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LASERS AND THEIR  
APPLICATIONS

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## The Influence of Collective Effects on Quantum Fluctuations of Laser Radiation

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**Abstract**—Quantum fluctuations in a laser with two different relaxation times are considered, i.e., transverse (polarization relaxation) and longitudinal (population relaxation) in the case in which the cavity transmission band half-width is much smaller than the transverse width and much larger than the longitudinal one. The lasing frequency detuning from the transition frequency of a two-level system is assumed to be arbitrary in this case, and it is necessary to take into account the contribution of two-particle correlators into the dispersion and laser linewidth. The results are considered as applied to a semiconductor laser.

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### INTRODUCTION

The contribution of quantum fluctuations of the electromagnetic field amplitude and phase and the medium into the semiconductor laser linewidth has been found in [1]. It has been shown that this contribution increases the laser linewidth by about an order of magnitude compared to the contribution of spontaneous radiation. The following simplifications were made during the problem solution. All calculations were carried out at the maximum of laser gain line; corrections quadratic in the density matrix elements were omitted during calculation of the photon diffusion coefficients [2], since their contribution is insignificant. Another simplification was connected with ignorance of two-particle correlators, though they should be taken into account if the cavity transmission linewidth is larger than the atom linewidth. The case in which  $\sigma/\Gamma \ll 1$ ,  $\sigma\tau_0 \gg 1$  ( $\sigma$  is the cavity transmission line half-width,  $\sigma \approx 10^{12} - 10^{13} \text{ s}^{-1}$ ;  $\Gamma$  is the inverse time of relaxation in semiconductor bands,  $\Gamma \approx 10^{13} \text{ s}^{-1}$ ; and  $\tau_0$  is the interband relaxation time,  $\tau_0 \approx 10^{-9} - 10^{-10} \text{ s}$ ) is implemented in a semiconductor. The effect of correlations for ordinary two-level systems was considered in [2] ( $\sigma\tau_0 \gg 1$ ,  $\Gamma^{-1} = \tau_0$ ) and in [3] ( $\sigma/\Gamma \ll 1$ ,  $\sigma\tau_0 \gg 1$ ). The lasing frequency detuning from the transition frequency was considered to be zero in works [2, 3]. The contribution of collective effects into the laser radiation amplitude fluctuations was found in [3]. Appearance of an additive term proportional to  $\sigma\tau_0$  in the photon distribution function was proved. The question of linewidth was not consid-

ered. In work [4], it has been mentioned but not proven that allowance for correlation effects does not contribute to the laser linewidth at  $\sigma/\Gamma \ll 1$ ,  $\sigma\tau_0 \gg 1$ .

Therefore, there is a need in solution of the problem about the effect of correlators on the amplitude and phase fluctuations in a laser if  $\sigma/\Gamma \ll 1$ ,  $\sigma\tau_0 \gg 1$  for ordinary two-level systems with allowance for arbitrary detuning of the lasing frequency from the transition frequency. Transition frequency  $\omega_{21}$  is fixed in this case; hence, the set of equations for corresponding density matrices can be reduced to algebraic.

The results are discussed as applied to a semiconductor laser. In this case, interband transition frequency  $\omega_l = (E_{2l} - E_{1l})/\hbar$ , where  $E_{2l}$  is the electron energy in the conduction ( $2l$ ) and valence ( $1l$ ) bands with wavenumber  $l$ . Summation of the corresponding parameters over  $l$  is reduced to integration with the density of the number of transitions with the frequency  $\omega_l$  [1]. Thus, the simple algebraic set of equations for calculating correlators [2–4] turns into a set of integral equations, and the calculation of corrections is difficult due to allowance for correlators. However, using the exact solution for ordinary two-level systems, one can ascertain how the correlators affect the semiconductor laser linewidth.

