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NONLINEAR AND QUANTUM OPTICS

Parametric Field Excitation in a Cavity with Oscillating Mirrors

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Abstract—The dynamics of the parametric excitation of an electromagnetic field in a cavity that is composed of two plane mirrors has been analyzed. One of the mirrors is immobile and has a finite reflectance spectral band, while the other mirror is oscillating with a wider spectral band. The field energy in the cavity may increase due to a transfer from the kinetic energy of the mirrors. Estimates and calculation results for a Lorentz profile of the spectral band of the reflectance show that, under conditions of a parametric resonance, the initial exponential increase in the field energy in the cavity is stopped and the stable pulse-periodic mode is formed.

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INTRODUCTION

Parametric generation of electromagnetic radiation in an empty cavity with oscillating mirrors is fundamentally possible due to a transfer of the kinetic energy of the mirrors to the field energy [1, 2]. At the same time, the only known experimental demonstration of this generation carried out for superconducting quantum interference devices [3] is an indirect proof of the effect existence. In the theory of this effect, both classical [1] and quantum (the so-called Casimir dynamic effect [2]), there are still some insufficiently studied fundamental questions. This circumstance hinders the searching for new, more efficient schematics of parametric generation of this type.

One of these questions (leaving the plane-wave approximation and consideration of cavities with spherical mirrors) was described in [4]. The purpose of this study was to analyze the manifestations of boundedness of the spectral band of the reflectance of cavity mirrors. Apparently, this question has not been discussed yet within the quantum approach; however, as it follows from the classical consideration [1], specifically this factor should determine the characteristics of stationary parametric generation. Indeed, with this boundedness neglected, the field energy in the cavity increases and the generation pulse width decreases with time under the conditions of parametric resonance. Due to the latter circumstance, the radiation spectrum becomes so wide that the mirrors effectively reflect only a part of the radiation corresponding to their reflection spectral band. Consideration of this question in [1] was qualitative to a greater extent, and it should be supplemented with quantitative data.

ESTIMATES OF THE DYNAMICS OF PULSE FORMATION

We will consider an electromagnetic field in a cavity composed of two plane mirrors, one of which periodically oscillates along the normal coinciding with the cavity axis, while the other is immobile and characterized by a high reflectance in a limited frequency range (characteristic width of the reflection spectral band is γ). In this case, the frequency dependence of the reflectance of the first mirror can be neglected, which is substantiated if its width of the reflection spectral band significantly exceeds γ. There is vacuum between mirrors; the period-averaged cavity length is denoted as L_0. The problem is considered within the approximation of plane waves propagating along the cavity axis in the positive and negative directions (with the transverse effects neglected).

The case in which the frequency dependence is neglected for the oscillating mirror corresponds to large widths of the spectral band of mirror reflection in comparison with the radiation spectral width. According to [1], when the conditions of parametric resonance are satisfied, the initial field pulse is compressed to a sharp peak propagating in the vacuum gap with speed of light c so that it collides with the periodically

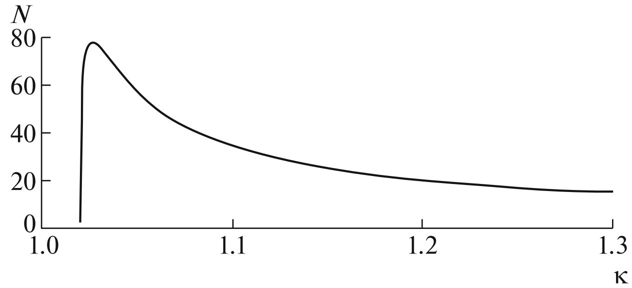

Fig. 1. Dependence of effective number N of pulse passages in the dynamic cavity on relativistic factor κ ($r_d r_0 = 0.98$, $\gamma/\omega_0 = 100$).

oscillating mirror when the latter achieves maximum velocity V_m. Having introduced the relativistic factor

$$\kappa = \frac{1 + \frac{V_m}{c}}{1 - \frac{V_m}{c}}, \quad (1)$$

we obtain that the instantaneous frequency (a time derivative of the radiation phase) is multiplied by κ ($\kappa > 1$) after each passage of the pulse through the cavity. Then, after the nth passage, the instantaneous frequency is

$$\omega_n = \omega_0 \kappa^n. \quad (2)$$

Initial frequency ω_0 can be chosen to be the lowest-mode frequency of the static cavity (with immobile mirrors): $\omega_0 = \pi c / L_0$.

Let r_s and r_d be the maximum (within the low-frequency limit) reflectances of the immobile and oscillating mirrors, respectively. The reflectance of the immobile mirror is assumed to decrease with an increase in frequency. The transformation of the field amplitude after the nth passage is reduced to the multiplication by factor $K_n = \kappa r_d r_s(\omega_n)$. The field energy increases while $K_n > 1$. Accordingly, it is convenient for estimations to introduce the effective number N of pulse passages in the cavity from the condition $K_N = 1$. One can assume that, after N passages of the pulse, its characteristics are stabilized (i.e., there are no further decrease in the width and increase in the energy).

The frequency dependence of the reflectance of the immobile mirror can be written as

$$r_s(\omega) = r_0 \left(1 - \frac{\omega^2}{\gamma^2}\right). \quad (3)$$

Here, γ characterizes the spectral-band width of the immobile mirror reflection. Then

$$K_n = \kappa r_d r_0 \left(1 - \frac{\omega_n^2}{\gamma^2}\right) \quad (4)$$

and

$$N = \frac{\ln\left[\frac{\gamma^2}{\omega_0^2}\left(1 - \frac{1}{r_d r_0 \kappa}\right)\right]}{2 \ln \kappa}. \quad (5)$$

The dependence of the effective number of passages on relativistic factor κ is shown in Fig. 1. At small κ, the function $N(\kappa)$ is increasing, which is caused by an increase in the field amplitude at reflections from the oscillating mirror due to the relativistic factor at almost invariable reflection loss for the mirrors. The function $N(\kappa)$ reaches a maximum with an increase in κ and then decreases due to a decrease in reflection coefficient (3) for increasing radiation frequency.

