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FREE VIBRATIONS OF HETEROGENEOUS ORTHOTROPIC CYLINDRICAL SHELLS REINFORCED BY ANNULAR RIBS AND FILLED BY FLUID

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Abstract: Natural vibrations of heterogeneous orthotropic cylindrical shells reinforced by annular ribs and filled with fluid are studied. Dependences of a frequency response on various geometric and physical parameters of the problem are described.

Keywords: free vibrations, reinforced inhomogeneous orthotropic cylindrical shell, fluid filling the shell.

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Shell structures are widely used in construction and engineering. The main permanent load on a shell is its weight. Shell strength can be improved by using light porous materials with low bulk density, but these materials have low strength. To compensate for this drawback, technological heterogeneity is developed. In designing shells, the choice of material is of great importance. Recently, various promising materials have appeared, particularly those obtained using nanotechnologies. Porous aluminum with the addition of polymer, carbon, or metal particles is widely used as a shell material. Heterogeneity is created in the supporting structures by adding another high-strength material to the surface layers of the material by means of diffusion or other technologies. Thus, there is technological heterogeneity in the structure with a clearly manifested front separating hardened and unhardened materials.

There is a need to develop methods for calculating the stress–strain state of heterogeneous shells and study the effect of heterogeneity on the frequencies of their own vibrations. Algorithms are required for determining resonant frequencies, leading to the fracture of the initial and hardened materials of inhomogeneous shells.

When using polymeric materials in engineering practice, particularly fiberglass, one should account for the anisotropy of elastic properties in studying low-frequency vibrations of shells. The thin-walled part of the shell can be made more rigid by reinforcement with ribs, which significantly improves its strength along with the mass of the structure.

It should be noted that, if a shell has geometric and physical nonlinearity, the equations describing its stress–strain state are nonlinear differential equations in partial derivatives. These equations are solved in [1] by a method of successive loads [2, 3]. Both the error caused by linearizing the equation and the calculation time are reduced by a two-step method of successive parameter perturbation [4]. The impact of the conditions of mounting along the contour on the stability of polymer concrete shells is described in [5]. In [6], the Hamilton–Ostrogradsky variational principle is used to study the free vibrations of a longitudinally supported orthotropic cylindrical shell of heterogeneous thickness, which is in contact with a moving fluid. The study of parametric vibrations in a viscoelastic medium of a rectilinear rod made of a nonlinear material of heterogeneous thickness using the Pasternak contact

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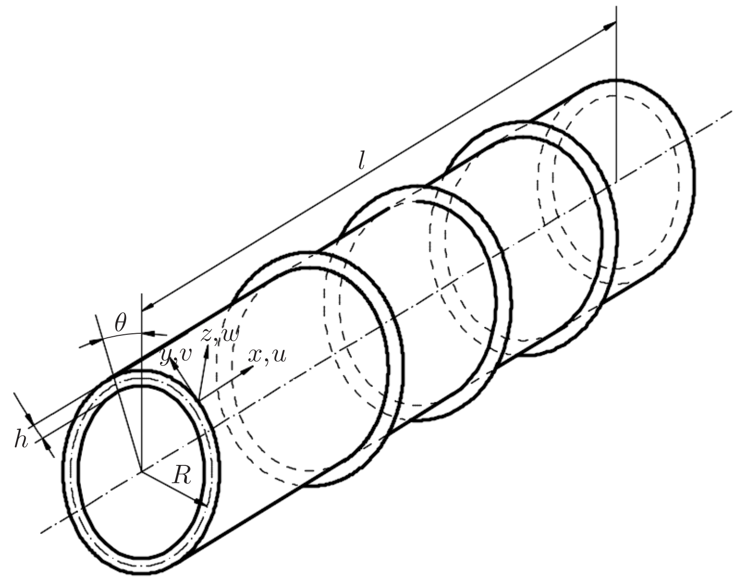

Fig. 1. Reinforced heterogeneous cylindrical shell filled with fluid.

model is given in [7–9]. The influence of the main factors (elastic foundation, damageability of the rod and shell materials, the dependence between a shear coefficient and a vibration frequency on the characteristics of longitudinal vibrations of the rod points in a viscoelastic medium) is investigated. In all the cases studied, the dependences of the size and position of the region of dynamic stability of the rod vibrations in a viscoelastic medium on the structural parameters in the load–frequency plane are constructed. In [10], the frequencies of free vibrations of a cylindrical anisotropic homogeneous fiberglass shell reinforced by annular ribs and filled with a fluid in the presence of Navier boundary conditions are determined. The results of calculating the natural frequency of vibrations are presented in the form of dependences of this value on the angle of fiberglass winding for the shell made of fiberglass fabric and on the fluid velocity for various values of the wave formation parameter and various ratios of geometric parameters of the shell.

