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FREQUENCY DISPERSION OF THE ELECTROELASTICITY MODULUS IN AQUEOUS ELECTROLYTE SOLUTIONS

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Friction coefficients β_a and β_b, relaxation times τ_a, τ_b, τ_{ab}, and $\in(\omega)$ for aqueous solutions of LiCl, NaCl, KCl, RbCl, CsCl are numerically calculated as functions of concentration c, density ρ, and temperature T in a wide range of frequencies ω using analytical expression for the electroelasticity modulus $\in(\omega)$ obtained previously in the case of exponentially relaxing flows for a specific form of potential energy corresponding to the interaction between structural units of solution $\Phi_{ab}(r)$ and the equilibrium radial distribution function $g_{ab}(r)$. The obtained theoretical data are presented in the form of tables and graphs and are shown to be in a satisfactory agreement with experimental results.

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INTRODUCTION

Studying elastic properties of liquids, along with other physicochemical properties, by experimental and theoretical methods is an important and relevant task. The knowledge of transport, elastic, acoustic, electroconductive and dielectric properties of liquids and solutions, which are closely related to the interactions of external and internal fields, allows for their efficient use in industry, hydroacoustics, medicine, and chemical technology. To date, significant progress has been achieved in studying interactions between external fields such as mechanical, thermal, etc. and electric fields in condensed matter. However, theoretical studies of electroconductive, electroelastic, and dielectric properties of electrolyte solutions are a complex problem that still remains open due to the necessity of taking into account the contributions of interaction energies between structural units of ion-molecular systems when determining transfer rates and other physical parameters.

The studies of elastic properties of solids are based on Hooke's law and, along with other research areas, are widely demanded in acoustics and design of piezoelectric materials [1-5] that are used to create various technical devices. Engineering calculations are based on the linear theory of electroelasticity to describe the properties of piezoceramic materials using a set of elastic and piezoelectric moduli and the dielectric permittivity, while the piezoceramic material is considered a homogeneous medium. Great progress has been made in studying interactions between mechanical and electric fields in piezoelectric materials. According to [5], electrical properties of these materials allow formulating electroelasticity

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problems in a somewhat similar way to the classical formulation in the theory of elasticity. Currently, there are numerous works describing the theory and methods of electro- and magnetoelastic materials as well as their application in industry to produce piezoelectric elements. Studying elastic properties of liquids and their solutions is quite a complicated problem.

Due to fluidity, the only elastic reaction of a liquid subject to slow influence of external forces is the resistance to the bulk compression. This resistance is characterized by isothermal compressibility β_T or by the corresponding isothermal elasticity modulus $K_T = \beta_T^{-1}$. In the presence of fast external influences [6, 7], liquids exhibit elastic properties similar to those of solids. For the stress tensor, which appears due to fast deformation of the liquid, Hooke's generalized law will be true in a linear approximation with respect to the displacement vector and its velocity. This law contains both the coefficients of bulk and shear viscosity and the corresponding bulk and shear elasticity moduli [8]. In the works [7, 9], analytical expressions for the moduli of bulk and shear elasticity of mono-atomic liquids were obtained from the potential energy of intermolecular interaction $\Phi(r)$ and the equilibrium radial distribution function $g(r)$. According to [9], elastic moduli of simple liquids remain constant (frequency independent) at high frequencies. High-frequency elastic and thermal moduli of the fluid were also determined in [10] using corresponding complex analytical expressions of transfer coefficients (bulk and shear viscosity, thermal conductivity). Along with viscosity and thermoelastic properties, liquids and solutions have electroconductive and dielectric properties which are determined, in the presence of the gradient of electric potential, by conductivity and permittivity coefficients and by the electroelasticity modulus.

Electrically conductive properties of highly dilute electrolyte solutions are theoretically described by the Debye–Hückel theory [11-13]. At equilibrium, both ionic and dipole atmospheres take place in solutions [11]. It was also noted that the general reason of the changes in mechanical, thermal, and other properties of electrolyte solutions, as well as the changes in their isothermal compressibility β_T, is due to the densification of the solvent in a strong electrostatic field of ions. The correct explanation of all phenomena may follow from a theory that solves the problem of the interaction of the solute with the solvent. According to [13], the applied external electrostatic or another field of ions primarily changes these atmospheres and hence the structure of the solvent while simultaneously linking the solvent molecules with ordered ions. In addition to the interaction between constant dipole moments, hydration in aqueous solutions of electrolytes is also affected by the polarization (induced dipole moment) of the molecules and by the dispersion effect. It is the interaction between these fields in solutions that results in the electroelasticity phenomenon.

