

Modeling the Deformation of a Plate Using Piezoelectric Elements Located on its Surface

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Abstract—One of the directions of using piezoelectric elements in modern technology is related to their use for controlling the shape of a structure under the influence of operational loads. The issue of a change in the shape (geometry) of a structure can be caused, for example, by the need for minimization of displacements in certain regions or, vice versa, for maximization of displacements to ensure the stability of a structural shape under operational loads. Due to the presence of the inverse piezoelectric effect in piezoelectric materials, this problem can be solved by applying a predetermined electric voltage to the electroded surfaces of piezoelectric elements. Elastic structures with elastic piezoelectric elements attached to their surfaces become electroelastic. It is necessary to first assess the abilities of piezoelectric elements to have an effect on deformations caused by external actions of different types in order to use different strategies of control over their mechanical behavior. This means that piezoelectric elements must provide a controlled form change depending not only on the characteristics of the elements themselves (the size, physico-mechanical properties of the material, and position) but also on the parameters of the structure (its geometry, size, boundary conditions, and physico-mechanical characteristics) and acting loads. The influences of different factors on the deformation of an electroelastic structure under the action of an electric voltage applied to the piezoelectric elements in this work was established numerically based on mathematical modeling by solving the static problem of electroelasticity. The numerical implementation was carried out by the finite-element method in the ANSYS software package. The possibilities for using piezoelectric elements for changing the shape of a structure for different numbers of elements and different options of their layout on the surface are demonstrated based on the example of a cantilevered fixed plate.

Keywords: piezoelectric elements, shape control, deformation, bending, torsion, numerical modeling

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1. INTRODUCTION

Creation of some elements of space vehicles, such as antennas and probes, automated manipulators, and other high-technology products, requires the development of highly effective lightweight structures due to strict requirements on weight. Light structures inherently possess low internal damping and higher flexibility; they are susceptible to vibrations with a long decay time and are easily deformable under the action of operation loads. Optimization of the mechanical behavior of such structures to ensure the best operational characteristic requires appropriate control tools that can operate in an automatic mode. This, in turn, gave rise to the idea of making structures of such a type that are capable of self-verification and self-control.

The use of this idea in practice turned out to be possible due to the creation and use of smart technologies. The main feature of structures formed based on such technologies, or smart structures, is the involvement of elements made of functional materials that can change their characteristics depending on external conditions and thus change the characteristics of the initial object. One can consider shape memory alloys, piezoelectric materials, magnetic fluids, and some other materials as such materials. Elements made of these materials can operate in a structure as sensors that register some of the parameters of a structure or actuators that act on structural properties in a required way. Piezoelectric materials became

most widely used in connection with their broad assortment and sufficiently wide range of physico-mechanical characteristics (piezoceramics, piezopolymers, and other types). Piezoelectric materials are distinguished by the ability to generate an electric field under deformation (the direct piezoeffect) and, on the contrary, be deformed under the action of an electric field (the inverse piezoeffect). These properties make them attractive for providing possibilities of control both for the shape of a structure and for oscillations of any its part without a significant increase in its weight.

It is well known that elements made of piezoelectric materials and integrated with structures can act as distributed sensors and actuators and are able to endow these structures with functions of self-checking and self-control. The ability of self-checking in smart structures opens new possibilities of correcting the shape and curvature of mirrors/antennas for high pointing accuracy or for maintaining desirable shapes of flexible aerospace structures, [1, 2], oscillation damping [3, 4], monitoring of structure states [5, 6], identification of damages during operation, and so on. In Irschik's survey [2], the corresponding applications of static and dynamic control for the shape of a structure by use of piezoelectric actuators were described.

Some fundamental problems following from the problem of control for the shape of a structure are common for the static and dynamic cases. Statistical control for the shape of flexible products with the use of piezoelectric actuators is of great importance for many applied areas, especially such as robotics, aerospace and aircraft industry [7–9]. Under static control for the shape of electroelastic structures, the main aim is to choose characteristics of the governing electric action for each piezoelectric actuator to make the obtained structure maximally correspond to the required one.

The direct and inverse piezoeffects are not mutually reversible; therefore, to send to the actuator a signal that is necessary, e.g., for the recovery of the initial shape of a structure, it is required to transform the signal from the sensor in a certain way. In connection with this, it is extremely important to understand the operation mechanism of the inverse piezoeffect, i.e., the mechanical response of the piezoelectric element to the applied electrical action.

One of the important problems that arise when using piezoactuators is their proper use that allow one to minimize their weight-and-dimensional characteristics and to raise efficiency. These results can be achieved owing to the optimum choice of their arrangement in the structure, geometry, quantity, and parameters of the material.

The surveys of works devoted to the problem of optimizing the topology of structures with piezoelectric elements were presented in [10–14]. However, the question of how to do it in the best way to control structural behavior still requires its solution for each specific construction with allowance for its final application.

The magnitude of the generated electric signal is significantly affected by the arrangement of piezoelectric elements both relative to the construction and relative to each other [15, 16]. The effect of boundary conditions imposed on the structure during the control for its shape was studied in [17] based on the example of a beam with one or two clumped ends.

As well, it is necessary to note the use of the prestress effect in structures. This can be important from the viewpoint of creating conditions for the control for their shape under the action of operation loads. However, researchers as a rule do not consider this possibility. Since the creation of prestress is a type of activation of the intrinsic mechanical stress it can be used in the context of methods for controlling shape, as mentioned in [18].