This paper is devoted to studying the frequency of natural vibrations of an inhomogeneous orthotropic cylindrical fiberglass shell reinforced by annular ribs and filled with a fluid in the presence of Navier boundary conditions. The results of calculating the natural vibration frequency are presented in the form of dependences of this quantity on the fluid velocity and the number of reinforcing elements for various values of wave formation and various ratios of elastic moduli.

FORMULATION OF THE PROBLEM

In solving the problem, we apply the Hamilton–Ostrogradsky variational principle. For this purpose, we write the expression for the total energy of the structure under study, which consists of an inhomogeneous cylindrical shell and reinforcing annular elements whose number varies. The inner surface of the structure is in contact with the moving fluid, and it is assumed that the fluid entirely fills the inner region of the shell (Fig. 1)

Inhomogeneity in the thickness of the cylindrical shell is accounted for by using a three-dimensional functional. In this case, the expression for the functional of the total energy of the cylindrical shell has the form

$$V = \frac{1}{2} \iint \int_{-h/2}^{h/2} \Big(\sigma_{11}\varepsilon_{11} + \sigma_{22}\varepsilon_{22} + \sigma_{12}\varepsilon_{12} + \rho\Big(\Big(\frac{\partial u}{\partial t}\Big)^2 + \Big(\frac{\partial v}{\partial t}\Big)^2 + \Big(\frac{\partial w}{\partial t}\Big)^2\Big)\Big)\, dx\, dy\, dz, \tag{1}$$

where u, v, and w are the displacement vector components of the points of the middle surface of the shell along the generatrix, in the tangential direction, and along the normal to the middle surface, respectively, h is the shell thickness, ε_{11}, ε_{22}, and ε_{12} denote the deformations of the middle surface of the shell, ρ is the density of the shell material, σ_{11}, σ_{22}, and σ_{12} are the stresses in the shell, and t is the time.

There are various ways to account for the heterogeneity of the shell material. One of these methods is as follows: Young's modulus and the density of the shell material are assumed to be functions of the normal and longitudinal coordinates [11]. It is considered that Poisson's ratio is constant. In this case, the strain–stress ratio has the form

$$\sigma_{11} = b_{11}(z,x)\varepsilon_{11} + b_{12}(z,x)\varepsilon_{22}, \quad \sigma_{22} = b_{12}(z,x)\varepsilon_{11} + b_{22}(z,x)\varepsilon_{22}, \quad \sigma_{12} = b_{66}(z,x)\varepsilon_{12}; \tag{2}$$

$$\varepsilon_{11} = \frac{\partial u}{\partial x}, \qquad \varepsilon_{22} = \frac{\partial v}{\partial y} + w, \qquad \varepsilon_{12} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \tag{3}$$

It is assumed that

$$\begin{gathered} b_{11}(z,x) = \tilde{b}_{11} f_1(z) f_2(x), \qquad b_{22}(z,x) = \tilde{b}_{22} f_1(z) f_2(x), \qquad b_{12}(z,x) = \tilde{b}_{12} f_1(z) f_2(x), \\ b_{66}(z,x) = \tilde{b}_{66} f_1(z) f_2(x), \qquad \rho(z,x) = \tilde{\rho} f_1(z) f_2(x). \end{gathered} \tag{4}$$

Here $\tilde{b}_{11} = E_1/(1-\nu_1\nu_2)$, $\tilde{b}_{22} = E_2/(1-\nu_1\nu_2)$, $\tilde{b}_{12} = \nu_1 E_1/(-\nu_1\nu_2) = \nu_2 E_2/(1-\nu_1\nu_2)$, and $\tilde{b}_{66} = G$ denote the main elastic moduli of the homogeneous orthotropic material of the shell, $\tilde{\rho}$ is the density of the material of the homogeneous shell, $f_1(z)$ and $f_2(x)$ are the heterogeneity functions in the direction to the normal and along the generatrix of the shell, respectively, ν_1 and ν_2 denote Poisson's ratios, and E_1 and E_2 stand for Young's moduli of the shell material in the direction of the x and y axes, respectively.