Transport phenomena, elastic, dielectric, and electroconductive properties of electrolyte solutions were studied using the method of kinetic equations in [14-19]. Within certain models (with the potential energy of interaction between solution's structural units $\Phi_{ab}(r)$ and the equilibrium radial distribution function $g_{ab}(r)$) these physical parameters were numerically calculated for aqueous solutions of electrolytes in wide density, concentration, temperature, and frequency ranges. The obtained results were in satisfactory agreement with experimental data. In this respect, it is of interest to determine the region of frequency dispersion and to calculate numerically the electroelasticity modulus of aqueous solutions of electrolytes depending on the thermodynamic parameters of the state.

INITIAL EXPRESSIONS FOR THE ELECTROELASTICITY MODULUS OF ELECTROLYTE SOLUTIONS

Based on kinetic equations for single-particle $f_a(\boldsymbol{x}_a, t)$ and two-particle $f_a(\boldsymbol{x}_a, \boldsymbol{x}_b, t)$ distribution functions (where $\boldsymbol{x}_a(\boldsymbol{q}_a, \boldsymbol{p}_a)$, $\boldsymbol{q}_a$ are coordinates and $\boldsymbol{p}_a$ are momenta of the particles, a and b are the type of the particles) analytical expressions for the dynamic modulus of electroelasticity $\in(\omega)$ were obtained in [19] depending on the nature of relaxing flows. The expressions for $\in(\omega)$ for two cases of power $\sim t^{-d/2}$ (d is the dimension of the space) and exponential $\sim e^{-\omega t}$ laws of flow relaxation were reported in [20] to have the following forms, respectively:

$$\in(\omega) = \sum_a \omega\hat{\sigma}_a(\omega)\left\{\omega\tau_a + \sum_b n_b^* q_{ab} \int_0^\infty \frac{\partial \Phi_{ab}(r)}{\partial r}\left[\int_0^\infty G_2^{ab}(r, r_1, \omega)\frac{\partial g_{ab}^0(r_1)}{\partial r_1} r_1 dr_1\right] r dr\right\}, \tag{1}$$

$$\in(\omega)=\sum_a \frac{(\omega\tau_a)^2 \in_{\infty a}}{1+(\omega\tau_a)^2}\left[1+\sum_b \frac{(1+\tau_{ab}/\tau_a)G_0^{ab}(r)}{1+(\omega\tau_a)^2(\tau_{ab}/\tau_a)^2}\right], \tag{2}$$

where

$$G_2^{ab}(r,r_1,\omega)=\left(\frac{\omega\tau_a}{2}\right)^{-1/2}[\mathrm{e}^{-\varphi_1}(\cos\varphi_1+\sin\varphi_1)-\mathrm{e}^{-\varphi_2}(\cos\varphi_1+\sin\varphi_1)], \tag{3}$$

$$G_0^{ab}(r)=2\pi n_b^* q_{ab}\int\frac{\partial\Phi^*_{ab}(r)}{\partial r}\frac{\partial g^0_{ab}(r)}{\partial r}r^2dr, \tag{4}$$

$$\left.\begin{aligned}
&\hat{\sigma}_a(\omega)=\frac{\sigma_a}{1+(\omega\tau_a)^2}; \quad \sigma_a=\frac{n_a^0e_a^2}{\beta_a}=\in_{\infty a}\tau_a; \quad \in_{\infty a}=\frac{n_a^0e_a^2}{m_a};\\
&\tau_a=\frac{m_a}{2\beta_a}; \quad \tau_{ab}=\frac{d_{ab}^2}{kT_0}\frac{\beta_a\beta_b}{\beta_a+\beta_b}; \quad n_b^*=\frac{\pi}{6}n_bd_{ab}^3;\\
&\Phi^*_{ab}(r)=\frac{\Phi_{ab}(r)}{kT_0}; \quad q_{ab}=\frac{4}{\pi}\frac{e_a\beta_b-e_b\beta_a}{e_a(\beta_a+\beta_b)};\\
&\varphi_{1,2}=\varphi_{1,2}^{ab}(r,r_1,\omega)=\left(\frac{\omega\tau_{ab}}{2}\right)^{1/2}(r\mp r_1); \quad \boldsymbol{r}=\boldsymbol{r}_{ab}/d_{ab};
\end{aligned}\right\} \tag{5}$$

