

Excitation of Copper Vapor Lasers by Storage Capacitor Direct Discharge via High-Speed Photothyristors

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Abstract—The possibility of using a “pulsed fiber laser–photothyristor” optocoupler as a switch in the excitation schemes of copper vapor lasers (CVLs) has been investigated. It has been shown that such a switch has a nanosecond performance and is able to form monopolar and alternating current pulses through CVLs with a power of up to 10 MW and a repetition rate of tens of kilohertz when an electrical efficiency of the excitation circuit is more than 95%. A simple but very accurate model of a photothyristor is proposed, which can be used in full-scale CVL modeling programs.

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INTRODUCTION

For the efficient operation of copper vapor lasers (CVL) and other metals vapor lasers, it is necessary that the excitation circuit provide the shortest possible (less than 10 ns) fronts of voltage pulses $U_R(t)$ at active component $R_f(t)$ of the impedance of the gas discharge tube [1–3]. The simplest excitation circuit is shown in Fig. 1 [1]. Pulsed hydrogen thyratrons [4], which have a number of disadvantages [1, 5], were used as the switch in most of the studies. First, the typical limit rate of rise of the current of the thyratrons is $dJ/dt \approx 4 \text{ A/ns} \ll U_{C0}/L \approx 50 \text{ A/ns}$ (U_{C0} is the initial voltage at storage capacitance C , L is the inductance of the discharge circuit) and the characteristic fall time of the voltage at the thyatron when the turning on is of the order of several tens of nanoseconds. Therefore, the duration of the excitation pulse front is determined not only by the parameters of the scheme, but also by the properties of the thyatron to a large extent. Second, the losses in the thyratrons reach $(0.4–0.6)CU_{C0}^2/2$ [5], which significantly reduces the practical efficiency of the entire device. Third, their service life (about 1000 h) is insufficient for a number of practical applications. Thus, the possibility of replacing thyratrons with more efficient switches has already been studied for the past dozen years, but without much success.

The authors of a recent paper [6] analyzing the causes of the stagnation in the development of metals vapor lasers over the past 20 years came to the conclusion that “the output power and generation efficiency of modern industrial lasers are limited by the imperfection of their power systems.” They have hopes for

the use of a new type of arresters—kivotrons [7, 8], which can dramatically reduce the duration of the front of the excitation pulses of the CVL and thus provide “an increase in the generation efficiency by more than an order of magnitude with a significant increase in output power.” However, this is not to solve other important problems—increasing the efficiency and durability of the excitation circuit, since, first, the use of kivotron introduces very significant additional losses and, second, kivotron only decreases the front of the pulse that is generated by the primary switch, which is necessary to be the same thyatron [6–8].

Meanwhile, the authors of [9] proposed as long as 38 years ago to use a switch that consisted of several series-connected silicon photothyristors of special

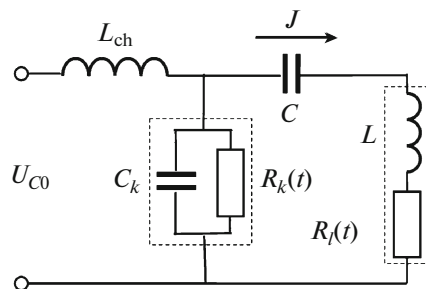


Fig. 1. Equivalent laser excitation scheme with direct discharge of storage capacity C through a switch with capacitance C_k and variable resistance $R_k(t)$. L_{ch} is the inductance of the charging choke, L is the full inductance of the discharge circuit, and $R_f(t)$ is the active component of the impedance of the gas-discharge tube of the laser.

design [10] controlled by pulses of neodymium laser radiation for the CVL excitation. However, neither experimental nor theoretical substantiation of the effectiveness of this technical solution has been presented in [9] and the development of such methods of metal vapor laser ignition has been suspended so far for two reasons. First, the reliability, pulse duration, and efficiency of the standard neodymium lasers of that time (LTIPCh type) were far from sufficient to provide the necessary operational characteristics of photothyristor switches. Secondly, there was actually no theory of switching high-voltage structures with $p-n$ transitions from blocking to conducting state under the influence of short pulses of ionizing irradiation until recently. This did not allow the optimization of the photothyristor design. Recently, this theory was built in [11–14], the results of which indicate the possibility of switching high-voltage silicon photothyristors of special design in the conducting state for 0.1–5 ns under the action of light pulses with a wavelength $\lambda \approx 1060$ nm, duration $t_{\text{ph}} = 0.1\text{--}5$ ns, energy $W_{\text{ph}} = 0.1\text{--}1$ mJ, and repetition rate f up to 80 kHz. Such parameters of control pulses can be easily obtained with the help of modern commercial fiber lasers with a practical efficiency of 10–30% [15].