In view of system (4), Eqs. (2) yield

$$\begin{gathered} \sigma_{11} = (\tilde{b}_{11}\varepsilon_{11} + \tilde{b}_{12}\varepsilon_{22}) f_1(z) f_2(x), \qquad \sigma_{22} = (\tilde{b}_{12}\varepsilon_{11} + \tilde{b}_{22}\varepsilon_{22}) f_1(z) f_2(x), \\ \sigma_{12} = \tilde{b}_{66}(z,x)\varepsilon_{12} f_1(z) f_2(x). \end{gathered} \tag{5}$$

An expression for the functional of the total energy of the cylindrical shell with account for Eqs. (5) has the form

$$\begin{aligned} V = \frac{1}{2} \int\limits_{-h/2}^{h/2} f_1(z)\,dz \iint (\tilde{b}_{11}\varepsilon_{11}^2 + 2\tilde{b}_{12}\varepsilon_{11}\varepsilon_{22} + \tilde{b}_{22}\varepsilon_{22}^2 + \tilde{b}_{66}\varepsilon_{12}^2) f_2(x)\,dx\,dy \\ + \int\limits_{-h/2}^{h/2} f_1(z)\,dz \iint \tilde{\rho}\Big(\Big(\frac{\partial u}{\partial t}\Big)^2 + \Big(\frac{\partial v}{\partial t}\Big)^2 + \Big(\frac{\partial w}{\partial t}\Big)^2\Big) f_2(x)\,dx\,dy. \end{aligned} \tag{6}$$

Expressions for the potential energy of elastic strain of the jth transverse rib are written as

$$\begin{aligned} \Pi_j = \frac{1}{2} \int\limits_0^{2\pi R} \Big[\tilde{E}_j F_j \Big(\frac{\partial v_j}{\partial y} - \frac{w_j}{R}\Big)^2 + \tilde{E}_j J_{yj} \Big(\frac{\partial^2 w_j}{\partial x^2} + \frac{w_j}{R^2}\Big)^2 \\ + \tilde{E}_j J_{zj} \Big(\frac{\partial^2 u_j}{\partial y^2} - \frac{\varphi_j}{R}\Big)^2 + \tilde{G}_j J_{\mathrm{tor},j} \Big(\frac{\partial \varphi_{\mathrm{tor},j}}{\partial y} + \frac{1}{R}\frac{\partial u_j}{\partial y}\Big)\Big]\,dy, \end{aligned} \tag{7}$$

and expressions for the kinetic energy of the rib have the form

$$K_j = \rho_j F_j \int\limits_0^{2\pi R} \Big[\Big(\frac{\partial u_j}{\partial t}\Big)^2 + \Big(\frac{\partial v_j}{\partial t}\Big)^2 + \Big(\frac{\partial w_j}{\partial t}\Big)^2 + \frac{J_{\mathrm{tor},j}}{F_j}\Big(\frac{\partial \varphi_{\mathrm{tor},j}}{\partial t}\Big)^2\Big]\,dy. \tag{8}$$

In Eqs. (7) and (8), F_j, J_{zj}, J_{yj}, and $J_{\mathrm{tor},j}$ denote the area and the moments of inertia of the cross section of the jth transverse rod respectively relative to the z axis and the axis that is parallel to the y axis and passing through the cross-sectional center of gravity, as well as its moment of inertia during torsion; $\tilde{E}_j$ and $\tilde{G}_j$ are the elastic and shear moduli of the material of the jth transverse rod, respectively; ρ_j is the density of the material of which the jth transverse rod is made; u_j, v_j, and w_j are the displacement vector components of the axis points of the jth transverse rod; φ_j and $\varphi_{\text{tor},j}$ are the angles of rotation and torsion of the cross section of the jth rod, which are expressed via the displacement of the shell as follows:

$$\varphi_j(y) = \varphi_2(x_j, y) = -\Big(\frac{\partial w}{\partial y} + \frac{v}{r}\Big)\Big|_{x=x_j}, \qquad \varphi_{\text{tor},j}(y) = \varphi_1(x_j, y) = -\frac{\partial w}{\partial x}\Big|_{x=x_j}.$$

