ISSN 0030-400X, Optics and Spectroscopy, 2017, Vol. 123, No. 4, pp. 610–614. © Pleiades Publishing, Ltd., 2017.
Original Russian Text © R.M. Arkhipov, M.V. Arkhipov, A.V. Pakhomov, I. Babushkin, N.N. Rosanov, 2017, published in Optika i Spektroskopiya, 2017, Vol. 123, No. 4, pp. 600–605.

NONLINEAR AND QUANTUM OPTICS

Collisions of Unipolar Subcycle Pulses in a Nonlinear Resonantly Absorbing Medium

R. M. Arkhipov[a, b, *], M. V. Arkhipov[a], A. V. Pakhomov[c, d], I. Babushkin[e, f], and N. N. Rosanov[b, g, h]

[a] *St. Petersburg State University, St. Petersburg, 199034 Russia*
[b] *ITMO University, St. Petersburg, 197101 Russia*
[c] *Samara National Research University, Samara, 443086 Russia*
[d] *P.N. Lebedev Physical Institute, Samara Branch, Russian Academy of Sciences, Samara, 443011 Russia*
[e] *Institute of Quantum Optics, Leibniz University Hannover, 30167 Hannover, Germany*
[f] *Max Born Institute, Berlin, 12489 Germany*
[g] *Vavilov State Optical Institute, St. Petersburg, 199053 Russia*
[h] *Ioffe Physical Technical Institute, Russian Academy of Sciences, St. Petersburg, 194021 Russia*
**e-mail: arkhipovrostislav@gmail.com*

Received April 24, 2017

Abstract—Collisions of unipolar subcycle pulses in a nonlinear resonant medium that coherently interact with it are studied theoretically. The dynamics of spatial polarization structures and population difference that the pulses induce in the medium are analyzed. A surprising feature is that the medium is capable of "remembering" the result of the interaction with the pulses and their collisions during times comparable to the polarization relaxation time. An analysis of the dynamics of light-induced structures makes it possible to judge the parameters of subcycle pulses at the times longer than the pulse duration, which, in the future, can be useful in their detection.

DOI: 10.1134/S0030400X17100046

INTRODUCTION

At present, ultrashort and extremely short pulses of femtosecond and attosecond durations have been obtained [1–3]. Their generation gave birth to attosecond science, which made it possible to investigate and control the dynamics of wave packets of matter [4, 5], to control electron beams with a subcycle resolution [6], etc.

Coherent interaction of extremely short pulses with a resonant medium is of independent interest [7–27]. Coherent interaction manifests itself in the appearance of Rabi oscillations, and it arises if duration of a light pulse τ_p is shorter than the relaxation times of the population difference and the polarization, T_1 and T_2, $\tau_p \ll T_1, T_2$ [28, 29]. At the times that are comparable with T_2, the state of the electromagnetic field, as well as of the matter, can be modified without losses. Coherent interaction of light with matter manifests itself in the effect of self-induced transparency, which consists of the possibility of loss-free propagation of a pulse in a resonantly absorbing medium (2π pulse) [30]. Another distinctive feature of the coherent interaction is the ability to store information in a medium in the phase of the field without losses at the times shorter than relaxation times. It is evident that the use of the coherent interaction opens new prospects in optics and photonics, since devices based on it may have comparatively high speeds of operation and low losses. In a recent review [31], these devices were called "coherent photonic devices." Indeed, in [32–40], the possibility of mode locking in lasers based on the coherent interaction (self-induced transparency mode-locking) was studied. The authors of [11–15] showed the possibility of generating attosecond solitons in single-mode optical fibers filled with amplifying and absorbing centers as a result of simultaneous action of self-induced transparency and generation of π pulses. Rabi oscillations have been observed experimentally in different systems, in particular, in experiments with single quantum dots and upon propagation of a short pulse in a medium of quantum dots [41–46]. At present, the coherent propagation of long [28–30, 47, 48] and extremely short pulses in resonant media has been well studied [7–27].