Such a radical improvement of the situation stimulated re-analyzing of the prospect of the replacement of thyristors with the “fiber laser-photothyristor” optocoupler in the schemes of excitation of CVL and other self-contained laser. The results of this analysis, which are presented below, show that such an optocoupler is an almost ideal switch, which has a subnanosecond performance close to 100% efficiency and high durability, which is characteristic of all solid-state devices.

MODELS OF CVL AND SWITCH

In the present work, the consideration was restricted to only electrical efficiency of photothyristor operation as a switch of the excitation circuit of a CVL. It is not necessary to use full-scale models of gas-discharge processes in CVL, which have been described in detail, for example, in monograph [1], to solve this problem. It is enough to know the plausible values of inductance and the active component of the impedance of the gas-discharge tube $R_i(t) = \Lambda/q\mu n\pi r_{\text{pl}}^2$, which depends on time t due to changes in mobility μ and concentration n of free electrons in the gas-discharge plasma during the excitation process (Λ is the interelectrode distance of the gas-discharge tube and r_{pl} is the effective radius of the plasma filament). An extremely simplified zero-dimensional model of CVL was used for this purpose, assuming that the electron energy distribution in the first approximation is quasi-stationary with respect to the change in the field strength $E = U_R/\Lambda$, where $U_R = R_i J$ and J is the con-

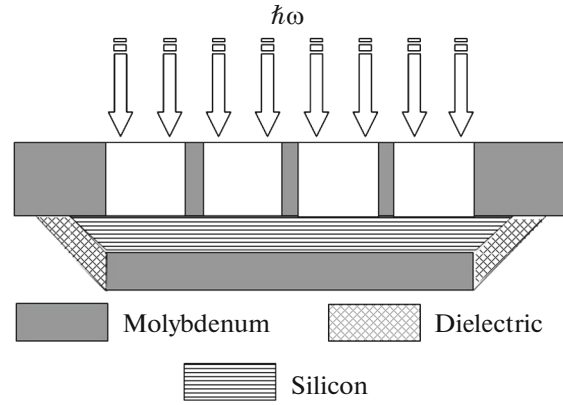


Fig. 2. Schematic representation of the cross section of the photothyristor.

tour current. If the recombination of electrons is negligible during the excitation pulse, then

$$R_i = \Lambda M(E)/q\mu_0 n S_{\text{pl}}, \quad (1)$$

$$\frac{dn}{dt} = n(N_{\text{Cu}} - n)K_{i0}A(E), \quad n(0) = n_0, \quad (2)$$

where n_0 and μ_0 are the concentration and mobility of free electrons at the beginning of the excitation pulse, N_{Cu} is the concentration of copper atoms, K_{i0} is the constant of their ionization, and $M(E)$ and $A(E)$ are instantaneous functions of the field strength. The following approximations were used for them

$$M(E) = [1 + (E/E_M)^4]^{1/16},$$

$$A(E) = \exp(-E_A/E).$$

They describe well the results of calculations [16] at values $\mu_0 N_{\text{Ne}} = 6.6 \times 10^{22}$ V/s/cm, $E_M/N_{\text{Ne}} = 2.12 \times 10^{-12}$ V/cm², $K_{i0} = 3 \times 10^{-8}$ cm³ s⁻¹, and $E_A/N_{\text{Ne}} = 1.36 \times 10^{-11}$ V cm².

The switch is an assembly of m series-connected silicon $n^+p-i-n-p^+$ structures of photothyristors with an area S_k that are placed between molybdenum electrodes (Fig. 2). Many (~ 100) windows with total area $S_0 \sim 0.5S_k$ were made in the cathode electrode and in the n^+ -layer metallization for quasi-uniform illumination. One possible variant of the design of such a photothyristor is described in [17].