The potential energy of external surface loads acting from the side of an ideal fluid to the shell is defined as the work performed by these loads when the system transfers from a deformed state to an initial undeformed state:

$$A_0 = -\int_0^l \int_0^{2\pi R} q_z w \, dx \, dy. \tag{9}$$

The total energy of the system is equal to the sum of the energy of elastic strains of the shell and all transverse ribs, as well as the potential energies of external loads acting from the side of an ideal fluid:

$$J = V + \sum_{j=1}^{k_2} (\Pi_j + K_j) + A_0 \tag{10}$$

(k_2 is the number of transverse ribs).

Radial pressure acting from the side of the fluid on the shell wall is determined by solving the wave equation written with respect to the perturbation velocity potential φ [12, 13]:

$$\Delta\varphi - \frac{1}{a_0^2}\Big(\frac{\partial^2\varphi}{\partial t^2} + 2U\,\frac{\partial^2\varphi}{R\,\partial x\,\partial t} + U^2\frac{\partial^2\varphi}{\partial x^2}\Big) = 0. \tag{11}$$

Here a_0 denotes the sound propagation velocity in the fluid, and U is the fluid flow velocity.

The expression for the total energy of system (10) and the fluid flow equation (11) are supplemented by contact conditions.

On the shell–fluid contact surface, the continuity condition of radial velocities and pressures is fulfilled. The impermeability condition of the shell wall has the form [12]

$$v_r\Big|_{r=R} = \frac{\partial\varphi}{\partial r}\Big|_{r=R} = -\Big(\omega_0\,\frac{\partial w}{\partial t_1} + U\,\frac{\partial w}{R\,\partial\xi}\Big), \tag{12}$$

where v_r is the radial velocity of the fluid points, $\omega_0 = \sqrt{E_1/[(1-\nu_1^2)R^2\tilde{\rho}]}$, and $t_1 = \omega_0 t$.

The equality condition of external surface loads and the radial pressure acting from the side of the fluid on the shell wall is written as

$$q_z = -p\Big|_{r=R}, \tag{13}$$

where pressure p is determined via potential φ according to the following expression [12]:

$$p = -\rho_m\Big(\frac{\partial\varphi}{\partial t} + U\,\frac{\partial\varphi}{\partial x}\Big) \tag{14}$$

(ρ_m is the fluid density). It is believed that the conditions of hard contact between the shell and the rods are fulfilled:

$$\begin{gathered} u_j(y) = u(x_j, y) + h_j\varphi_1(x_j, y), \qquad v_j(y) = v(x_j, y) + h_j\varphi_2(x_j, y), \qquad w_j(y) = w(x_j, y), \\ \varphi_j(y) = \varphi_2(x_j, y), \qquad \varphi_{\text{tor},j}(y) = \varphi_1(x_j, y), \qquad h_j = 0.5h + H_j^1. \end{gathered} \tag{15}$$

Here H_j^1 is the distance from the axes of the jth rod to the surface of the cylindrical shell, h_j is the thickness of the jth transverse rod. It is assumed that, on lines $x = 0$ and $x = l$, the following Navier boundary conditions are satisfied:

$$v = 0, \qquad w = 0, \qquad T_{11} = 0, \qquad M_{11} = 0. \tag{16}$$

Here l is the shell length, T_{11} and M_{11} are the forces and moments acting in the cross sections of the cylindrical shell.

The problem of natural vibrations of the heterogeneous cylindrical shell reinforced by annular ribs and filled with fluid is reduced to joint integration of the expressions for the total energy of system (10), the fluid flow equations (11) with the conditions (13) and (15) on their contact surface and boundary conditions (16).

SOLVING THE PROBLEM OF NATURAL VIBRATIONS OF THE SHELL

First, the radial pressure p acting from the side of the fluid on the shell wall is determined.

The perturbation velocity potential φ is sought in the form

$$\varphi(\xi, r, \theta, t_1) = f(r)\cos(n\theta)\sin(\chi\xi)\sin(\omega_1 t_1), \tag{17}$$

where $\omega_1 = \omega/\omega_0$, $\chi = kR = m\pi R/l$ and n are wave numbers in the longitudinal and circumferential directions, respectively, $\xi = x/R$, and ω is the desired frequency.

