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STATISTICAL MODEL OF THE EXPLOSIVE FRAGMENTATION OF A RING

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Abstract: A model of the explosive fragmentation of a thin ring is developed which takes into account the statistical dispersion of the relative fracture deformation along the length of the ring. A formula is proposed for calculating the velocity of the boundary of the region near a plastic rupture in which the plastic flow of the ring material ceases. Methods for the numerical and analytical calculation of the average number of fragments of the ring are developed. The calculation results are compared with available experimental data.

Keywords: thin ring, explosion, velocity of the boundary of the plastic flow region, number of fragments.

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1. FORMULATION OF THE PROBLEM

An overview of papers dealing with the evaluation of the length and number of fragments of a ring fractured during rapid symmetric radial expansion can be found in [1], together with Kuznetsov's formula for estimating the characteristic length of fragments of a ring fractured by the explosion of a coaxially placed, internal explosive charge:

$$l = 2\delta_0 \sinh\,(\varepsilon_r)\, c/v. \tag{1}$$

Here l is the length of a fragment, δ_0 is the initial thickness of the ring, ε_r is the relative fracture strain of the ring material, v is the expansion velocity of the ring, and c is the propagation velocity of one-dimensional elastic waves in the ring material.

In [2], in the derivation of formula (1), it was assumed that fragmentation of the ring takes place at the stage of plastic flow of the material. Plastic rupture occurs when the relative hoop strain ε_r reaches the limiting value first on the inner fiber in the cross-section of the ring, and then on the outer fiber. The characteristic fragment length is equal to the product of the time of propagation of the rupture from the inner fiber to the outer fiber and the unloading rate of the stressed material near the rupture, taken equal to the one-dimensional velocity of the wave of elastic unloading. The fragmentation of the whole ring is not considered. Formula (1) can be used to obtain engineering estimates of the parameters of structural failure as a result of explosion of internal explosives.

Below we consider a similar problem of fragmentation of a thin narrow ring made of a ductile structural material of the type of low-carbon steel with high-velocity radial expansion. As in [2], it is assumed that plastic fracture occurs when the relative hoop deformation reaches the value ε_r at which failure occurs. It is assumed that the parameter ε_r is a random function of the angular coordinate of the cross section of the ring. Fragmentation during expansion of the ring begins with a rupture in the cross section with minimum strain ε_{r1} at which failure occurs. Decrease in stress leads to the formation and enlargement of regions on both sides of the rupture in which plastic flow ceases (the material hardens). Further expansion of the ring is accompanied by the occurrence of ruptures in sections with larger fracture strain $\varepsilon_{ri} > \varepsilon_{r1}$ $(i = 2, 3, \ldots)$ in its unhardened part, with hardening

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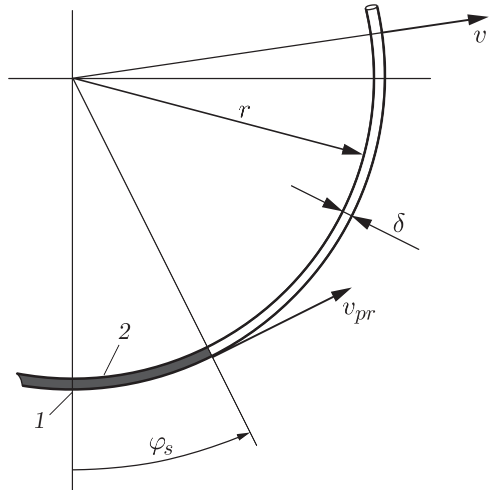

Fig. 1. Problem geometry: (*1*) rupture line; (*2*) zone of flow cessation (hardening) of the material.

regions spreading from these ruptures. The fragmentation process is completed when the entire ring material is hardened. The length of a fragment is equal to the sum of the lengths of two hardening regions spreading from neighboring ruptures and is a random quantity. We will assume that plastic rupture is instantaneous and the flow of the ring material is one-dimensional.

The geometry of the problem is shown in Fig. 1 [v_{pr} is the velocity of the boundary of the hardening region with respect to the rupture (plastic unloading rate)].

2. PLASTIC UNLOADING RATE NEAR A RUPTURE

The length of the hardened region near a rupture and its expansion velocity can be obtained from the plastic flow condition at the boundary of the hardening region and in the flow region.

