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PLANE HARMONIC WAVES IN THERMOELASTIC MATERIALS WITH A MEMORY-DEPENDENT DERIVATIVE

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Abstract: The Lord–Shulman theory of generalized thermoelasticity based on a memory-dependent derivative is employed to study the propagation of plane harmonic waves in a two-dimensional semi-infinite thermoelastic medium. The numerical solution is analyzed for various values of the time delay parameter.

Keywords: Lord–Shulman model, memory-dependent derivative, time delay, kernel function, displacement potential.

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INTRODUCTION

Considerable interest in fractional calculus (mathematical analysis based on fractional derivatives) has been inspired by applications in various areas of physics and engineering [1]. It is well known that fractional calculus is still in infancy, although it has been invented for three centuries. Recently, some efforts have been applied to modify the classical Fourier law of heat conduction by using fractional calculus [2–5].

The memory-dependent derivative is defined in an integral form of a common derivative with a kernel function. The kernels in physical laws are important in many models that describe physical phenomena including the memory effect. Wang and Li [6] introduced the concept of a memory-dependent derivative (MDD). Yu et al. [7] introduced the MDD instead of fractional calculus into the rate of the heat flux in the Lord–Shulman (LS) theory of generalized thermoelasticity [8] to denote memory dependence and established a memory-dependent LS model. This novel model [7] might be beneficial to the fractional models owing to the following arguments. First, the new model is unique in its form, while the fractional-order models have different modifications (Riemann–Liouville, Caputo, and other models). Second, the physical meaning of the new model is more clear due to the essence of the MDD definition. Third, the new model is depicted by integer-order differentials and integrals, which is more convenient in numerical calculation as compared to the fractional models. Finally, the kernel function and time delay of the MDD can be arbitrarily chosen; thus, the model is more flexible in applications than the fractional models, in which the significant variable is the fractional-order parameter [2–5]. Some solutions of one-dimensional problems obtained with the use of the memory-dependent LS model of generalized thermoelasticity can be found in [9–12].

The goal of the present work is to solve a two-dimensional problem of propagation of plane harmonic waves in a two-dimensional semi-infinite thermoelastic medium.

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1. MATHEMATICAL MODEL

The system of the governing differential equations for a homogenous and isotropic generalized thermoelastic material described by the MDD model [8–11] consists of the stress–strain relation

$$\sigma_{ij} = 2\mu e_{ij} + (\lambda e_{kk} - \gamma\theta)\delta_{ij}, \tag{1}$$

equation of motion

$$\sigma_{ij,j} = \rho\ddot{u}_i, \tag{2}$$

and heat conduction equation

$$k_1\nabla^2\theta = (1 + \varkappa D_\varkappa)(\rho C_E\dot{\theta} + \gamma T_0\dot{e}). \tag{3}$$

In Eqs. (1)–(3), σ_{ij} are the stress tensor components, $e_{ij} = (u_{i,j} + u_{j,i})/2$ are the strain tensor components, u_i are the components of the displacement vector $\boldsymbol{u}$, λ and μ are the Lamé constants, $\gamma = (3\lambda + 2\mu)\alpha_T$ is a material constant characteristic of the theory, α_T is the coefficient of linear thermal expansion, δ_{ij} is the Kronecker delta, ρ is the mass density, $k_1 > 0$ is the thermal conductivity, C_E is the specific heat at constant strain, $\theta = T - T_0$ is the small temperature increment, T is the medium temperature, T_0 is the reference temperature, $|\theta/T_0| \ll 1$, and $e = e_{kk}$ is the cubical dilatation.

For the differentiable function $f(t)$, Wang and Li [6] introduced the first-order MDD with respect to the time delay $\varkappa > 0$ for a fixed time t:

$$D_\varkappa f(t) = \frac{1}{\varkappa}\int\limits_{t-\varkappa}^{t} K(t-\xi)f'(\xi)\,d\xi.$$

The choice of the kernel function $K(t-\xi)$ and the time delay parameter $\varkappa$ is determined by the material properties. The kernel function $K(t-\xi)$ is differentiable with respect to the variables t and ξ. The motivation for such a new definition is that it provides more insight into the memory effect (the instantaneous change rate depends on the past state) and also better physical meaning, which might be superior to the fractional models.