### BASIC EQUATIONS

Let us consider a model of a single-mode laser. It is represented by a quantum oscillator with the frequency  $\omega$ , which interacts with a system of  $N$  fixed two-level atoms. The quantum laser theory was devel-

oped in [2–5]. We follow the method used in [2] (see also [5]). According to [2, 5], there is a set of equations of combined density matrices of the oscillator and atoms:

$$i \frac{\partial D(z, t)}{\partial t} = \nabla v_- D(z, t) - \bar{\nabla} v_+ D(z, t), \quad (1)$$

$$v_- = gP_- + i\sigma z + \tilde{\delta} z, v_+ = \bar{v}_-,$$

$$\begin{aligned} \dot{r}_{21k} + i(v_- \nabla - \text{c.c.})r_{21k} + \tau_2^{-1}(1 - i\varepsilon)r_{21k} = & -igzr_{3k} \\ + igr_{22k} \frac{\bar{\nabla} D}{D} - ig \sum_{i \neq k} \left( \delta r_{21i, 21k} \frac{\nabla D}{D} - \delta r_{12i, 21k} \frac{\bar{\nabla} D}{D} \right), \\ \dot{r}_{3k} + i(v_- \nabla - \text{c.c.})r_{21k} & \\ + \tau_0^{-1} r_{3k} = \tau_0^{-1} q_k - 2ig(\bar{z}r_{21k} - \text{c.c.}) & \\ + ig \left( r_{21k} \frac{\nabla D}{D} - \text{c.c.} \right) - ig \sum_i \left( \delta r_{21i, 3k} \frac{\nabla D}{D} - \text{c.c.} \right). & \end{aligned} \quad (2)$$

The bar indicates complex conjugation,  $P_- = \sum_k r_{21k}$ ,  $P_+ = \sum_k r_{12k}$ ,  $\tilde{\delta} = \omega - \omega_n$ ,  $\varepsilon = (\omega - \omega_{21})\tau_2$ ,  $\omega$  is the lasing frequency,  $\omega_n$  is the cavity frequency,  $\omega_{21}$  is the atom transition frequency, and  $r_{3k} = r_{22k} - r_{11k}$ ,  $q_k$  is the overpopulation in the absence of field.

Set of equations (1) and (2) is written in the coherent state representation. In this representation, photon annihilation operator  $a$  is diagonal,  $a|z\rangle = z|z\rangle$ ,  $D(z, t)$  is the quantum oscillator density matrix in the Glauber representation, and  $r_{\alpha\beta k}$  is the density matrix of the  $k$ th atom ( $\alpha, \beta = 1, 2$ ) for states  $\alpha$  and  $\beta$ . Actually,  $r_{\alpha\beta k}$  is the combined density matrix for the corresponding atom transition and quantum oscillator, since it depends on  $D$ ;  $\delta r_{\alpha\beta i, \gamma\delta k}$  is the irreducible part of the combined two-particle density matrix of the  $i$ th and  $k$ th atoms and quantum oscillator. Again,  $2\sigma$  is the inverse time of photon relaxation in the cavity,  $g$  is the coupling constant:  $g = d\mathcal{E}/\hbar$ ,  $\mathcal{E} = \sqrt{4\pi\hbar\omega/V}$  is the photon electric field strength,  $d$  is the dipole matrix element of a resonance transition,  $V$  is the cavity field volume,  $\tau_0$  is the longitudinal time of overpopulation relaxation,  $\tau_2$  is the transverse time which corresponds to the relaxation of an off-diagonal element of the atom density matrix. Set (2) includes only terms proportional to  $\nabla D/D$  and  $\bar{\nabla} D/D$ ; terms neglected do not affect the photon diffusion coefficients. A set of equations for  $\delta r_{\alpha\beta i, \gamma\delta k}$  has been derived in [2] and is given below. In the case  $\sigma\tau_{\alpha\beta} \ll 1$  ( $\alpha, \beta = 1, 2$ ,  $\tau_{21} = \tau_2$ ,  $\tau_{11} = \tau_{22} = \tau_0$ ), it is sufficient to find a correlator in the photon and atom distribution  $r_{\alpha\beta k}$ ; if the condition is violated, correlators in distributions of different atoms are also to be found. In this case, the relaxation in the atom system runs more slowly than in the oscillator and the equation for the (radial) photon distribution function is not reduced to the diffusion equation. If