The rate of increase in the length of the ring sector during symmetric radial expansion of this ring is

$$v_\varphi = \frac{(r + \Delta r)\,\varphi - r\varphi}{\Delta t} = \frac{\Delta r\,\varphi}{\Delta t} = v\varphi \tag{2}$$

(r is the current radius of the rings and Δt and Δr are the small increments of time and the current radius of the ring, respectively). The mass of the hardened part of the ring is given by the formula

$$m_s = \rho A r \varphi_s, \tag{3}$$

where ρ is the density of the ring material and A is the cross-sectional area of the ring. In view of (2), the velocity of the center of mass of the hardened part of the ring with respect to the section in which rupture occurred is

$$v_s = v\varphi_s/2. \tag{4}$$

If the length of the hardened part of the ring is much smaller than its radius, we can write the equation of motion of the hardened part of the ring as a body of variable mass in the projection onto the tangent at the point $\varphi = 0$ (see Fig. 1):

$$\frac{d}{dt}\,(m_s v_s) = \sigma_{\text{p.f.}} A \tag{5}$$

($\sigma_{\text{p.f.}}$ is the plastic flow stress of the material).

Taking into account (3) and (4) and neglecting the change in the parameters A, r, v, and $\sigma_{\text{p.f.}}$ in a short fragmentation time, we can write Eq. (5) as

$$\frac{d}{dt}(\varphi_s^2) = \frac{2\sigma_{\text{p.f.}}}{\rho r v}.$$

The solution of this equation with the initial condition $\varphi_s = 0$ for $t = 0$ has the form

$$\varphi_s = \sqrt{\frac{2\sigma_{\text{p.f.}}}{\rho}\frac{t}{rv}}\,.$$

The length of the hardened part of the ring is

$$l_s = r\varphi_s = \sqrt{\frac{2\sigma_{\text{p.f.}}}{\rho}\frac{rt}{v}}\,. \tag{6}$$

From (6) we obtain the instantaneous and average velocities of the boundaries of the hardening region of the material relative to the section in which rupture occurred:

$$v_{pr} = \frac{dl_s}{dt} = \sqrt{\frac{\sigma_{\text{p.f.}}}{2\rho}\frac{r}{vt}}\,, \qquad v_{prc} = \frac{l_s}{t} = \sqrt{\frac{2\sigma_{\text{p.f.}}}{\rho}\frac{r}{vt}} = 2v_{pr}. \tag{7}$$

The velocity of the boundary of the hardened region of the ring relative to the section in which rupture occurred is called the plastic unloading rate. It follows from formulas (7) that the plastic unloading rate decreases with time.

The increment of the relative strain in the ring material during plastic flow $\Delta\varepsilon$ in time t after rupture is calculated by the formula

$$\Delta\varepsilon = vt/r, \tag{8}$$

where v and r are the radial velocity and radius of the ring at the moment of rupture.

Taking into account (8), formulas (6) and (7) can be represented in dimensionless form

$$\frac{l_s}{r} = \sqrt{\frac{2\sigma_{\text{p.f.}}\,\Delta\varepsilon}{E}}\,\frac{c}{v}; \tag{9}$$

$$\frac{v_{pr}}{c} = \sqrt{\frac{\sigma_{\text{p.f.}}}{2E\,\Delta\varepsilon}}\,, \qquad \frac{v_{prc}}{c} = \sqrt{\frac{2\sigma_{\text{p.f.}}}{E\,\Delta\varepsilon}}\,; \tag{10}$$

$$c = \sqrt{E/\rho}\,, \tag{11}$$

where c is the propagation velocity of one-dimensional elastic waves in the ring material.

It follows from formulas (9)–(11) that the instantaneous and average plastic unloading rates are independent of the radius of the ring r and are determined only by the material characteristics ρ, E, and $\sigma_{\text{p.f.}}$ and the increment of the relative strain during the time interval from the moment of rupture $t = 0$ to the current time t.

It is not difficult to show that the formulas obtained for the plastic unloading rate in the vicinity of the rupture are also valid in the case of a stretched rectilinear rod whose material is in the state of one-dimensional plastic flow.

Table 1 gives instantaneous and average rates of plastic unloading near rupture for 20 steel and St.3 steels calculated from formulas (10) for various values of $\Delta\varepsilon$ and for $\rho = 7800$ kg/m^3, $E = 210$ GPa, and $\sigma_{\text{p.f.}} = 1$ GPa [3, 4]. It follows from Table 1 that the plastic unloading rate is a decreasing function of the strain increment and for $\Delta\varepsilon > 0.05$, it is significantly smaller than the elastic-unloading wave velocity $c = 5189$ m/s.

