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NONLINEAR AND QUANTUM OPTICS

Generation of a Sequence of Frequency-Modulated Pulses in Longitudinally Inhomogeneous Optical Waveguides

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Abstract—The conditions for the generation and efficient amplification of frequency-modulated soliton-like wave packets in longitudinally inhomogeneous active optical waveguides have been studied. The possibility of forming a sequence of pico- and subpicosecond pulses from quasi-continuous radiation in active and passive optical waveguides with the group-velocity dispersion (GVD) changing over the waveguide length is considered. The behavior of a wave packet in the well-developed phase of modulation instability with a change in the waveguide inhomogeneity parameters has been investigated based on the numerical analysis.

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INTRODUCTION

A great deal of attention has invariably been paid to the dynamics of optical soliton during the past 30 years [1–5]. One of the most important problems in this field concerns the optimal amplification of soliton pulses at which their shape and character of their elastic interaction are retained. It is known that the incoherent soliton amplification cannot be implemented in a nonlinear active optical waveguide with constant parameters over its length. As soon as the soliton energy increases by a factor of about e, independent of the amplification method, the soliton shape and spectrum become significantly distorted because of the rise in the nonsoliton pulse component. As a result, the nonlinear wave packet ceases to be structurally stable and, correspondingly, loses its soliton properties. This dynamic scenario of soliton amplification had been considered for a long time as the only possible one. However, it was found in [6, 7] that an optical soliton can be amplified as a single whole provided that the phase of a soliton-like pulse at the optical waveguide input is a parabolic function of time and the gain increment is a hyperbolic function of distance. The interaction of these frequency-modulated (FM) pulses becomes completely elastic when their phases and the gain of the medium are self-consistent.

One of the main difficulties in the experimental implementation of the proposed scenario of the ideal amplification of wave packets retaining their shape is the necessity of forming the corresponding inhomogeneity of the gain increment in an optical waveguide. It was shown in [8, 9] that this problem can be solved using optical waveguides having not only a hyperbolic gain profile (along their length) but practically any desired dependence of a gain increment on the longitudinal coordinate if a corresponding group velocity dispersion (GVD) profile is formed along the waveguide length.

Currently, optical waveguides with anomalous GVD, gradually decreasing in modulus along the waveguide length, and a *W*-profile refractive index in their cross section appear to be most promising in this context. The necessary longitudinal GVD distribution along the waveguide length at ultralow values of the third-order dispersion parameter can be implemented in these optical waveguides. The necessary dispersion profile in these waveguides is generally obtained due to the controlled change in their transverse sizes. The up-to-date waveguide drawing technique provides a significant controlled change in dispersion even at a small variation in the waveguide diameter along the entire waveguide length (generally, by no more than 3 μm at an average waveguide diameter of about 100 μm) [10].

In this paper, we analyze the possibility of generating a sequence of short frequency-modulated laser pulses due to the development of the modulation instability (MI) mode. It is shown that sequences of pico- and subpicosecond laser pulses with a THz repetition rate can be formed directly from a quasi-continuous wave in dispersion-decreasing optical waveguides.

BASIC EQUATIONS AND RELATIONS

Let us consider the dynamics of an optical wave packet propagating in an inhomogeneous amplifying optical waveguide. In this case, the field can be presented in the form

$$\mathbf{E}(t,r,z) = \frac{\mathbf{e}}{2}U(r,z)\left\{A(t,z)\exp\left[i\left(\omega_0 t - \int_0^z \beta'(\xi)d\xi\right)\right] + \text{K.C.}\right\}, \quad (1)$$

where $\mathbf{e}$ is the polarization unit vector, $U(r,z)$ is the radial field distribution in the optical waveguide, ω_0 is the wave packet's carrier frequency, and β' is the real part of the complex propagation constant. Time envelope $A(t,z)$ is described by the nonlinear Schrödinger equation [1, 2] with the coefficients dependent on the longitudinal coordinate:

$$\frac{\partial A}{\partial z} - i\frac{D(z)}{2}\frac{\partial^2 A}{\partial \tau^2} + iR(z)|A|^2 A = \gamma(z)A. \quad (2)$$

The following parameters are introduced here: time in a moving coordinate system, $\tau = t - \int_0^z d\xi/u(\xi)$; wave packet group velocity $u(z) = (\partial\beta'(z)/\partial\omega)_0^{-1}$; GVD $D(z) = (\partial^2\beta'(z)/\partial\omega^2)_0$; Kerr nonlinearity coefficient $R(z)$; and effective gain increment, which is given by the expression

$$\gamma(z) = g(z) - (\partial S_m/\partial z)/2S_m, \quad (3)$$

where $g(z)$ is the gain increment of the waveguide material. The second term on the right-hand side of (3) determines the contribution related to the possible change in the effective area of the mode

$$S_m(z) = 2\pi\int_0^\infty |U(r,z)|^2 rdr, \quad (4)$$

where $U(r,z)$ is the profile mode function of the optical waveguide under consideration.

