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CRITICAL VELOCITIES IN FLUID-CONVEYING SINGLE-WALLED CARBON NANOTUBES EMBEDDED IN AN ELASTIC FOUNDATION

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Abstract: The problem of stability of fluid-conveying carbon nanotubes embedded in an elastic medium is investigated in this paper. A nonlocal continuum mechanics formulation, which takes the small length scale effects into consideration, is utilized to derive the governing fourth-order partial differential equations. The Fourier series method is used for the case of the pinned–pinned boundary condition of the tube. The Galerkin technique is utilized to find a solution of the governing equation for the case of the clamped–clamped boundary. Closed-form expressions for the critical flow velocity are obtained for different values of the Winkler and Pasternak foundation stiffness parameters. Moreover, new and interesting results are also reported for varying values of the nonlocal length parameter. It is observed that the nonlocal length parameter along with the Winkler and Pasternak foundation stiffness parameters exert considerable effects on the critical velocities of the fluid flow in nanotubes.

Keywords: critical velocity, SWCNT, nonlocal elasticity theory, two-parameter foundation.

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INTRODUCTION

In recent times, a number of researchers have focused their investigations on various aspects of carbon nanotubes. Carbon nanotubes have very good mechanical, electrical, and chemical properties, with potential applications as bio-sensors, nano-oscillators, drug delivery systems, or as purely structural components in nano-devices.

In the area of fluid transport and particularly in the field of dynamics of fluid-conveying carbon nanotubes, the research started in 2005 has been accelerated in the last years. At the beginning, researchers applied the theory of the classical continuum mechanics [1–3] and did not consider the effect of small length scales involved. The diameter of a single-walled carbon nanotube (SWCNT) is in the range of 1–7 nm and the length is in the range of 20–140 nm (of the order of the C—C bond length). At such small length scales, the properties of the material at the atomic level may affect its dynamic behaviour. Hence, application of classical continuum mechanics models to carbon nanotubes is questionable. To address this problem, researchers are increasingly using Eringen's nonlocal continuum mechanics theory [4, 5], wherein the stress at any point is defined to be a functional of the strain field at every point in the body.

The first to apply nonlocal mechanics to consider vibrations of a fluid-conveying SWCNT were Lee and Chang [6], who later studied vibrations of fluid-conveying carbon nanotubes embedded in a Winkler-type elastic medium [7]. Simha [8] used the formulation of Lee and Chang [7] to model a fluid-conveying carbon nanotube

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embedded in a Pasternak-type elastic medium. However, Tounsi et al. [9] pointed out an error in the formulation of Lee and Chang [6] and derived a correct governing equation. Wang [10] also formulated a consistent model, perhaps independently of Tounsi, but did not consider any embedding elastic medium. Farshidianfar et al. [11] considered a two-parameter elastic embedding medium in their analysis of fluid-conveying carbon nanotubes. Both the Pasternak and viscoelastic two-parameter foundation models were used. However, their formulation of the problem was based on the classical continuum mechanics and did not include the most important nonlocal elasticity effects. Very recently, utilising a nonlocal viscoelastic sandwich-beam model, Liang and Bao [12] presented an analysis of a fluid-conveying carbon nanotube embedded in the Kelvin–Voigt two-parameter foundation. However, in formulating the governing equation for the fluid-conveying pipe, they ignored the nonlocal effects due to the presence of the Kelvin–Voigt parameters; therefore, their results on stability regions for the case of simply supported boundary conditions may be incorrect predictions.

Ghorbanpour Arani and Amir [13] studied free vibrations of double carbon nanotubes conveying a fluid and embedded in a visco-Pasternak medium based on the nonlocal elasticity theory. Their study utilised the Euler–Bernoulli beam (EBB) model for idealising the carbon nanotubes, which were placed in uniform temperature and magnetic fields. Hosseini et al. [14] analysed nonlocal free vibrations and stability of a single-walled carbon nanotube (SWCNT) conveying a nanoflow and embedded in a biological soft tissue. Considering nonlinear vibrations of fluid-conveying beams, Reddy and Wang [15] derived the governing equations of motion for the fluid-conveying beams by using the kinematic assumptions of both the Euler–Bernoulli and Timoshenko beam theories (the system of nonlinear differential equations was solved by the finite element method).

Ghorbanpour Arani et al. [16] studied the nonlinear flow-induced flutter instability related to double-bonded Reddy beams by considering the effect of a longitudinal magnetic field. Zhang et al. [17] derived nonlinear equations of three-dimensional motion for the case of straight fluid-conveying pipes with general boundary conditions represented by multiple springs at the two ends of the pipes. In that study, the flexural displacements were effectively expressed as a superposition of a Fourier cosine series with four additional supplementary functions to satisfy the boundary conditions.