3. NUMERICAL CALCULATION OF THE NUMBER OF FRAGMENTS OF A RING UNDER RADIAL EXPANSION

In the numerical simulation, the input parameters are the initial radius r_0 and the thickness δ_0 of the ring, the elastic modulus E, the density ρ, the plastic flow stress $\sigma_{\text{p.f.}}$, the mean error $m_{\varepsilon r}$ and the root-mean-square

Table 1. Plastic unloading rate for St. 20 and St. 3 steels

$\Delta\varepsilon$	v_{pr}, m/s	v_{prc}, m/s
0.01	2530	5060
0.05	1130	2260
0.10	800	1600
0.20	565	1130

error $\sigma_{\varepsilon r}$ of the relative strain at which fracture occurs, and the radial expansion velocity v at the fragmentation moment. In the determination of a random change in the fracture strain along the length of the ring, this ring consists of a finite number N_e of elements, in each of which the fracture strain ε_{ri} is a statistically independent random variable.

In the calculation, the method of finite increments (MFI) of the strain ε is used. The reference values ε are calculated from the formulas

$$\varepsilon_1 = 0, \qquad \varepsilon_{j+1} = \varepsilon_j + \Delta\varepsilon, \quad j = 1, 2, 3, \ldots, \qquad \Delta\varepsilon \approx 0.2\sigma_{\varepsilon r}. \tag{12}$$

Assuming a normal distribution of random strain values at which failure occurs, the average number of ruptures occurring in the zone of plastic flow during expansion of the ring (when the strain changes from ε_j to ε_{j+1}) is equal to

$$\Delta N_{rj} = (N_e - N_{sj})\, f_{\varepsilon r}(\varepsilon_j)\, \Delta\varepsilon, \tag{13}$$

where $f_{\varepsilon r}(\varepsilon_r)$ is the probability density function of the normal distribution of the fracture strain ε_r with parameters $m_{\varepsilon r}$ and $\sigma_{\varepsilon r}$:

$$f_{\varepsilon r}(\varepsilon_r) = \frac{1}{\sqrt{2\pi}\,\sigma_{\varepsilon r}} \exp\Big(-\frac{1}{2}\Big(\frac{\varepsilon_r - m_{\varepsilon r}}{\sigma_{\varepsilon r}}\Big)^2\Big),$$

N_{sj} is the number of hardened elements of the ring near all ruptures at the moment of propagation of strain ε_j.

The average number of ruptures that appear when the strain changes from ε_1 to ε_j is determined by summation of the increments (13):

$$N_{rj} = \sum_{i=1}^{i=j} \Delta N_{ri}.$$

The number of hardened elements of the ring N_{sj} is calculated as follows. In view of (9), the length of the hardened zone formed on both sides of the rupture in the interval from ε_i to ε_{i+1} during the time when the strain changes from ε_i to ε_j is equal to

$$l_{sij} = 2r\sqrt{\frac{2\sigma_{\text{p.f.}}(\varepsilon_j - \varepsilon_i)}{E}}\;\frac{c}{v}. \tag{14}$$

The number of hardened elements of the ring that corresponds to this length is

$$N_{sij} = l_{sij}/\Delta l. \tag{15}$$

Here Δl is the arc length of the elementary region:

$$\Delta l = 2\pi r/N_e.$$

In view of (13), the average number of the hardened ring elements corresponding to all ruptures that occurred in the interval of strain change from ε_i to ε_{i+1} during expansion of the ring corresponding to the change in strain from ε_i to ε_j is equal to

$$\Delta N_{sij} = \Delta N_{ri} N_{sij}.$$

The average number of the hardened elements of the ring corresponding to all ruptures that occurred when the relative strain increased from ε_1 to ε_j is determined from the formula

$$N_{sj} = \sum_{i=1}^{j} \Delta N_{sij}.$$

The calculation is carried out with an increase in ε_i according to (12) starting from $\varepsilon_1 = 0$ (or from the larger value which corresponds to the $\Delta N_{ri} \approx 0.01$) and ends when the number of unhardened elements of the ring decreases to zero. In the first step of the calculation ($i = 1$), it is assumed that $N_{si} = 0$, and in the subsequent steps for ε_i, it is assumed that the value of N_{si} is equal to the value of N_{si} in the previous step. The final value of N_{rj} corresponds to the average number of ruptures m_n. In calculations by formula (15), rounding to rounding to the nearest whole number was not carried out.