$\boldsymbol{r}_{ab}=\boldsymbol{q}_b-\boldsymbol{q}_a=\boldsymbol{q}_2-\boldsymbol{q}_1$ is the relative distance; $d_{ab}=(d_{aa}+d_{bb})/2$ is the average diameter of ions a and b; $n_a^0=N_a/V$ is the equilibrium numerical density of ions a; $\omega=2\pi\nu$ is the cyclic frequency; ν is the process frequency; k is the Boltzmann constant; T_0 is the equilibrium temperature; τ_{ab} is a phenomenological parameter analogous to the structural relaxation time; $e_a=z_ae$, z_a, m_a, β_a, β_b are the ion charge, valency, mass, and friction coefficients; $\Phi_{ab}(r)$ is the potential energy of interparticle interaction, and $g_{ab}^0(r)$ is the equilibrium radial distribution function. Function $G_2^{ab}(r,r_1,\omega)$ determined from (3) is the fundamental solution (Green's function) of the Smolukhowski equation for a binary density in the configuration space $n_{ab}(\boldsymbol{q}_1,\boldsymbol{q}_2,t)$.

Equation (1) (the power law of relaxing flows) is the dynamic modulus of electroelasticity $\in(\omega)$ and, by analogy with bulk $K(\omega)$, shear $\mu(\omega)$, and thermal $Z(\omega)$ elasticity moduli in electrolyte solutions obtained in [14-16], the integral in this expression contains Green's function $G_2^{ab}(r,r_1,\omega)$ from the Smoluchowski equations for the binary density in the configuration space $n_{ab}(\boldsymbol{q}_1,\boldsymbol{q}_2,t)$. Within specific solution model (i.e. specific expressions $\Phi_{ab}(r)$ and $g_{ab}^0(r)$ depending on the thermodynamic parameters of the state such as density, concentration, and temperature), equation (1) allows studying frequency dispersion of the modulus of electroelasticity in electrolyte solutions, as well as to perform numerical calculations. However, due to the complexity of function $G_2^{ab}(r,r_1,\omega)$ and of the inner integration with respect to dr_{10}, we will below use the analytical expression for $\in(\omega)$ in the form of Eq. (2) obtained according to the exponential law of relaxing flows, which has a simpler form than Eq. (1).

Thus, we take formula (2) as the initial one for numerical calculations of the dynamic modulus of electroelasticity $\in(\omega)$ of electrolyte solutions. According to expressions (5), this formula should contain friction coefficients β_a and β_b, relaxation times τ_a, τ_b, τ_{ab}, and function $G_0^{ab}(r)$ in the expression under the integral sign is determined by $\Phi_{ab}(r)$ and $g_{ab}(r)$. To determine β_a, β_b, τ_a,, τ_b, τ_{ab}, $G_0^{ab}(r)$ and to calculate $\in(\omega)$, functions $\Phi_{ab}(r)$ and $g_{ab}(r)$ should be taken in the explicit form which is considered known in literature.

CHOICE OF THE SOLUTION MODEL AND NUMERICAL CALCULATIONS

According to (2) and (4), the dynamic modulus of electroelasticity $\in(\omega)$ is determined by the potential energy of interaction $\Phi_{ab}(r)$ and the radial distribution function $g^0_{ab}(r)$ as well as by the times τ_a, τ_b, τ_{ab} of ion relaxation. The latter are calculated using friction coefficients of ions β_a and β_b which, in turn, depend on the structure of the solution and its concentration c, density ρ, and temperature T. In [15], dependences β_a and β_b were calculated using the following analytical expressions:

$$\begin{aligned}\beta_a^2 &= \sum_a \frac{4\pi}{3}\rho_a kT \sum_b d_{ab} \int_0^\infty \nabla^2\Phi^*_{ab}(r) g^0_{ab}(r) r^2 dr,\\ \beta_b^2 &= \sum_b \frac{4\pi}{3}\rho_b kT \sum_a d_{ab} \int_0^\infty \nabla^2\Phi^*_{ab}(r) g^0_{ab}(r) r^2 dr,\end{aligned} \tag{6}$$

where $\rho_a = m_a n_a$, $\rho_b = m_b n_b$ is the mass density of ions a and b; $\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$ is the radial part of the Laplace operator; k is the Boltzmann constant.