Recently, another interesting phenomenon has been predicted, which arises as a result of the coherent interaction of extremely short pulses with a resonantly absorbing medium [16, 17]. It consists of the possibility of inducing, erasing, and ultrafast controlling spatial gratings of the population difference by means of a sequence of extremely short pulses that do not overlap in the medium. Commonly, these gratings are formed in the medium as a result of interference of two or more overlapping quasi-monochromatic light beams [49]. However, if the interaction with matter is coherent, no beam overlap is required. Gratings may arise due to the fact that the medium "remembers" the action of a preceding pulse; i.e., when a field is no longer present, the polarization of the medium still behaves itself as if the field were present. In this case, the medium acts as an "information keeper" of a sort about passed pulses.

Coherent interaction of extremely short pulses with a resonant medium has been studied mainly in the case of bipolar pulses. Bipolar pulses have zero electric area (integral of the electric-field strength with respect to time at a given point in space). Recently, the possibility of obtaining unipolar subcycle pulses attracts attention, see, e.g., [7, 11–15] and reviews [50, 51]. Each of these pulses contains only one half-wave of the field, and their electrical area can be nonzero. Application of unipolar pulses, compared to bipolar ones, has a number of advantages, not only because of their short duration, but also due to the fact that the occurrence of a constant component makes it possible to efficiently transfer momentum to charged particles. This makes important the use of unipolar pulses for controlling the dynamics of wave packets and acceleration of charged particles (see recent reviews [50, 51] and references therein). Although it is sometimes doubted whether unipolar and subcycle pulses can be obtained, in a number of cases, such pulses can exist in the form of soliton solutions of equations of nonlinear optics, see references in [50, 51] upon reflection of a single-cycle pulse from a thin metal layer or dielectric in the one-dimensional case [52] and in a quadratically nonlinear medium [53]. Almost unipolar pulses have been obtained experimentally in the terahertz range [54], and they have been used to control the dynamics of Rydberg atoms [55]. Recently, new methods have been proposed for generating unipolar pulses in Raman-active media [56–58] and in media with nonlinear field coupling [59, 60].

Despite of these advantages of unipolar impulses, their coherent interaction with the medium has been poorly studied in the literature. Investigations that have been performed [7–27] were mainly aimed at studying coherent interaction of bipolar pulses with a resonant medium and a possibility of obtaining unipolar subcycle pulses in the form of solitons upon propagation of bipolar pulses in a nonlinear medium [7, 11–15, 18, 19, 50]. In a number of works, collisions of bipolar extremely short pulses of self-induced transparency in a resonant medium have been examined (see [23–26] and references therein). In [27], collisions of unipolar subcycle pulses in a single-mode optical fiber containing amplifying and absorbing particles were studied.

Taking into account the aforesaid, in this paper, we will study collisions of unipolar subcycle pulses in a resonantly absorbing medium under conditions when their interaction with the medium is coherent. Recently, it was shown that such pulses, if they do not overlap in the medium, are capable of inducing in it spatial polarization structures and population differences that can exist in the medium during times comparable to T_2 [61, 62]. In this work, we will study the dynamics of these structures in a medium upon collisions of unipolar subcycle pulses in it. As will be shown below, the study of the dynamics of these structures can be used in the future to extract information on passed pulses.