The material parameters of an optical waveguide are related to the effective transverse sizes of the mode propagating in it. For example, the material gain increment for a radially symmetric waveguide is given by the expression

$$g(z) = 2\pi k_0 S_m^{-1}(z)\int_0^\infty n''(r,z)|U(r,z)|^2 rdr, \quad (5)$$

where $k_0 = \omega_0/c$, c is the speed of light in vacuum, and n'' is the imaginary part of the refractive index of the waveguide material. Note that the material gain increment is related to the energy of a pulse propagating through the optical waveguide:

$$W(z) = W_0\exp\left(2\int_0^z g(\xi)d\xi\right), \quad (6)$$

where W_0 is the energy of a pulse introduced into the waveguide.

In the general case, the nonlinearity parameter is given by the expression

$$R(z) = 2\pi k_0 S_m^{-2}(z)\int_0^\infty \tilde{n}(r,z)|U(r,z)|^4 rdr, \quad (7)$$

where $\tilde{n}$ is the nonlinear refractive index of the optical waveguide material. If the linear and nonlinear refractive indices n' and $\tilde{n}$ and profile function U are independent of coordinate z, parameters g and R are also independent of this variable and correspond to a longitudinally homogeneous optical waveguide.

AMPLIFICATION OF FM SOLITON-LIKE PULSES

Particular Case of Inhomogeneity

It is known [3, 9] that soliton-type solutions to Eq. (2) can be obtained for hyperbolic secant pulses propagating in a optical waveguide with a longitudinally constant anomalous GVD and nonlinearity and a gain increment of the waveguide material varying along the waveguide length according to the law

$$g(z) = g_0/(1 - 2g_0 z), \quad (8)$$

In particular, at $DR < 0$, one has the following solution in the form of a wave packet, amplified at $2g_0 z < 1$:

$$A(\tau,z) = \frac{A_0}{1 - 2g_0 z}\operatorname{sec}h\left(\frac{\tau}{\tau_s}\right)\exp\left(i\frac{\alpha_0\tau^2 - \Gamma z}{1 - 2g_0 z}\right), \quad (9)$$

where $\tau_s = \tau_0(1 - 2g_0 z)$ is the wave packet width; and parameter $\Gamma = g_0/2\alpha_0\tau_0^2$. The parameters entering (9) are assumed to satisfy the relations $2\Gamma = |D|/\tau_0^2 = R|A_0|^2$. Nonlinear wave packets of type (9), which are referred to as bright FM solitons, have a property of elastic interaction, which is important for practical applications [1–3].

General Case

Now let the dispersion and nonlinearity parameters be functions of coordinate z. For the convenience of the further analysis, we present them in the form $D(z) = D_0 d(z)$ and $R(z) = R_0 r(z)$, where D_0 and R_0 are considered to be the values of the corresponding parameters at the optical waveguide input. Then we introduce variable $\eta(z) = \int_0^z d(\xi)d\xi$ and pulse envelope

$C(\tau, z) = \sqrt{r(z)/d(z)}A(\tau, z)$. As a result of these transformations, we pass from Eq. (2) to the equation

$$\frac{\partial C}{\partial \eta} - i\frac{D_0}{2}\frac{\partial^2 C}{\partial \tau^2} + iR_0|C|^2 C = \gamma_{\mathrm{ef}}(\eta)C. \quad (10)$$

Thus, the problem of nonlinear pulse propagation through an optical waveguide with longitudinally inhomogeneous material parameters is reduced to the problem of pulse propagation through an optical waveguide with homogeneous dispersion D_0 and nonlinearity R_0 but an inhomogeneous effective gain $\gamma_{\mathrm{ef}}(\eta)$:

$$\gamma_{\mathrm{ef}}(\eta) = \frac{g(\eta)}{d(\eta)} - \frac{1}{2}\frac{\partial}{\partial \eta}\ln\frac{\tilde{S}_m(\eta)d(\eta)}{r(\eta)}, \quad (11)$$

where $\tilde{S}_m = S_m(\eta)/S_m(0)$ is the normalized effective mode area.