In this paper, the governing equations are derived for a fluid-conveying single-walled carbon nanotube modelled as an Euler-Bernoulli beam embedded in a Pasternak-type elastic medium by using the consistent formulation of Tounsi et al. [9]. The solution for the pinned–pinned boundary situation is obtained by using the Fourier series approach, and that for the clamped–clamped end condition is obtained by utilizing the Galerkin method. Closed-form expressions are derived for the critical velocity for both boundary conditions considered here and are solved for different values of the nonlocal length parameter and the Winkler and Pasternak foundation stiffness parameters.

1. EQUATIONS OF MOTION AND METHOD OF THE SOLUTION

Let us formulate the equations of motion of the fluid in a carbon nanotube embedded in an elastic medium.

1.1. Nonlocal Relations

As discussed by Eringen [5], the nonlocal constitutive relations for the elastic medium have the form

$$[1-(e_0a)^2\nabla^2]\sigma_{kl}=\tau_{kl}, \tag{1}$$

where $\sigma_{kl}(x)$ is the nonlocal stress tensor at any point x, $\sigma_{kl}(x')$ is the local (classical) stress tensor at any point x', e_0 is a material constant, which depends on experimental results, and a is an internal characteristic length, which can be the C—C bond length or the lattice parameter. Equation (1) for a one-dimensional structure can be written as

$$\sigma_{xx}-(e_0a)^2\,\frac{\partial^2\sigma_{xx}}{\partial x^2}=E\varepsilon_{xx}, \tag{2}$$

where σ_{xx} is the axial stress, ε_{xx} is the axial strain, and E is Young's modulus of the carbon nanotube.

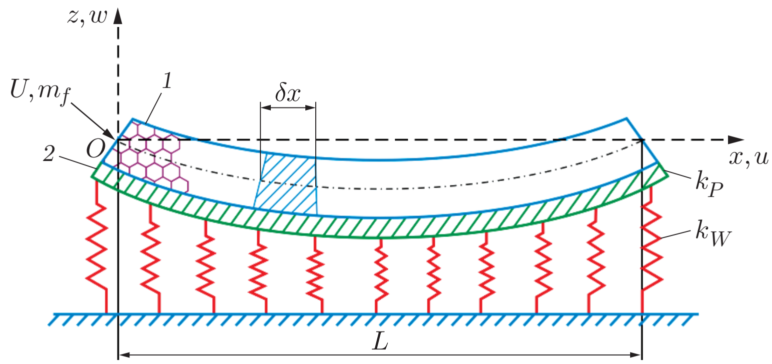

Fig. 1. Model of a fluid-conveying carbon nanotube embedded in an elastic Pasternak medium: (*1*) SWCNT; (*2*) foundation.

1.2. Model of the Pasternak Medium

The Pasternak model of the foundation is a two-parameter model, which implies that the elastic medium is mathematically not only a series of closely spaced linear springs (Winkler model), but also that there exist shear interactions between the linear spring elements of the Winkler model. This condition is realized in the model by introducing another parameter called the shear or the Pasternak stiffness parameter in addition to the Winkler stiffness parameter.

A fluid-conveying carbon nanotube embedded in the Pasternak medium is schematically shown in Fig. 1. The SWCNT of length L is clamped, or simply supported, or pinned at both ends. It has a mass per unit length m_c and flexural rigidity EI.

The SWCNT conveys an incompressible fluid of mass m_f with a uniform flow velocity U in the x direction through the SWCNT cross-sectional area A. In this model, the two foundation parameters are the Winkler modulus k_W and the Pasternak modulus k_P. Kerr [18] derived the following relation for the resisting force applied to the SWCNT by the Pasternak foundation per unit length:

$$k_P \frac{\partial^2 \sigma_{xx}}{\partial x^2} - k_W w = R_P(x,t). \tag{3}$$

1.3. Mathematical Model of the Fluid-Conveying SWCNT

The relation between the longitudinal strain, bending, shearing force Q, and bending moment M_b for the Euler–Bernoulli beam can be written as

$$\varepsilon_{xx} = z \frac{\partial^2 \sigma_{xx}}{\partial x^2}; \tag{4}$$

$$Q = -\frac{\partial M_b}{\partial x}, \qquad M_b = \int_A z\sigma_{xx}\, dA.$$

Equations (2) and (4) yield

$$M_b(x,t) = EI \frac{\partial^2 w}{\partial x^2} + (e_0 a)^2 \frac{\partial^2 M_b}{\partial x^2}. \tag{5}$$