In recent years, researcher attention [1, 2, 18–21] has been focused on the optimal spatial distribution of the controlling electrical action applied to elements made of a piezoelectric material to obtain effective and accurate control over the shape of a structure.

As mentioned in [19], successful control of the shape of a structure requires determination of the following parameters: the actuator position corresponding to the desired goal, the required size of the piezoelectric element, that is, the actuator, and the magnitude of the applied controlling electric voltage. The optimum positions and lengths of piezoactuators that allow one to achieve the maximum or minimum deflections of the beam were found. The authors demonstrated that an optimum electric voltage exists that can be applied to actuators; it depends on their length and position. In that work, it was also demonstrated that a single actuator is sufficient for simpler cases of loading, when the deflection curve is either convex or concave for the entire beam. However, for more complicated cases of loading, in which a part of the beam can be bent downward and a part of it is bent upward, a single actuator with application of electric voltage on this bounded area ceases to be effective. Numerically, based on the example of a plate with two actuators, it was shown that the maximum deflection can be decreased substantially by placing the actuators optimally and applying oppositely directed electric fields. When comparing the results in

cases of using a single actuator or double actuators of the same length, a reduction of 174% in the maximum deflection was observed.

The results of the investigation presented in [19] indicate that optimization of piezoelectric element configurations from the viewpoint of the geometry and characteristics of the electric field is an effective way to increase the performance of piezoactuators and must be a part of the development of adaptive structures for technical applications with restrictions on weight.

It should be noted that the efforts of researchers in the consideration of beams [22] or plates [23, 24] were directed as a rule at achieving their bending without regard to the possibility of obtaining a torsion effect. Versions of nonstandard arrangements of the piezoelectric element (in particular, at an angle to the axial line) were also not considered.

In this work, based on the example of a cantilevered fixed plate, we obtained results that demonstrate the influence of different arrangements of piezoelectric elements, their number, and different polarization electric voltages applied to them on structural deformation.

2. THE MATHEMATICAL STATEMENT OF THE PROBLEM

The geometric model of real objects in this work is a piecewise homogeneous body with a volume $V = V_1 + V_2$. In this body, the volume $V_1 = \sum_{k=1}^N V_1^k$ consists of N homogeneous electroelastic (piezoelectric) elements; the volume $V_2 = \sum_{l=1}^M V_2^l$, of M homogenous elastic elements. The total surface of the piecewise homogeneous body S bounding the volume V can be represented as a sum $S = S_u + S_\sigma + S_q + S_\phi + S_{p0}$. The sum includes surface portions on which the following quantities are specified: displacements u_i^0 (S_u), surface forces p_i (S_σ), charge density q_s (S_q), electric potential ϕ^0 (S_ϕ), and the part of the body surface not covered by electrodes (nonelectroded surface) S_{p0} .

Based on the principle of virtual displacements, Maxwell's differential equations in the quasi-static approximation, and the relations of linear elasticity theory, the variational equilibrium equation in the case of quasi-static deformation of a piecewise homogeneous electroelastic body with a volume V under the action of volumetric forces f_i and surface forces p_i [25–27] is formulated:

$$\begin{aligned} \sum_{n=1}^N \left(\int_{V_1^n} (\sigma_{ij} \delta \varepsilon_{ij} - D_i \delta E_i) dV - \int_{V_1^n} f_i \delta u_i dV \right) + \sum_{m=1}^M \left(\int_{V_2^m} (\sigma_{ij} \delta \varepsilon_{ij}) dV - \int_{V_2^m} f_i \delta u_i dV \right) \\ = \int_{S_\sigma^n} p_i \delta u_i dS + \int_{S_q^n} q_s \delta \phi dS + \int_{S_\phi^m} p_i \delta u_i dS. \end{aligned} \quad (1)$$

In Eq. (1), the following notation is used: D_i and E_i are components of the electric induction and electric field strength vectors; σ_{ij} are components of the symmetric Cauchy stress tensor; ε_{ij} are components of the linear strain tensor; u_i are components of the displacement vector; ρ_n and ρ_m are the specific densities of materials of the n -th and m -th components of the piecewise homogeneous body (materials of the volumes V_1^n and V_2^m , respectively); S_σ^n and S_ϕ^m are the parts of surfaces of the volumes V_1^n and V_2^m on which the surface forces p_i are specified; S_q^n is the portion of the surface of the volume V_1^n on which the surface charge density q_s is specified; and ϕ is the electric potential.

The relationship between the components of the displacement vector and strain tensor is described by the Cauchy differential relations

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \quad (2)$$

For the electric field, the potentiality condition is satisfied:

$$E_j = -\phi_{,j}. \quad (3)$$

The system of equations is closed by the physical relations for the electroelastic components:

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}^n \varepsilon_{kl} - \beta_{ijk}^n E_k, \\ D_i &= \beta_{ijk}^n \varepsilon_{jk} + \varepsilon_{ik}^n E_k, \end{aligned} \quad (4)$$

and for elastic components in the case of an isotropic material [28, 29],

$$\begin{aligned} \sigma_{ij} - \sigma\delta_{ij} &= 2G^{(m)}\left(\varepsilon_{ij} - \frac{1}{3}\theta\delta_{ij}\right), \\ \sigma &= B^{(m)}\theta, \end{aligned} \tag{5}$$

or anisotropic material [28, 29],

$$\sigma_{ij} = C_{ijkl}^m \varepsilon_{kl}. \tag{6}$$

Here, C_{ijkl}^n and C_{ijkl}^m are components of elastic constant tensors; $G^{(m)}$ and $B^{(m)}$ are the elastic shear and bulk moduli; σ is the mean stress; θ is the volumetric strain; and β_{ijk}^n and ε_{ik}^n are components of tensors of piezoelectric and dielectric coefficients.