This kind of the definition can reflect the memory effect on the delay interval $[t-\varkappa, t]$, which varies with time. They also suggested that the kernel form $K(t-\xi)$ can also be chosen freely, e.g., as 1, $\xi - t + 1$, $[(\xi - t)/\varkappa + 1]^2$, $[(\xi - t)/\varkappa + 1]^{1/4}$, etc. The kernel function can be understood as the degree of the past effect on the present. Therefore, the forms $[(\xi - t)/\varkappa + 1]^2$ and $[(\xi - t)/\varkappa + 1]^{1/4}$ may be more practical because they are monotonic functions: $K(t-\xi) = 0$ for the past time $t - \varkappa$ and $K(t-\xi) = 1$ for the present time t, i.e., it is easily concluded that the kernel function $K(t-\xi)$ is a monotonic function increasing from zero to unity with time. The right side of the MDD definition given above can be understood as a mean value of $f'(\xi)$ on the past interval $[t-\varkappa, t]$ with different weights. Generally, from the viewpoint of applications, the function $K(t-\xi)$ should satisfy the inequality $0 \leqslant K(t-\xi) < 1$ for $\xi \in [t-\varkappa, t]$. Therefore, the magnitude of the MDD $D_\varkappa f(t)$ is usually smaller than that of the common derivative $f'(t)$. It can also be noted that the common derivative d/dt is the limit of $D_\varkappa$ as $\varkappa \to 0$. Following [6, 9–12], the kernel function $K(t-\xi)$ is taken here in the form

$$K(t-\xi) = 1 - \frac{2b}{\varkappa}(t-\xi) + \frac{a^2}{\varkappa^2}(t-\xi)^2 = \begin{cases} 1, & a = 0, \quad b = 0, \\ 1 + (\xi - t)/\varkappa, & a = 0, \quad b = 1/2, \\ \xi - t + 1, & a = 0, \quad b = \varkappa/2, \\ [1 + (\xi - t)/\varkappa]^2, & a = 1, \quad b = 1, \end{cases} \tag{4}$$

where a and b are constants. It should be also mentioned that the kernel in the fractional sense is singular, while that in the MDD model is nonsingular. The kernel can be now simply considered as a memory manager [6, 9, 10]. The comma is further used to indicate the derivative with respect to the space variable, and the superimposed dot represents the time derivative.

2. FORMULATION OF THE PROBLEM

We consider an isotropic and homogeneous thermoelastic solid occupying the half-space $x \geqslant 0$. We also assume that the bounding surface of the half-space is traction-free and is subjected to a time-dependent thermal shock. For two-dimensional deformation, all the considered functions depend only on the space variables x and y and on the time variable t. Hence, the displacement vector $\boldsymbol{u}$ has the components $u \equiv u(x, y, t)$, $v \equiv v(x, y, t)$, and $w \equiv w(x, y, t) = 0$.

Under the above-made assumptions, Eqs. (1)–(3) are simplified to

$$\sigma_{xx} = (\lambda + 2\mu)u_{,x} + \lambda\, v_{,y} - \gamma\theta, \quad \sigma_{yy} = \lambda u_{,x} + (\lambda + 2\mu)\, v_{,y} - \gamma\theta, \quad \sigma_{xy} = \mu(v_{,x} + u_{,y}),$$

$$(\lambda + 2\mu)u_{,xx} + (\lambda + \mu)v_{,xy} + \mu u_{,yy} - \gamma\theta_{,x} = \rho\ddot{u},$$

$$(\lambda + 2\mu)v_{,yy} + (\lambda + \mu)u_{,xy} + \mu v_{,xx} - \gamma\theta_{,y} = \rho\ddot{v},$$

$$k_1(\theta_{,xx} + \theta_{,yy}) = \rho C_E \frac{\partial\theta(x,t)}{\partial t} + \gamma T_0 \frac{\partial e(x,t)}{\partial t} \tag{5}$$

$$+ \int\limits_{t-\varkappa}^{t} K(t-\xi)\Big(\rho C_E \frac{\partial^2\theta(x,\xi)}{\partial\xi^2} + \gamma T_0 \frac{\partial^2 e(x,\xi)}{\partial\xi^2}\Big)\, d\xi.$$

In the dimensionless variables

$$(x', y', u', v') = c_1\eta(x, y, u, v), \quad t' = c_1^2\eta t, \quad \sigma'_{ij} = \frac{\sigma_{ij}}{\rho c_1^2}, \quad \theta' = \frac{\gamma}{\rho c_1^2}\,\theta, \quad \eta = \frac{\rho C_E}{k_1},$$