As well as Eq. (2), Eq. (10) has a solution in the form of an amplified FM soliton if $D(\eta)R(\eta) < 0$ and the effective gain increment (11) can be described by the dependence $\gamma_{\mathrm{ef}}(\eta) = q/(1-2q\eta)$, where parameter $q = \gamma_{\mathrm{ef}}(0)$. In this case, the solution to Eq. (10) is given by

$$C(\tau,\eta) = \frac{C_0}{1-2q\eta}\operatorname{sech}\frac{\tau}{\tau_s}\exp\left(i\frac{\alpha_0\tau^2 - \Gamma_0\eta}{1-2q\eta}\right), \quad (12)$$

and the parameters entering this equation must satisfy the relations $2\Gamma_0 = |D_0|/\tau_0^2 = R_0|A_0|^2$ and $q = \alpha_0|D_0|$. The formation energy of a soliton-like wave packet can be written as $W_s = \tau_0|A_0|^2 = |D_0|/R_0\tau_0$.

The condition for the existence of FM soliton (9) in an inhomogeneous optical waveguide with an anomalous GVD can be presented as

$$\gamma_{\mathrm{ef}}(z) = -\frac{\alpha_0 D_0}{1 + 2\alpha_0 D_0 \eta} = \frac{q}{1-2q\eta}. \quad (13)$$

With allowance for (11), relation (13) for the effective amplification takes the form

$$\left(1 + 2\alpha_0\int_0^z D(\xi)d\xi\right)\exp\left(2\int_0^z g(\xi)d\xi\right) = \frac{D(z)}{D_0}\frac{S_m(z)}{S_m(0)}\frac{R_0}{R(z)}. \quad (14)$$

The GVD profile necessary to form an FM soliton-like pulse is determined by the relation

$$D(z) = D_0 f(z)\exp\left(-2q\int_0^z f(\xi)d\xi\right), \quad (15)$$

with the following parameters introduced: $f(z) = F(z)\exp\left(2\int_0^z g(\xi)d\xi\right)$ and $F(z) = R(z)S_m(0)/R_0S_m(z)$.

In the simplest case, $g(z) = 0$ and $F(z) = 1$, which is valid for passive W-type optical waveguides, the dispersion profile must satisfy the condition

$$D(z) = D_0\exp(-2qz),$$

where $D_0 < 0$ and $\alpha_0 > 0$. In this case, the exact solution for the duration of a hyperbolic secant soliton-like pulse in an anomalous-dispersion medium has the form

$$\tau_s(z) = \frac{\tau_0}{F(z)}\frac{D(z)}{D_0}\exp\left(-2\int_0^z g(\xi)d\xi\right) = \tau_0\exp\left(-2q\int_0^z f(\xi)d\xi\right). \quad (16)$$

In addition, one can assume that the relation $\tau_s(z)\alpha(z) = \mathrm{const} = \tau_{s0}\alpha_0$ holds true for FM hyperbolic secant solitons. Proceeding from this relation, one can easily derive the following expression for the pulse chirp:

$$\alpha(z) = \alpha_0\exp\left(2q\int_0^z f(\xi)d\xi\right). \quad (17)$$

With allowance for relation (6), the desired dispersion profile and pulse duration can be written as

$$D(z) = D_0 F(z)\frac{W(z)}{W_0}\exp\left(-\frac{2q}{W_0}\int_0^z F(\xi)W(\xi)d\xi\right),$$
$$\tau(z) = \tau(0)\exp\left(-\frac{2q}{W_0}\int_0^z F(\xi)W(\xi)d\xi\right). \quad (18)$$

In the general case, when describing the dynamics of a subpicosecond pulse in an inhomogeneous optical waveguide, one must also take into account the third-order dispersion, which significantly affects the pulse shape and may lead to pulse decay at the aforementioned pulse durations. Even when parameter D_3 (which characterizes the influence of the third-order dispersion effects) is small at the input of the waveguide, its influence becomes significant at some distance from the input of the waveguide. Therefore, to implement FM soliton amplification, the condition $|D_3(z)| < |D(z)|/\Delta\omega(z)$ ($\Delta\omega(z)$ is the wave packet spectral width) should be satisfied throughout the entire waveguide length. A decrease in the pulse duration (i.e., increase in its spectral width) and a decrease in the GVD's absolute value along the waveguide's length may violate the conditions for the amplification of the FM soliton.