THE THEORETICAL MODEL AND THE SYSTEM UNDER CONSIDERATION

To study collisions of subcycle unipolar pulses in a resonant medium, we used the system of Maxwell–Bloch equations. Because exciting pulses are of a short duration, we do not use the approximations of slowly varying amplitudes and rotating waves. The medium is described in the two-level approximation using the density-matrix formalism. This system of equations has the form

$$\frac{\partial \rho_{12}(z,t)}{\partial t} = -\frac{\rho_{12}(z,t)}{T_2} + i\omega_0\rho_{12}(z,t) - \frac{i}{\hbar}d_{12}E(z,t)n(z,t), \quad (1)$$

$$\frac{\partial n(z,t)}{\partial t} = -\frac{n(z,t) - n_0(z)}{T_1} + \frac{4i}{\hbar}d_{12}E(z,t)\,\mathrm{Im}\,\rho_{12}(z,t), \quad (2)$$

$$P(z,t) = 2N_0 d_{12}\,\mathrm{Re}(\rho_{12}), \quad (3)$$

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P(z,t)}{\partial t^2}. \quad (4)$$

Here ω_0 is the frequency of the resonant transition of the medium ($\lambda_0 = 2\pi c/\omega_0$ is the wavelength of the resonant transition), d_{12} is the transition dipole moment, n_0 is the population difference in the absence of an electric field (for the absorbing medium, $n_0 = 1$), P is the polarization of the medium, N_0 is the concentration of active centers, E is the electric-field strength, c is the speed of light in a vacuum, and $\hbar$ is the reduced Planck constant.

Equations (1) and (2) describe the evolution of the off-diagonal element of the density matrix, ρ_{12} and the difference between the diagonal elements of the density matrix, n. Quantity n has the meaning of the population difference (inversion) between the ground and

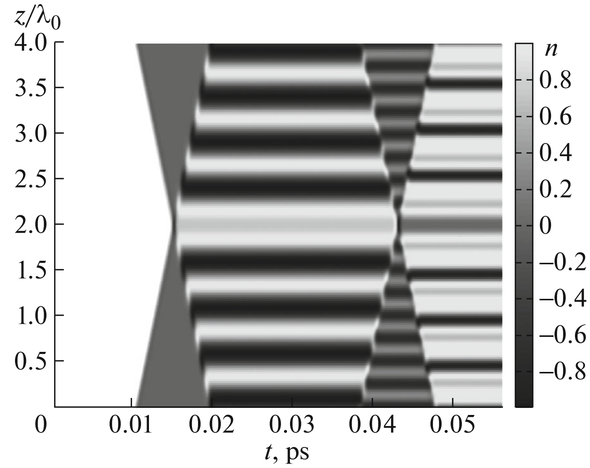

Fig. 1. Dynamics of population difference $n(z, t)$ under the action of unipolar subcycle pulses colliding at the center of the medium (at the point $z/\lambda_0 = 2$). The parameters of the calculation were as follows: The wavelength of the resonant transition was $\lambda_0 = 700$ nm (the period of eigenoscillations was $T_0 = \lambda_0/c = 2.33$ fs), the length of the medium was $L_m = 4\lambda_0$, the transition dipole moment was $d_{12} = 20$ dB, the concentration of two-level atoms was $N_0 = 5\times10^{14}$ cm^{-3}, the relaxation times were $T_1 = T_2 = 1$ ns, the amplitude of unipolar pumping pulses was $E_0 = 9.55\times10^4$ esu, the duration of each pulse was $\tau_p = 0.38$ fs ($T_0/6$), and the delays were $\tau_1 = \tau_2 = 2.5\tau_p$.

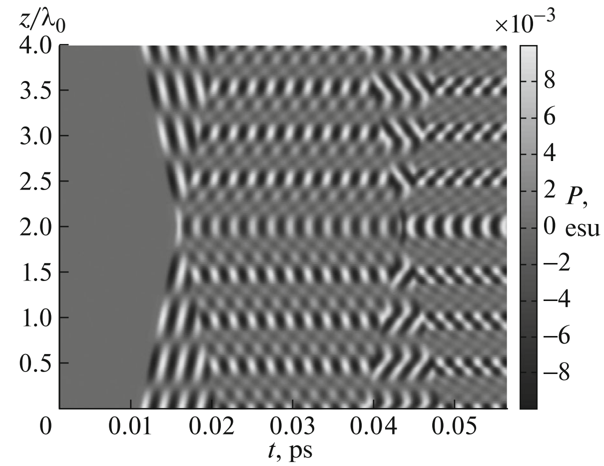

Fig. 2. Dynamics of polarization $P(z, t)$ under the action of unipolar subcycle pulses colliding at the center of the medium (at the point $z/\lambda_0 = 2$). The parameters of the calculation are the same as in Fig. 1.