Eqs. (5) take the following form (the primes at the dimensionless variables are omitted for convenience):

$$\sigma_{xx} = u_{,x} + (1 - 2\beta^2)v_{,y} - \theta, \quad \sigma_{yy} = v_{,y} + (1 - 2\beta^2)u_{,x} - \theta, \quad \sigma_{xy} = \beta^2(v_{,x} + u_{,y}); \tag{6}$$

$$u_{,xx} + (1 - \beta^2)v_{,xy} + \beta^2 u_{,yy} - \theta_{,x} = \ddot{u}, \qquad v_{,yy} + (1 - \beta^2)u_{,xy} + \beta^2 v_{,xx} - \theta_{,y} = \ddot{v},$$
$$\theta_{,xx} + \theta_{,yy} = (1 + \varkappa D_\varkappa)(\dot{\theta} + \varepsilon\dot{e}). \tag{7}$$

Here $\beta^2 = \mu/(\lambda + 2\mu)$, $\varepsilon = \gamma^2 T_0/[\rho C_E(\lambda + 2\mu)]$, and $c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ is the speed of the longitudinal waves.

Let us introduce the displacement potentials ϑ and ψ:

$$u = \vartheta_{,x} + \psi_{,y}, \qquad v = \vartheta_{,y} - \psi_{,x}, \qquad e = \nabla^2\vartheta. \tag{8}$$

It follows from Eqs. (7) and (8) that

$$\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2}\Big)\vartheta - \theta = 0; \tag{9}$$

$$\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{1}{\beta^2}\frac{\partial^2}{\partial t^2}\Big)\psi = 0; \tag{10}$$

$$\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\partial}{\partial t}\,(1 + \varkappa D_\varkappa)\Big)\theta - \varepsilon\,\frac{\partial}{\partial t}\,(1 + \varkappa D_\varkappa)\nabla^2\vartheta = 0. \tag{11}$$

Clearly, Eqs. (9) and (11) are coupled in terms of ϑ and θ, while Eq. (10) is uncoupled in terms of the potential ψ.

3. PLANE HARMONIC WAVES

The expressions for plane harmonic waves are sought in the form

$$[\vartheta, \psi, \theta, u, v, \sigma_{ij}](x, y, t) = [\hat{\vartheta}, \hat{\psi}, \hat{\theta}, \hat{u}, \hat{v}, \hat{\sigma}_{ij}](x)\, \mathrm{e}^{im(y-ct)}, \tag{12}$$

where $\hat{\vartheta}(x)$, … are the amplitudes of the corresponding variables, i is the imaginary unit, $c = \omega/m$ is the phase velocity, $\omega > 0$ is the assigned angular frequency, and m is the wave number.

Substituting Eq. (12) into Eqs. (9)–(11), we obtain

$$\Big(\frac{d^2}{dx^2} - m^2 - \omega^2\Big)\hat{\vartheta}(x) - \hat{\theta}(x) = 0; \tag{13}$$

$$\Big(\frac{d^2}{dx^2} - m^2 - \frac{\omega^2}{\beta^2}\Big)\hat{\psi}(x) = 0; \tag{14}$$

$$\Big(\frac{d^2}{dx^2} - m^2 + i\omega(1+\Gamma)\Big)\hat{\theta}(x) + i\varepsilon\omega(1+\Gamma)\Big(\frac{d^2}{dx^2} - m^2\Big)\hat{\vartheta}(x) = 0, \tag{15}$$

where Γ is a constant depending on the time delay parameter $\varkappa$, angular frequency ω, and constants a and b. The constant $\Gamma(\varkappa, \omega, a, b)$ for the kernel function (4) is computed with the help of the Mathematica software.

After some simple transformations, Eqs. (13) and (15) can be written as

$$\frac{d^2\hat{\theta}}{dx^2} = \alpha_1\hat{\theta} + \alpha_2\hat{\vartheta}, \qquad \frac{d^2\hat{\vartheta}}{dx^2} = \beta_1\hat{\theta} + \beta_2\hat{\vartheta}, \tag{16}$$

where $\alpha_1 = m^2 - i\omega(1+\varepsilon)(1+\Gamma)$, $\alpha_2 = i\omega^3(1+\Gamma)$, $\beta_1 = 1$, and $\beta_2 = m^2 - \omega^2$.