The aforementioned condition can be satisfied for optical waveguides with a transverse W-profile refractive index [11, 12]. The necessary longitudinal GVD distribution can be implemented in these waveguides at ultralow values of the third-order dispersion parameter. The effective mode area and nonlinearity coefficient in them can be considered practically constant along the entire waveguide length; hence, one can assume (with a rather high accuracy) function $F(z)$, introduced when deriving expression (15), to be unity: $F(z) = 1$. As an example, we will consider the case of the longitudinally constant material gain increment. If $g(z) = g_0$, $f(z) = \exp(2g_0z)$, and the expressions for the GVD necessary for FM pulse amplification and the pulse duration take the form

$$D(z) = -|D_0|\exp\left[-\frac{\alpha_0|D_0|}{g_0}(\exp(2g_0z)-1)+2g_0z\right], \quad (19)$$

$$\tau(z) = \tau_0\exp\left[-\frac{\alpha_0|D_0|}{g_0}(\exp(2g_0z)-1)\right]. \quad (20)$$

Note that the technology of the optical waveguides with a GVD profile of type (19) appears to be easily implemented. In particular, the fabrication of inhomogeneous optical waveguides with an exponential dispersion profile was discussed even in [13, 14].

FM SOLITONS IN A PASSIVE OPTICAL WAVEGUIDE

Note that the amplification of a subpicosecond FM soliton meets a number of technical difficulties. Primarily, this is related to the influence of MI and stimulated Raman scattering, whose development may destroy a stable wave packet. Therefore, it is expedient to use the light-waveguiding systems under consideration not to amplify the chirped pulses but to strongly modulate and compress them to subpicosecond durations. To form a strong frequency modulation of a soliton-like pulse, it is reasonable to use a passive optical waveguide with the minimal possible loss, for which one can assume highly accurately that $g(z) = 0$ over the entire waveguide length. In real passive optical waveguides with a W-radial distribution of the refractive index, the optical loss is less than 0.5 dB/km. If a waveguide shorter than 1 km is used for wave packet modulation and time compression, one can assume with a high degree of accuracy that $g(z) = 0$. In this case, expressions (21)–(23), with allowance for the equality $F(z) = 1$ (characteristic of the W-profile optical waveguides), are significantly simplified:

$$D(z) = D_0\exp(-2qz), \quad \tau_s = \tau_0\exp(-2qz), \quad \alpha = \alpha_0\exp(2qz). \quad (21)$$

Here, $q = \alpha_0|D_0|$ can be considered an inhomogeneity parameter. It follows from (21) that the existence of anomalous GVD and a positive input chirp lead to the temporal compression of a pulse and additional frequency modulation. For example, at input values $D_0 = -10^{-26}$ s^2/m and $\alpha_0 = 10^{24}$ s^{-2}, the pulse duration can be reduced from the input value $\tau_0 = 10^{-12}$ s to $\tau_s(L) \approx 10^{-13}$ s at the output for a waveguide length of $L \approx 100$ m. At this length, the GVD value, in correspondence with (21), may exponentially change practically by an order of magnitude.

The above-considered optical waveguides can be used to generate pulses with a strong frequency modulation and an almost linear change in the instantaneous frequency. Under these conditions, an FM pulse obtained on the output from an inhomogeneous optical waveguide can be additionally compressed using a medium with normal effective dispersion. This procedure can be implemented both on a pair of diffraction gratings and in a photonic-crystal waveguide with low Kerr nonlinearity. In the latter case, the system can be made an all-fiber one.

DEVELOPMENT OF MI AND GENERATION OF FM SOLITONS

The effects considered above are closely related to the characteristic property of nonlinear dispersive MI systems, which can generally be defined as the amplification of some spectral components at the expense of the other components, which finally leads to wave packet deformation. In a homogeneous optical waveguide (the anomalous GVD of which lies in the range of $-4RP_0/\Omega^2 < D < 0$) at the initial development stage of MI (while the wave packet is propagated), the harmonic perturbation exponentially increases with an increment [1–5]

$$g = |\Omega|\sqrt{4RP_0|D| - D^2\Omega^2}, \quad (22)$$

where $P_0 = |A_0|^2$ is the radiation power introduced into the optical waveguide, $\Omega = \omega_0 - \omega_v$ is the perturbation frequency, and ω_v is the frequency of the signal perturbing wave, or spontaneous noise perturbation. It follows from (22) that a spontaneous modulation of the stationary state occurs in the detuning range $|\Omega| < \Omega_c = \sqrt{4RP_0/|D|}$.