consider this function to be already relaxed to the steady state, then the diffusion equation can be derived for the phase, since the mean field attenuation can be considered as a slow quasi-stationary process, with the characteristic time much longer than  $\tau_{\alpha\beta}$ . The problem has been solved in [2] for the case  $\sigma\tau_{\alpha\beta} \gg 1$ ,  $\tau_{\alpha\beta} = \tau_0$ . In this work, we consider an intermediate case in which  $\sigma\tau_2 \ll 1$ ,  $\sigma\tau_0 \gg 1$ . This problem was considered in [3] (see also [4]) for the case of zero detuning, where the contribution of two-atom correlators into the laser linewidth is exactly zero, and the contribution into the photon distribution function variance linearly depends on  $\sigma\tau_0$ . Below, we considered the detuning nonzero and assume that all atoms are in equal conditions in the upper state in the absence of radiation; therefore, summation over atoms is reduced to multiplication by  $N$  ( $N$  is the number of atoms in the system). One can prove [3] that  $\delta P_- = \sum_{k \neq l} \delta r_{21k, 21l}$  and  $\delta P_{+-} = \sum_{k \neq l} \delta r_{12k, 21l}$  are negligible if  $\tau_2 \ll \tau_0$ , and the corresponding set of correlator equations includes only equations for

$$\delta P_{-3} = \sum_{k \neq l} \delta r_{21k, 3l}, \quad \delta P_{33} = \sum_{k \neq l} \delta r_{3k, 3l},$$

where

$$\begin{aligned} \sum_{k \neq l} \delta r_{21k, 3l} &= \sum_{k \neq l} (\delta r_{21k, 22l} - \delta r_{21k, 11l}), \\ \sum_{k \neq l} \delta r_{3k, 3l} &= \sum_{k \neq l} (\delta r_{22k, 22l} - 2\delta r_{22k, 11l} + \delta r_{11k, 11l}). \end{aligned}$$

In the steady state, density matrix  $D(z, t)$  depends on  $\xi = |z|^2$ , i.e.,  $D = D_0(\xi)$  (see expansion (13) below and [1]). Under this condition,  $v_{\pm} = 0$  or  $gP_{\pm} = -i\sigma z - \tilde{\delta} z$  follow from Eq. (1), and the set of equations for the macroscopic parameters  $P_-$ ,  $P_3 = \sum_k (r_{22k} - r_{11k})$  can be written in the form

$$P_3 = N - 2ig\tau_0 z (\bar{z}P_- - \text{c.c.}) + ig\tau_0 \left( P_- \frac{\nabla D}{D} - \text{c.c.} \right) - ig\tau_0 \left( \delta P_3 - \frac{\nabla D}{D} - \text{c.c.} \right), \quad (3)$$

$$\begin{aligned} P_- &= -\frac{ig\tau_2 z}{1 - i\varepsilon} P_3 + \frac{ig\tau_2}{1 - i\varepsilon} P_2 \frac{\bar{\nabla} D}{D} \\ &- i \frac{g\tau_2}{1 - i\varepsilon} \left( \delta P_- - \frac{\nabla D}{D} - \text{c.c.} \right), \end{aligned} \quad (4)$$

where

$$\nabla D_0/D_0 = \bar{z} \frac{d \ln D_0}{d\xi}, \quad \bar{\nabla} D_0/D_0 = z \frac{d \ln D_0}{d\xi}$$

in the steady state. Taking into account the above said, the set of equations for two-particle correlators can be reduced to the following:

$$\begin{aligned} (1 - i\varepsilon)\delta P_{3-} + ig\tau_{2z}\delta P_{33} &= ig\tau_{2z}P_2P_3', \\ \frac{4g'\tau_0}{1+i\varepsilon}\delta P_{3-} + \delta P_{33} &= ig\tau_0(\bar{z}P_- - \text{c.c.})P_3', \\ P_3' &= \frac{dP_3}{d\xi}, \quad g' = g\left(1 + \frac{1}{2}P_3'\right). \end{aligned} \quad (5)$$