Equations (16) are written in the vector–matrix form as follows [13]:

$$\frac{d\boldsymbol{V}}{dx} = A\boldsymbol{V}, \qquad \boldsymbol{V} = \begin{pmatrix} \hat{\theta} \\ \hat{\vartheta} \\ d\hat{\theta}/dx \\ d\hat{\vartheta}/dx \end{pmatrix}, \qquad A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & 0 & 0 \\ \beta_1 & \beta_2 & 0 & 0 \end{pmatrix}. \tag{17}$$

The matrix A has the eigenvalues λ_1, $-\lambda_1$, λ_2, and $-\lambda_2$, where

$$\lambda_j^2 = \frac{\alpha_1 + \alpha_2 + (-1)^{j+1}\sqrt{(\alpha_1+\alpha_2)^2 - 4(\alpha_1\beta_2 - \alpha_2\beta_1)}}{2}, \quad \operatorname{Re}(\lambda_j) > 0, \quad j = 1, 2. \tag{18}$$

Let $\boldsymbol{X}_{(\lambda)}$ be an eigenvector corresponding to the eigenvalue of the matrix A. Then, from [13], we find

$$\boldsymbol{X}_{(\lambda)} = \begin{pmatrix} \lambda^2 - \beta_2 \\ 1 \\ \lambda(\lambda^2 - \beta_2) \\ \lambda \end{pmatrix}. \tag{19}$$

The general solution of Eq. (17) is written in the form

$$\boldsymbol{V}(x) = C_1\boldsymbol{X}_2\,\mathrm{e}^{-\lambda_1 x} + C_2\boldsymbol{X}_4\,\mathrm{e}^{-\lambda_2 x}, \qquad x \geqslant 0, \tag{20}$$

where the vectors $\boldsymbol{X}_2 = \boldsymbol{X}_{(-\lambda_1)}$ and $\boldsymbol{X}_4 = \boldsymbol{X}_{(-\lambda_2)}$ can be obtained from Eq. (19).

Using Eqs. (14), (17)–(20), one can write the solutions for the functions $\hat{\theta}(x)$, $\hat{\vartheta}(x)$, and $\hat{\psi}(x)$ as

$$\hat{\theta}(x) = (\lambda_1^2 - \beta_2)C_1\,\mathrm{e}^{-\lambda_1 x} + (\lambda_2^2 - \beta_2)C_2\,\mathrm{e}^{-\lambda_1 x}; \tag{21}$$

$$\hat{\vartheta}(x) = C_1\,\mathrm{e}^{-\lambda_1 x} + C_2\,\mathrm{e}^{-\lambda_1 x}, \qquad \hat{\psi}(x) = C_3\,\mathrm{e}^{-\lambda_3 x}, \tag{22}$$

where C_j ($j = 1, 2, 3$) are constants and $\lambda_3 = \sqrt{m^2 + \omega^2/\beta^2}$. Due to the regularity conditions, we discard the exponentially growing terms in the solutions derived above.

Substituting Eqs. (12), (21), and (22) into Eqs. (6) and (8), we obtain

$$\hat{u}(x) = -(\lambda_1 C_1\,\mathrm{e}^{-\lambda_1 x} + \lambda_2 C_2\,\mathrm{e}^{-\lambda_2 x}) + imC_3\,\mathrm{e}^{-\lambda_3 x},$$

$$\hat{v}(x) = im(C_1\,\mathrm{e}^{-\lambda_1 x} + C_2\,\mathrm{e}^{-\lambda_2 x}) + \lambda_3 C_3\,\mathrm{e}^{-\lambda_3 x};$$

$$\hat{\sigma}_{xx}(x) = g_{11}C_1\,\mathrm{e}^{-\lambda_1 x} + g_{12}C_2\,\mathrm{e}^{-\lambda_2 x} + g_{13}C_3\,\mathrm{e}^{-\lambda_3 x}; \tag{23}$$

$$\hat{\sigma}_{yy}(x) = g_{21}C_1\,\mathrm{e}^{-\lambda_1 x} + g_{22}C_2\,\mathrm{e}^{-\lambda_2 x} + g_{23}C_3\,\mathrm{e}^{-\lambda_3 x};$$

$$\hat{\sigma}_{xy}(x) = g_{31}C_1\,\mathrm{e}^{-\lambda_1 x} + g_{32}C_2\,\mathrm{e}^{-\lambda_2 x} + g_{33}C_3\,\mathrm{e}^{-\lambda_3 x}, \tag{24}$$

where

$$g_{1j} = \beta_2 - m^2 + 2\beta^2 m^2 \quad (j = 1, 2), \qquad g_{13} = -2i\beta^2\lambda_3 m,$$

$$g_{2j} = \beta_2 - m^2 - 2\beta^2\lambda_j^2 \quad (j = 1, 2), \qquad g_{23} = -g_{13} = 2i\beta^2\lambda_3 m,$$

$$g_{3j} = -2i\beta^2\lambda_j m \quad (j = 1, 2), \qquad g_{33} = -\beta^2(\lambda_3^2 + m^2).$$