Differentiating Eq. (5) twice with respect to x, we obtain

$$\frac{\partial^2 M_b}{\partial x^2} = EI \frac{\partial^4 w}{\partial x^4} + (e_0 a)^2 \frac{\partial^2}{\partial x^2}\Big(\frac{\partial^2 M_b}{\partial x^2}\Big). \tag{6}$$

The equation of motion for the fluid-conveying SWCNT embedded in the Pasternak medium is written as

$$-\frac{\partial^2 M_b}{\partial x^2} = -R_P + m_f a_{fz} + m_c a_{cz}. \tag{7}$$

The terms $m_c a_{cz}$ and $m_f a_{fz}$ in the right side of Eq. (7) are the inertial forces due to SWCNT element and fluid element acceleration, respectively. The fluid flow is considered to be a simple plug flow, where the expressions for the SWCNT element and fluid element accelerations are described by the following expressions [8]:

$$a_{cz} = \frac{\partial^2 w}{\partial t^2}, \qquad a_{fz} = \Big(\frac{\partial^2 w}{\partial t^2} + U^2 \frac{\partial^2 w}{\partial x^2} + 2U \frac{\partial^2 w}{\partial x\, \partial t}\Big). \tag{8}$$

Substituting Eq. (6) into Eq. (7) and using Eqs. (3) and (8), we obtain the following differential equation for the deflection w of the fluid-conveying SWCNT:

$$EI \frac{\partial^4 w}{\partial x^4} + M \frac{\partial^2 w}{\partial t^2} + (m_f U^2 - k_P) \frac{\partial^2 w}{\partial x^2} + 2m_f U \frac{\partial^2 w}{\partial x\, \partial t} + k_W$$

$$- (e_0 a)^2 \Big[M \frac{\partial^4 w}{\partial x^2\, \partial t^2} + (m_f U^2 - k_P) \frac{\partial^4 w}{\partial x^4} + 2m_f U \frac{\partial^4 w}{\partial x^3\, \partial t} + k_W \frac{\partial^2 w}{\partial x^2}\Big] = 0. \tag{9}$$

1.4. Solution of Eq. (9) for the pinned–pinned Case

Following the procedure given in the paper by Rao and Simha [19], the solution of Eq. (9) for the pinned–pinned case is considered as

$$w(x,t) = \sum_{n=1,3,5,\ldots} \frac{a_n \sin(n\pi x)}{L} \sin\omega_j + \sum_{n=2,4,6,\ldots} \frac{a_n \sin(n\pi x)}{L} \cos\omega_j, \qquad j = 1,2,3,\ldots \tag{10}$$

(ω_j is the natural frequency of the jth mode of vibrations). Solution (10) must satisfy the boundary conditions

$$w(0,t) = w(L,t) = 0, \qquad \frac{\partial^2 w(0,t)}{\partial x^2} = \frac{\partial^2 w(L,t)}{\partial x^2} = 0.$$

Substituting Eq. (10) into Eq. (9) and expanding the resultant expression into a Fourier series, we obtain the system of algebraic equations

$$[K - \omega_j^2 M I]\boldsymbol{a} = 0,$$

where K is the stiffness matrix whose elements are enumerated in [8], I is an identity matrix, and $\boldsymbol{a}^{\mathrm{t}} = (a_1, a_2, a_3, \ldots, a_n)$.

The SWCNT stability condition is defined by a particular value of the flow velocity parameter, called the critical flow velocity parameter V_{cr}. At the critical flow velocities, the natural frequencies of the system become zero. Retaining only the first two terms in the expansion and setting the determinant of the algebraic system and the natural frequencies to zero, we obtain the critical velocity equation

$$[4\pi^4 + 20\pi^6 e_n^2 + 16\pi^8 e_n^4]V^4 + [(-20\pi^6 - 8\pi^4\gamma_P - 5\pi^2\gamma_W) - e_n^2(320\pi^8 + 40\pi^6\gamma_P + 25\pi^4\gamma_W)$$

$$- e_n^4(20\pi^6\gamma_W + 32\pi^8\gamma_P)]V^2 + [(16\pi^8 + 20\pi^6\gamma_P + 17\pi^4\gamma_W + 4\pi^4\gamma_P^2 + 5\pi^2\gamma_P\gamma_W + \gamma_W^2)$$

$$+ e_n^2(20\pi^6\gamma_W + 5\pi^2\gamma_W^2 + 32\pi^8\gamma_P + 20\pi^6\gamma_P^2 + 25\pi^4\gamma_P\gamma_W)$$

$$+ e_n^4(16\pi^6\gamma_P^2 + 20\pi^6\gamma_P\gamma_W + 4\pi^4\gamma_W^2)] = 0, \tag{11}$$

where

$$V = UL\sqrt{\frac{m_f}{EI}}, \quad \beta = \frac{m_f}{m_c + m_f} = \frac{m_f}{M}, \quad M = m_c + m_f,$$

$$\gamma_W = \frac{k_W L^4}{EI}, \quad \gamma_P = \frac{k_P L^2}{EI}, \quad e_n = \frac{e_0 a}{L}, \quad \Omega_j = \omega_j \sqrt{\frac{ML^4}{EI}}.$$