excited states of the two-level system. Using the off-diagonal element of the density matrix, polarization (3) of the medium is calculated, which serves as a source of the field in the wave equation (4). The system (1)–(4) of the Maxwell–Bloch equations was solved numerically. The Bloch equations (1) and (2) for the density matrix were solved by the fourth-order Runge–Kutta method. The wave equation (4) was solved by the method of finite differences.

The length of the total integration range was $L = 12\lambda_0$. The resonant medium was arranged along the z axis in the center of the range between the points at $z_1 = 4\lambda_0$ and $z_2 = 8\lambda_0$. To create a sequence of unipolar pulses, zero boundary conditions for the values of the field at the ends of the integration range were taken in the numerical calculation so that the pulses reflected from the boundaries and propagated back.

Subcycle pulses of the same amplitude were sent from the left and right to the medium. The field at the left end of the integration range ($z = 0$) was taken in the form

$$E_1(t,0) = E_0 \exp\left(-\frac{[t-\tau_1]^2}{\tau_p{}^2}\right). \qquad (5)$$

At the right end ($z = L$), the field of the pulse sent from the right to the left also had a Gaussian profile,

$$E_2(t,L) = E_0 \exp\left(-\frac{[t-\tau_2]^2}{\tau_p{}^2}\right). \qquad (6)$$

Here, $\tau_{1,2}$ are the delays which were chosen such that the pulses would collide in the center of the medium. The amplitude of all pulses, E_0, was chosen to be the same.

THE DYNAMICS OF LIGHT-INDUCED STRUCTURES FORMED AS A RESULT OF PULSE COLLISIONS

For simplicity, as in [16, 17, 59, 60], we consider the case in which the amplitude of all the pulses is such that the pulse saturates the medium ($\pi/2$ pulse). For extremely short pulses, the notion of the “pulse area,” which was introduced for long pulses, for which the approximations of slowly varying amplitudes and rotating waves are valid, is no longer applicable [16, 17, 20–26]. Figures 1 and 2 show the evolution of the dynamics of the population difference and the polarization that form upon propagation of subcycle pulses (5) and (6) in the medium.

The results of our numerical simulation showed the following dynamics of the system: Both pulses saturated the medium and left traveling polarization waves behind them (Figs. 1, 2). In the center of the medium, near the point at $z/\lambda_0 = 2$, the pulses collided, causing a burst of the population difference near it. Near this point, the medium was transferred to the ground state (Fig. 1). In the collision region of the pulses, a region was formed in which particles were transferred

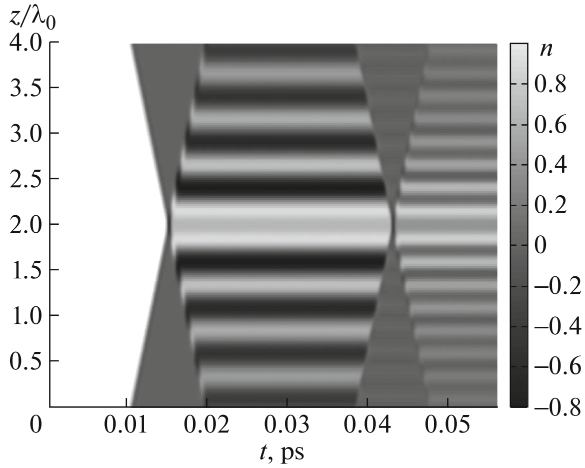

Fig. 3. The same as in Fig. 1. Dynamics of population difference $n(z, t)$ under the action of unipolar subcycle pulses colliding at the center of the medium (at the point $z/\lambda_0 = 2$). $T_2 = 10$ fs. The remaining parameters of the calculation are the same as in Figs. 1 and 2.

to the ground state (the white region near the point at $z/\lambda_0 = 2$ in Fig. 1). The length of this region is on the order of the spatial length of the pulses. Therefore, by determining the length of the inversion structure in this region, one can obtain information on the duration of passed pulses.