4. INITIAL AND BOUNDARY CONDITIONS

We consider the problem in a semi-infinite medium

$$\Omega = \{(x, y, z): \ 0 \leqslant x < \infty, \ -\infty < y < \infty, \ -\infty < z < \infty\}.$$

The homogeneous initial conditions are

$$t = 0: \qquad u = v = \theta = \varphi = \dot{u} = \dot{v} = \dot{\theta} = \dot{\varphi} = 0.$$

The constants C_j $(j = 1, 2, 3)$ are determined from the boundary conditions for the temperature (the boundary of the half-space is subjected to a time-dependent thermal shock with a constant amplitude $\tilde{\theta}$)

$$\theta(x, y, t) = \tilde{\theta}\, \mathrm{e}^{im(y-ct)}, \qquad x = 0 \tag{25}$$

and for the stresses (both the normal and shearing stresses vanish on the free surface of the medium Ω)

$$\sigma_{xx}(x, y, t) = \sigma_{xy}(x, y, t) = 0, \qquad x = 0. \tag{26}$$

Substituting Eqs. (21), (23), and (24) into the boundary conditions (25) and (26), we obtain the system of equations

$$(\lambda_1^2 - \beta_2)C_1 + (\lambda_2^2 - \beta_2)C_2 = \tilde{\theta}, \quad g_{11}C_1 + g_{12}C_2 + g_{13}C_3 = 0, \quad g_{31}C_1 + g_{32}C_2 + g_{33}C_3 = 0,$$

from which the constants C_j $(j = 1, 2, 3)$ can be found as

$$C_1 = \frac{\tilde{\theta}(g_{12}g_{33} - g_{13}g_{32})}{\Delta}, \qquad C_2 = \frac{-\tilde{\theta}(g_{11}g_{33} - g_{13}g_{31})}{\Delta}, \qquad C_3 = \frac{\tilde{\theta}(g_{11}g_{32} - g_{12}g_{31})}{\Delta},$$

where $\Delta = (\lambda_1^2 - \beta_2)(g_{12}g_{33} - g_{13}g_{32}) - (\lambda_2^2 - \beta_2)(g_{11}g_{33} - g_{13}g_{31})$.

5. NUMERICAL RESULTS

The numerical results of the analytical expressions obtained in Section 4 are presented below. The computations were performed for copper with the following characteristics [9–11]: $\rho = 8954$ kg/m^3, $\lambda = 7.76 \cdot 10^{10}$ N/m^2, $\mu = 3.86 \cdot 10^{10}$ N/m^2, $\alpha_T = 1.78 \cdot 10^{-5}$ K^{-2}, $T_0 = 293$ K, $k_1 = 386$ N/(K·s), $C_E = 383.1$ m^2/K, $\eta = 8886.73$ s/m^2, $\varepsilon = 0.0168$, $c_1 = 4158$ m/s, and $\beta = 0.4994$. The other constants of the problem may be taken as $\omega = 3$, $m = 4 + 5i$, and $\tilde{\theta} = 50$. The programming code for numerical computations was prepared using the MATLAB software, and the accuracy maintained during the computations was six digits.

The above-listed hypothetical numerical values were used to calculate the real parts of the dimensionless temperature θ, displacement components u and v, and stress tensor components σ_{xx}, σ_{yy}, and σ_{xy} for different values of the time delay parameter $\varkappa$ at a fixed time parameter t and kernel function $K(t - \xi)$ on the surface $y = 0$.

Figure 1 exhibits the temperature θ, displacement component u, and stress tensor component σ_{xx} for different values of the time delay parameter $\varkappa$ at $y = 0$, $t = 0.1$, and $K(t - \xi) = (1 - (t - \xi)/\varkappa))^2$.

We notice from these figures that the temperature θ, displacement component u, and stress tensor component σ_{xx} increase with an increase in the time delay parameter $\varkappa$. The time needed for the thermal wave to reach a steady state depends on the value of $\varkappa$, which actually means that the particles transport the heat to other particles easily. The magnitudes of all the field variables in the LS model of generalized thermoelasticity are smaller than those predicted by the memory-dependent LS model. All field variables attain their maximum values at $\varkappa = 0.5$.

