative theorem, the authors of the works mentioned above modeled the localized moments' dynamics by the phenomenological Landau–Lifshitz–Gilbert equation. As to the spin density dynamics in a nonmagnetic material, it is described by the Bloch equation with phenomenological spin relaxation frequencies. The response of weakly nonequilibrium systems to the thermal perturbations can be universally constructed through the method of the nonequilibrium statistical operator (NSO) and its modifications [2].

Developing new methods for generating and detecting spin currents has been the central task of spintronics. The two most used methods to generate spin currents in metallic or insulating ferromagnets

<sup>1</sup> The article is published in the original.

employ the spin pumping effect (SPE) and the spin Seebeck effect (SSE). It is known that the spin orbit interaction, a fundamental relativistic effect that couples the motion of electrons to their spins and is also an important mechanism for spin relaxation in solids. The coupling between the configurational and spin motions makes it impossible to separate the quantum transitions into purely configurational and purely spin ones; we can speak only of predominantly configurational and predominantly spin transitions. But this changes appreciably the conditions for the excitation of the different transitions. Namely, it becomes possible to excite spin transitions by the electric component of the electro-magnetic field. This effect is a combined resonance (CR) [4, 5].

In the first part of the given review, we consider the spin-thermal effects in a normal conductor–magnetic insulator, using the NSO method to describe thermal perturbations. We conducted an analysis of macroscopic equations describing the spin-thermal effects [3].

In the second part of our review we look at how the electric-dipole excitation of the electron spin subsystem of a semiconductor affects the spin-wave current generation in a nonconducting ferromagnetic material in the hybrid structure semiconductor/ferromagnetic insulator. We consider the excitation of the spin-wave (magnonic) current at a saturation of the combined electron-resonance and a temperature gradient [6, 7].

The propagation of magnons in a magnetic insulator is described by two characteristic quantities: mean free path and spin diffusion length that are governed, in turn, by various magnon relaxation mechanisms. A series of experiments determine the range of the diffusion lengths as being quite wide: from 4 to 120  $\mu\text{m}$ . As to the temperature dependence of the Seebeck coefficient, it is non-monotonic and reaches its maximum within the range of 50–100 K. And as the investigations have shown, it is affected by strength of a magnetic field, dimensions of the samples, and quality of the interface. To explain the low temperature enhancement the phonon-drag [8] SSE scenario based on a some theoretical models was proposed.

We have developed an “enhanced model” of the drag effect in spin currents, which is based on the formation of two interacting magnon flows. There are two magnon groups with different energies, when SSE occurs in such structures. The first group consists of magnons produced by an inhomogeneous temperature field applied to the magnetic insulator “thermal” magnons. The energy of the magnons is of the order of a temperature ( $k_B T$ ). Along with them, there are the magnons injected into the magnetic insulator due to inelastic scattering of spin-polarized electrons of the metal by localized spins located in the vicinity of the interface. The energy of the “injected” magnons is of the order of spin accumulation energy of conduction electrons of the metal  $\Delta_s \gg k_B T$ . Thus, it can be said

that another subsystem of the “injected” electrons is formed in the magnetic system and is actually responsible for the SSE. As a consequence, in the presence of a nonuniform temperature field, there are three flows inside the magnetic insulator, namely, phonon and two magnon ones. Consideration of the magnon–phonon drag effect in an “enhanced model” is the content of the third part of the review [8].

## THERMO-SPIN EFFECTS

Using nonequilibrium statistical operator method (NSO) we studied the response of weakly nonequilibrium systems to the thermal perturbations. To analyze the kinetics of the spin-thermal effects, we employ a scheme developed in the NSO method applied to the case of a small deviation of the system from the equilibrium Gibbs distribution  $\rho_0 = \exp\{-S_0\}$ .  $S_0$  is the entropy of the equilibrium system. In the linear approximation the deviation from the equilibrium, the NSO (or the density matrix)  $\rho(t)$  can be written as follows [11]

$$\rho(t) = \rho_q(t) + \int_{-\infty}^0 dt_1 e^{\epsilon t_1} \int_0^1 d\lambda \rho_0^\lambda \hat{S}(t + t_1, t_1) \rho_0^{1-\lambda}. \quad (1)$$