The uniqueness of the solution is provided by the boundary conditions. When posing boundary value problems of electroelasticity at each point of the surface bounding the body it is necessary to specify the boundary conditions on the mechanical and electrical variables. Electric energy is delivered to a deformed piezoelectric body and taken from it using electrode coatings deposited on areas of the surface of the body (electroded surfaces). In what follows, we assume that electrode coatings are thin ideal conductors with a negligibly low mass. Covering of a portion of the piezoelectric body surface with a conducting layer makes it equipotential, i.e., the electric potential value is the same throughout the entire electroded surface.

The boundary conditions are stated in the following form:

$$u_i|_{S=S_u^n} = u_i^0, \tag{7}$$

$$\sigma_{ij}n_j|_{S=S_\sigma^n} = p_i, \tag{8}$$

$$\varphi|_{S=S_\varphi^n} = \varphi_0. \tag{9}$$

The potential φ is determined up to an additive constant. For this reason, it is taken that a zero potential is specified on a certain area of the electroded surface; φ_0 then has the meaning of the potential difference.

The posed problem was implemented numerically by the finite-element method using possibilities of the ANSYS proprietary application program package [30].

According to the finite-element method, the equilibrium equation for an electroelastic body (1) can be written in the following matrix form:

$$\begin{aligned} \sum_{n=1}^N \left[\int_{V_1^n} (\delta\{\varepsilon_1\}^T [D_1^n] \{\varepsilon_1\}) dV - \int_{V_1^n} \delta\{u^{V_1^n}\}^T \{f_1\} dV \right] + \sum_{m=1}^M \left[\int_{V_2^m} (\delta\{\varepsilon_2\}^T [D_2^m] \{\varepsilon_2\}) dV \right. \\ \left. - \int_{V_2^m} \delta\{u^{V_2^m}\}^T \{f_m^{V_2}\} dV \right] = \int_{S_\sigma^{V_1}} \delta\{u^{V_1}\}^T \{p_n^{V_1}\} dS + \int_{S_\sigma^{V_2}} \delta\{u^{V_2}\}^T \{p_m^{V_2}\} dS. \end{aligned} \tag{10}$$

Here, the following notation is used:

$$\begin{aligned} \{u^{V_k}\} &= \{u_1, u_2, u_3\}^T, \quad \{u^{V_2}\} = \{u_1, u_2, u_3, \varphi\}^T, \\ \{\varepsilon_1\} &= \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23}\}^T, \quad \{\varepsilon_2\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{12}, \varepsilon_{13}, \varepsilon_{23}, E_1, E_2, E_3\}^T, \\ \{\sigma_1\} &= \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}\}^T, \quad \{\sigma_2\} = \{\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{13}, \sigma_{23}, D_1, D_2, D_3\}^T, \\ \{\rho_k\} &= \{\rho_k, \rho_k, \rho_k\}^T, \quad \{\rho\} = \{\rho, \rho, \rho, 0\}. \end{aligned}$$

The behavior of electroelastic components of the volume V_1 and elastic components of the volume V_2 was modeled using 20-node solid finite elements in the form of tetrahedrons with quadratic approximation of nodal unknowns; in V_1 , the SOLID226 element; in V_2 , the SOLID186 element from the library of the ANSYS package. Each node of the SOLID226 element contains four nodal unknowns, the displacements u_i, v_i, w_i and the electric potential; each node of the SOLID186 element contains three nodal displacements u_i, v_i, w_i .

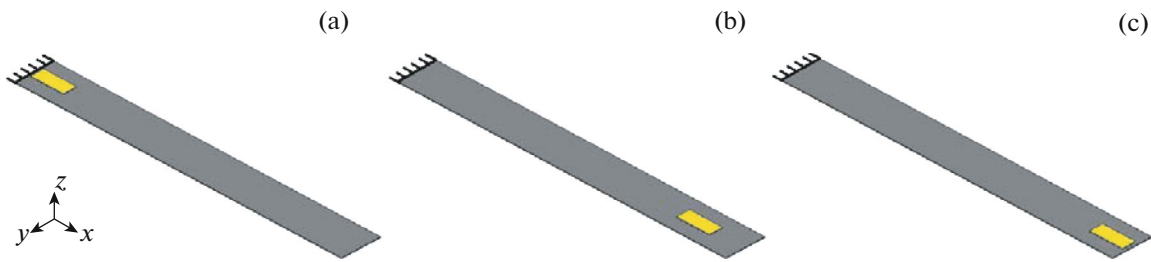


Fig. 1. The computation schemes for different positions of the piezoelectric element on the plate surface at $\alpha = 0$: (a) near from the fixed edge and (b, c) at different distances from the free edge.