Figure 2 depicts θ, u, and σ_{xx} as functions of the dimensionless variable x for different values of the time delay parameter $\varkappa$ at $y = 0$, $t = 0.1$, and $K(t - \xi) = 1 - (t - \xi)/\varkappa$. In this case, the temperature θ and the displacement vector component u are weakly affected by the parameter $\varkappa$, but the stress component σ_{xx} shows a significant dependence on this parameter. Comparing Figs. 1 and 2, we can see that different kernels produce different effects on the field variables for the same value of the time delay parameter $\varkappa$. The temperature and all field variables attain their maximum values at $\varkappa = 0.5$. Figures 1 and 2 also clearly display that the boundary conditions (25) and (26) are identically satisfied.

Figure 3 depicts the three-dimensional distributions of the displacement vector component v and the stress tensor components σ_{xx}, σ_{yy}, and σ_{xy} for $t = 0.1$, $\varkappa = 0.05$, and $K(t - \xi) = 1 - (t - \xi)$.

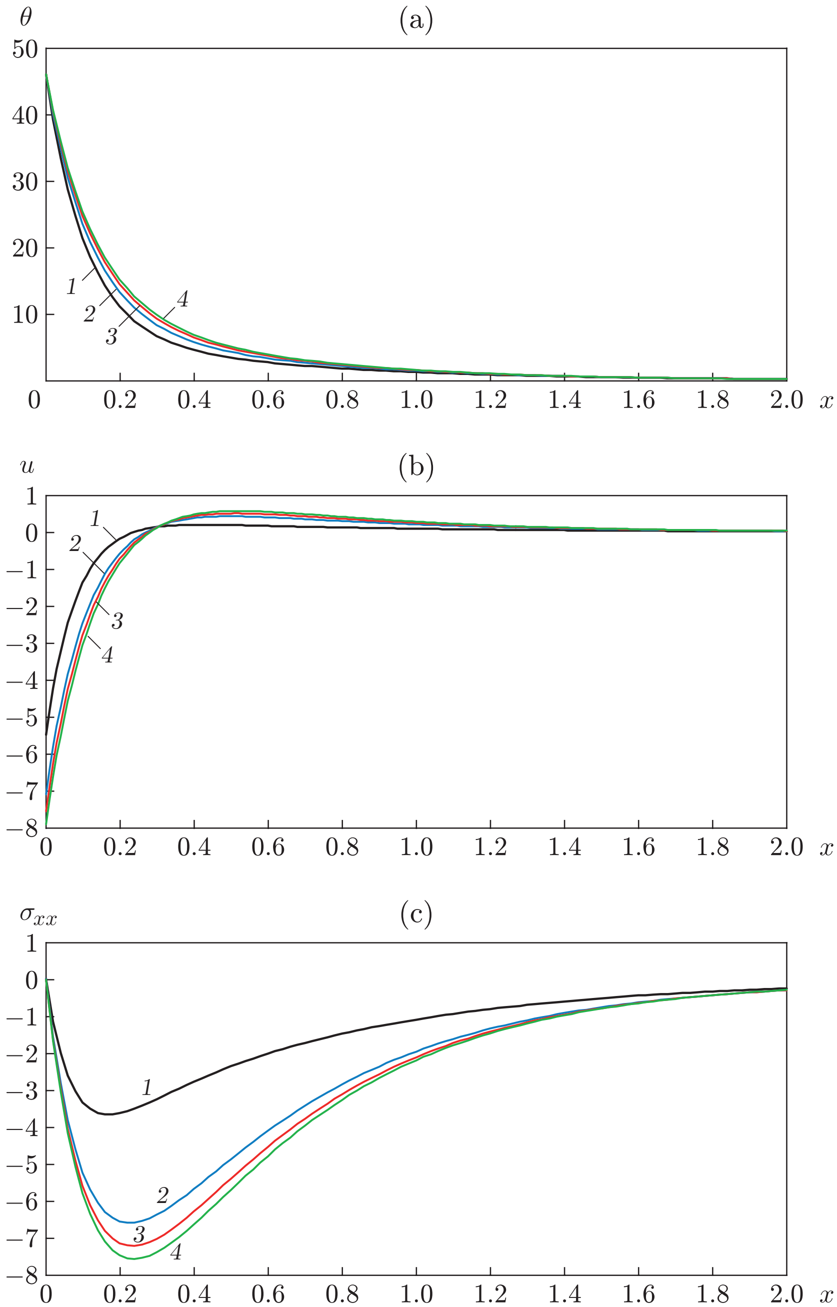

Fig. 1. Temperature θ (a), displacement vector component u (b), and stress tensor component σ_{xx} (c) versus the dimensionless coordinate x for $K(t-\xi) = (1-(t-\xi)/\varkappa)^2$ and different values of the time delay parameter $\varkappa$: results predicted by the LS theory [8] (*1*); results obtained by using the memory-dependent LS theory for $\varkappa = 0.005$ (*2*), 0.05 (*3*), and 0.5 (*4*).