The number of elements of the ring can be related to the minimum length of the failure zone. In explosive experiments with thin-walled shells, the angle of inclination of the rupture surface to the surface of the sheet is approximately is 45°, so that the length of the failure zone can be taken to be approximately equal to $\sqrt{2}\,\delta$ and the number of elements of the ring can be calculated by the formulas

$$N_e \approx 2\pi r/(\sqrt{2}\,\delta) \approx 4.44r/\delta; \tag{16}$$

$$r \approx r_0(1 + m_{\varepsilon r} - k_1\sigma_{\varepsilon r}); \tag{17}$$

$$\delta \approx k_2\delta_0/(1 + m_{\varepsilon r} - k_1\sigma_{\varepsilon r}), \tag{18}$$

where r and δ are the average radius and thickness of the ring in the fragmentation period, $k_1 \approx 1$ is a variable coefficient that takes into account the weak dependence of the calculation results on the average strain deformation at which failure occurs, and $k_2 \approx 0.5$ is the coefficient that takes into account the ductility of the inner layer of the ring material, due to the pressure of the detonation products on it. The coefficient k_2 is introduced to take into account that cracks arise in the surface layer of the material of explosively fractured tubes and then extend to its entire thickness [4]. It should be noted that the value of N_e has an insignificant effect on the results of calculation of the number of fragments.

The stress at which plastic flow of steel occurs for the conditions of explosive experiments is selected on the basis of experimental data [3, 5] or can be calculated from the formulas

$$\sigma_{\text{p.f.}} = \sigma_{\text{st}} + \Delta\sigma_d, \quad \sigma_{\text{st}} \approx \sigma_b = \sigma_{br}(1 + \psi_b), \quad \Delta\sigma_d = \sigma_{\text{t}0}(\varepsilon'/C)^{1/P}, \quad \varepsilon' = v/r, \tag{19}$$

where σ_{st} is the static component of the plastic flow stress at the moment of fragmentation, $\Delta\sigma_d$ is the dynamic component of the plastic flow stress due to the yield lag, σ_{br} is the breaking strength of the material, σ_b and ψ_b are the true stress and the reduction in area at the moment the tensile force in tests of standard samples reaches a maximum value, $\sigma_{\text{t}0} = 0.11$ GPa is the yield strength of pure iron, $C = 40$ s^{-1} and $P = 5$ are the characteristic parameters of dynamic hardening, ε' is the strain rate of the material, and r is the radius of the ring in the fragmentation period [see (17)].

Dependences (19) are constructed for low-carbon steels using the results of available experimental works, in particular, the results of [6], where the depth of penetration of a cylindrical impactor with a conical tip into pure iron was measured, and the results of [5], where the dynamic yield stress of structural materials was determined in experiments on the impact of rods on a rigid target. The results of measuring the compression of plastic dampers made of 20 steel upon impact of experimental assemblies on a rigid target with a velocity of 13 m/s. In all these cases, the results of numerical calculations obtained using the above dependences and dynamic hardening parameters are in satisfactory agreement with the results of experiments.

Some data on the fracture strain of St.3 and 30 KHGSA steel and D16 aluminum alloy are given in [4]. In explosive experiments on fracture of St.3 steel tubes with an inner diameter of 63 mm, a length of 200 mm, and a thickness of 3–8 mm by a spherical charge of 50/50 TNT/RDX with a diameter of 62 mm, that ruptures occurred in the outer layers of the tubes at a relative strain of 5–10%. The propagation of ruptures over the entire thickness of the wall occurred at a significantly higher relative strain of 13–70%. The delay in the propagation of ruptures along the thickness is obviously due to the effect of the high pressure of detonation products on the inner surface of the samples, which leads to an increase in the ductility of the inner layer of the material. Estimates of $m_{\varepsilon r}$ and $\sigma_{\varepsilon r}$ obtained using the results of measuring the thickness of fragments of 20 steel model shells fractured by explosion of a compact explosive charge are given in Table 2 (L is the shell length, N is the number of fragments). Thickness measurements were carried out with a MK micrometer with a division value 0.01 mm, fitted with nozzles providing a diameter of the contact area equal to 1 mm.

Table 2. Average relative strains at which failure occurs and its standard deviation for 20 steel

L, mm	$m_{\varepsilon r}$	$\sigma_{\varepsilon r}$	N
140	0.073	0.027	17
200	0.076	0.032	14

The values of $m_{\varepsilon r}$ given in Table 2 are consistent with the data of [6]. Standard deviations of the relative strain from the average value $\sigma_{\varepsilon r}$ are obtained from the results of two explosive experiments and need to be refined in further experiments.