Furthermore, as a result of the interference of pulses with the polarization waves, the pulses propagating on the right and on the left of the center of the medium induced a population grating. Complex polarization structures in the form of standing waves also appeared (Fig. 2). After that, the pulses left the medium and the "recorded information" remained in the medium in the form of the light-induced spatial structures. Then the pulses were reflected from the boundary of the integration range and were sent to the medium again. The medium still remembers the result of their preceding action. The pulse that propagates on the left of the point at $z/\lambda_0 = 2$ in the direction from the left to the right erases the population grating (see weak oscillations in Fig. 1). The same occurs on the right of the point at $z/\lambda_0 = 2$ upon propagation of the second pulse. Then, the pulses collided again at the point at $z/\lambda_0 = 2$, after which they propagated to the right and to the left of this point, now inducing nonharmonic inversion and polarization structures. Therefore, it is seen that the use of the coherent interaction makes it possible to create light-induced polarization and inversion structures, which can be stored in the medium within times until the coherence of the medium becomes destroyed (times shorter than T_1 and T_2). It is evident that a decrease in the polarization relaxation time leads to a smearing of light-induced structures. This circumstance is illustrated by Fig. 3, which shows the dynamics of spatial inversion structures at $T_2 = 10$ fs.

CONCLUSIONS

In this work, we showed theoretically the existence of the dynamics of light-induced polarization and inversion structures in a resonant medium upon collision of unipolar subcycle pulses coherently interacting with the medium. These polarization and population difference gratings can exist in the medium within times comparable to relaxation times, until the coherence of the medium is destroyed.

Traveling waves of the medium polarization can be considered as relativistic mirrors and can be used to convert the frequency of reflected light pulses [63], because these mirrors can move with relativistic velocities, unlike ordinary "material" mirrors. It can be expected that, experimentally, the effect may be studied in gases or quantum dots. Quantum dots possess large transition dipole moments (dozens of Debyes). At low temperatures, the relaxation polarization times may reach tens to hundreds of nanoseconds [64].

The examined effect can be used to create optical switches and sources of short-term information storage (memory cells). Reading of light-induced structures may be used in the future to extract data on parameters of extremely short pulses.

ACKNOWLEDGMENTS

This work was supported by the Russian Science Foundation, project no. 17-19-01097.

REFERENCES

1. F. Krausz and M. Ivanov, Rev. Mod. Phys. **81**, 163 (2009).
2. C. Manzoni et al., Laser Photon. Rev. **9**, 129 (2015).
3. F. Calegari et al., J. Phys. B: At. Mol. Opt. Phys. **49**, 062001 (2016).
4. L. Gallmann, C. Cirelli, and U. Keller, Ann. Rev. Phys. Chem. **63**, 447 (2012).
5. K. Ramasesha, S. R. Leone, and D. M. Neumark, Ann. Rev. Phys. Chem. **67**, 41 (2016).
6. M. Kozak, J. McNeur, K. J. Leedle, et al., Nat. Commun. **8**, 1 (2017).
7. V. P. Kalosha and J. Herrmann, Phys. Rev. Lett. **83**, 544 (1999).
8. J. Xiao, Z. Wang, and Z. Xu, Phys. Rev. A **65**, 031402 (2002).
9. X. Cai, J. Zhao, Z. Wang, and Q. Lin, J. Phys. B: At. Mol. Opt. Phys. **46**, 175602 (2013).
10. Y. Lin, I. H. Chen, and R. K. Lee, Phys. Rev. A **83**, 043828 (2011).
11. N. V. Vysotina, N. N. Rozanov, and V. E. Semenov, JETP Lett. **83**, 279 (2006).
12. N. N. Rosanov, V. E. Semenov, and N. V. Vysotina, Quantum Electron. **38**, 137 (2008).