In the case of an inhomogeneous dispersion profile along the waveguide length, the gain increment is also a function of the longitudinal coordinate $g(z)$. The perturbation gain per waveguide length L is given by the expression

$$G(\Omega) = \int_0^L g(z,\Omega)dz. \quad (23)$$

Figure 1 shows the dependences of the integral increment of gain G of a weak harmonic perturbation at the detuning frequency Ω in the initial development stage of MI, which were obtained for an exponentially decreasing (in modulus) anomalous dispersion (24); the optical waveguide length $L = 300$ m; and the following values of the input GVD and the parameter characteriz-

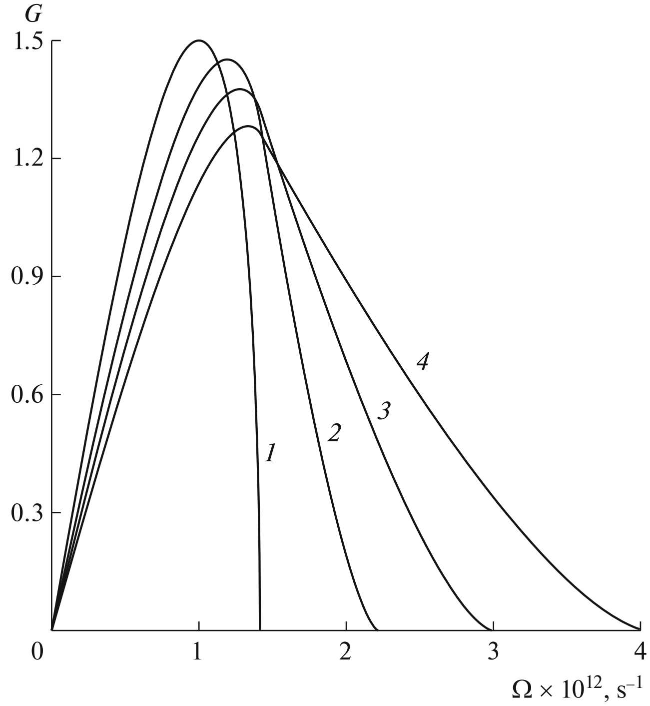

Fig. 1. Dependences of the integral increment of the gain on the detuning frequency at an optical waveguide length $L = 300$ m and inhomogeneity parameter q = (*1*) 0, (*2*) 3×10^{-3}, (*3*) 5×10^{-3}, and (*4*) 7×10^{-3} m^{-1}.

ing the change in dispersion: $D_0 = -10^{-26}$ s^2/m and $q = (0, 3, 5, 7)$ m^{-1} (curves *1*–*4*). One can see that, at a fixed waveguide length, the MI frequency range expands and the maximum increment decreases with an increase in parameter q. The frequency at which the gain is maximum is

$$\Omega_m = \sqrt{\frac{2RP_0}{|D_0|}\frac{\exp(qL)-1}{\sinh(qL)}}. \qquad (24)$$

An analysis of the growth of the small harmonic perturbations of a continuous wave in an optical waveguide with exponentially decreasing (in modulus) anomalous dispersion shows that the bandwidth of unstable frequencies $\omega_m = \omega_0 + \Omega_c$ and the maximum-gain frequency $\omega_m = \omega_0 + \Omega_m$ exponentially increase along the waveguide length [15–18].

Significant differences occur in the phase of a developed MI, which leads to a continuous-wave being decomposed into a sequence of soliton-like pulses at a linear frequency modulation rate. The spectrum of a continuous pump wave with two initial harmonics ω_0 and ω_v, where ω_0 is the carrier frequency of a quasi-continuous wave packet and ω_v is the perturbing signal frequency, evolves into a temporal sequence of pulses. Below we report the results of analyzing the development of MI based on the numerical solution of Eq. (2), which determines the dynamics of a wave packet in an inhomogeneous optical waveguide.

Let a weakly modulated wave packet introduced into an optical waveguide be set by the relation

$$A(0,\tau) = \sqrt{P_0}\left[1 + m\cos(\Omega_{\mathrm{mod}}\tau)\right], \qquad (25)$$

where m is the modulation depth. Figure 2 shows the results of the numerical solution of Eq. (2) using the split-step Fourier method (SSFM) [3], with the following values of parameters in use: $D_0 = -10^{-26}$ s^2/m, $P_0 = 1$ W, and $q = (0, 3, 5, 7) \times 10^{-3}$ m^{-1} (a, b, c, d). The modulation depth was taken to be $m = 0.01$, and the wave packet modulation frequency was chosen to be equal to the frequency at which the gain of a weak harmonic perturbation is maximum ($\Omega_{\mathrm{mod}} = \Omega_m$). These dependences demonstrate that the initial phase of the development of MI, in which the growing perturbation can still be considered harmonic, covers a rather large part of the optical waveguide length. Note that autonomous ultrashort pulses with a large (relative to the initial value) peak amplitude are formed on the length of a significantly inhomogeneous optical waveguide, which can roughly be estimated as $z_{cr} \geq 1/2q$.