In the blocking state, almost all voltage U_{k0} falls on the built-in inversely displaced $p-i-n$ diodes, thickness of which d is slightly less than the thickness of the entire silicon structure. A leakage current flows through the photothyristors prior to lighting

$$J_0 = qn_{k0}v(E_{k0})S_k, \quad (3)$$

where q is the elementary charge; n_{k0} is the concentration of electrons and holes in the exhausted i layer, which are supplied by injection of emitters and ther-

mogeneration in the i -layer; $E_{k0} = U_{k0}/md$ and $v(E)$ is the total drift velocity of electrons and holes that depends on the field strength by the law

$$v(E) = v_{sn} \frac{E}{|E| + E_{sn}} + v_{sp} \frac{E}{|E| + E_{sp}}, \quad (4)$$

$E_{sn,sp} \ll E_{k0}$ are the characteristic field strengths, above which the drift velocities approach the values $v_{sn,sp} \approx 10^7$ cm/s. For certainty, each photothyristor is assumed to be illuminated by a light pulse with power (the final results are very weakly dependent on the shape of the radiation pulse)

$$P(t) = P_M \frac{\sqrt{2e}}{m} \frac{t}{t_{ph}} \exp\left[-\left(\frac{t}{t_{ph}}\right)^2\right], \quad (5)$$

where P_M is peak power. The energy m of such impulses is $W_{ph} = \sqrt{e/2} P_M t_{ph}$. After the beginning of the lighting, the concentration \bar{n}_k that is averaged over the thickness of the structure increases with time in the illuminated areas according to the law

$$\begin{aligned} \bar{n}_k(t) &= n_{k0} + n_{k1} \{1 - \exp[-(t/t_{ph})^2]\}, \\ n_{k1} &= W_{ph} \frac{1 - R_{ph}}{\hbar\omega S_0} \frac{1 - e^{-\kappa d}}{md}, \end{aligned} \quad (6)$$

where $\hbar\omega$ is the energy of the light quantum, R_{ph} is the reflection coefficient from the window surface, and κ is the light absorption coefficient in silicon. It can be shown that the voltage U_k drop at the switch is related to current J that flows through the load by the relation [11]

$$C_k \frac{dU_k}{dt} = J - S_0 \bar{j}, \quad (7)$$

where $C_k = \epsilon S_k/md$ is the capacity of the switch in the blocking state, ϵ is the dielectric permittivity of silicon, and \bar{j} is current density of electrons and holes in the illuminated area that was averaged over the thickness of the i layer. Strictly speaking, it is necessary to solve Eq. (7) together with the Poisson equation and continuity equations for electron and hole concentrations as it was done in [11] to calculate the function $\bar{j}(t)$. However, in this paper a simplified formula is used

$$\bar{j} = q\bar{n}_k v(E_k), \quad (8)$$

which is approximately true at $t \ll d/v$ that is the time of flight of electrons and holes through the i layer, if the field strength in the i layer is weakly dependent on the coordinate (i.e., $E_k \approx U_k/md$) and $\kappa d < 1$. This simplified model is justified (the exact numerical simulation (see below) confirms its applicability) not only because it qualitatively correctly describes the switching of photothyristors to the conducting state, but also because of the weak influence of the parameters of the

“almost ideal switch” on the excitation process of the CVL. It should be emphasized that the above applies only to devices of special design [10, 17] with proper selection of the parameters of the control pulses of light, but it is not applicable to conventional high-voltage photothyristors [18].

