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THEORY OF LARGE DEFORMATIONS OF METALS

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Abstract: This paper presents the theory of irreversible deformations that allows one to analyze large deformations of metals and determine the characteristics of the stress-strain state, deformation damage, and structural characteristics at various structural levels.

Keywords: plasticity, creep, long-term strength, dislocation density, large and intense deformations.

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INTRODUCTION

Mathematical simulation is widely used today in developing the manufacturing technology of forging and stamping [1, 2].

In the middle of the 20th century, the mathematical theory of plasticity [3, 4] and the deformability theory [5, 6] were developed, which made it possible to formulate and solve initial-boundary-value problems of plastic shaping under small deformations and simple loading.

Most forging and stamping processes and modern plastic structure formation processes (obtaining metal with a grain size of the order of 10^{-6} m) occur under large deformations ($\varepsilon > 0.2$) and intense deformations ($\varepsilon > 3.0$) [7]. The development of the physical theory of large and intense deformations of metals is touched upon in [8]. When the mathematical theory of small deformations (flow theory) is generalized for the case of large (finite) elastoplastic and elastoviscoplastic deformations, the main problem is to separate the large strain tensor into a reversible (elastic) and irreversible (plastic) components [9–13]. Under the conditions of complex loading of elastoviscoplastic material, it turns out to be quite difficult to correctly separate elastic from irreversible components.

The mathematical theory of large and intense deformations is developed using a structural and phenomenological approach [14–18]. In strength and plasticity physics, the value of plastic deformation induced by dislocation motion and structural changes is unambiguously determined:

$$\varepsilon_p = \lambda\rho b$$

(λ refers to the mean free path of mobile dislocations, ρ denotes the scalar density of the dislocations that passed through a volume under consideration, and b stands for the Burgers vector modulus of dislocations).

The purpose of this paper is describe the main provisions of the physical-mathematical theory of large and intense deformations.

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1. MATHEMATICAL MODEL OF LARGE AND INTENSE IRREVERSIBLE DEFORMATIONS OF METALS

The original system of equations comprises a motion equation and the kinetic equations of structural characteristics (scalar densities of immobile dislocations and microcrack nuclei):

$$\frac{d\varepsilon}{dt} = \dot{\varepsilon}_* \exp\Big(-\frac{\beta G b^3 - \sigma b^2/(m\sqrt{\rho_s}\,)}{kT}\Big),$$

$$\frac{d\rho_s}{dt} = \rho_g \nu_{gs} - \rho_s \nu_{s0}, \qquad \frac{dN_m}{dt} = \xi_0 \rho_s \nu_{sm} - N_m \nu_{md}, \qquad \nu_{s0} = \nu_0 \exp\Big(-\frac{U - \sigma V_{s0}/m}{kT}\Big), \tag{1}$$

$$\nu_{sm} = \nu_{sm}^0 \exp\Big(-\frac{U_{sm} - \sigma V_{sm}/m}{kT}\Big), \qquad \nu_{md} = \nu_{md}^0 \exp\Big(-\frac{U_{md} - pV_{md}/M}{kT}\Big),$$

$$\dot{\varepsilon} = \rho_g b v, \qquad v/\nu_{gs} = \lambda.$$

Here t denotes time, $\dot{\varepsilon}_* = 10^{12}$ s^{-1} is the constant, $\beta G b^3$ is the self-diffusion activation energy, T is the temperature, $m = 3.1$ is the coefficient, ρ_s is the immobile dislocation density, k is the Boltzmann constant, ρ_g is the density of mobile dislocations, ν_{gs} is the transformation frequency of mobile dislocations into immobile ones, ν_{s0} is the frequency of vanishing of immobile dislocations, N_m is the scalar density of strain microcracks, $\xi_0 = 10^{-5}$ is the coefficient, ν_{sm} is the frequency of transformation of a set of stationary dislocations into a microcrack, ν_{md} is the "healing" frequency of microcracks, and G is the shear modulus.