In the case of a homogeneous optical waveguide in the phase of a developed MI, the compression of pulses cyclically alternates with their broadening. This process is periodic, because the newly formed pulses collapse again into the initial state (a continuous modulated wave) (a). In an inhomogeneous optical waveguide, because of the decrease in the dispersion modulus along the waveguide length, the processes of the generation of a pulse sequence and its decay are not reversible (b, c). Because of the constant spectral broadening, the generated sequence of pulses cannot return to the state of a modulated continuous wave. As a result, the pulse duration constantly decreases (with some oscillations). The amplitude and period of these oscillations also decrease. An increase in the inhomogeneity parameter leads to a more significant temporal pulse compression. However, at a large value of the inhomogeneity parameter, the dispersion decreases so rapidly (d) that the modulated wave fails to decompose into a sequence of ultrashort pulses.

Figure 3 shows the dependences of the energy maximum of the modulated wave temporal profile, $P_{\max} = |A|^2_{\max}$, on the optical waveguide length, obtained at $D_0 = -10^{-26}$ s^2/m, $R = 10^{-2}$ W^{-1} m^{-1}, $P_0 = 1$ W, and $q = (0, 3, 5, 7) \times 10^{-3}$ m^{-1} (curves *1*–*4*). One can see an oscillating increase in the peak power of the generated soliton-like pulses along the waveguide length, which depends on the rate of GVD variation in the waveguide, and the formation of an almost linear chirp for each individual pulse.