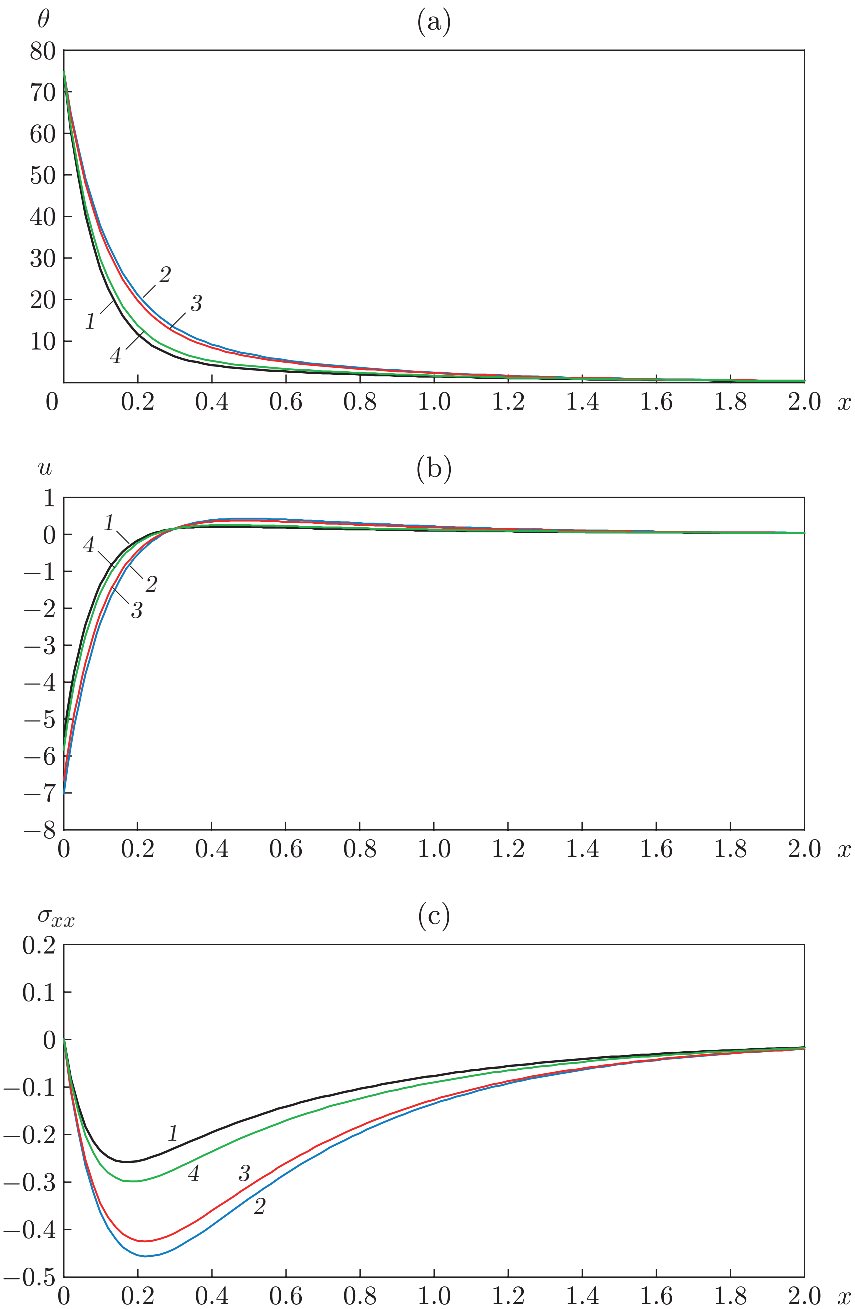

Fig. 2. Temperature θ (a), displacement vector component u (b), and stress tensor component σ_{xx} (c) versus the dimensionless coordinate x for $K(t-\xi) = 1 - (t-\xi)/\varkappa$ and different values of the time delay parameter $\varkappa$ (notation the same as in Fig. 1).