The application of a numerical step-by-step method for solving the nonlinear system (1) allows solving nonstationary problems of large and intense deformations with account for the loading history and structural evolution.

An algorithm for solving the initial-boundary-value problem of irreversible shaping using the proposed theory is described below, including the determination of the stress-strain state and the prediction of strain damage and material structure at different structural levels.

2. PROBLEM SOLUTION ALGORITHM

System (1) is supplemented with relations between strain $d\varepsilon = \dot{\varepsilon}\, dt$, strain rate $\dot{\varepsilon} = d\varepsilon/dt$, and time $dt = d\varepsilon/\dot{\varepsilon}$, thereby obtaining the mathematical models of plasticity [17], creep [14], long-term strength [16], and viscous fracture, respectively.

The law of the flow of a hardened viscoplastic material, which is general for the above-mentioned processes and obtained using the generalized Mises maximum principle, has the following form [17, 18]:

$$d\varepsilon_{ij(g)} = \frac{3}{2}\,\frac{d\varepsilon_{(g)}}{\sigma^T_{(g)} + d\sigma^u_{(g)} - d\sigma^r_{(g)}}\,(s^T_{ij(g)} + ds^u_{ij(g)} - ds^r_{ij(g)}) \tag{2}$$

[subscript $g = 1, 2, 3, \ldots, n$ refers to the number of the computation step in the numerical solution of system (1), (2), at which the increment of irreversible deformations acquires a small but finite value $d\varepsilon_{(g)} = 0.002$–$0.020$]. Here

$$\sigma^T_{(g)} = \Big(\beta m G b - \frac{kT_{(g)} m}{b^2} \ln \frac{\dot{\varepsilon}_* b \sqrt{\rho_{s(g-1)}}}{\dot{\varepsilon}_{(g)}}\Big)\sqrt{\rho_{s(g-1)}}$$

is the yield limit of the material at step g, β is the coefficient in the expression for the self-diffusion activation energy $\beta G b^3$, $m = 2.9$–3.1, $T_{(g)}$ is the thermodynamic temperature at the computation step g, $\dot{\varepsilon}_{(g)}$ is the plastic strain rate at the computation step g, $\rho_{s(g-1)}$ is the scalar density of immobile dislocations at the computation step $g-1$,

$$d\sigma^u_{(g)} = \frac{\beta m G b}{2\sqrt{\rho_{s(g)}}\, b\lambda}\, d\varepsilon_{(g)}$$

is the increment in the stress intensity at the computation step g, induced by the strain hardening, and

$$d\sigma^r_{(g)} = \Big\{\frac{\beta m G b^2 \rho^2_{s(g)} \nu_D}{2\dot{\varepsilon}_{(g)}} \exp\Big(-\frac{\beta G b^3 - \sigma_{(g-1)} b^2/(m\sqrt{\rho_{s0}}\,)}{kT_{(g)}}\Big)$$

$$+ \frac{mkT_{(g)}}{2b^3\lambda\sqrt{\rho_{s(g)}}}\Big(1 + \ln\frac{\dot{\varepsilon}_* b\sqrt{\rho_{s(g)}}}{\dot{\varepsilon}_{(g)}}\Big)\Big[1 - \frac{\rho^2_{s(g)} b^2 \nu_D \lambda \sqrt{\rho_{s(g)}}}{\dot{\varepsilon}_{(g)}} \exp\Big(-\frac{\beta G b^3 - \sigma_{(g-1)} b^2/(m\sqrt{\rho_{s0}}\,)}{kT_{(g)}}\Big)\Big]\Big\}\, d\varepsilon_{(g)}$$

is the increment of the stress intensity at the computation step g, induced by thermodynamic softening; $s^T_{ij(g)}$, $ds^u_{ij(g)}$, and $ds^r_{ij(g)}$ denote the deviators of the tensors $\sigma^T_{ij(g)}$, $d\sigma^u_{ij(g)}$, and $d\sigma^r_{ij(g)}$, respectively.