Resolving set (5) for  $\delta P_{3-}$ , we derive

$$\delta P_{3-} = \frac{1}{\det}(ig\tau_{2z})P_3'[P_2 - ig\tau_0(\bar{z}P_- - \text{c.c.})],$$

where the determinant of set (5)

$$\det = (1 - i\varepsilon)\left(1 + \frac{4gg'\tau_0\tau_2|z|^2}{1 + \varepsilon^2}\right).$$

Substituting the equality  $gP_- = -i\sigma z - \tilde{\delta}z$  in Eq. (4) and distinguishing the real and imaginary parts, two equations can be derived:

$$\frac{g^2\tau_2}{1 + \varepsilon^2}P_3 - \frac{g^2\tau_2}{1 + \varepsilon^2}P_2 \frac{d \ln D_0}{d\xi} = \sigma, \quad (6)$$

$$-\frac{g^2\tau_2}{1 + \varepsilon^2}\varepsilon P_3 + \frac{g^2\tau_2}{1 + \varepsilon^2}\varepsilon \frac{d \ln D_0}{d\xi} = \tilde{\delta}. \quad (7)$$

Comparison of Eqs. (6) and (7) provides the equation for lasing frequency  $\omega$ :

$$-\varepsilon\sigma = \tilde{\delta} \quad \text{or} \quad \omega = \frac{\sigma\omega_{21} + \omega_n/\tau_2}{\tau_2^{-1} + \sigma}.$$

If  $\sigma\tau_2 \ll 1$ , then  $\omega = \omega_n$ ; i.e.,  $\tilde{\delta} \equiv 0$ .

Let us introduce the designations  $P_3 = q/N$ ,  $\eta = \frac{g^2\tau_2N}{(1 + \varepsilon^2)\sigma}$ , where  $\eta$  is the lasing parameter, and assume that  $gP_- = -i\sigma z$ ; then set (3) and (4) can be rewritten as

$$\eta q = 1 + A \frac{d \ln D_0}{d\xi}, \quad (8)$$

$$\eta q = \eta - \frac{4g^2\tau_0\tau_2}{1 + \varepsilon^2}\xi + B \frac{d \ln D_0}{d\xi}, \quad (9)$$

where

$$\begin{aligned} P_2 &= N \frac{1+q}{2}, \quad A = \frac{\eta+1}{2}, \\ B &= \frac{2g^2\tau_0\tau_2}{1 + \varepsilon^2}\xi - \frac{ig\tau_0\eta}{N}(\bar{z}\delta P_{3-} - \text{c.c.}). \end{aligned}$$

Solving Eqs. (8) and (9) in the principal order [2] above the threshold, we have

$$q_0 = \frac{1}{\eta}, \quad \xi_0 = \langle n \rangle = \frac{\eta-1}{4g^2\tau_0\tau_2}(1 + \varepsilon^2).$$

In this region, the photon distribution function is Gaussian

$$D_0(\xi) \sim \exp\left(-\frac{(\xi - \xi_0)^2}{2d\xi_0}\right),$$

and

$$\frac{dD_0}{D_0 d\xi} = -\frac{\delta\xi}{d}, \quad \delta\xi = \frac{\xi - \xi_0}{\xi_0}. \quad (10)$$

Since  $\delta\xi$  is small, then, suggesting

$$q = q_0 + q_1\delta\xi, \quad \xi = \xi_0(1 + \delta\xi), \quad (11)$$

the set of algebraic equations

$$\eta q_1 = -\frac{A}{d}, \quad (12)$$

$$-\eta q_1 = \eta - 1 + (B/d)$$

can be derived instead of Eqs. (8) and (9) in the first order with respect to  $\delta\xi$ .