To study viscoelastic and electroconductive properties of electrolyte solutions and to calculate β_a, β_b τ_a, τ_b, τ_{ab}, $\in(\omega)$ according to [15, 20] within the McMillan–Mayer solution theory, the following models were chosen for $\Phi_{ab}(r)$ and $g^0_{ab}(r)$.

$\Phi_{ab}(r)$ was represented by the following expression consisting of the sum of the Lennard-Jones potential and the generalized Debye potential and taking into account the configuration of ion sizes

$$\Phi_{ab}(r) = \frac{4\varepsilon_{ab}}{\varepsilon_{ss}}(r^{-12} - r^{-6}) + \frac{R_{ab}}{r} e^{-\kappa^* r}, \tag{7}$$

where $\varepsilon_{ab} = \sqrt{\varepsilon_{aa}\varepsilon_{bb}}$, $d_{ab} = (d_a + d_b)/2$ are the parameters of the Lennard-Jones potential which were given in [21], $R_{ab} = \frac{f z_a z_b e^2}{kT\varepsilon_{ss} d_{ab}} \frac{\exp(\kappa^*)}{1+\kappa^*}$; $f = \frac{1}{4\pi\varepsilon_0} = 9\cdot 10^9\,\mathrm{m/F}$; ε_0 is the electric constant; ε_{SS} is the dielectric constant of the solvent; e is the elementary charge; z_a, z_b are valence numbers of ions a and b; $\kappa^* = d_{ab}\kappa_a$ is the reduced reciprocal Debye shielding radius $\kappa^2 = \frac{\sum_a n_a e_a^2}{\varepsilon\varepsilon_0 kT}$ [22], where $n_a = \frac{N_a}{V}$. According to [23], the radial distribution function of the ion subsystem is written as follows:

$$g_{ab}(r) = y(\rho^*)\, e^{-\frac{\Phi_{ab}(r)}{kT}}, \tag{8}$$

where $\Phi_{ab}(r)$ is the interaction potential of the basic system in the form (7); $y(\rho^*)$ is the binary distribution function for two cavities, which at a distance $r = 1$ ($r_{ab} = d_{ab}$) has the form of the Karnahan–Starling function [23]

$$y(\rho^*) = \frac{(2-\rho^*)}{2(1-\rho^*)^3}, \tag{9}$$

where $\rho^* = \frac{\pi}{6} n d_{ab}^3 = \frac{\pi}{6}\rho \frac{d_{ab}^3 N_0}{M}$ is the reduced density; ρ is the density of the solution; N_0 is the Avogadro number; M is the molar mass.

The semiphenomenological model (7)-(9) can be used to calculate coefficients β_a, β_b, relaxation times τ_a, τ_b, τ_{ab}, and the dynamic modulus of electroelasticity $\in(\omega)$ for electrolyte solutions in a wide range of thermodynamic parameters and frequencies.

Tables 1, 2 and Figs. 1-3 present the values of friction coefficients of ions β_a, β_b, relaxation times τ_a, τ_b, τ_{ab}, and the dynamic modulus of electroelasticity $\in(\omega)$ numerically calculated from (2) with the account of (5)-(8) for aqueous solutions of LiCl, NaCl, KCl, RbCl, CsCl. The molecular parameters (d_{aa}, d_{bb}, ε_{aa}/k, ε_{bb}/k, κ^*) from (5) as well as the Lennard-Jones potential and the generalized Debye potential were taken from [21], and the corresponding experimental values ρ, c_a, c_b, and T were taken from [24, 25].