It is seen that Eq. (11) is a simple quadratic equation with respect to V^2. Solving this equation, we obtain the critical flow velocity for the pinned–pinned case.

1.5. Solution of Eq. (9) *for the Clamped–Clamped Case*

Again, the procedure given in [8, 19] is adopted to obtain results for the clamped–clamped case:

$$w(x,t) = \mathrm{Re}\,[\varphi_n(x/L)\,\mathrm{e}^{i\omega t}]. \tag{12}$$

Here Re denotes the real part of the expression; $\varphi_n(x/L)$ are the finite expansions of the beam eigenfunctions $\psi_r(\xi)$ given by

$$\varphi_n(\xi) = \sum_{r=1}^{n} a_r \psi_r(\xi),$$

$\xi = x/L$, $\psi_r = \cosh(\lambda_r\xi) - \cos(\lambda_r\xi) - \sigma_r(\sin(\lambda_r\xi) - \sin(\lambda_r\xi))$, and λ_r is the frequency parameter of the clamped–clamped beam. For $r = 1, 2$, we have $\lambda_1 = 4.730\,041$ and $\lambda_2 = 7.853\,205$ [20].

Substitution of Eq. (12) into the equation of motion (9) yields

$$L_n(\varphi) = L_n\Big(\sum_{r=1}^{n} a_r\psi_r(\xi)\Big),$$

where L_n is the differential operator given by

$$L_n = \Big(EI - e_n^2L^2V^2\,\frac{EI}{L^2} + e_n^2L^2\gamma_P\,\frac{EI}{L^2}\Big)\frac{\partial^4}{\partial x^4} - 2e_n^2L^2M\beta\,\frac{V}{L}\sqrt{\frac{EI}{M\beta}}\,i\omega\,\frac{\partial^3}{\partial x^3}$$

$$+ 2M\beta\,\frac{V}{L}\sqrt{\frac{EI}{M\beta}}\,i\omega\,\frac{\partial}{\partial x} + \Big(\frac{V^2EI}{L^2} - \gamma_P\,\frac{EI}{L^2} - e_n^2L^2\gamma_W\,\frac{EI}{L^4}\Big)\frac{\partial^2}{\partial x^2} + e_n^2L^2M\omega^2\,\frac{\partial^2}{\partial x^2} = M\omega^2 + \gamma_W\,\frac{EI}{L^4}.$$

Using the Galerkin method, we obtain the following system of equations:

$$\int_0^L L\Big(\sum_{r=1}^{N} a_r\psi_r(\xi)\Big)\psi_s(\xi)\,dx = 0, \qquad s = 1, 2, 3, \ldots, N.$$

Retaining only two terms in the expansion and following the procedure detailed in [8], we obtain the equation for the critical flow velocity of the clamped–clamped fluid-conveying SWCNT:

$$\begin{aligned}
&[C_{11}C_{22} - e_n^2(\lambda_1^4C_{22} + \lambda_2^4C_{11}) + e_n^2(\lambda_1^4\lambda_2^4)]V^4 + \{[\lambda_1^4C_{22} + \lambda_2^4C_{11} + (C_{11} + C_{22})\gamma_W - 2C_{11}C_{22}\gamma_P]\\
&+ e_n^2[2\gamma_P(\lambda_1^4C_{22} + \lambda_2^4C_{11}) - 2\lambda_1^4\lambda_2^4 - (\lambda_1^4 + \lambda_2^4)\gamma_W - 2C_{11}C_{22}\gamma_W]\\
&- e_n^4[\gamma_W(\lambda_1^4C_{22} + \lambda_2^4C_{11}) + 2\lambda_1^4\lambda_2^4\gamma_P]\}V^2 + \{[\lambda_1^4\lambda_2^4 - \gamma_P(\lambda_1^4C_{22} + \lambda_2^4C_{11}) + (\lambda_1^4 + \lambda_2^4)\gamma_W\\
&+ C_{11}C_{22}\gamma_P^2 + \gamma_W^2 - (C_{11} + C_{22})\gamma_P\gamma_W] + e_n^2[2\gamma_P\lambda_1^4\lambda_2^4 + 2C_{11}C_{22}\gamma_P\gamma_W\\
&- \gamma_W(\lambda_1^4C_{22} + \lambda_2^4C_{11}) - \gamma_P^2(\lambda_1^4C_{22} + \lambda_2^4C_{11}) + (\lambda_1^4 + \lambda_2^4)\gamma_P\gamma_W - (C_{11} + C_{22})\gamma_W^2]\\
&+ e_n^4[\lambda_1^4\lambda_2^4\gamma_P^2 + C_{11}C_{22}\gamma_W^2 - \gamma_P\gamma_W(\lambda_1^4C_{22} + \lambda_2^4C_{11})]\} = 0.
\end{aligned} \tag{13}$$