Let us write  $q$  and  $\xi$  instead of  $q_0$  and  $\xi_0$ , and change  $\xi \frac{dq}{d\xi}$  to  $q_1$ ,  $P_3' = \frac{N}{\xi_0}q_1$ ,  $P_2 = N \frac{1+q_0}{2}$  in the calculations of  $A$  and  $B$ . Let us now calculate  $\delta P_{3-}$ :

$$\det = (1 - i\varepsilon)\eta(1 + 2\sigma\tau_0q_1),$$

$$\delta P_{3-} = \frac{ig\tau_{2z}}{(1 - i\varepsilon)\eta(1 + 2\sigma\tau_0q_1)} \frac{N^2 q_1}{\eta\xi}.$$

We derive

$$B = \frac{\eta-1}{2} + \frac{2\sigma\tau_0q_1}{1 + 2\sigma\tau_0q_1}.$$

Substituting  $B$  in Eqs. (12), we find

$$q_1 = -\frac{(\eta-1)A}{\eta + 2\sigma\tau_0(\eta-1)A}, \quad d = \frac{1}{\eta-1} + \sigma\tau_0 \frac{\eta+1}{\eta}.$$

From this (since  $\frac{\langle(\Delta n)^2\rangle}{\langle n \rangle} = d + 1$ ),

$$\frac{\langle(\Delta n)^2\rangle}{\langle n \rangle} = \frac{\eta}{\eta-1} + \sigma\tau_0 \frac{\eta+1}{\eta},$$

where  $\langle \dots \rangle$  means averaging with the density matrix  $D_0(\xi)$ .

## CALCULATION OF LASER LINEWIDTH

The mean field attenuation time due to quantum fluctuations of the phase is much longer than  $\tau_0$ ; hence, this time can be calculated by the diffusion

equation. For this, let us assume that the generation reaches the steady-state region, i.e., the radial distribution function relaxed to the state  $D_0(\xi)$ . Expanding  $D(z, t)$  in a Fourier series, we derive [1, 2]:

$$D(z, t) = \pi^{-1} \sum_{m=-\infty}^{m=\infty} D_m(\xi, t) e^{-im\varphi}, \quad (13)$$

and represent  $P_-$  as

$$P_- = P_-^{(st)} + P_-^{(l)} \quad (|P_-^{(l)}| \ll P_-^{(st)}),$$

where  $P_-^{(st)}$  has been found from Eqs. (3) and (4) with  $D(z, t) = D_0(\xi)$ .

Let us represent  $P_-^{(l)}$  as

$$P_-^{(l)} = \tilde{a} \frac{\bar{\nabla} D}{D} + \tilde{b} \frac{z}{\bar{z}} \frac{\nabla D}{D}. \quad (14)$$

Substituting Eqs. (13) and (14) into Eq. (1), we derive for  $D_1(\xi, t)$

$$\frac{\partial D_1}{\partial t} = -2\sigma \frac{B_2}{\xi_0} D_1, \quad (15)$$

where

$$B_2 = \frac{g}{4\sigma} \text{Im}(\tilde{a} - \tilde{b}) \quad (16)$$

or

$$B_2 = \frac{1}{4} \left[ \frac{\eta+1}{2} + \frac{(\eta-1)\varepsilon^2}{2} \left( 1 + \frac{2(\eta+1)}{\eta} \sigma\tau_0 \right) \right],$$

$$\varepsilon = (\omega_n - \omega_{21})\tau_2.$$

Variations in  $\varepsilon$  are limited to the generation region:  $\eta_0 \geq 1 + \varepsilon^2$ , where  $\eta_0$  is the gain parameter at the line center  $\left( \eta = \frac{\eta_0}{1 + \varepsilon^2} \right)$ . Let us find the solution of Eq. (15) in the form  $D_1 = c_1 e^{i\Delta v t}$ ,  $\Delta v = i \frac{2\sigma}{\xi_0} B_2$ ;  $c_1$  is the constant determined by the initial condition. The mean field  $\langle a \rangle \sim e^{-\Delta v t}$ ,

$$\Delta v = \frac{\sigma}{4\xi_0} \left[ \eta + 1 + \varepsilon^2 (\eta - 1) \left( 1 + \frac{2(\eta+1)}{\eta} \sigma\tau_0 \right) \right] \quad (17)$$

is the laser linewidth.