Here  $\rho_q(t) = \exp\{-S(t)\}$  is the quasi-equilibrium statistical operator,  $\hat{S}(t)$  the entropy production operator. A further algorithm for constructing the operator  $\rho(t)$  reduces to finding the entropy production operator [5]. Let us construct the equations for the averages, which have the meaning of local conservation laws of the average energy density for the electron spin (s) and magnetic (m) subsystems and of the particle number density. Inserting the entropy production operator into the expression for the NSO (1), we average the operator equation for the energy change in the magnetic subsystem  $\langle A \rangle^t = Sp(A \rho(t))$

$$\langle \dot{H}_m(R) \rangle^t = \int dR' \left\{ \sum_{\gamma} D_{H_m H_m}^{\gamma}(R, R') \nabla^{\gamma} \beta_m(R') - \beta \mu_s(R') / (\hbar \omega_m) L_{m,ms}^m(R, R') + (\beta_m(R') - \beta) \right. \quad (2)$$

$$\left. \times L_{m,mp}^m(R, R') + (\beta_m(R') - \beta_s(R')) (\omega_s / \omega_m) L_{m,ms}^m(R, R') \right\},$$

where

$$L_{m,mi}^m(R, R') = \int_{-\infty}^0 dt' e^{\epsilon t'} (\dot{H}_{m,mi}(R), \dot{H}_{m,mi}(R', t'))_0,$$

$$D_{H_m H_m}^{\gamma}(R, R') = \int_{-\infty}^0 dt' e^{\epsilon t'} \sum_{\lambda} \nabla^{\lambda} (I_{H_m}^{\lambda}(R), I_{H_m}^{\gamma}(R', t'))_0.$$

Here  $(A, B)_0 = \int_0^1 d\lambda Sp\{A \rho_0^{\lambda} (B - \langle B \rangle_0) \rho_0^{1-\lambda}\}$ ,  $H_{i,ik} = (i\hbar)^{-1} [H_i, H_{ik}]$ , and  $I_{H_m}(R) = -\hbar \omega_m I_{S^z}(R)$  is the

flux density of the magnon energy in the (m)-subsystem.  $\beta^{-1} = T$  is the equilibrium temperature of the system,  $\beta_m^{-1} = T_m$ ,  $\beta_s^{-1} = T_s$  are the temperature of magnetic and spin electron subsystems.

So we have derived the generalized Bloch equation that describes the motion of the spin magnetization density: the spin diffusion and relaxation processes. The first term on the right-hand side of the Eq. (2) is the energy density change in the magnon subsystem due to the temperature gradient, which in turn leads to the magnon flux. This term describes the magnon diffusion. The second and third terms of the expression (2) are responsible for the impact of the electronic (spin) and lattice (phonon) subsystems on the magnon energy change through their interaction. The role of the electron spin subsystem reduces to the generation and annihilation of the magnons by the inelastic spin-flip electron scattering at the normal metal/ferroelectric interface. The phonon subsystem proves to affect the magnon energy change in a twofold manner. On the one hand, the magnon-phonon scattering processes make themselves felt in the energy relaxation behavior of the magnon subsystem, on the other hand, the phonon subsystem often acts as a ‘‘heating’’ of the heat/charge transfer processes by means of drag effects. As can easily be seen from (2), a non-zero contribution to the spin-wave current is the consequence of the difference in the temperatures of the above subsystems. Equation (2) implies that, depending upon its direction, the magnon flux produced by a temperature gradient can give rise to the angular momentum transfer from the magnon system to the electron.

### DYNAMIC GENERATION OF SPIN-WAVE CURRENT

Our model consists of a semiconductor (S) and a ferromagnetic insulator (F). The system of conduction electrons in the semiconductor consist of two subsystems: the kinetic and the spin one.  $H_{ks}$  is the spin-orbit interaction in semiconductor system. A typical expression for the interaction between the kinetic and spin degrees of freedom of the electrons can be written in the form

$$H_{ks} = \sum_i f(p_i) s_i = D \sum_i p_i^{\alpha_1} p_i^{\alpha_2} \dots p_i^{\alpha_n} S_i^m,$$

where  $D$  is a constant depending on the spin-orbital interaction intensity.  $f(p_i)$  is a pseudo-vector whose components are a formulas of order  $s$  of the kinetic impulse components  $p_i^\alpha$ . As a rule, the terms of the Hamiltonian  $H_{ks}$  are small. In this case, to eliminate the interaction of the spin and kinetic degrees of freedom of the electrons in the linear approximation, we can perform a momentum-dependent canonical transformation of the Hamiltonian [5]. As a result, we

obtain a new Hamiltonian with autonomous subsystems ( $k$ ) and ( $s$ ), the electronic system effectively interacting with the electromagnetic field, which determines the resonant energy absorption. The renormalized interaction with the alternating electric field has the form

$$\bar{H}_{ef} = [r, T(p)] e E(t), \quad T(p) = D \sum \frac{T^{\alpha_1 \dots \alpha_n; m}}{\hbar \Omega_{\alpha_1 \dots \alpha_n; m}},$$

where  $T(p)$  is the operator of the canonical transformation;  $\Omega_{\alpha_1 \dots \alpha_n; m}$  is a linear combination of the cyclotron  $\omega_0$ , and the Zeeman  $\omega_s$  frequencies of electrons. Averaging the operator equations, we can construct the macroscopic equations for the density of the spin magnetization of conduction electrons and localized spins

$$\begin{aligned} (\beta_s - \beta_m(x)) L_{s,sm}(x) + Q_s &= 0, \\ (\beta_m - \beta_s(x)) L_{m,sm}(x) + (\beta_m - \beta_p(x)) L_{m,mp}(x) &= 0. \end{aligned}$$