Using Kirchhoff's laws for the circuit and formulas (1)–(8), it is easy to obtain the following system of equations and the corresponding initial conditions:

$$\frac{dJ}{dt} = \frac{Q}{\tau^2} - \frac{U_R + U_k}{\rho\tau}, \quad J(0) = J_0, \quad (9)$$

$$\frac{dQ}{dt} = -J, \quad Q(0) = CU_{C0}, \quad (10)$$

$$\frac{dn}{dt} = n(N_{Cu} - n)K_{i0}A(U_R/\Lambda), \quad \eta(0) = 0, \quad (11)$$

$$\frac{dU_R}{dt} = R_0 M \frac{\frac{dJ}{dt} - \frac{J}{n} \frac{dn}{dt}}{\frac{n}{n_0} - \frac{JR_0}{\Lambda} \frac{dM}{dE}}, \quad U_R(0) = J_0 R_0, \quad (12)$$

$$\frac{dU_k}{dt} = C_k^{-1} \left[J - q\bar{n}_k v \left(\frac{U_k}{md} \right) S_0 \right], \quad U_k(0) = U_{k0}, \quad (13)$$

where $\tau = \sqrt{LC}$, $\rho = \sqrt{L/C}$, $R_0 = \Lambda/q\mu_0 n_0 S_{pl}$, Q is the storage capacity charge, and $U_{C0} = (U_{k0} + U_{R0})$ is the initial voltage at the storage capacity.

CALCULATION RESULTS AND DISCUSSION

The results of solving system of approximate equations (9)–(13) by the Runge–Kutta method and accurate numerical simulation of processes in the photothyristor using the program “NVESTIGATION” [19] are presented in this section as an example. The values of the CVL and circuit parameters were taken from [3]:

$\Lambda = 48$ cm, $r_{pl} = 1$ cm, $N_{Ne} = 2 \times 10^{18}/\text{cm}^3$, $N_{Cu} = 1.5 \times 10^{-3} N_{Ne}$, $n_0 = (2-20) \times 10^{13}/\text{cm}^3$, inductance of the gas-discharge tube with a coaxial reverse conductor $L = 160$ nH, and storage capacity $C = 1.5$ nF is charged to initial voltage $U_{C0} = 15$ kV. It was assumed that the switch consists of $m = 3$ series-connected silicon photothyristors with the following parameters of each of them: $S_k = 1$ cm², $S_0 = 0.5S_k$, $d = 600$ μm , breakdown voltage $U_b = 6.34$ kV, leakage current $J_0 = 1.7$ mA at $U_{k0} = 5$ kV, and a blocking ability recovery time of 10 μs .¹ Thyristors are switched to a conductive

¹ For this, it is necessary to reduce lifetime of nonequilibrium charge carriers τ to about 1 μs . The usual high-voltage photothyristors [18] are inoperable at such τ due to the low efficiency of the injection mechanism of conductivity modulation. However, a decrease in τ to 1 μs has practically no effect on the pulse characteristics of the photothyristor at photoionization almost uniform in crystal volume and current pulse duration < 0.1 μs , but ensures its operation at frequencies up to tens of kilohertz.

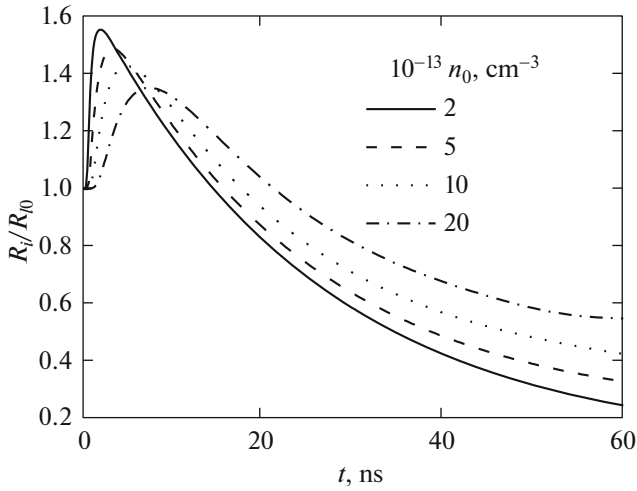


Fig. 3. Dependences of the active component of the laser impedance R_l on time that were obtained by solving system of approximate equations (9)–(13) at $W_{ph} = 3 \times 108 \mu\text{J}$, $t_{ph} = 5 \text{ ns}$, and various prepulse electron concentrations n_0 .

state under the action of light pulses with a wavelength of $1.064 \mu\text{m}$ (the silicon absorption coefficient $\kappa \approx 32 \text{ cm}^{-1}$ at a crystal operating temperature of 100°C).