In the estimation of the length of the hardening zone by formula (14), it is assumed that the boundary of this zone extends from any rupture into the region of unhardened material and it is not taken into account that the boundaries of the hardening zones from neighboring ruptures meet each other and the expansion of the zones is terminated. Therefore, the length of the hardening zone near a rupture obtained using the above algorithm may be overestimated and the number of fracture (and fragments) underestimated.

4. ANALYTICAL ESTIMATION OF AVERAGE LENGTH AND NUMBER OF FRAGMENTS OF THE RING

According to the ring fragmentation model described in Section 1, the length of a fragment is equal to the sum of the lengths of the plastic unloading zones near neighboring ruptures at the moment of vanishing of the length of the plastic flow zone between them:

$$l = l_{s1} + l_{s2}.$$

Here

$$l_{s1} = r\sqrt{\frac{2\sigma_{\text{p.f.}}(\varepsilon_k - \varepsilon_{r1})}{E}}\,\frac{c}{v}, \qquad l_{s2} = r\sqrt{\frac{2\sigma_{\text{p.f.}}(\varepsilon_k - \varepsilon_{r2})}{E}}\,\frac{c}{v},$$

ε_{r1} and ε_{r2} are the fracture strains in sections 1 and 2, in which ruptures occurred, and ε_k is the strain at the moment of meeting of the plastic unloading zones (in this case, $\varepsilon_k > \varepsilon_{r1}$ and $\varepsilon_k > \varepsilon_{r2}$).

Let rupture 1 be formed earlier than fracture 2. The lower bound of the fragment length is obtained for $\varepsilon_k = \varepsilon_{r2}$ and zero value of l_{s2}:

$$l_m = l_{s1} = \frac{rc}{v}\sqrt{\frac{2\sigma_{\text{p.f.}}\,\Delta\varepsilon_m}{E}}\,; \tag{20}$$

$$\Delta\varepsilon_m = |\varepsilon_{r2} - \varepsilon_{r1}|. \tag{21}$$

The sign of the module in (21) was introduced to take into account the possibility of the occurrence of the first rupture both in section 1 and in section 2.

In a short stage of fragmentation, all the quantities included in formula (20) can be assumed to be constant. In this case, the length of a fragment is a random variable whose statistical characteristics are determined by the statistical characteristics of the fracture strain ε_r. The mathematical expectation and root-mean-square error in determining the length of a fragment can be estimated on the basis of formulas (20), (21) using the linearization method [7]. Let the fracture strains ε_{r1} and ε_{r2} be independent normally distributed random numbers with mathematical expectation $m_{\varepsilon r}$ and root-mean-square error $\sigma_{\varepsilon r}$. Then, the difference of the values of $\Delta\varepsilon = \varepsilon_{r2} - \varepsilon_{r1}$ for which failure occurs has a normal distribution with zero mathematical expectation and dispersion $2\sigma_{\varepsilon r}^2$ [7]. The modulus of this difference $\Delta\varepsilon_m$ has a truncated normal distribution with the probability density function

$$f(\Delta\varepsilon_m) = \frac{1}{\sqrt{\pi}\,\sigma_{\varepsilon r}}\exp\Big(-\frac{\Delta\varepsilon_m^2}{4\sigma_{\varepsilon r}^2}\Big), \qquad \Delta\varepsilon_m > 0.$$

The mathematical expectation $\Delta\varepsilon_m$ is given by the formula

$$m_{\Delta\varepsilon m} = \int_0^\infty f(\Delta\varepsilon_r)\,\Delta\varepsilon_r\,d(\Delta\varepsilon_r) = \frac{2}{\sqrt{\pi}}\,\sigma_{\varepsilon r}.$$

Replacing $m_{\Delta\varepsilon m}$ in formula (20) by $\Delta\varepsilon_m$, we obtain the lower bound of the mathematical expectation of the fragment length [7]:

$$m_l \approx 1.5r\sqrt{\frac{\sigma_{\text{p.f.}}\,\sigma_{\varepsilon r}}{E}\,\frac{c}{v}}.$$

Results in better agreement with the results of calculation by the MFI and experimental data can be obtained using the refined formula for the average fragment length

$$m_l \approx 2.8r\sqrt{\frac{\sigma_{\text{p.f.}}\sigma_{\varepsilon r}}{E}\,\frac{c}{v}}. \tag{22}$$

The average number of fragments is estimated as

$$m_n \approx \frac{2\pi r}{m_l} = 2.25\sqrt{\frac{E}{\sigma_{\text{p.f.}}\sigma_{\varepsilon r}}\,\frac{v}{c}}. \tag{23}$$

As in most papers devoted to the study of fragmentation of a ring (see [1]), formula (23) represents the linear dependence of the number of fragments on the radial velocity of the ring v. The coefficient before the fraction v/c is expressed in terms of the material parameters E and $\sigma_{\text{p.f.}}$ determining the plastic unloading rate, and in terms of the error of determining the fracture strain $\sigma_{\varepsilon r}$. According to (22) and (23), an increase in the stress at which plastic flow occurs and an increase in the error of determining the fracture strain leads to an increase in the average length of fragments and a decrease in their number.