With account for the condition (12), we use Eq. (17) to obtain

$$\varphi = -\frac{f(r)}{f'(R)}\Big(\omega_0\,\frac{\partial w}{\partial t_1} + U\,\frac{\partial w}{R\,\partial\xi}\Big). \tag{18}$$

The unknown function $f(r)$ is determined by substituting the solution of Eq. (17) into Eq. (11). Thus, there is a Bessel equation whose solutions are the Bessel functions. Depending on $M_1 = (U + \omega_0\omega_1/k)/a_0$, different functions become the solution of the Bessel equation: the modified Bessel functions of the first and second kind of the order n $I_n(\beta_2 r)$ and $-K_n(\beta_2 r)$ for $M_1 < 1$, the Bessel functions of the first and second kind of the order n $J_n(\beta_1 r)$ and $Y_n(\beta_1 r)$ for $M_1 > 1$ ($\beta_1^2 = -\beta_2^2 = R^{-2}(M_1^2 - 1)k^2$), and functions r^n and r^{-n} for $M_1 = 1$. In all the given cases, if $r = 0$, the modified Bessel function $K_n(\beta_2 r)$ and the Bessel function $Y_n(\beta_1 r)$ have a specific feature. Therefore, the Bessel equations are solved using the modified Bessel function of the order n $[I_n(\beta_2 r)]$, the Bessel function of the first kind of the order n $[J_n(\beta_1 r)]$, and function r^n. We introduce the denotation

$$\varphi_{\alpha n} = \begin{cases} I_n(\beta_2 r)/I_n'(\beta_2 R), & M_1 < 1, \\ J_n(\beta_1 r)/J_n'(\beta_1 R), & M_1 > 1, \\ r^n/(nR^{n-1}), & M_1 = 1. \end{cases} \tag{19}$$

Then, expressions (14) and (18) are written as

$$\varphi = -\varphi_{\alpha n}\Big(\omega_0\,\frac{\partial w}{\partial t_1} + U\,\frac{\partial w}{R\,\varphi\xi}\Big),$$

$$p = \tilde{\varphi}_{\alpha n}\rho_m\Big(\omega_0^2\,\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\,\frac{\partial^2 w}{R\,\partial\xi\,\partial t_1} + U^2\,\frac{\partial^2 w}{R^2\,\partial\xi^2}\Big), \tag{20}$$

where $\tilde{\varphi}_{\alpha n} = \varphi_{\alpha n}|_{r=R}$.

In expression (10), the varied quantities are u, v, and w. Their unknown values are approximated as follows:

$$u = u_0\cos(\chi\xi)\cos(n\theta)\sin(\omega_1 t_1), \qquad v = v_0\sin(\chi\xi)\sin(n\theta)\sin(\omega_1 t_1),$$

$$w = w_0\sin(\chi\xi)\cos(n\theta)\sin(\omega_1 t_1). \tag{21}$$

Here u_0, v_0, and w_0 are the unknown constants. When calculating energy J in expression (10), we accept the dependences

$$f_1(z) = 1 + \alpha z/l, \qquad f_2(x) = 1 + \beta x/l, \tag{22}$$

where α and β denote the constant heterogeneity parameters in the direction along the normal and along the generatrix of the shell, respectively, while $\alpha \in [0, 1]$ and $\beta \in [0, 1]$.

The solution of Eqs. (21) are substituted into Eq. (10), and, with account for expressions (22) and (10), a second-order polynomial with respect to constants u_0, v_0, and w_0 are obtained:

$$J = (\varphi_{11}u_0^2 + \varphi_{22}v_0^2 + \varphi_{33}w_0^2 + \varphi_{44}u_0v_0 + \varphi_{55}u_0w_0 + \varphi_{66}v_0w_0)\sin^2(\omega t).$$

Expressions for φ_{11}, φ_{22}, φ_{33}, φ_{44}, φ_{55}, and φ_{66} are cumbersome, so they are omitted in this paper.