## A SEMICONDUCTOR LASER

Let us consider how all that was discussed above can be applied to a semiconductor laser. The further consideration supplements the results of work [1]. Therefore, we are to exactly follow designations from [1] (see also [6]) without their explanation, except for this is necessary. As has been already mentioned, it is hard to find corrections to the diffusion coefficients due to collective effects, since algebraic set of equa-

tions (5) turns into a set of integral equations. However, it is clear that these corrections are proportional to  $\sigma\tau_0$ ; i.e., we suppose that the terms that include correlators  $\delta r_{2l,2lk}$  and  $\delta r_{2l,12k}$  are small with respect to  $\tau_2/\tau_0 \ll 1$  ( $\tau_2 = \Gamma^{-1}$ ).

Let us trace how the diffusion coefficient for phase (see Eq. (16)) depends on the lasing frequency detuning relative to the semiconductor laser gain maximum. For this, let us introduce the parameters  $N$  and  $P_-$  [1]:

$$N = V^{-1} \sum_l r_{22l} \quad (18)$$

is the density of electrons injected into the transmission band,

$$P_- = \sum_l r_{21l}, \quad (19)$$

where, according to [1],

$$r_{21l} = \frac{-igz}{\Gamma - i\delta_l} (f_{2l} - f_{1l}) + \frac{ig}{\Gamma - i\delta_l} r_{22l} \frac{\bar{\nabla} D}{D}, \quad (20)$$

$\delta_l = \omega - \omega_l$ ,  $\omega$  is the lasing frequency equal to  $\omega_n$  and  $f_{1,2l}$  is the electron distribution function in the valance and transmission bands. According to [6], we have

$$\sum_l \frac{f_{2l} - f_{1l}}{\Gamma - i\delta_l} = L(\delta - \varepsilon_m)\Delta N,$$

$\Delta N = N - N_0$ , where  $N_0$  is the concentration under transparency condition,  $L = L_1 + iL_2$ ,

$$L_1 = 0.06 \frac{2\sqrt{\pi}\hbar}{k_B T} \frac{(\Delta + \gamma)\Delta}{(\delta - \varepsilon_m)^2 + (\Delta + \gamma)^2},$$

$$L_2 = 0.06 \frac{2\sqrt{\pi}\hbar}{k_B T} \frac{(\delta - \varepsilon_m)\Delta}{(\delta - \varepsilon_m)^2 + (\Delta + \gamma)^2}, \quad (21)$$

$\gamma = \frac{\hbar}{k_B T} \Gamma$ ,  $\delta = \frac{\hbar}{k_B T} (\omega - \omega_g)$ ,  $\hbar\omega_g$  is the band gap,  $\Delta$  is the gain line half-width, and  $\varepsilon_m$  is the gain line maximum. All estimates and calculations in [6] were carried out for *AsGa*, where the ratio of carrier masses  $m_2/m_1 \approx 0.14$ , and room temperature.

Again, the equations for  $\Delta N$  and  $P_-$  [1] with allowance for the correlators can be written as

$$\Delta N = \Delta j\tau_0 - \frac{ig\tau_0}{V} (\bar{z} P_- - \text{c.c.}) + \frac{ig\tau_0}{V} \sum_{k \neq l} (\delta r_{12k,22l} \frac{\bar{\nabla} D}{D} - \text{c.c.}), \quad (22)$$

$$P_- = -igzVL\Delta N + ig \frac{2\hbar\sqrt{\pi}N_2}{k_B T} \alpha \frac{\bar{\nabla} D}{D}, \quad (23)$$

$\Delta j = j - \frac{N_0}{\tau_0}$ ,  $j = J/eV$ , where  $J$  is the pumping current,  $e$  is the electron charge,  $V$  is the semiconductor volume,  $N_2 = 2(m_2 k_B T / 2\pi\hbar^2)^{3/2}$ ,  $m_2$  is the carrier mass in the transmission band,  $k_B$  is the Boltzmann constant,  $T$  is the temperature, and

$$\alpha = \int_0^\infty d\varepsilon \sqrt{\varepsilon} \frac{1}{e^{\varepsilon - \varepsilon_{2F}} + 1} \frac{1}{\pi(\gamma - i(\delta - \varepsilon))}.$$