TABLE 1. Calculated Coefficients of Friction β_1, β_2 Calculated Depending on the Concentration, Density, and Temperature of LiCl, NaCl, KCl, RbCl, and CsCl Aqueous Solutions

c,% [24]	t = 20 °C			t = 30 °C			t = 35 °C			t = 55 °C		
	ρ, kg/m^3 [24]	$\beta_1 \cdot 10^{13}$, kg/s	$\beta_2 \cdot 10^{13}$, kg/s	ρ, kg/m^3 [24]	$\beta_1 \cdot 10^{13}$, kg/s	$\beta_2 \cdot 10^{13}$, kg/s	ρ, kg/m^3 [24]	$\beta_1 \cdot 10^{13}$, kg/s	$\beta_2 \cdot 10^{13}$, kg/s	ρ, kg/m^3 [24]	$\beta_1 \cdot 10^{13}$, kg/s	$\beta_2 \cdot 10^{13}$, kg/s
LiCl												
0.442	1001	4.107	6.984	998	4.134	7.012	997	4.150	7.029	988	4.223	7.114
4.07	1021	1.061	2.280	1018	1.060	2.293	1017	1.059	2.298	1009	1.057	2.330
11.30	1063	0.640	1.266	1060	0.641	1.271	1059	0.641	1.273	1055	0.641	1.280
14.50	1082	0.560	1.088	1079	0.561	1.092	1077	0.562	1.095	1070	0.564	1.105
20.30	1116	0.462	0.879	1113	0.463	0.882	1112	0.464	0.884	1105	0.466	0.891
25.30	1148	0.402	0.756	1145	0.403	0.759	1143	0.404	0.761	1136	0.406	0.767
29.80	1178	0.360	0.672	1174	0.361	0.674	1173	0.361	0.675	1166	0.364	0.681
NaCl												
0.994	1002	4.014	5.432	1000	4.021	5.459	998	4.026	5.475	990	4.051	5.552
5.53	1038	1.567	2.216	1034	1.568	2.227	1032	1.570	2.233	1023	1.576	2.261
10.50	1072	1.119	1.529	1070	1.120	1.533	1068	1.122	1.537	1058	1.129	1.554
14.90	1105	0.918	1.234	1103	0.919	1.127	1101	0.921	1.240	1091	0.928	1.254
19.00	1137	0.793	1.055	1135	0.794	1.058	1132	0.796	1.060	1122	0.803	1.073
22.60	1166	0.709	0.938	1164	0.710	0.940	1161	0.712	0.943	1110	0.747	0.993
KCl												
0.733	1003	2.661	6.309	1000	2.667	6.327	999	2.671	6.338	990	2.690	6.394
6.93	1043	0.823	2.006	1040	0.825	2.013	1038	0.826	2.017	1029	0.832	2.035
13.00	1083	0.584	1.403	1080	0.586	1.408	1078	0.587	1.411	1068	0.591	1.423
18.30	1118	0.479	1.145	1114	0.481	1.149	1120	0.478	1.144	1104	0.484	1.159
23.00	1154	0.416	0.990	1150	0.417	0.994	1148	0.418	0.996	1138	0.421	1.004
RbCl												
1.19	1007	5.849	6.165	1004	5.862	6.182	1003	5.869	6.191	994	5.907	6.241
10.8	1082	1.817	1.946	1039	1.877	2.015	1037	1.879	2.019	1068	1.834	1.972
19.5	1159	1.280	1.358	1155	1.284	1.362	1153	1.285	1.364	1144	1.294	1.375
26.6	1231	1.042	1.101	1226	1.045	1.104	1224	1.047	1.106	1214	1.054	1.115
32.6	1295	0.899	0.948	1291	0.902	0.951	1288	0.904	0.953	1277	0.911	0.961
42.1	1412	0.730	0.768	1403	0.735	0.773	1404	0.734	0.773	1392	0.740	0.779
CsCl												
1.66	1011	6.540	6.066	1008	6.552	6.081	1007	6.560	6.089	998	6.599	6.133
14.40	1120	2.051	1.917	1117	2.055	1.923	1115	2.058	1.925	1106	2.070	1.939
25.20	1232	1.443	1.341	1227	1.447	1.345	1225	1.449	1.347	1215	1.459	1.357
33.60	1332	1.174	1.088	1328	1.177	1.092	1325	1.179	1.093	1314	1.187	1.101
40.20	1425	1.015	0.939	1420	1.018	0.942	1417	1.020	0.944	1405	1.027	0.951
50.30	1589	0.824	0.762	1583	0.827	0.765	1580	0.829	0.767	1566	0.835	0.773
57.40	1729	0.713	0.659	1723	0.716	0.661	1719	0.717	0.663	1705	0.723	0.668