The calculation results are shown in Figs. 3–5. Dependences $R(t)$, examples of which are shown in Fig. 3, were used in the program “NVESTIGATION” [19] for accurate modeling of processes in thyristor $n^{++}-p^+-p-n-n^+-p^{++}$ structures, the main parameters of which are given above, and the realistic distribution of doping impurities in the thickness of the crystals is described in [12]. The thus-obtained “approximate” and “accurate” dependences $U_k(t)$ and $U_R(t)$ are in good agreement between themselves (Fig. 4). A noticeable relative discrepancy is observed only for very small (about 1 V) $U_k(t)$ values. The reason for this is apparently that the simplified model of thyristors does not take into account the voltage drop (about 0.7 V) at the forward biased emitter transitions. However, this discrepancy practically does not affect the results of calculations of energy losses in thyristors W_k (Fig. 5). As can be seen, W_k begins to increase sharply with the increase in the duration of the control pulse of light at $t_{ph} > 10 \text{ ns}$. Therefore, the light pulses with $t_{ph} \leq 5 \text{ ns}$, which provide switching to the conductive state during a time of about 1 ns or less, should be used to control photothyristors.

Energy of losses W_k in thyristors decreases with growth in W_{ph} , but the losses increase in the control laser at the same time. The result is a nonmonotonic dependence of total energy of losses in the optocoupler W_{sum} on W_{ph} , which has a minimum (Fig. 6). Thus,

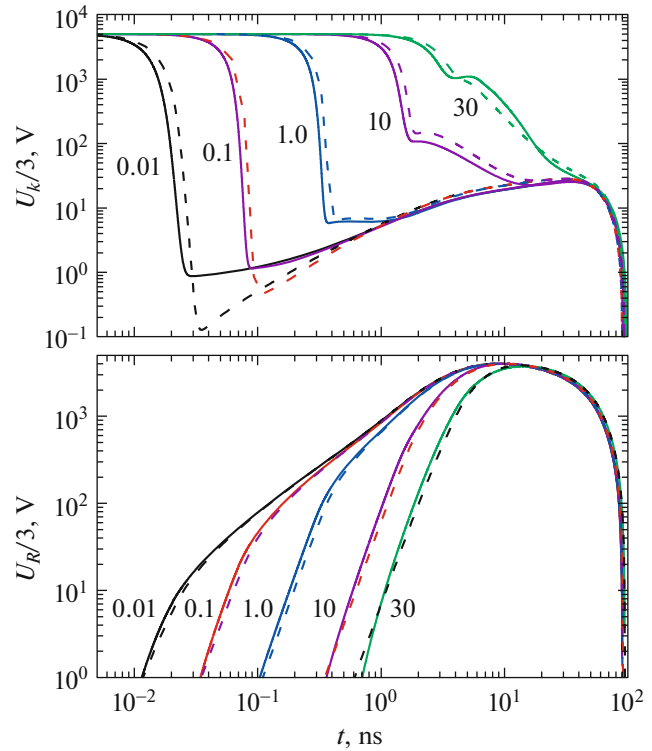


Fig. 4. Dependences of the voltages on the switch U_k and on the active component of the laser impedance U_R on time that were obtained by accurate numerical modeling (solid lines) and solving system of equations (9)–(13) (dashed lines) at $W_{ph} = 3 \times 122 \mu\text{J}$ and $n_0 = 10^{14} \text{ cm}^{-3}$. The numbers by the curves indicate the duration of the control light pulse t_{ph} in nanoseconds.