Standard procedures of the finite-element method reduce problems described by Eq. (10) to systems of linear algebraic equations (SLAEs). These systems can be represented for the problem about quasi-static deformation in the matrix form as follows:

$$[K]\{\delta\} = \{F\}. \quad (11)$$

Using the presented mathematical statement, we obtained numerical results that demonstrate the possibility of control for the geometry of a structure by use of piezoelectric elements to electroded surfaces, on which an electric potential is applied.

3. NUMERICAL RESULTS

An elastic isotropic rectangular cantilevered aluminum plate with dimensions of 500×60 mm was the object of study. The mechanical characteristics of the plate material are $E = 7.1 \times 10^{10}$ Pa; $\nu = 0.31$; and $\rho = 2750$ kg/m³. The plate thickness was taken to be equal to 0.5 mm. Piezoelectric elements with dimensions of $50 \times 20 \times 0.3$ mm made of PZT-19 piezoceramics were attached to the plate surface. The upper and the under surface of the piezoelectric elements were electroded and the polarization axis coincided with the direction of the normal to the electroded surfaces. For the calculations, the material of the piezoelectric elements was assumed to be elastic. Its physico-mechanical characteristics were as follows: the elasticity moduli $C_{11} = C_{22} = 13.9 \times 10^{10}$ N/m², $C_{12} = 7.78 \times 10^{10}$ N/m², $C_{13} = C_{23} = 7.43 \times 10^{10}$ N/m², $C_{33} = 11.5 \times 10^{10}$ N/m², $C_{44} = 3.06 \times 10^{10}$ N/m², and $C_{55} = C_{66} = 2.56 \times 10^{10}$ N/m²; the piezoelectric coefficients $\beta_{31} = \beta_{32} = -5.2$ C/m², $\beta_{33} = 15.1$ C/m², and $\beta_{52} = \beta_{61} = 12.7$ C/m²; the dielectric coefficients $e_{11} = e_{22} = 6.45 \times 10^{-9}$ F/m, and $e_{33} = 5.62 \times 10^{-9}$ F/m; the density $\rho = 7700$ kg/m³; and the dielectric permeability of vacuum $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. This material was chosen as most typical and most widespread both in theoretical and applied investigations.

As an illustration of possibilities for the control for the geometry of a structure, the influence of the arrangement of piezoelectric elements, their number, and electrical voltage applied to electroded surfaces of the piezoelectric elements on plate displacements in the direction perpendicular to the plate surface U_z was studied.

Let us estimate the effect of the rotation angle (α) of the piezoelectric element about the longitudinal axis of symmetry of the plate on the value of displacements U_z of the free end of the plate for the following positions of a single piezoelectric element: its center of mass is at a distance of 30 mm from the fixed end of the plate (Fig. 1a); its center of mass is 70 (Fig. 1b) and 30 mm from the free end (Fig. 1c).

The lower electroded surface of the piezoelectric element is grounded (the electric potential on it has a zero value); on the upper electroded surface the value of the electric potential is 200 V. The results are presented in Table 1 and in Fig. 2.

Analyzing the distribution patterns of displacements U_z which are presented in Fig. 2, one can say that:

—arrangement of the piezoelectric element in the region of fixation allows one to use it most effectively for changing the shape of the structure because such a position provides the maximum values of the displacements of the free edge of the plate;

—the farther the piezoelectric element is from the fixation point, the stronger the observed effect of torsion of the transverse cross section is when its orientation (angle of rotation) changes relative to the axes of symmetry of the plate.

Table 1. The maximum values U_z^{\max} as functions of the angle of rotation of the piezoelectric element relative to the rolling axis of symmetry of the plate in different positions

α , deg	Position of the piezoelectric element		
	Fig. 1a	Fig. 1b	Fig. 1c
	U_z^{\max} , mm		
0	0.925	0.124	0.056
30	0.844	0.120	0.060
45	0.758	0.109	0.056
60	0.674	0.096	0.050
90	0.586	0.074	0.036

Therefore, these results make it possible to conclude that the transverse bending of the plate can be controlled by a displacement of the piezoelectric element along the axis of symmetry of the plate and its torsion can be formed using the angle of rotation (α) of the piezoelectric element with respect to the longitudinal axis of symmetry of the plate.

Further, we consider the possibility to control the geometry of a structure due to arrangement of several piezoelectric elements. In this section, it is shown how the shape of the initial object can be changed depending on the number of piezoelectric elements arranged on the surface of the object and different combinations of controlling actions (an electric signal applied to the piezoelectric elements).

The first series of numerical experiments was carried out with two piezoelectric elements. The piezoelectric elements were placed at equal distances from the plate edges parallel to each other and their long sides were oriented along the longitudinal axis of symmetry of the plate. The centers of mass of both piezoelectric elements were at a distance of 30 mm from the fixation. An electric signal $\varphi_1 = \varphi_2 = +200$ V was applied to both piezoelectric elements. Here, φ_1 and φ_2 are the potential differences applied to the first and second piezoelectric element, respectively. In this process, pure bending was observed; however, the bending magnitude U_z was 1.3 mm, which is larger than with the use of a single piezoelectric element by a factor of 1.4.

In further calculations, the configuration of piezoelectric elements remained the same as in the previous series but the characteristics of the controlling electrical action were varied. For calculations of this series, it was taken that signals applied to each of the piezoelectric elements have different polarities (a positive electric potential was applied to one piezoelectric element and a negative one to the other).