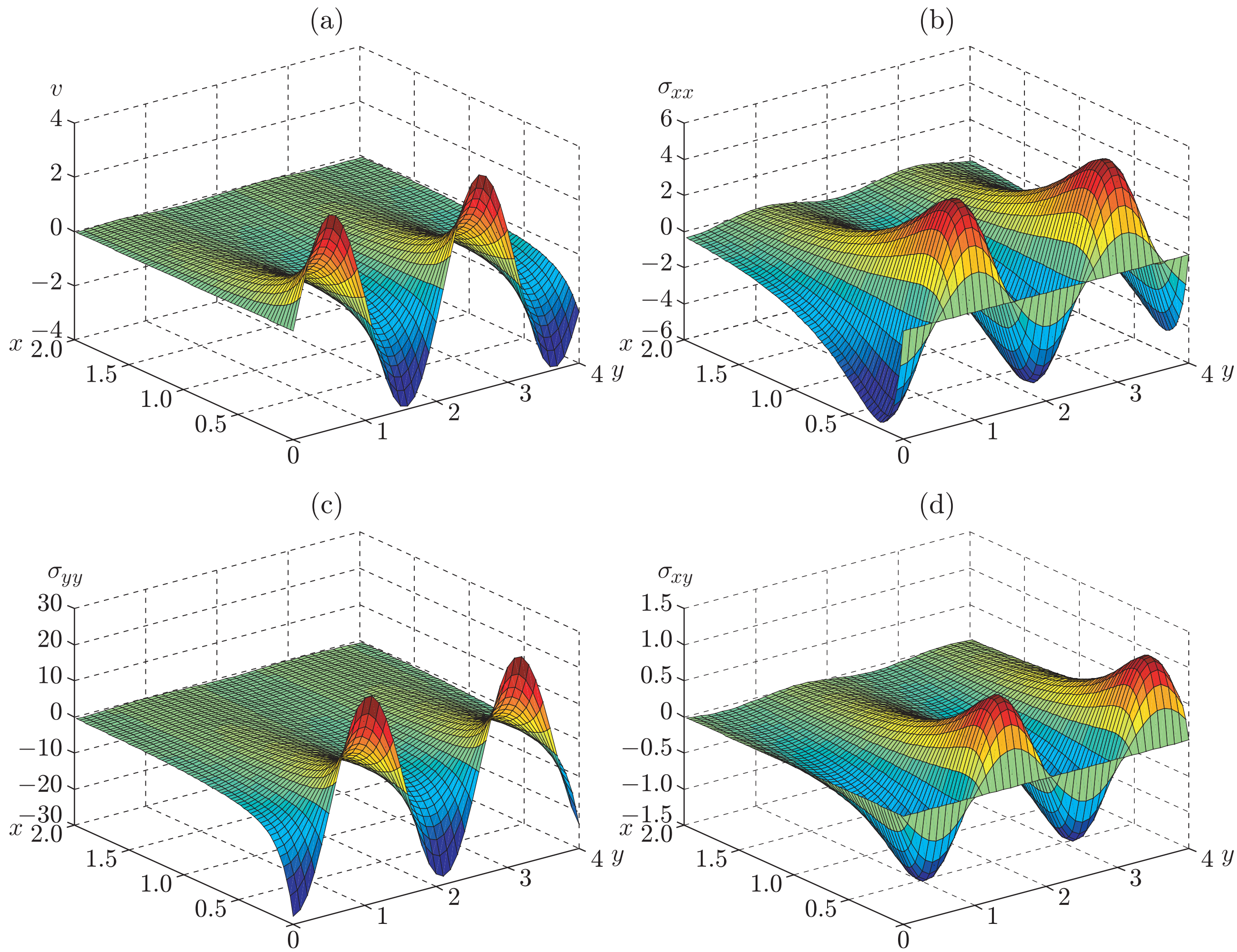

Fig. 3. Displacement vector component (a) v and stress tensor components σ_{xx} (b), σ_{yy} (c), and σ_{xy} (d) versus the dimensionless coordinates x and y for $\varkappa = 0.05$ and $K(t-\xi) = 1 - (t-\xi)$.

CONCLUSIONS

An analytical solution of a two-dimensional thermoelasticity problem was derived. The derivation procedure included the Fourier law of heat conduction and the governing equations with the instantaneous change rate depending on the past state. The dependence of the solution on the time delay parameter and on the form of the kernel used for averaging the derivative over the time interval was studied.

The new model can be applied to solve thermoelasticity problems in geophysics, biology, biophysics, electrochemistry, and other fields of science.

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