TABLE 2. Numerical Calculation of the Dynamic Electroelasticity Modulus $\in(\omega)$ Depending on the Concentration and Frequency of LiCl, NaCl, KCl Aqueous Solutions at t = 18 °C and RbCl, CsCl Aqueous Solutions at t = 20 °C

c,% [25]	ρ, kg/m^3 [25]	$\in$, S/m·s ($\nu^*=10^{-6}$)	$\in$, 10^2 S/m·s ($\nu^*=10^{-5}$)	$\in$, 10^4 S/m·s ($\nu^*=10^{-4}$)	$\in$, 10^6 S/m·s ($\nu^*=10^{-3}$)	$\in$, 10^8 S/m·s ($\nu^*=10^{-2}$)	$\in$, 10^{10} S/m·s ($\nu^*=10^{-1}$)	$\in$, 10^{11} S/m·s ($\nu^*=1$)
				LiCl				
2.50	1012	19.093	19.093	19.093	19.093	19.087	18.575	6.528
5.00	1026	37.639	37.639	37.639	37.639	37.629	36.683	13.207
10.00	1056	73.450	73.450	73.450	73.450	73.433	71.803	27.038
20.00	1117	149.121	149.121	149.121	149.120	149.089	146.086	56.929
30.00	1183	233.020	233.020	233.020	233.020	232.973	228.461	90.251
40.00	1251	325.443	325.443	325.443	325.442	325.379	319.218	127.094
				NaCl				
5.00	1034	10.275	10.275	10.275	10.275	10.273	10.115	4.152
10.00	1070	20.400	20.400	20.400	20.400	20.397	20.103	8.497
15.00	1109	31.036	31.036	31.036	31.036	31.031	30.598	13.122
20.00	1148	42.353	42.353	42.353	42.353	42.347	41.765	18.055
25.00	1189	54.399	54.399	54.399	54.399	54.392	53.653	23.310
26.00	1197	56.912	56.912	56.912	56.912	56.904	56.133	24.407
				KCl				
5.00	1030	6.705	6.705	6.705	6.705	6.704	6.591	2.545
10.00	1063	13.596	13.596	13.596	13.596	13.594	13.370	5.228
15.00	1097	22.700	22.700	22.700	22.700	22.697	22.328	8.800
20.00	1132	28.569	28.569	28.569	28.569	28.564	28.102	11.090
21.00	1140	30.174	30.174	30.174	30.174	30.169	29.682	11.720
				RbCl				
[24]	[24]							
1.19	1007	0.496	0.496	0.496	0.496	0.496	0.491	0.239
10.80	1082	4.954	4.954	4.954	4.954	4.953	4.900	2.352
19.50	1159	9.441	9.441	9.441	9.441	9.440	9.339	4.518
26.60	1231	13.593	13.593	13.593	13.593	13.592	13.448	6.525
32.60	1295	17.485	17.485	17.485	17.485	17.483	17.298	8.407
42.10	1412	24.538	24.538	24.538	24.538	24.536	24.277	11.818
				CsCl				
1.66	1011	0.366	0.366	0.366	0.366	0.366	0.363	0.194
14.40	1120	3.565	3.565	3.565	3.565	3.565	3.533	1.874
25.20	1232	6.791	6.791	6.791	6.791	6.791	6.731	3.586
33.60	1332	9.762	9.762	9.762	9.762	9.761	9.675	5.163
40.20	1425	12.469	12.469	12.469	12.469	12.468	12.359	6.600
50.30	1559	17.363	17.363	17.363	17.363	17.361	17.210	9.199
57.40	1729	21.530	21.530	21.530	21.530	21.528	21.340	11.413