Figure 3 shows the deformation patterns according to the following combinations of controlling signals applied to the piezoelectric elements: the electric potential at the first piezoelectric element was constant and equal to $\varphi_1 = +200$ V; an electric potential equal to -200 (Fig. 3a), -199 (Fig. 3b), -195 (Fig. 3c), -170 (Fig. 3d), and -150 V (Fig. 3e) was successively applied to the second piezoelectric element.

These results indicate that applying an electric signal with different polarity to both piezoelectrical elements when using two elements allows one to achieve different degrees on plate torsion. In this process, depending on the relation between magnitudes of the signals, one can control both the torsion angle and the position of the torsion axis.

In the case where the piezoelectric elements are displaced relative to the fixation but their positions relative to each other are preserved, torsion also appears in a certain local region in a neighborhood of the piezoelectric element (Fig. 3f, parameters of the controlling signal were $\varphi_1 = +200$ V and $\varphi_2 = -200$ V).

In the next series of calculations, two piezoelectric elements were initially positioned parallel to each other and the centers of mass were distant from the fixation area by 30 mm. One element remained in its place and the second one was displaced along its longitudinal axis of symmetry parallel to the longitudinal

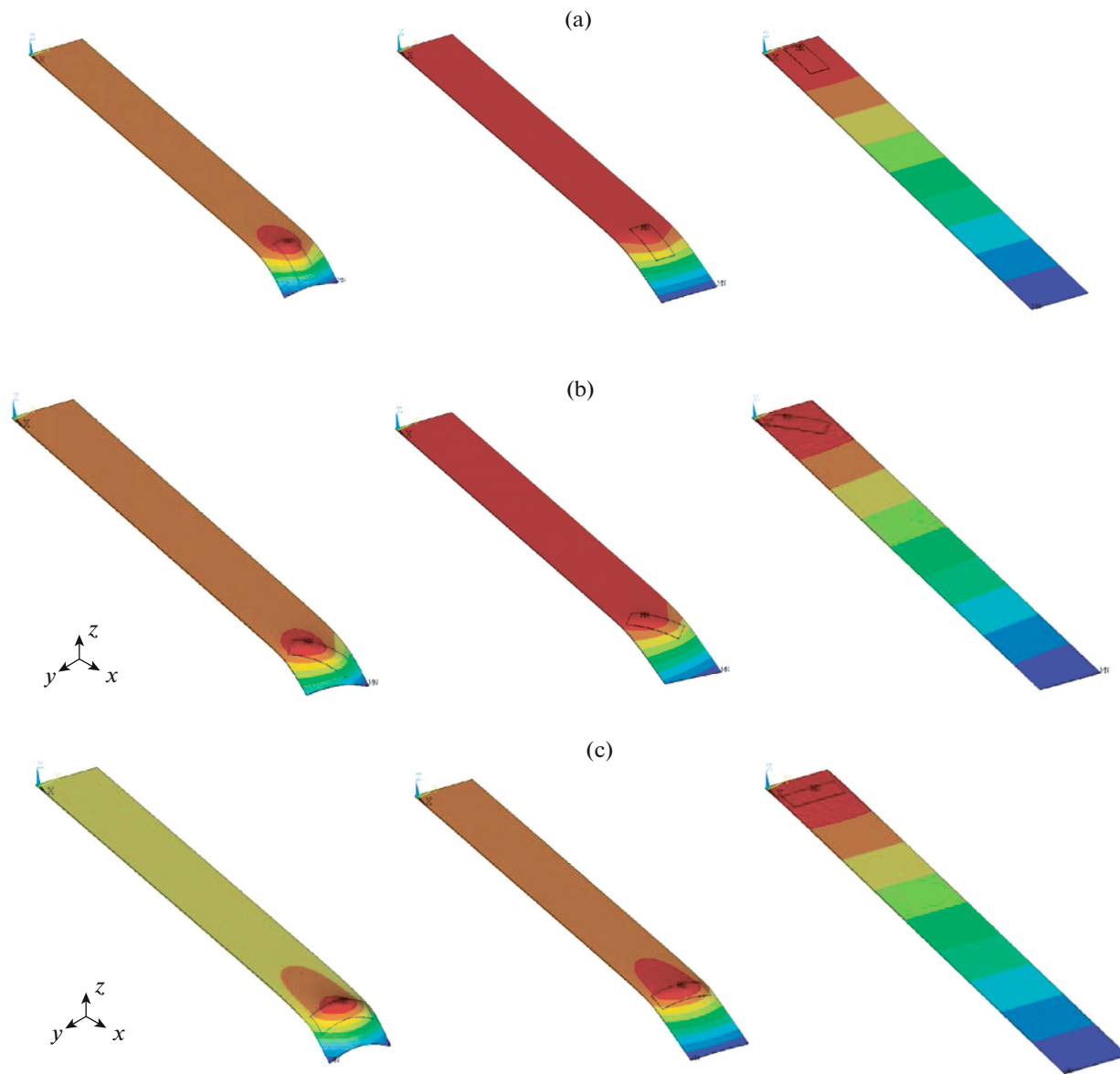


Fig. 2. The distribution of the displacements U_z depending on the position of the center of mass of the piezoelectric element at different values of the angle α , deg: (a) 0, (b) 45, and (c) 90.

axis of symmetry of the plate. For the controlling signal, an electric potential of different polarity but similar in its modulus was applied to the piezoelectric elements: $\varphi_1 = +200$ V and $\varphi_2 = -200$ V.