This equation is again a quadratic equation with respect to V^2; C_{11} and C_{22} are the constants of integration [21, 22].

2. RESULTS AND DISCUSSION

The critical flow velocity V_{cr} is an important parameter for studying stability of fluid-conveying SWCNTs. At the critical flow velocity, the natural frequency becomes zero, leading to divergence instability of the SWCNT. The smallest roots of Eqs. (11) and (13) are the critical flow velocities V_{cr}. For the clamped–clamped and pinned–pinned boundary conditions, this parameter was evaluated for different values of the Winkler stiffness parameter γ_W, Pasternak stiffness parameter γ_P, and nonlocal parameter e_n.

Table 1. Critical flow velocity parameter V_{cr} for the pinned–pinned fluid-conveying SWCNT for different values of the nonlocal parameter e_n, Pasternak stiffness parameter γ_P, and Winkler stiffness parameter γ_W

γ_P	γ_W	V_{cr}						
		$e_n = 0$	$e_n = 0.05$	$e_n = 0.10$	$e_n = 0.15$	$e_n = 0.20$	$e_n = 0.25$	$e_n = 0.30$
10^{-6}	10^{-6}	3.1416	3.1035	2.9972	2.8419	2.6601	2.4707	2.2862
	1	3.1577	3.1198	3.0140	2.8596	2.6791	2.4911	2.3083
	10^2	4.4723	4.4457	4.3721	4.2671	4.1483	3.7308	3.3472
	10^4	17.1109	17.0069	16.7812	16.5593	16.3893	16.2693	16.1856
	10^6	159.2789	159.2678	159.2438	159.2206	159.2030	159.1907	159.1822
1	10^{-6}	3.2969	3.2607	3.1596	3.0127	2.8418	2.6654	2.4954
	1	3.3122	3.2762	3.1756	3.0294	2.8596	2.6843	2.5156
	10^2	4.5828	4.5568	4.4850	4.3827	4.2671	3.8625	3.4934
	10^4	17.1401	17.0363	16.8109	16.5895	16.4198	16.3000	16.2165
	10^6	159.2821	159.2709	159.2470	159.2237	159.2062	159.1938	159.1853
10^2	10^{-6}	10.4819	10.4705	10.4395	10.3960	10.3478	10.3007	10.2580
	1	10.4867	10.4754	10.4443	10.4008	10.3527	10.3056	10.2629
	10^2	10.9545	10.9437	10.9140	10.8724	10.8263	10.6733	10.5453
	10^4	19.8187	19.7290	19.5348	19.3445	19.1992	19.0968	19.0256
	10^6	159.5925	159.5814	159.5575	159.5343	159.5168	159.5045	159.4960
10^4	10^{-6}	100.0493	100.0481	100.0449	100.0404	100.0354	100.0305	100.0261
	1	100.0498	100.0487	100.0454	100.0409	100.0359	100.0310	100.0266
	10^2	100.1000	100.0988	100.0955	100.0910	100.0860	100.0696	100.0560
	10^4	101.4533	101.4359	101.3983	101.3618	101.3341	101.3148	101.3014
	10^6	188.0685	188.0591	188.0388	188.0192	188.0043	187.9938	187.9866
10^6	10^{-6}	1000.0049	1000.0048	1000.0045	1000.0040	1000.0035	1000.0030	1000.0026
	1	1000.0050	1000.0049	1000.0045	1000.0041	1000.0036	1000.0030	1000.0027
	10^2	1000.0100	1000.0099	1000.0096	1000.0091	1000.0086	1000.0070	1000.0056
	10^4	1000.1464	1000.1446	1000.1408	1000.1371	1000.1343	1000.1320	1000.1310
	10^6	1012.6054	1012.6037	1012.5999	1012.5963	1012.5935	1012.5910	1012.5902