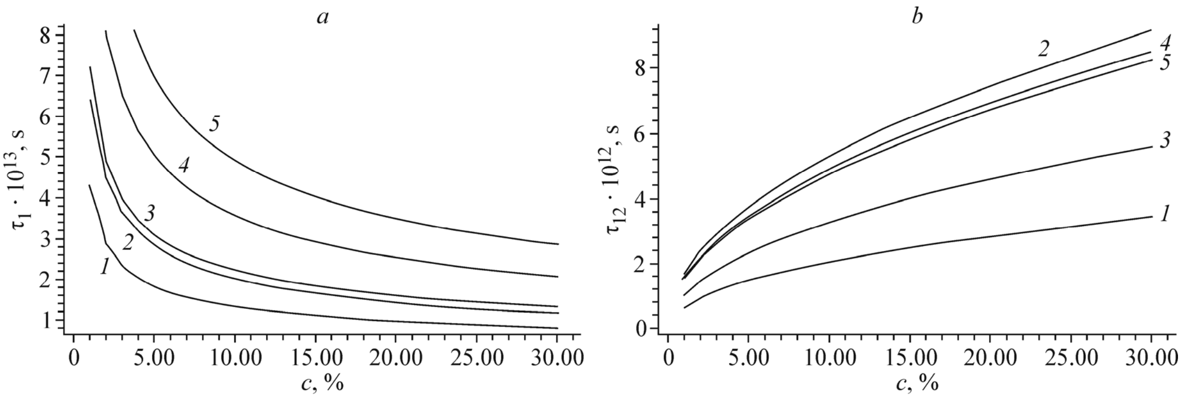

Fig. 1. Relaxation times τ_1 (*a*) and τ_{12} (*b*) as functions of concentration at $t = 30$ °C for aqueous solutions of LiCl (*1*), NaCl (*2*), KCl (*3*), RbCl (*4*), and CsCl (*5*).

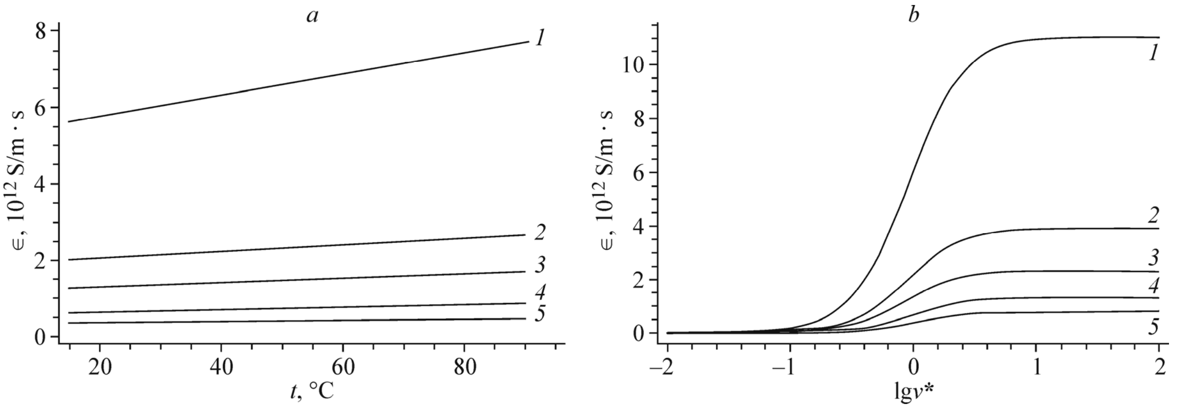

Fig. 2. Dynamic modulus of electroelasticity $\in(\omega)$ as a function of temperature (*a*) and frequency (*b*) for aqueous solutions of LiCl (*1*), NaCl (*2*), KCl (*3*), RbCl (*4*), and CsCl (*5*) at a fixed concentration *c*, %: 20.3 (*1*); 22.6 (*2*); 23 (*3*), 26.6 (*4*); 25.2 (*5*).