A comparison of formula (22) with the Kuznetsov formula (1) shows that they are significantly different. In formula (1), the characteristic fragment length depends on the thickness of the ring and the relative strain at which fracture occurs. In formula (22), the average fragment length depends on the radius of the ring, the stress at which plastic flow occurs and the standard deviation the relative strain at which fracture occurs. These differences follow from the fragmentation model used in the present work, which takes into account the dependence of the plastic unloading rate near a rupture on time and the statistical dispersion of strain values at which fracture along the length of the ring occurs.

Formula (23) does not directly include the radius of the ring, i.e., the scale effect is not taken into account explicitly. However, according to the above formulas for estimating the plastic flow stress (19), an increase in the radius of the ring leads to a decrease in the strain rate and the additional dynamic stress at which plastic flow occurs . Therefore, according to (23), the number of fragments increases with increasing radius of the ring. In view of (19), the factor of increase is

$$k_m = \frac{m_n(r_2, v)}{m_n(r_1, v)} = \Big(\frac{\sigma_{\text{p.f.}}(r_1)}{\sigma_{\text{p.f.}}(r_2)}\Big)^{1/2}. \tag{24}$$

The value of k_m depends on both parameters r_1 and r_2.

5. COMPARISON OF RESULTS OF NUMERICAL CALCULATIONS WITH EXPERIMENTAL DATA

In evaluating the above methods of calculation of the number of fragments of a ring, we used the results of studies [8, 9] of the fracture of steel tubes by a coaxial long explosive charge initiated from the end, since their fracture is determined by hoop strain.

The results of an experimental study of the fragmentation of cylindrical shells of 20 steel are given in [8]. The diameter of the main part of the samples was $2r_0 = 42$ mm, the shell length was equal to five diameters, and the wall thickness $\delta_0 \approx 0.9$ mm. The number of fragments was determined by flash radiography.

Table 3. Number of fragments obtained in experiments [8] for 20 steel shells

v, m/s	n_{Ei}	$\sigma_{\text{p.f.}}$, GPa	m_n	
			Calculation using MFI	Calculation by formula (23)
175	10	0.76	9	7
290	21	0.79	14	12
365	30	0.81	17	15
395	22	0.81	19	16
810	50	0.87	34	31
905	33	0.88	37	35
1370	60	0.92	51	52
1690	63	0.94	60	63
1810	70	0.95	63	67
2040	80	0.96	70	75
2075	80	0.96	70	77
2075	67	0.96	70	77
2425	69	0.98	78	89

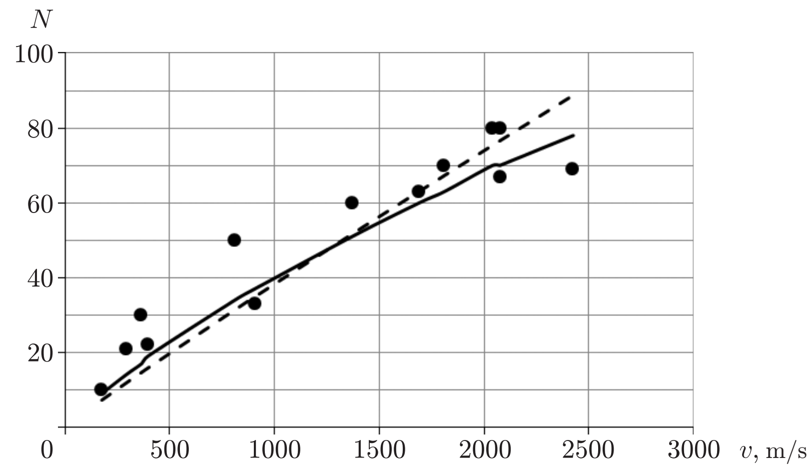

Fig. 2. Number of fragments versus expansion velocity for 20 steel shells: points correspond to the experimental data [8], the solid curve corresponds to the calculation using the MFI, and the dashed curve to the calculation by formula (23).