According to [5], we have

$$Q_s = \beta \omega_s^2 D^2 \sum_{q,\omega} |E^-(\omega)|^2 \frac{\Gamma(q, \omega) \omega^2 (s^+(q), s^-(-q))_0}{(\omega - \omega_s)^2 + \Gamma^2(q, \omega)}.$$

Here

$$\Gamma(q, \omega) = v(q, \omega) + q^\alpha q^\gamma D_{\alpha,\gamma}^\pm(q, \omega).$$

where  $v(q, \omega)$  is the known formula for the frequency of the transverse electron spin relaxation [10], and  $D_{\alpha,\gamma}^\pm(q, \omega)$  is the diffusion tensor for the transverse spin magnetization components.

Thus, the expression for spin-wave current in the ferromagnetic insulator is due to non-equilibrium magnon system can be written as

$$\delta M_z(q, \omega) = \frac{\chi_0 [q^\alpha q^\gamma D_{\alpha,\gamma}^{zz}(q, \omega) + Q_s(q, \omega) (\omega_m / \omega_s)^2]}{L_{m,sm}(q, \omega) + L_{m,mp}(q, \omega)},$$

where  $\chi_0$  is the static susceptibility of localized spins.

### DRAG EFFECT

Under the influence of a nonuniform temperature field (a temperature gradient) applied to the system, the magnons and phonons begin travelling; their macroscopic drift affects the propagation of the spin-wave current. The problem to be solved reduces to constructing and analyzing a set of macroscopic momentum balance equations for the ‘‘coherent’’ and thermal magnon ( $i = 1, 2$ ) and phonon ( $i = p$ ) subsystems.

We obtain [9]:

$$\begin{aligned}
 & \frac{d}{dt} \langle P^i(r) \rangle^t \\
 &= - \int_{-\infty}^0 dt_1 e^{\varepsilon t_1} \int dr [D_i^\beta(r, r', t - t_1) \nabla^\beta V_i(r', t - t_1) \\
 &+ L_{mp}^i(r, r', t - t_1) \delta V_{i,p}(r, t) + L_{12}^i(r, r', t - t_1) \delta V_{i,j}(r, t)] \beta, \\
 & \frac{d}{dt} \langle P^p(r) \rangle^t \\
 &= \int_{-\infty}^0 dt_1 e^{\varepsilon t_1} \int dr [D_p^\beta(r, r', t - t_1) \nabla^\beta V_p(r', t - t_1) \\
 &+ L_{ip}^p(r, r', t - t_1) \delta V_{p,i}(r, t) + L_{pp}^p V_p(r', t - t_1)] \beta.
 \end{aligned}$$

Here  $D_\beta^\alpha(r, r', t)$  is the diffusion coefficient.  $\delta V_{ik} = V_i - V_k$ .  $V_i$  is the drift velocity of subsystem  $i$ . The correlation functions  $L_{\alpha\gamma}^\alpha(r, r', t)$  describe relaxation processes.

Thus, we have come to the system of equations describing the dynamics of the momentum of the considered system within the model of three flows: “coherent” and thermally excited magnon ones, and phonon one.

The analysis of the macroscopic momentum balance equations of the systems of interest conducted for different ratios of the drift velocities of the magnon and phonon currents: (a)  $V_1 = V_2, V_p$ ; (b)  $V_1 = V_p, V_2$ ; (c)  $V_1, V_2, V_p$  show that the “injected”-thermal magnons relaxation is possible to be dominant over its relaxation on the phonons. This interaction will be the defining in the forming of the temperature dependence of the spin-wave current under SSE conditions, and inelastic part of the magnon–magnon interaction is the dominant spin relaxation mechanism.

## ACKNOWLEDGMENTS

The given work has been done as the part of the state task on the theme Electron 01201463330 (project no. 12-T-2-1011) with the support of the Ministry of Education of the Russian Federation (grant 14.Z50.31.0025, 16-02-00044) and project no. 15-17-2-17).

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