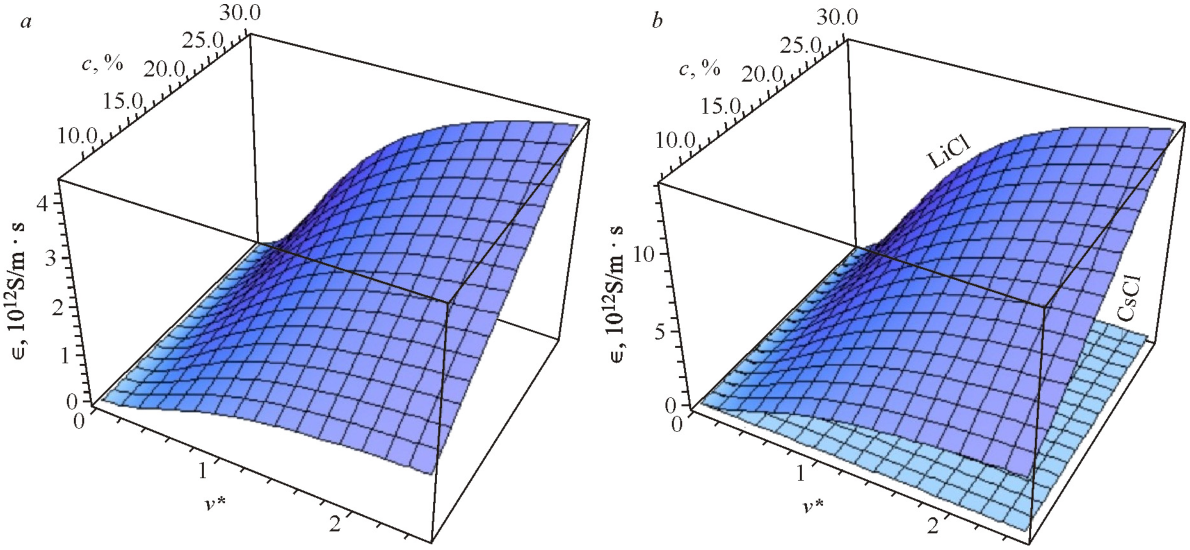

Fig. 3. Dynamic modulus of electroelasticity $\in(\omega)$ as a function of reduced frequency ν* and concentration *c* for NaCl aqueous solution (*a*) and LiCl, CsCl aqueous solutions (*b*) at a fixed temperature $t = 30$ °C.

Table 1 summarizes the values of coefficients for LiCl, NaCl, KCl, RbCl, CsCl for four temperatures $t = 20$ °C, 30 °C, 35 °C, 55 °C and the corresponding concentrations and densities taken from [24]. As can be seen, the $\beta \sim 10^{-13}$ kg/s values remain of the same order, while β_1 and β_2 slightly change as the temperature increases and decrease as the concentration (density) increases.

Table 2 presents calculated values of the dynamic modulus of electroelasticity $\in(\omega)$ depending on the reduced frequency ν^* for aqueous solutions of LiCl, NaCl, KCl at $t = 18$ °C and corresponding concentration values taken from [25], as well as the values for RbCl and CsCl at $t = 20$ °C and concentration values from [24]. It can be seen that the modulus of electroelasticity $\in(\omega)$ for these aqueous solutions increases together with the concentration (density) and reduced frequency ν^*. Fig. 1*a*,*b* shows numerically calculated relaxation times τ_1, τ_2, τ_{12} as functions of concentration for the same aqueous solutions.

Fig. 2*a* shows the temperature dependences of the isofrequency $\nu^* = 1$ ($\nu \sim 10^{12}$ Hz) electroelasticity modulus $\in$ at the corresponding concentrations c, Fig. 2*b* shows the frequency dispersion $\in(\omega)$ as a function of $\lg\nu^*$ at $t = 30$ °C. As can be seen, the modulus of electroelasticity $\in(\omega)$ of electrolytes increases together with the frequency and remains constant at high frequencies corresponding to high-frequency elastic moduli of liquids according to [9, 10].

Fig. 3*a* shows the modulus of electroelasticity $\in(\nu^*, c)$ plotted as a function of reduced frequency ν^* and concentration c at $t = 30$ °C for the aqueous solution of NaCl; Fig. 3*b* shows the same dependences for aqueous solutions of LiCl and CsCl. As can be seen from Fig. 2*b*, frequency and concentration dependencies $\in(\nu^*, c)$ of electrolytes NaCl, KCl and RbCl are confined between those of LiCl and CsCl.

According to the obtained data, the dispersion region of the frequency of electroelasticity modulus $\in(\omega)$ is $\sim 10^2$ Hz in the case of exponential flow relaxation, which is consistent with the conclusions of the general relaxation theory [2, 3].

CONFLICT OF INTERESTS

The authors declare that they have no conflict of interests.

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