The frequency equation of the ribbed heterogeneous shell filled with fluid is obtained on the basis of the Hamilton–Ostrogradsky stationary action principle

$$\delta W = 0, \tag{23}$$

where $W = \int_{t'}^{t''} J\,dt$ is the Hamiltonian action, and t' and t'' are the set arbitrary instances.

As the expression of W is varied with respect to u_0, v_0, and w_0 and the coefficients for independent variations are equated to zero, we obtain

$$\begin{gathered} 2\varphi_{11}u_0 + \varphi_{44}v_0 + \varphi_{55}w_0 = 0, \\ \varphi_{44}u_0 + 2\varphi_{22}v_0 + \varphi_{66}w_0 = 0, \\ \varphi_{55}u_0 + \varphi_{66}v_0 + 2\varphi_{33}w_0 = 0. \end{gathered} \tag{24}$$

As system (24) is a homogeneous system of linear algebraic equations, a necessary and sufficient condition for the existence of its nonzero solution is the equality of its principal determinant to zero. As a result, we have the frequency equation

$$\begin{vmatrix} 2\varphi_{11} & \varphi_{44} & \varphi_{55} \\ \varphi_{44} & 2\varphi_{22} & \varphi_{66} \\ \varphi_{55} & \varphi_{66} & 2\varphi_{33} \end{vmatrix} = 0, \tag{25}$$

which can be written as

$$4\varphi_{11}\varphi_{22}\varphi_{33} + \varphi_{44}\varphi_{55}\varphi_{66} - \varphi_{55}^2\varphi_{22} - \varphi_{66}^2\varphi_{11} - \varphi_{44}^2\varphi_{33} = 0. \tag{26}$$

ANALYZING THE RESULTS OF NUMERICAL CALCULATIONS

Equation (26) is solved numerically with the following parameter values: $\tilde{b}_{11} = 18.3$ GPa, $\tilde{b}_{12} = 2.77$ GPa, $\tilde{b}_{22} = 25.2$ GPa, $\tilde{b}_{66} = 3.5$ GPa, $\tilde{\rho} = \rho_j = 1850$ kg/m^3, $\tilde{E}_j = 6.67 \cdot 10^9$ N/m^2, $m = 1$, $h_j = 1.39$, $R = 160$ cm, $\alpha = 0.4$, $I_{\text{tor},j} = 0.48$ mm^4, $I_{xj} = 19.9$ mm^4, $F_j = 0.45$ mm^2, $h = 0.45$ mm, $\nu_1 = \nu_2 = 0.35$, $l/R = 3$, $h/R = 1/6$, $a_0 = 1450$ m/s, and $\rho_m = 1000$ kg/m^3.

Figures 2–5 show the dependences of the frequency on the number of reinforcing rods k_2 on the shell surface, the heterogeneity in the direction of the generatrix of the shell β, the fluid velocity U, and the wave number n in the circumferential direction, respectively, for different values of the ratio of the elastic moduli of the shell material.

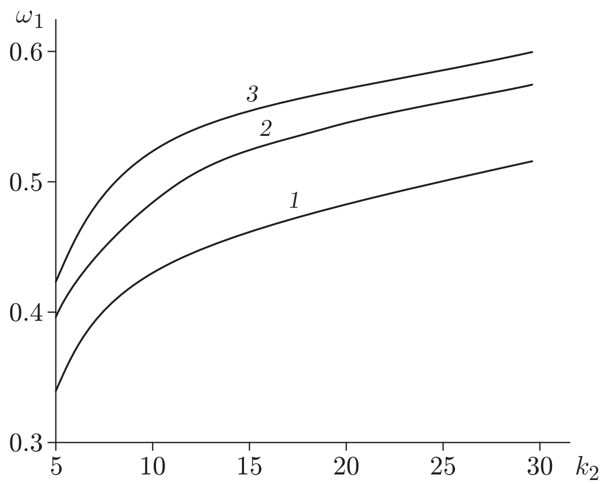

Fig. 2.