Substituting Eq. (23) in Eq. (22), we derive

$$\begin{aligned} \Delta N = \Delta j \tau_0 \lambda + \frac{g^2 \tau_0 2\hbar \sqrt{\pi} N_2 \lambda}{k_B T} \left( \alpha \bar{z} \frac{\bar{V} D}{D} + \text{c.c.} \right) \\ + \frac{ig \tau_0 \lambda}{V} \sum_{k \neq l} \left( \delta r_{12k,22l} \frac{\bar{V} D}{D} - \text{c.c.} \right), \quad (24) \\ \lambda = (1 + g^2 \tau_0 L_1 |z|^2)^{-1}. \end{aligned}$$

Finally,

$$P_- = \tilde{P}_- + g^2 \tau_0 z \lambda L \sum_{k \neq l} \left( \delta r_{12k,22l} \frac{\bar{V} D}{D} - \text{c.c.} \right). \quad (25)$$

According to the above results, the correction connected with the correlation effects can be considered proportional to  $\sigma \tau_0$ . Since we assume that the lasing frequency detuning relative to the gain maximum is small, we consider it only in the terms that include correlators. Other terms are designated as  $\tilde{P}_-$  and have been calculated in [1]. The second term in Eq. (25) contributes into the phase diffusion coefficient proportional to (see Eq. (16))

$$\text{Im}(\tilde{a} - \tilde{b}) = g^2 \tau_0 \lambda L_2 \sum_{k \neq l} (z \delta r_{12k,22l} + \text{c.c.}), \quad (26)$$

where  $L_2 \sim (\delta - \varepsilon_m)$ . The term that determines the phase diffusion should be even in detuning; therefore, we can take

$$\begin{aligned} \sum_{k \neq l} (z \delta r_{12k,22l} + \text{c.c.}) \sim \sigma \tau_0 (\delta - \varepsilon_m) \quad \text{and} \\ B_2 \sim \sigma \tau_0 (\delta - \varepsilon_m)^2, \quad (27) \end{aligned}$$

where  $B_2$  is the phase diffusion coefficient. Thus, the contribution of collective effects into the semiconductor laser linewidth at the gain line maximum is exactly zero, like for the case of ordinary two-level systems.

## CONCLUSIONS

The semiconductor laser linewidth connected with quantum fluctuations of the electromagnetic field amplitude and phase and the medium was found in [1] with the use of quantum laser theory [2–4] neglecting the correlation effects. The corresponding diffusion coefficients were calculated for the lasing frequency at the laser gain maximum. The laser linewidth calculated turned out to be ten times higher than the contribution due to spontaneous radiation. The linewidth should be twice as high so as to coincide with the experiment [7]. Though the correction due to correlation effects is exactly zero at the gain line maximum, the parameter  $\sigma \tau_0$  is extremely high ( $10^3 - 10^4$ ). Therefore, it should be interesting to determine how the semiconductor laser linewidth depends on the lasing frequency detuning from the gain maximum.

Using Eq. (17) for the radiation linewidth for two-level systems, let us make the following rough estimate. Let us determine detuning values at which the term independent of the detuning becomes equal to the term that depends on it. We find  $2\varepsilon^2(\eta - 1)\sigma \tau_0 / \eta \approx 1$  or  $|\varepsilon| \approx (\eta / 2\sigma \tau_0 (\eta - 1))^{1/2}$ . Assuming that  $\eta - 1 = 0.1$ ,  $\sigma \tau_0 \leq 10^3 - 10^4$ , we find  $|\varepsilon| \leq 10^{-1} - 10^{-2}$ . Thus, to exclude the contribution of collective effects, extremely accurate tuning to the gain line maximum is required.

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