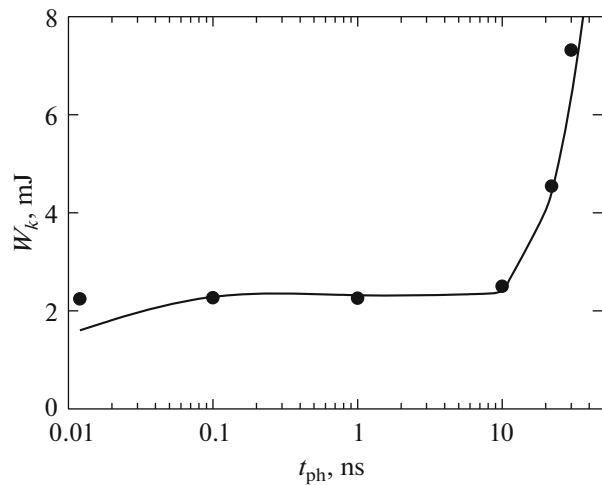


Fig. 5. Dependences of switching loss energy W_k in three photothyristors on the duration of control light pulse t_{ph} that were obtained by accurate numerical modeling (symbols) and by solving system of approximate equations (9)–(13) (line) at $W_{ph} = 3 \times 200 \mu\text{J}$ and $n_0 = 10^{14} \text{ cm}^{-3}$.

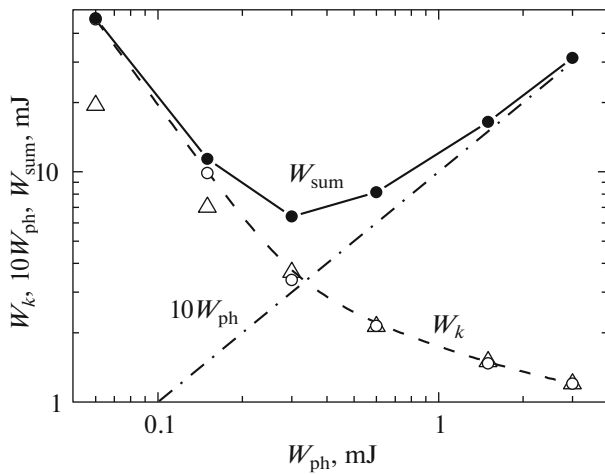


Fig. 6. Dependences of energy W_k dissipated by the three thyristors of the switch that were obtained by accurate numerical simulation (light circles + dashed line) and by solving system of approximate equations (9)–(13) (light triangles) and by the power source of the control laser with an efficiency of 10% ($10W_{ph}$, dash-dotted line) and of total optocoupler loss energy W_{sum} per excitation pulse (dark circles + solid line) on energy W_{ph} of control laser radiation pulse at $t_{ph} = 5$ ns and $n_0 = 10^{14}$ cm $^{-3}$.

there is an optimal value W_{ph} , which is equal to 0.33 mJ at $t_{ph} = 5$ ns and the efficiency of the control laser being 10% when $W_{sum} \approx 2W_k \approx 6.6$ mJ in our particular case. Since the stored energy C in the storage capacity $CU_0^2/2 = 169$ mJ, the efficiency of the excitation circuit is more than 96% without taking into account the losses in the charge circuit. Pulse heating of thyristors in one cycle of excitation is approximately equal to 0.01 K, so in fact they operate in the mode of constant dissipation of average power $P_k = W_k f$. Each of the three thyristors will dissipate the average power $P_k/m \approx 22$ W at $f = 20$ kHz, so it is sufficient to use a simple cooling system with thermal resistance of crystal-environment 4 K/W to maintain the operating temperature of the crystals of 100°C.

The shape and amplitude of current $J(t)$ and voltage $U_R(t)$ pulses at the active component of the CVL impedance remain practically constant when the duration and energy of the control light pulses change within a reasonable range, but significantly depend on the prepulse electron concentration n_0 . As a result, the time dependence of power $P_R = U_R J$ and energy W_R that is absorbed by the CVL at the initial stage of excitation change as shown in Figs. 7 and 8. In addition, the reverse effect of CVL on the switch is observed: the energy losses in photothyristors increase with n_0 growth (Fig. 8).