The calculation was performed for material parameters $E = 210$ GPa, $c = 5189$ m/s, $\sigma_{br} = 0.44$ GPa, and $\psi_b = 0.2$ [10]. In view of data of Table 2, approximate values $m_{\varepsilon r} = 0.08$ and $\sigma_{\varepsilon r} = 0.03$ were used. According to (16)–(18), the number of elementary regions is $N_e = 226$. The shell velocity at the moment of fragmentation was taken to be the velocity of fragments measured in experiments [8]. Experimental values of the fragment velocity were taken from [8]. The number of fragments determined in the experiments and calculations is given in Table 3 and in Fig. 2. The residual values were calculated from the formula

$$R = \Big(\frac{1}{n}\sum_{i=1}^{n}(m_{ni} - n_{Ei})^2\Big)^{1/2},$$

where m_{ni} and n_{Ei} are the numbers of ring fragments obtained in calculations and experiments; the subscripts i and n correspond to the experiment number and the number of experiments. The residual for m_n calculated using the MFI is equal to 8.5, and that calculated using formula (23) is 10.0.

From Table 3 and Fig. 2, it follows that the numbers of fragments calculated by the MFI and using formulas (23) are close to the experimental values. The calculation results show that the residual R reaches a minimum for $\sigma_{\varepsilon r} = 0.03$. Calculations for a larger value $m_{\varepsilon r} = 0.15$ and the unchanged remaining parameters yielded almost the same results as for $m_{\varepsilon r} = 0.08$, i.e., this parameter has little effect on the calculation results. From Table 3, it follows that for high expansion velocity of the ring ($v \approx 2000$ m/s), the stress leading to plastic flow calculated by formulas (19) is close to an experimental of 1 GPa [3, 4], and at a lower velocity, it is much less than this value.

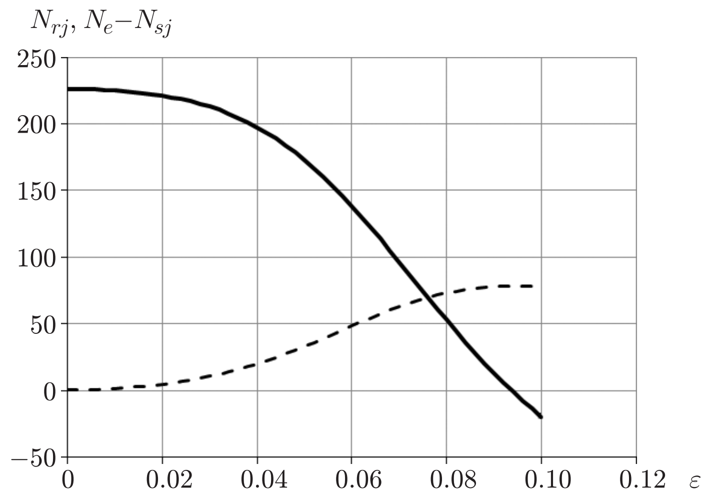

Fig. 3. Number of ruptures N_{rj} (dashed curve) and number of unhardened elements $N_e - N_{sj}$ (solid curve) versus relative strain at $v = 2425$ m/s.

Table 4. Values of the scaling factor of increase in the number of fragments k_m for $r_1 = 0.021$ m

r_2/r_1	k_m				
	$v = 200$ m/s	$v = 400$ m/s	$v = 600$ m/s	$v = 800$ m/s	$v = 1000$ m/s
4	1.050	1.055	1.057	1.060	1.061
10	1.079	1.087	1.092	1.095	1.098
20	1.100	1.110	1.116	1.120	1.124
40	1.118	1.131	1.138	1.144	1.148

It should be noted that in the numerical calculations, we used one-dimensional arrays of values of ε_i, $f_{\varepsilon r}(\varepsilon_i)$, and ΔN_{ri} containing 50 numbers and a two-dimensional array of values of N_{sij}, containing 50×50 numbers. For each velocity value, the calculation was carried out by selecting the step size $\Delta\varepsilon$ for which the fragmentation ends at the boundary of the grid of reference values of ε_i.

Figure 3 shows curves of the number of fracture and the number of unhardened elements of a ring versus relative strain at a ring expansion velocity $v = 2425$ m/s calculated using the MFI. It can be seen that the fragmentation of the ring ends when the relative strain reaches $\varepsilon \approx m_{\varepsilon r} + 0.4\sigma_{\varepsilon r} = 0.093$. The calculation for $v = 175$ m/s shows that the fragmentation ends when the relative strain reaches $\varepsilon \approx m_{\varepsilon r} - 1.3\sigma_{\varepsilon r} = 0.04$.