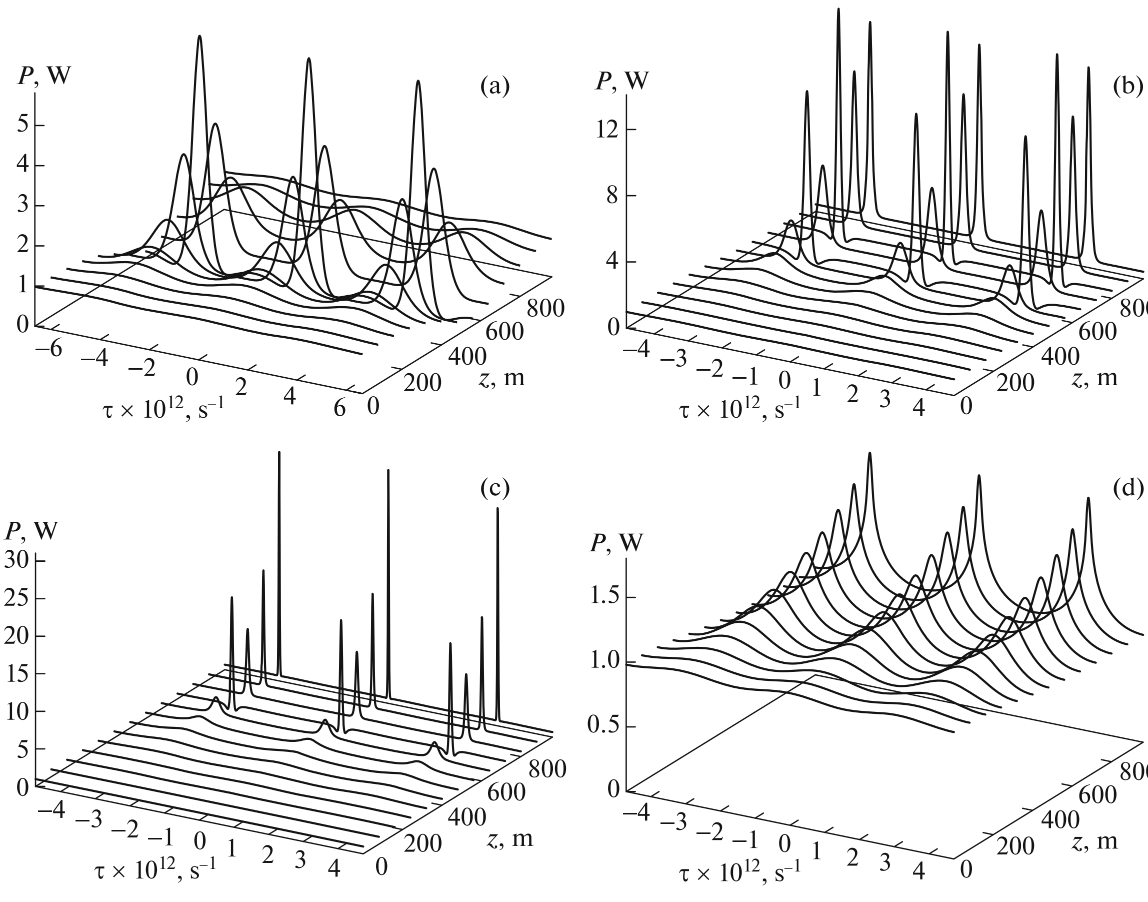

Fig. 2. Temporal profiles of a modulated wave in an inhomogeneous optical waveguide; $D_0 = 10^{-26}$ s^2/m; $R = 10^{-2}$ W^{-1}m^{-1}; $P_0 = 1$ W; and q = (a) 0, (b) 3×10^{-3}, (c) 5×10^{-3}, and (d) 7×10^{-3} m^{-1}.

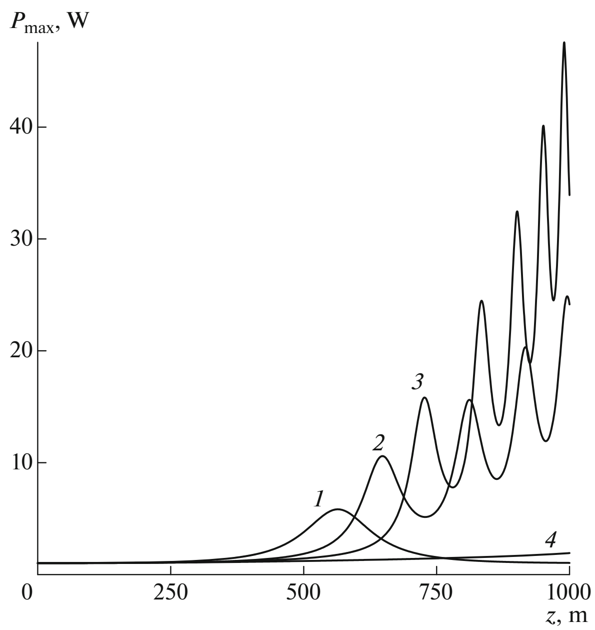

Fig. 3. Dependences of the energy maximum of the modulated-wave temporal profile on the optical waveguide length at q = (*1*) 0, (*2*) 3×10^{-3}, (*3*) 5×10^{-3}, and (*4*) 7×10^{-3} m^{-1}.

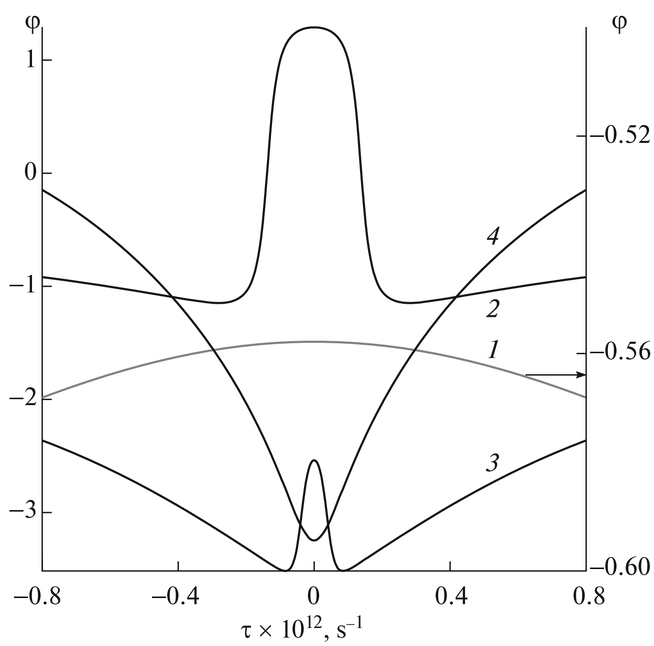

Fig. 4. Temporal profiles of a modulated-wave phase at q = (*1*) 0, (*2*) 3×10^{-3}, (*3*) 5×10^{-3}, and (*4*) 7×10^{-3} m^{-1}.