When solving the initial-boundary-value problems related to the irreversible deformations, it is reasonable to use three equations [see Eq. (2)]:

$$d\varepsilon_{ij(g)} = \frac{3}{2}\,\frac{d\varepsilon_{(g)}}{\sigma^T_g}\, s^T_{ij(g)};$$

$$d\varepsilon_{ij(g)} = \frac{3}{2}\,\frac{d\varepsilon_{(g)}}{d\sigma^u_{(g)}}\, ds^u_{ij(g)}; \qquad (3)$$

$$d\varepsilon_{ij(g)} = \frac{3}{2}\,\frac{d\varepsilon_{(g)}}{d\sigma^r_{(g)}}\, s^r_{ij(g)}.$$

The problem formulation starts with using Eq. (3) because the relationship $\sigma^u(\varepsilon)$ describing the strain hardening always monotonically increases and the local plasticity modulus $d\sigma^u_{(g)}/d\varepsilon_{(g)}$ required for the numerical solution of the problem is always positive.

An increase in the scalar density of dislocations at the loading computation step g in the microvolume under consideration is determined by the expression

$$\rho_{s(g)} = \rho_{s(g-1)} + d\rho_{s(g)},$$

where

$$d\rho_{s(g)} = \Big[\frac{1}{b\lambda} - \frac{\rho^{3/2}_{s(g-1)} \nu_D b}{\dot{\varepsilon}_{(g)}} \exp\Big(-\frac{\beta G b^3 - \sigma_{(g-1)} b^2/(m\sqrt{\rho_{s0}}\,)}{kT_{(g)}}\Big)\Big] d\varepsilon_{(g)}$$

is the increment of the scalar density of dislocations at the computation step g and $\rho_{s(g-1)}$ is the scalar density of dislocations, accumulated for $g-1$.

The linear grain size of the material after deformation is estimated using the equation

$$d_{(g)} = B/\sqrt{\rho_{s(g)}}\,,$$

where $B = 10$.

The degree of strain damage (possibility of microfracture in a microvolume) is calculated as follows:

$$\psi_{(g)} = N_{m(g)}/N^*_{(g)}.$$

Here $N_{m(g)} = N_{m(g-1)} + dN_{m(g)}$ denotes the scalar density of microcracks at the computation step g, $N_{m(g-1)}$ is the microcrack density accumulated for $g-1$ steps, and $dN_{m(g)}$ is the increment of the scalar density of microcracks at the computation step g:

$$dN_{m(g)} = \Big\{\xi_0 \rho_{s(g)} - N_{m(g-1)} \exp\Big[-\frac{\beta G b^3}{kT_{(g)}}\Big(1 + \frac{K_{(g)}}{M}\Big)\Big]\Big\} d\varepsilon_{(g)}.$$

Here $N^*_{(g)}$ is the critical density of microcracks, at which they are combined into a macrocrack: $N^*_{(g)} = 10^7$ cm^{-2} for $K_{(g)} = \sigma_{0(g)}/\sigma_{(g)} < -2.5$ (compressive stresses are dominant), $N^*_{(g)} = 10^6$ cm^{-2} for $K_{(g)} > 0.58$ (tensile stresses are dominant), or $N^*_{(g)} = -60.2532 \cdot 10^4 K^3_{(g)} - 3 \cdot 10^6 K^2_{(g)} + 8 \cdot 10^6$ for $K_{(g)} \in [-2.50; 0.58]$); $\sigma_{0(g)} = \sigma_{ii(g)}/3$ is the average normal stress.

Criteria for macrofracture and deformation without macrofracture have respective forms

$$\psi_{(g)} = N_{m(g)}/N^*_{(g)} = 1.0, \qquad \psi_{(g)} = N_{m(g)}/N^*_{(g)} < 1.0.$$

CONCLUSIONS

The use of the theory of irreversible deformations of metals in this paper allows for a detailed analysis of the large and intense nonmonotonous deformations under nonstationary complex loading. In this case, it is possible to determine the stress-strain state characteristics and the possibility of a macrofracture under a certain degree of strain damage and for a linear grain size.

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