From Table 4, which shows the values of the scale factor k_m calculated using formula (24) for $r_1 = 0.021$ m and the above parameters of dynamic hardening of 20 steel, it follows that with an increase in the radius of the ring by a factor of four, the number of fragments increases by no more than 6%, and with an increase in the radius by a factor of 40, by no more than 15%.

The results of an experimental study of the fragmentation of cylindrical shells (inner diameter 38.1 mm, wall thickness 2.29 mm, and length 24.5 mm) by explosion of explosives are given in [9]. Steel shells with a mass fraction of carbon of 0.07% and other materials were tested. The number of fragments was estimated for shells of steel with a mass fraction of carbon of 0.07%. The characteristics of domestic 08 steel of similar composition were used: $\sigma_{br} = 0.39$ GPa and $\psi_b = 0.20$. The dynamic hardening parameters are the same as for 20 steel. The values of the error in determining the fracture strain were chosen equal to $m_{\varepsilon r} = 0.08$ and $\sigma_{\varepsilon r} = 0.04$. According to (16)–(18), the number of elementary regions is $N_e = 89$. The results of the calculations are given in Table 5 and in Fig. 4. Because shells of short length were tested, the experimentally obtained number of longitudinal ruptures were used for comparison. From Table 5 and Fig. 4, it follows that the estimates of the number fragments are close to the experimentally determined number of longitudinal ruptures. The residual for the value of m_n calculated using the MFI, is equal to 2.0, and that using formula (23) is 1.2.

Table 5. The number of longitudinal gaps, determined in experiments [9] for shells of steel with the mass fraction of carbon 0.07%

v, m/s	n_E	$\sigma_{\text{p.f.}}$, GPa	m_n	
			Calculating using MFI	Calculation using formula (23)
150	4	0.77	7	5
390	12	0.84	15	13
510	18	0.86	18	17

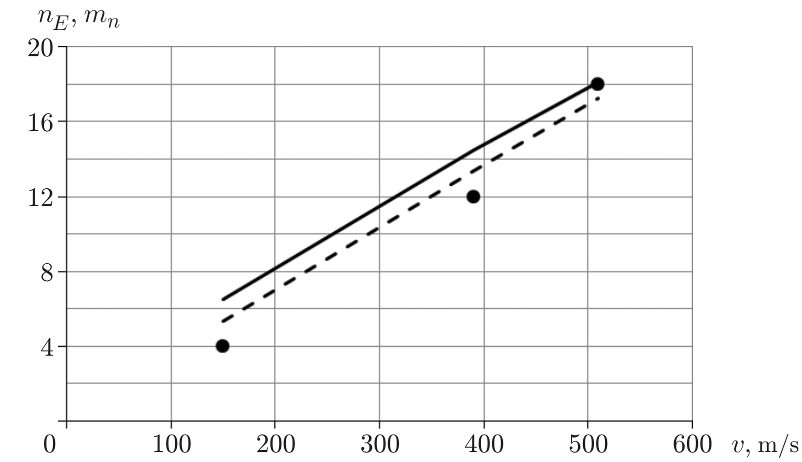

Fig. 4. Dependence of the number of longitudinal fracture on the expansion velocity for shells of steel with a mass fraction of carbon 0.07%: points refer to the experimental data [9], solid curve refers to the calculation using MFI, and dashed curve to the calculation by the formula (23).

CONCLUSIONS

The fragmentation of a thin narrow ring whose material is in the state of plastic flow under high-velocity radial expansion was considered.

The condition for the occurrence of plastic flow at the boundary of the hardening zone was used to obtain an analytical expression for the velocity of this boundary depending on the plastic flow stress and material density.

An algorithm for the numerical calculation of the average number of fragments of the ring was developed using the method of finite increments of plastic strain. Formulas for the analytical estimation of the average length of fragments of the ring and the average number of fragments were obtained. In the developed fragmentation model, the main material parameters determining the number of fragments of the ring are the plastic flow stress, material density, and the standard deviation of the relative fracture strain. From the results of calculations, it follows that with an increase in the radius of the rings, the number of fragments increases due to a decrease in the strain rate and the mean stress at which the plastic flow occurs.

Comparison of the number of ring fragments calculated by the methods described in the paper with available experimental data on the explosive fracture of cylindrical shells of low-carbon steels by explosive charges of great length showed that they are in satisfactory agreement if the plastic flow stress is specified taking into account the dynamic hardening of the material and the standard deviation of the fracture strain is adopted on the basis of the results of explosive experiments.

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