Figure 4 presents the time dependence of the pulse phase for different rates of GVD variation in an optical waveguide with a length $L = 1000$ m (the other parameters are the same as in the previous figures). It can be seen that the pulses generated in an inhomogeneous optical waveguide acquire quadratic positive phase modulation (with maximal parabolicity at the center of the generated pulse), which finally provides a sufficiently stable compression of the generated FM solitons along the waveguide length. The oscillations of the duration occurring in this case are caused by the nonideal parabolicity of the generated-pulse phase.

CONCLUSIONS

Our analysis showed that the dynamics of MI evolution in a longitudinally inhomogeneous optical waveguide with anomalous (exponentially decreasing in modulus) GVD depends on the rate of variation in dispersion along the waveguide length. With a properly chosen waveguide length, one can use the effect of the induced MI to generate THz sequences of pico- and subpicosecond optical pulses, whose repetition rate can be controlled. As follows from the presented dependences, with quite realistic values of all parameters, sequences of pico- and subpicosecond pulses with a THz repetition rate are formed in inhomogeneous optical waveguides with an exponentially varying dispersion. One can also conclude that the pulses evolving by means of only nonlinear effects and that are non-chirped in the initial state (or obtained in the phase of a developed MI), can be compressed to durations of up to 100 fs by correctly choosing the parameters of the optical waveguide with the dispersion decreasing in modulus. These compressed pulses exhibit a strong (close to linear) frequency modulation. In this case, one can speak about the formation of the so-called FM solitons.

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REFERENCES

1. N. N. Akhmediev and A. Ankevich, *Solitons: Non-Linear Pulses and Beams* (Fizmatlit, Moscow, 2003; Chapman Hall, London, 1997), rus. p. 304.
2. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic, San Diego, CA, 2003; Fizmatgiz, Moscow, 2005), p. 648.
3. S. A. Akhmanov, V. A. Vysloukh, and A. S. Chirkin, *The Optics of Femtosecond Pulses* (Nauka, Moscow, 1988; Springer, Berlin, 1992).
4. I. O. Zolotovskii, V. A. Lapin, and D. I. Sementsov, Opt. Spectrosc. **121**, 95 (2016).
5. I. O. Zolotovskii, D. A. Korobko, V. A. Lapin, and D. I. Sementsov, Opt. Spectrosc. **121**, 256 (2016).
6. V. N. Serkin and T. L. Belyaeva, JETP Lett. **74**, 573 (2001).
7. V. N. Serkin and T. L. Belyaeva, Quantum Electron. **31**, 1007 (2001).
8. D. A. Korobko, O. G. Okhotnikov, and I. O. Zolotovskii, J. Opt. Soc. Am. B **30**, 2377 (2013).
9. I. O. Zolotovskii, D. A. Korobko, O. G. Okhotnikov, A. A. Sysolyatin, and A. A. Fotiadi, Quantum Electron. **42**, 828 (2012).
10. U. G. Akhmetshin, V. A. Bogatyrev, A. K. Senatorov, A. A. Sysolyatin, and M. G. Shalygin, Quantum Electron. **33**, 265 (2003).
11. A. A. Sysolyatin and D. A. Nolan, J. Nonlin. Opt. Phys. Mat. **16** (4), 171 (2007).
12. A. Yu. Plotski, A. A. Sysoliatin, A. I. Latkin, V. F. Khopin, P. Harper, J. Harrison, and S. K. Turitsyn, JETP Lett. **85**, 319 (2007).
13. V. Chernikov and P. V. Mamyshev, J. Opt. Soc. Am. B **8**, 1633 (1991).
14. V. N. Serkin, A. Hasegawa, and T. L. Belyaeva, Phys. Rev. Lett. **92**, 199401 (2004).
15. I. O. Zolotovskii and D. I. Sementsov, Opt. Spectrosc. **102**, 594 (2007).
16. Li Qian, K. Senthilnathan, K. Nakkeeran, and P. K. A. Wai, J. Opt. Soc. Am. B **26**, 432 (2009).
17. A. V. Zhukov, I. O. Zolotovskii, O. G. Okhotnikov, D. I. Sementsov, A. A. Sysolyatin, and I. O. Yavtushenko, Opt. Spectrosc. **113**, 75 (2012).
18. E. M. Dianov, I. A. Bufetov, V. M. Mashinsky, A. V. Shubin, O. I. Medvedkov, A. E. Rakitin, M. A. Mel'kumov, V. F. Khopin, and A. N. Gur'yanov, Quantum Electron. **35**, 435 (2005).

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