13. N. V. Vysotina, N. N. Rosanov, and V. E. Semenov, Opt. Spectrosc. **106**, 713 (2009).
14. V. V. Kozlov, N. N. Rosanov, C. D. Angelis, and S. Wabnitz, Phys. Rev. A **84**, 023818 (2011).
15. N. N. Rosanov, *Dissipative Optical Solitons: From Micro- to Nano- and Atto- Scales* (Fizmatlit, Moscow, 2011), Chap. 17 [in Russian].
16. R. M. Arkhipov, M. V. Arkhipov, I. Babushkin, and N. N. Rosanov, Opt. Spectrosc. **121**, 758 (2016).
17. R. M. Arkhipov, M. V. Arkhipov, I. V. Babushkin, A. Demircan, U. Morgner, and N. N. Rosanov, Opt. Lett. **41**, 4983 (2016).
18. X. Song, W. Yang, Z. Zeng, R. Li, and Z. Xu, Phys. Rev. A **82**, 053821 (2010).
19. X. Song, Z. Hao, M. Yan, M. Wu, and W. Yang, Laser Phys. Lett. **12**, 105003 (2015).
20. S. Hughes, Phys. Rev. Lett. **81**, 3363 (1998).
21. A. V. Tarasishin, S. A. Magnitskii, and A. M. Zheltikov, Opt. Commun. **193**, 187 (2001).
22. A. V. Tarasishin, S. A. Magnitskii, V. A. Shuvaev, and A. M. Zheltikov, Opt. Express **8**, 452 (2001).
23. D. V. Novitsky, Phys. Rev. A **84**, 013817 (2011).
24. D. V. Novitsky, Phys. Rev. A **85**, 043813 (2012).
25. D. V. Novitsky, J. Phys. B: At., Mol. Opt. Phys. **47**, 095401 (2014).
26. D. V. Novitsky, Opt. Commun. **358**, 202 (2016).
27. N. N. Rosanov, V. E. Semenov, and N. V. Vyssotina, Laser Phys. **17**, 1311 (2007).
28. P. G. Kryukov and V. S. Letokhov, Sov. Phys. Usp. **12**, 641 (1970).
29. L. Allen and J. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).
30. S. L. McCall, Phys. Rev. **183**, 457 (1969).
31. R. M. Arkhipov, M. V. Arkhipov, A. V. Pakhomov, I. Babushkin, and N. N. Rosanov, Opt. Spectrosc. **122**, 949 (2017).
32. V. V. Kozlov, Phys. Rev. A **56**, 1607 (1997).
33. M. A. Talukder and C. R. Menyuk, Phys. Rev. Lett. **102**, 023903 (2009).
34. M. A. Talukder and C. R. Menyuk, Phys. Rev. A **79**, 063841 (2009).
35. V. V. Kozlov, N. N. Rosanov, and S. Wabnitz, Phys. Rev. A **84**, 053810 (2011).
36. V. V. Kozlov and N. N. Rosanov, Phys. Rev. A **87**, 033808 (2013).
37. R. M. Arkhipov, M. V. Arkhipov, and I. V. Babushkin, JETP Lett. **101**, 149 (2015).
38. R. M. Arkhipov, M. V. Arkhipov, and I. V. Babushkin, Opt. Commun. **361**, 73 (2016).
39. R. M. Arkhipov, M. V. Arkhipov, I. Babushkin, and N. N. Rosanov, Opt. Lett. **41**, 737 (2016).
40. R. M. Arkhipov, M. V. Arkhipov, A. A. Shimko, and I. Babushkin, JETP Lett. **101**, 232 (2015).
41. A. Zrenner et al., Nature **418**, 612 (2002).