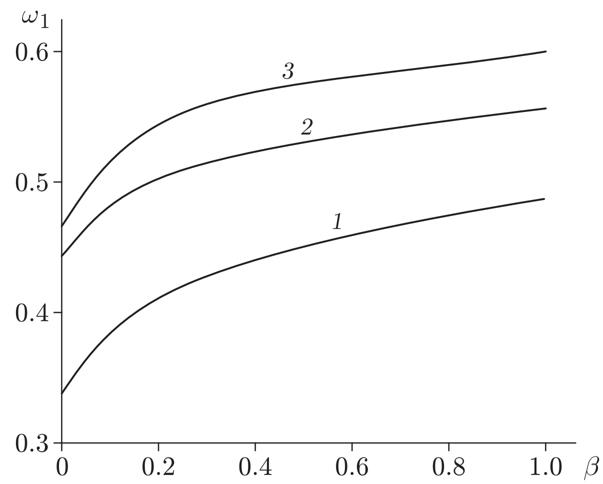

Fig. 3.

Fig. 2. Frequency versus the number of reinforcing rods k_2 for $n = 8$, $\beta = 0.6$, $U/a_0 = 0.1$ and $E_1/E_2 = 0.75$ (*1*), 1.0 (*2*), and 1.25 (*3*).

Fig. 3. Frequency versus the heterogeneity in the direction of the generatrix of the shell β for $n = 8$, $\alpha = 0.6$, $U/a_0 = 0.1$, $k_2 = 15$ and $E_1/E_2 = 0.75$ (*1*), 1.0 (*2*), and 1.25 (*3*).

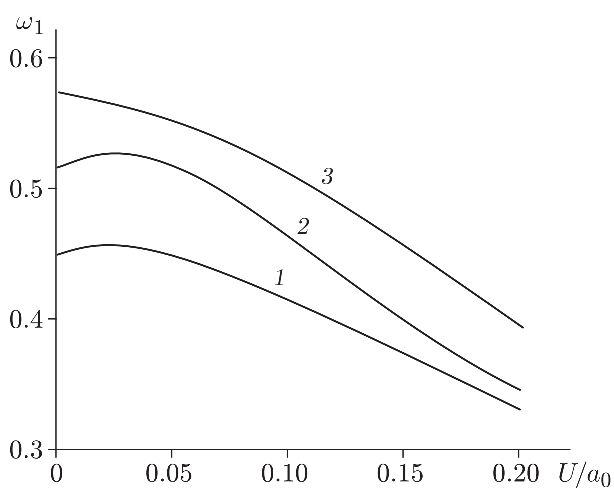

Fig. 4.

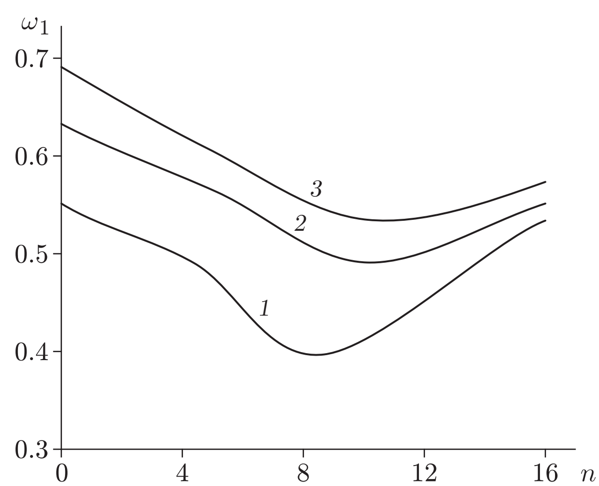

Fig. 5 .

Fig. 4. Frequency versus the fluid velocity U/a_0 for $n = 8$, $\beta = 0.4$, $k_2 = 15$ and $E_1/E_2 = 0.75$ (*1*), 1.0 (*2*), and 1.25 *3*).

Fig. 5. Frequency versus the wave number n in the circumferential direction for $\beta = 0.4$, $k_2 = 15$, and $E_1/E_2 = 0.75$ (*1*), 1.0 (*2*), and 1.25 (*3*).

Figure 2 shows that, as the number of transverse ribs increases, so does the frequency value. It follows from Fig. 3 that, as the heterogeneity increases in the direction of the generatrix of the shell β, so does the frequency value. Moreover, the frequency value becomes larger along with an increase in ratio E_1/E_2 and decreases with an increase in the fluid velocity (see Fig. 4). It can be seen in Fig. 5 that, as the number of waves n in the circumferential direction becomes larger, the system vibration frequency becomes smaller initially and then begins to increase upon reaching a minimum.

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