Figure 4 presents the plate deformation patterns for three versions of the arrangement relative to fixation of the second piezoelectric element: its center of mass is spaced from the fixation by 80 (Fig. 4a), 130 (Fig. 4b), and 250 mm (Fig. 4c). The presented deformation patterns indicate that when the piezoelectric elements are not parallel to each other and the electric signal applied to them has different polarity but is similar in its modulus, the shape of the obtained deformed structure is more complicated; it is a combination of bending and torsion.

This series of numerical experiments demonstrates that one can obtain more complicated shapes of the deformed structure when using several piezoelectric elements via their different arrangements.

Let us consider this based on the example of the generation of shapes of the transverse bending of a plate using three and five piezoelectric elements that are positioned one after another on the longitudinal axis of the plate. Figure 5 presents the plate deformation patterns when controlling the shape with the use

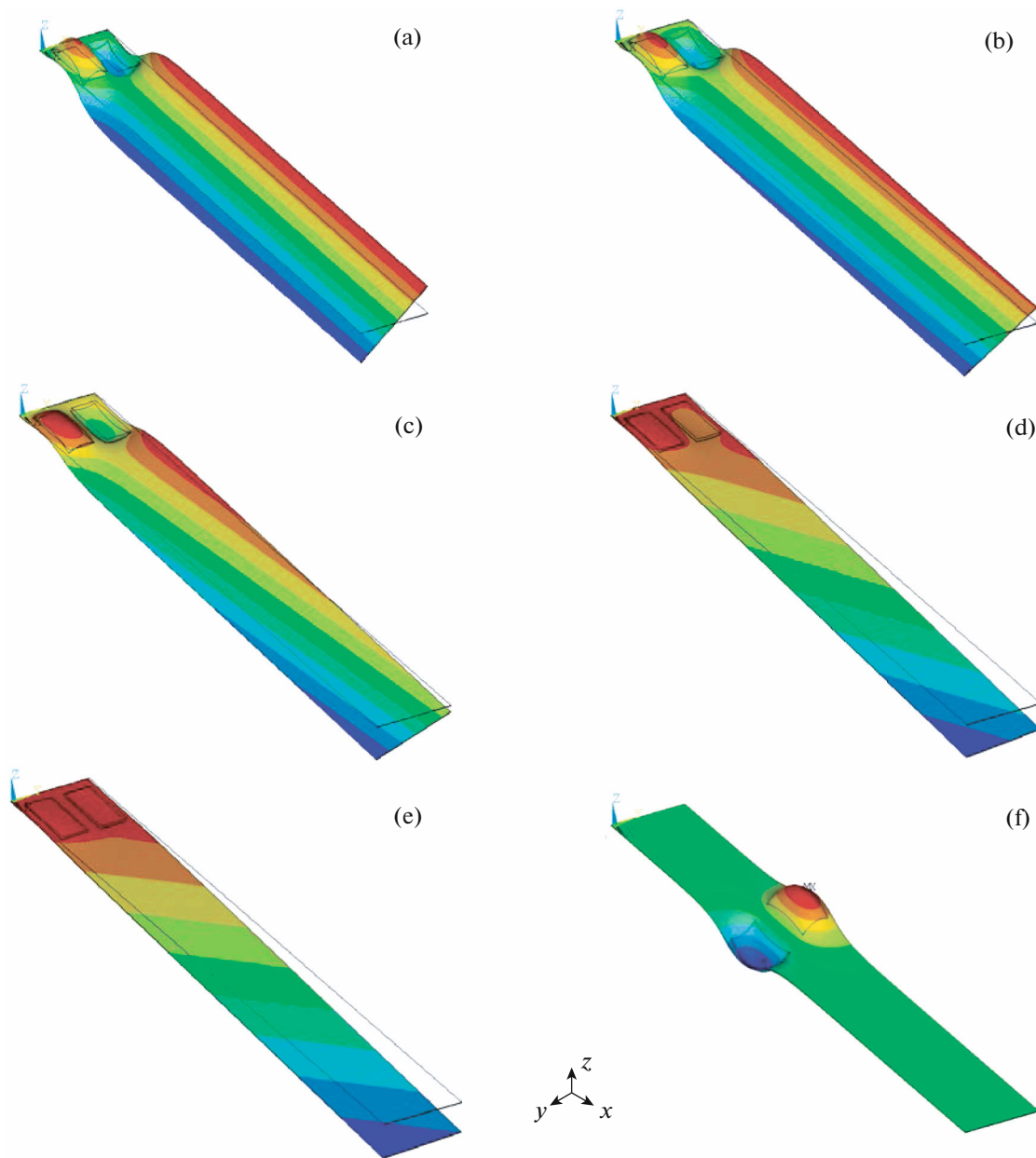


Fig. 3. The patterns of plate deformation corresponding to the following control signals applied to the piezoelectric elements: (a) $\varphi_1 = +200$ V, $\varphi_2 = -200$ V; (b) $\varphi_1 = +200$ V, $\varphi_2 = -199$ V; (c) $\varphi_1 = +200$ V, $\varphi_2 = -195$ V; (d) $\varphi_1 = +200$ V, $\varphi_2 = -170$ V; (e) $\varphi_1 = +200$ V, $\varphi_2 = -150$ V, the centers of mass of the piezoelectric elements are displaced by 30 mm from the fixation (a–e); and (f) $\varphi_1 = +200$ V, $\varphi_2 = -200$ V, the centers of mass of the piezoelectric elements are displaced by 200 mm from the fixation.

of three piezoelectric elements. The controlling signal was specified as follows: $\varphi_1 = \varphi_3 = +100$ V (elements 1 and 3) and $\varphi_2 = -200$ V (element 2). As a result, a local deflection in the middle of the plate was obtained for this configuration.