42. M. Kolarczik et al., Nat. Commun. **4**, 1 (2013).
43. M. Hosseini, B. M. Sparkes, G. Hétet, et al., Nature **461** (7261), 241 (2009).
44. T. H. Stievater, L. Xiaoqin, and D. G. Stee, Phys. Rev. Lett. **87**, 133603 (2001).
45. P. Borri, W. Langbein, S. Schneider, et al., Phys. Rev. B **66**, 081306 (2002).
46. O. Karni, A. Capua, G. Eisenstein, et al., Opt. Express **21**, 26786 (2013).
47. J. Schuttler, I. Babushkin, and W. Lange, Phys. Rev. A **78**, 035802 (2013).
48. A. A. Klyukanov, N. A. Loiko, I. V. Babushkin, and V. Gurau, in *Proceedings of the 17th International Conference on Coherent and Nonlinear Optics ICONO'2001* (2002), p. 301.
49. H. J. Eichler, E. Gunter, and D. W. Pohl, *Laser-Induced Dynamic Gratings* (Springer, Berlin, Heidelberg, New York, Tokyo, 1981).
50. R. M. Arkhipov, A. V. Pakhomov, I. Babushkin, Yu. A. Tolmachev, and N. N. Rosanov, JETP Lett. **105** (6), 408 (2017).
51. R. M. Arkhipov, A. V. Pakhomov, M. V. Arkhipov, I. Babushkin, Yu. A. Tolmachev, and N. N. Rosanov, Laser Phys. **27**, 053001 (2017).
52. M. V. Arkhipov, R. M. Arkhipov, A. V. Pakhomov, I. V. Babushkin, A. Demircan, U. Morgner, and N. N. Rosanov, Opt. Lett. **42** (11), 2189 (2017).
53. V. V. Kozlov, N. N. Rosanov, C. D. Angelis, and S. Wabnitz, Phys. Rev. A **84**, 023818 (2011).
54. K. Reimann, Rep. Prog. Phys. **70**, 1597 (2007).
55. A. Wetzels, A. Gjrtler, L. D. Noordam, et al., Phys. Rev. Lett. **89**, 273003 (2002).
56. R. M. Arkhipov, Opt. Spectrosc. **120**, 756 (2016).
57. R. M. Arkhipov, M. V. Arkhipov, P. A. Belov, Yu. A. Tolmachev, and I. Babushkin, Laser Phys. Lett. **13**, 046001 (2016).
58. R. M. Arkhipov, A. V. Pakhomov, I. V. Babushkin, M. V. Arkhipov, Yu. A. Tolmachev, and N. N. Rosanov, J. Opt. Soc. Am. B **33**, 2518 (2016).
59. A. V. Pakhomov, R. M. Arkhipov, I. V. Babushkin, M. V. Arkhipov, and N. N. Rosanov, Laser Phys. Lett. **13**, 126001 (2016).
60. A. V. Pakhomov, R. M. Arkhipov, I. V. Babushkin, M. V. Arkhipov, Yu. A. Tolmachev, and N. N. Rosanov, Phys. Rev. A **95**, 013804 (2017).
61. R. M. Arkhipov, M. V. Arkhipov, I. V. Babushkin, A. V. Pakhomov, and N. N. Rosanov, Quant. Electron. **47** (7), 589 (2017).
62. R. M. Arkhipov, M. V. Arkhipov, A. V. Pakhomov, I. V. Babushkin, and N. N. Rosanov, Las. Phys. Lett. (2017) (in press).
63. S. V. Bulanov, T. Zh. Esirkepov, M. Kando, A. S. Pirozhkov, and N. N. Rosanov, Phys. Usp. **56**, 429 (2013).
64. M. Bayer and A. Forchel, Phys. Rev. B **65**, 041308 (2002).