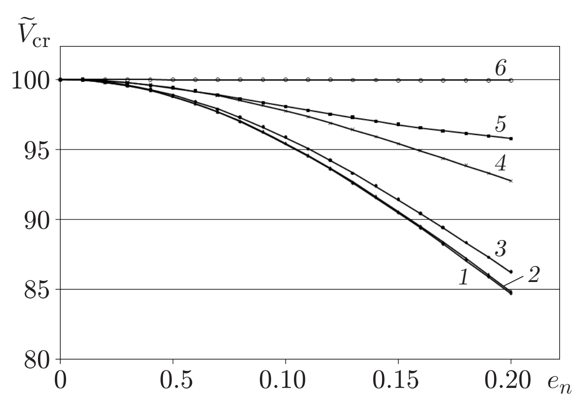

Fig. 2. Critical velocity parameter $\tilde{V}_{cr}$ for the pinned–pinned fluid-conveying SWCNT versus the nonlocal parameter e_n for $\gamma_P = 0$ and different values of the Winkler stiffness parameter: $\gamma_W = 0$ (*1*), 1 (*2*), 10 (*3*), 10^2 (*4*), 10^4 (*5*), and 10^6 (*6*).

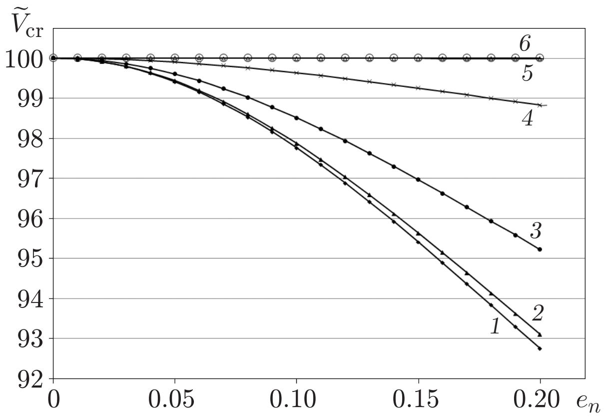

Fig. 3.

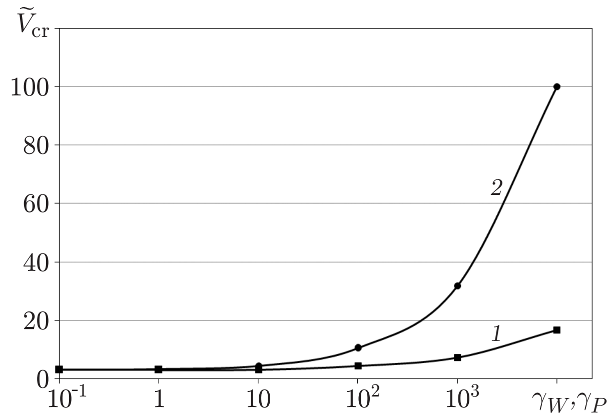

Fig. 4.

Fig. 3. Critical velocity parameter $\tilde{V}_{cr}$ for the pinned–pinned fluid-conveying SWCNT versus the nonlocal parameter e_n for $\gamma_W = 100$ and different values of the Pasternak stiffness parameter: $\gamma_P = 0$ (*1*), 1 (*2*), 10 (*3*), 10^2 (*4*), 10^4 (*5*), and 10^6 (*6*).

Fig. 4. Critical velocity parameter $\tilde{V}_{cr}$ for the pinned–pinned fluid-conveying SWCNT versus the stiffness parameters γ_W (*1*) and γ_P (*2*) for $e_n = 0.1$.

2.1. Pinned–Pinned Case

In the pinned–pinned case, the critical velocity is found by solving Eq. (11). The values of V_{cr} for different values of the stiffness parameters γ_W and γ_P and the nonlocal parameter e_n are listed in Table 1. It is seen that the critical velocity decreases with increasing e_n, and this decrease is more pronounced for lower values of the Pasternak stiffness parameter γ_P. The value $\gamma_W = \gamma_P = 10^{-6}$ can be considered to be zero stiffness, while the value $\gamma_W = \gamma_P = 10^6$ is considered to be very rigid. It is also seen from Table 1 that both stiffness parameters contribute to the SWCNT stiffness. In the formulation of Lee and Chang [6], the critical velocity is independent of the nonlocal parameter, due to the inherent error in the formulation. However, it can be noticed that the nonlocal parameter does affect the critical velocity, which is more pronounced at low values of both stiffness parameters of the embedding medium. At $e_n = 0$, the results are consistent with those of Lee and Chang [6]. The data in Table 1 agree with those of Wang [10], which confirms the validity of the present results.