LORENTZ SPECTRAL PROFILE OF THE MIRROR REFLECTANCE

Now, we describe a mirror with a finite width of the reflection band by the integral relation between the incident (E_i) and reflected (E_r) radiations:

$$E_r(t) = \int_{-\infty}^{t} D(t - t')E_i(t')dt' = \int_0^{\infty} D(\tau)E_i(t - \tau)d\tau. \quad (6)$$

A simple version of the response function is the exponential dependence of the kernel in (6)

$$D(\tau) = D_0 e^{-\gamma\tau}. \quad (7)$$

As above, parameter γ determines the reflection-band width. For monochromatic radiation $E_i \sim \cos(\omega t + \varphi)$, the spectral dependence of the mirror reflectance with respect to the intensity is described by the Lorentz profile

$$r_s^2(\omega) = \frac{D_0^2}{\gamma^2 + \omega^2}. \quad (8)$$

In this case, $r_0 = -|D_0|/\gamma < 0$.

The field dynamics in the vacuum gap in the accepted one-dimensional gap is described by the d'Alembert solution (i.e., by the sum of waves propagating in the positive and negative directions of the cavity axis [5]). Relations (6) and (7) are used as the boundary condition at the immobile mirror. The condition of zero electric field strength in the coordinate system related with the oscillating mirror is set on the moving mirror. A transition to the laboratory coordinate system can be performed using the Lorenz transform at an instantaneous mirror velocity [1]; this approximation is justified at fairly small mirror accelerations.

Figure 2 shows the dynamics of change in the electromagnetic field energy inside the dynamic cavity at two values of parameter γ. The immobile mirror was

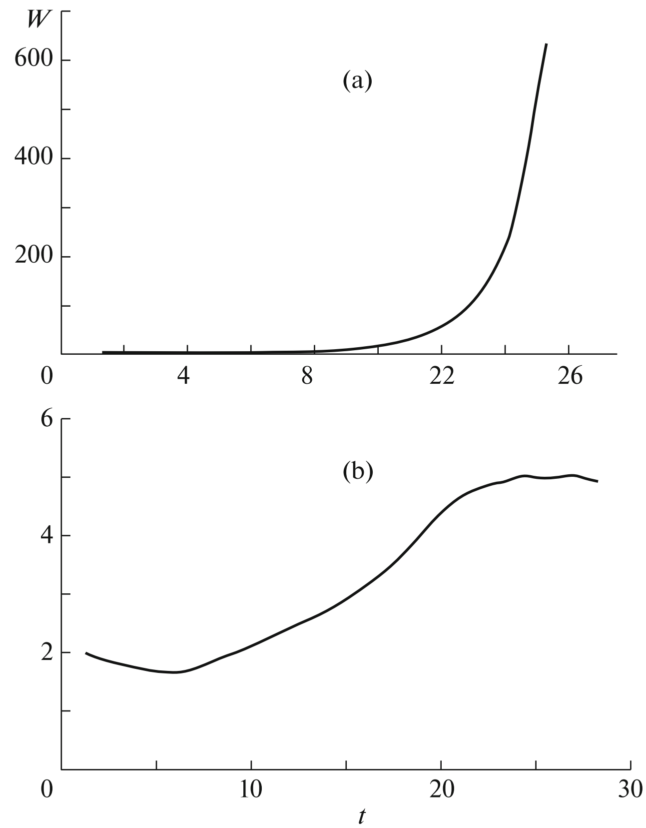

Fig. 2. Dependence of electromagnetic field energy W in the dynamic cavity on time t at γ/ω_0 = (a) 100 and (b) 25 ($r_d r_0 = 0.99$, the cavity-length modulation depth is $\mu = 0.01$, and the modulation frequency is $\Omega = 2\omega_0$).

placed at the coordinate origin, while the coordinate of the oscillating mirror was set in the form

$$L_r(t) = L_0[1 + \mu\cos(\Omega t)], \quad (9)$$

where μ and Ω are the modulation depth and frequency, respectively. The initial field distribution corresponded to the lowest-order mode with frequency $\omega_0 = \pi c/L_0$, and the modulation frequency was chosen from the parametric resonance condition $\Omega = 2\omega_0$. It can be seen in Fig. 2a that the field energy increases exponentially with time and the saturation for the shown time intervals is still absent at such a large γ. At a smaller γ value, the initial stage of exponential energy increase is replaced by the stage of energy stabilization (Fig. 2b).

DISCUSSION

Thus, the pulse energy and spectral characteristics in a dynamic cavity can be controlled by choosing the spectral dependence of the reflectivity of cavity mirrors. At a rather large width of the reflection spectral band, the effective number of pulse passages in the cavity is fairly large. The multiple increase in the pulse energy is accompanied by the corresponding increase in the characteristic radiation frequency, which allows for generation of teraherz radiation with the cavity length modulated at a significantly lower frequencies. The smallness of loss at the mirrors required for generation at a really achievable modulation depth can be obtained due to the relativistic factor and the use of the Doppler parametric effect with radiation reflection from inhomogeneities moving with relativistic velocities, which are induced in the medium by external pulses [6].

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