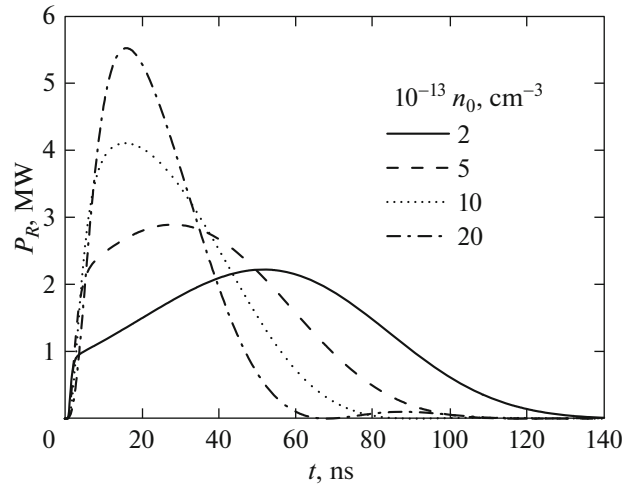


Fig. 7. Dependences of power P_R dissipated by the active component of the laser impedance R_l on time that were obtained by solving system of approximate equations (9)–(13) at $W_{ph} = 3 \times 108$ μ J, $t_{ph} = 5$ ns, and various prepulse electron concentrations n_0 .

In the example that was considered, the excitation circuit was almost exactly matched with the load: the amplitude of the reflected pulse was about 10 times less than that of the main one. However, it is not always possible to achieve such matching in practice and the emergence of a powerful reflected pulse leads to an additional tightening of the thyristors operation mode [1, 5]. In this regard, another advantage of the described switch should be noted: photothyristors are

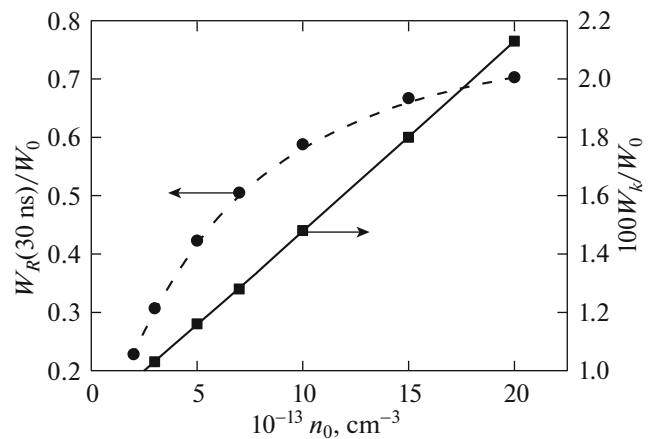


Fig. 8. Dependences of the normalized values of energy W_R dissipated by the active component of the laser impedance R_l for 30 ns after the start of the excitation pulse and energy W_k dissipated by the switch on prepulse electron concentration n_0 that were obtained by solving system of approximate equations (9)–(13) at $W_{ph} = 3 \times 108$ μ J and $t_{ph} = 5$ ns.

able to pass large reverse currents for hundreds of nanoseconds without any harm to themselves. Of course, this increases the losses, but a slight increase in the energy of the control light pulse allows the full compensate for this effect. Therefore, photothyristors can be used even for the excitation of coaxial CVL by a pulse-periodic induction discharge almost as effectively [20], which is impossible to implement when using thyristors.

CONCLUSIONS

In conclusion, three problems that are associated with the possibility of using the described switch will be noted. The first of them is the complexity of creating a low-inductive assembly of series-connected photothyristors [9] and the lack of industrial technology for the manufacture of silicon structures with breakdown voltage U_b of more than 10 kV [18]. In principle, it can be solved by using silicon carbide photothyristors, as prototypes of bipolar keys based on 4H-SiC with $U_b = 27$ kV have already been created [21]. However, the industrial technology of such devices has not yet been mastered and control lasers with a suitable wavelength $\lambda = 375$ nm are difficult to access. Therefore, the described switch can presently be used to excite either a not very powerful CVL with a short gas-discharge tube or a multisection CVL [22, 23] and CVL with transverse discharge [24, 25]. The second problem is the high cost of fiber lasers, which can be solved in the course of further progress in the technology of pulse semiconductor lasers. The third problem is that the above results, which illustrate the advantages of the “fiber laser–photothyristor” optocoupler, do not allow the answering of the question of how much this method can improve the output optical characteristics of the CVL: the average radiation power and total practical efficiency. It is necessary to solve the problem beyond the scope of this work to do this. Namely, it is necessary to optimize the mode of operation of the CVL using one of the full-scale models [1], replacing in it the phenomenological description of a thyristor (see Section 8.4 in [1]) with a very exact and physically based equation (13).

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