Figure 2 shows the behaviour of $\tilde{V}_{cr} = (V_{cr}|_{e_n}/V_{cr}|_{e_n=0}) \cdot 10^2$ as a function of e_n for $\gamma_P = 0$ and different values of γ_W. It is clearly seen from Fig. 2 that the critical velocity decreases as the nonlocal parameter increases. The greater the stiffness parameters, the smaller the influence of the nonlocal parameter on the critical velocity.

Figure 3 shows the remarkable effect of the Pasternak stiffness parameter γ_P on the critical flow velocity of the pinned–pinned fluid-conveying SWCNT for $\gamma_W = 100$ and different values of the nonlocal parameter e_n. It is seen that the Pasternak parameter has a more pronounced stabilizing effect on the SWCNT–fluid system than the Winkler parameter. The effect of the nonlocal parameter is also diminished.

Figure 4 brings out the effects of both stiffness parameters on the critical velocity parameter. It is seen that the critical flow velocity increases more rapidly with an increase in the Pasternak parameter, as compared to the Winkler parameter.

2.2. Clamped–Clamped Case

The critical flow velocity is obtained by solving Eq. (13). Table 2 shows the values of V_{cr} for different values of the stiffness parameters γ_W and γ_P and the nonlocal parameter e_n. It is noticed that the critical flow velocity is higher than that for the pinned–pinned case for lower values of both stiffness parameters. It also can be observed that the effect of the nonlocal parameter on the critical velocity is quite significant as compared to the pinned–pinned case (see Table 1 and Figs. 5 and 6). The character of the critical velocity parameter as a function of the nonlocal parameter in the clamped–clamped case differs from that in the pinned–pinned case.

Table 2. Critical flow velocity parameter $V_{\rm cr}$ for the clamped–clamped fluid-conveying SWCNT for different values of the nonlocal parameter e_n, Pasternak stiffness parameter γ_P, and Winkler stiffness parameter γ_W

γ_P	γ_W	$V_{\rm cr}$						
		$e_n = 0$	$e_n = 0.05$	$e_n = 0.10$	$e_n = 0.15$	$e_n = 0.20$	$e_n = 0.25$	$e_n = 0.30$
10^{-6}	10^{-6}	6.3787	6.0771	5.3778	4.6089	3.9351	3.3888	2.9543
	1	6.3851	6.0833	5.3838	4.6147	3.9410	3.3948	2.9605
	10^2	6.9868	6.6735	5.9505	5.1633	4.4834	3.8432	3.3344
	10^4	17.3133	16.4064	14.7962	13.5492	12.7510	12.2502	11.9272
	10^6	147.6417	141.9136	131.9639	124.4889	119.8285	116.9607	115.1358
1	10^{-6}	6.4566	6.1588	5.4700	4.7161	4.0602	3.5333	3.1189
	1	6.4629	6.1650	5.4759	4.7218	4.0659	3.5390	3.1248
	10^2	7.0580	6.7480	6.0340	5.2593	4.5936	3.9712	3.4811
	10^4	17.3422	16.4368	14.8299	13.5861	12.7902	12.2910	11.9691
	10^6	147.6451	141.9172	131.9677	124.4930	119.8327	116.9650	115.1402
10^2	10^{-6}	11.8612	11.7018	11.3543	11.0110	10.7464	10.5586	10.4273
	1	11.8646	11.7050	11.3572	11.0134	10.7486	10.5605	10.4290
	10^2	12.1990	12.0223	11.6365	11.2543	10.9591	10.7131	10.5413
	10^4	19.9938	19.2138	17.8585	16.8399	16.2046	15.8135	15.5646
	10^6	147.9799	142.2655	132.3423	124.8899	120.2451	117.3874	115.5693
10^4	10^{-6}	100.2032	100.1845	100.1445	100.1062	100.0774	100.0574	100.0436
	1	100.2036	100.1849	100.1448	100.1064	100.0776	100.0576	100.0438
	10^2	100.2438	100.2224	100.1769	100.1332	100.1005	100.0738	100.0556
	10^4	101.4877	101.3369	101.0887	100.9137	100.8097	100.7475	100.7088
	10^6	178.3201	173.6073	165.5732	159.6794	156.0733	153.8825	152.5000
10^6	10^{-6}	1000.0200	1000.0190	1000.0150	1000.0110	1000.0080	1000.0060	1000.0040
	1	1000.0200	1000.0190	1000.0150	1000.0110	1000.0080	1000.0060	1000.0040
	10^2	1000.0240	1000.0220	1000.0180	1000.0130	1000.0100	1000.0070	1000.0060
	10^4	1000.1500	1000.1350	1000.1100	1000.0920	1000.0810	1000.0750	1000.0710
	10^6	1010.8400	1010.0200	1008.6700	1007.7190	1007.1540	1006.8170	1006.6060