When using five piezoelectric elements, the goal was to obtain two plates areas that were symmetric in their displacements U_z . As a result of the performed computational experiments, it was found that obtaining the required shape required application of a controlling signal with the following characteristics: $\varphi_1 = \varphi_5 = +100$ V, $\varphi_2 = \varphi_4 = -200$ V, and $\varphi_3 = 200$ V. As a result, the pattern of plate bending presented in Fig. 6 was obtained.

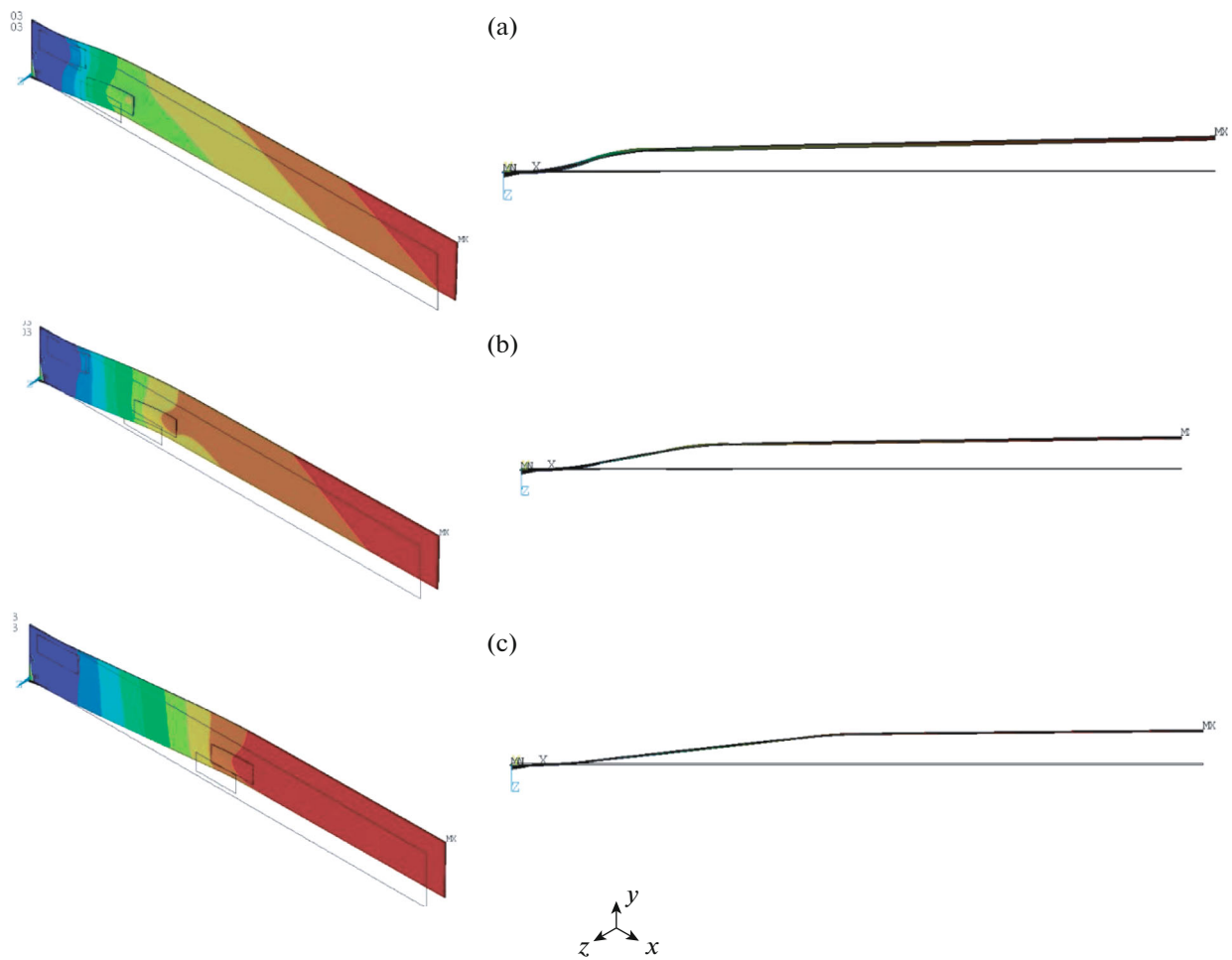


Fig. 4. The shapes of the deformed plate and the view in the (xz) plane for different versions of displacement of one of the piezoelectric elements relative to the fixation, mm: (a) 80, (b) 130, and (c) 250.