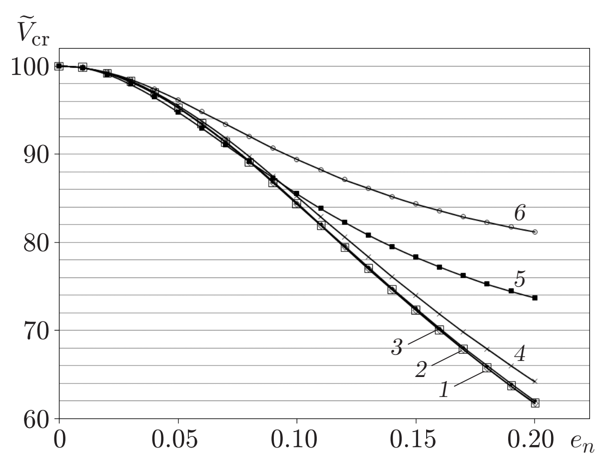

Fig. 5. Critical velocity parameter $\widetilde{V}_{\rm cr}$ for the clamped–clamped fluid-conveying SWCNT versus the nonlocal parameter e_n for different values of the Winkler stiffness parameter γ_W: $\gamma_W = 0$ (*1*), 1 (*2*), 10 (*3*), 10^2 (*4*), 10^4 (*5*), and 10^6 (*6*).

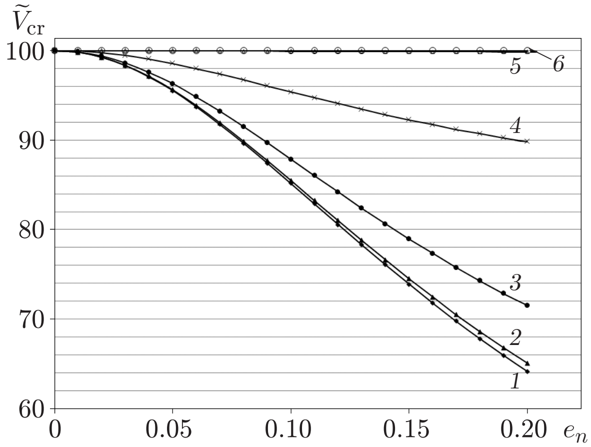

Fig. 6.

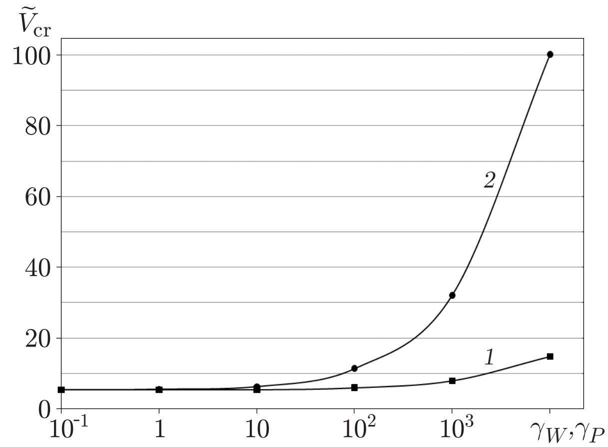

Fig. 7.

Fig. 6. Critical velocity parameter $\tilde{V}_{cr}$ for the clamped–clamped SWCNT versus the nonlocal parameter e_n for different values of the Pasternak stiffness parameter: $\gamma_P = 0$ (*1*), 1 (*2*), 10 (*3*), 10^2 (*4*), 10^4 (*5*), and 10^6 (*6*).

Fig. 7. Critical velocity parameter $\tilde{V}_{cr}$ for the clamped–clamped SWCNT versus the stiffness parameters γ_W (*1*) and γ_P (*2*) for $e_n = 0.1$.

Figure 7 shows the relative effects of the Winkler and Pasternak stiffness parameters on the critical flow velocity. As expected, the Pasternak stiffness parameter exerts a more pronounced effect on the critical velocity than the Winkler stiffness parameter.

CONCLUSIONS

This study has attempted to address the gaps in the literature by presenting numerical results for the stability of a fluid-conveying SWCNT embedded in a two-parameter elastic medium like the Pasternak medium. The governing equations are formulated based on the concept of the nonlocal elasticity theory. The two-term Fourier series solution procedure is utilized for dealing with the pinned–pinned end condition. The two-term Galerkin procedure is used for the case of the clamped–clamped boundary condition. It is found that the SWCNT becomes less stable as the nonlocal parameter increases.

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