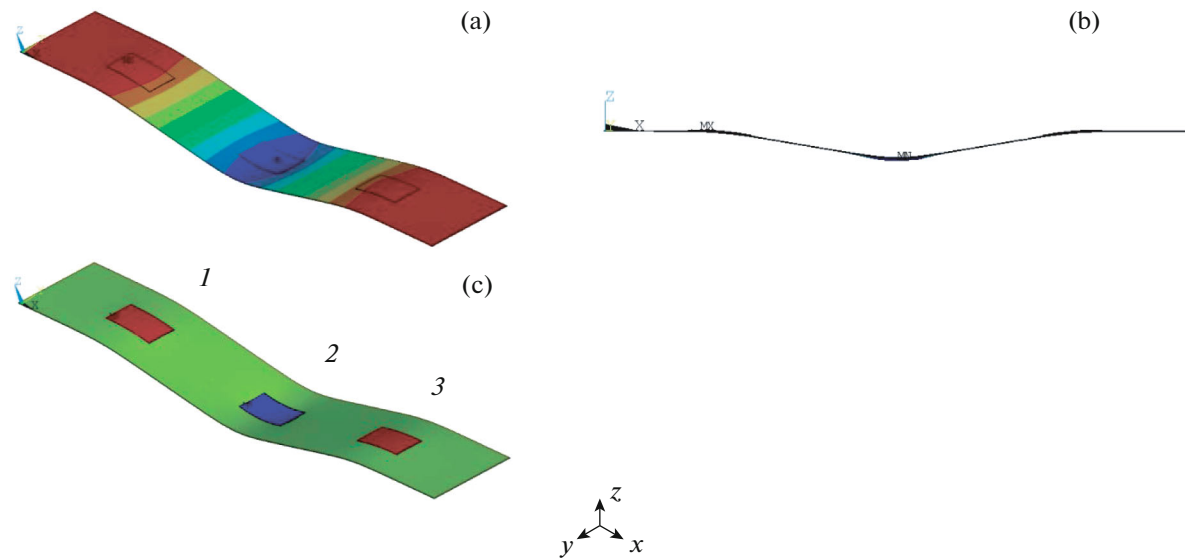


Fig. 5. The patterns of structural deformation when modeling bending using three piezoelectric elements: (a) isometric view and (b) view in the (xz) plane; (c) the scheme of applying the control action to the piezoelectric elements.

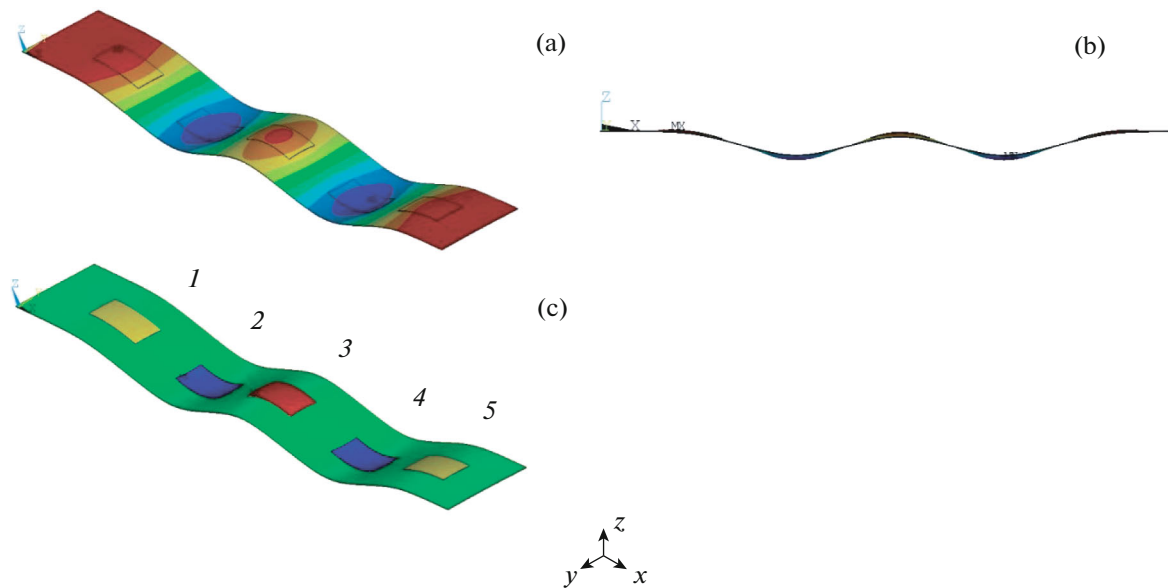


Fig. 6. The patterns of structural deformation when modeling bending using five piezoelectric elements: (a) isometric view and (b) view in the (xz) plane; (c) the scheme of applying the control action to the piezoelectric elements.

4. CONCLUSIONS

In this work, based on the example of a cantilevered fixed plate, the influence of piezoelectric elements on its deformation in different arrangements relative to the main structure and to each other has been demonstrated, as well as the influence of a varying electric potential applied to them. It has been shown that bending, torsion, and a complex deformed state that is a combination of the bending with torsion can occur depending on the combination of these factors.

These investigations can be useful both when using a form change to ensure the stability of the geometry of an item under the action of different loads and when using active strategies for control of the dynamic behavior of constructions by forming a prestress state leading to changes in the shape of oscillations of a structure at given modes.

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