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SIMULATION OF DYNAMIC PROCESSES IN THREE-DIMENSIONAL LAYERED FRACTURED MEDIA WITH THE USE OF THE GRID-CHARACTERISTIC NUMERICAL METHOD

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Abstract: This paper touches upon the computer simulation of the propagation of elastic waves in three-dimensional multilayer fractured media. The dynamic processes are described using the defining system of equations in the partial derivatives of the deformed solid mechanics. The numerical solution of this system is carried out via the grid-characteristic method on curvilinear structural grids. The fractured nature of the medium is accounted for by explicitly selecting the boundaries of individual cracks and setting special boundary conditions in them. Various models of heterogeneous deformed media with a fractured structures are considered: a homogeneous medium, a medium with horizontal boundaries, a medium with inclined boundaries, and a medium curvilinear boundaries. The wave fields detected on the surface are obtained, and their structures are analyzed. It is demonstrated that it is possible to detect the waves scattered from fractured media even in the case of nonparallel (inclined and curvilinear) boundaries of geological layers.

Keywords: fractured media, mathematical simulation, numerical methods, parallel algorithms, direct seismic prospecting tasks, composite materials.

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INTRODUCTION

The numerical methods and algorithms for correct simulation of dynamic processes occurring in layered structures, composite materials, and geological media during seismic prospecting are actively developed at the current moment. It should be noted that the ray-tracing methods widely used in the industry are approximate methods and do not make it possible to understand the behavior of most real fractured geological media. There are a great number of numerical methods for the simulation of seismic processes in geological media (the finite difference method, the Galerkin method, etc) [1–3]. There are also various numerical methods actively used to construct hybrid calculation algorithms [4]. Also note that, as the defining system of equations of elasticity that describes the propagation of seismic waves is hyperbolic, its numerical solution can be carried out by the grid-characteristic method.

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Apparently, the characteristic method was proposed for the first time in [5]. It was described in detail for a one-dimensional case in [6] and later generalized for a multidimensional case in [7]. Because the points at which the solution is calculated might become concentrated and, accordingly, the accuracy of the calculation could decrease, this method did not become widely used. The interpolation procedure was introduced in [8, 9], and it allowed converting this method into an inverse characteristic method (grid-characteristic method) [8, 9].

Earlier, the characteristic method was used in the numerical solution of the problems of gaseous dynamics [10]. It was adapted to solve the problems of deformed solid mechanics only in 1980s [11, 12]. At first, seismic fields were described by the characteristic method on unstructured triangular grids. It was used in [13] for the numerical simulation of seismic response in multilayer geological media in a two-dimensional case. Later, the grid-characteristic method was generalized for the case of the presence of a fluid-saturated crack in a uniform elastic medium [14]. In [15], the numerical simulation of wave response was carried out with account for stratification and cracking. In [16], the response of a cluster (set) of fluid-saturated cracks was studied. Aside from the series of calculations in a two-dimensional case, a three-dimensional test calculation was carried out on unstructured grids. The modification of the method with the use of unstructured grids was described in detail in [17]. The main restriction in three-dimensional calculations is the computational complexity of the problem. However, if the grid-characteristic method proposed in [18] is used on hexahedral grids, then it is possible to increase the speed of calculations significantly and simulate the three-dimensional problem of the formation of the response of the cluster of fluid-saturated cracks located in a uniform medium.

This paper describes the numerical simulation of propagation of elastic waves in a three-dimensional fractured medium with the curvilinear layer interface and the analysis of the wave fields recorded on the surface.

STATEMENT OF THE PROBLEM

The computational domain was a $10 \times 10 \times 3$ km rectangular parallelepiped, whose upper boundary was considered to be free of the applied forces (daylight surface), and the consumption condition preventing the formation of the reflected waves was set on the rest of the edges. The medium model consisted of a host array and a system (cluster) of subvertical cracks. At a depth of 2 km, in the middle of the host array, there was a cluster of 31 fluid-saturated macrocracks. The length of each crack was 3 km, the height was 100 m, and the interval between cracks was 100 m. The angle of slope of the crack plane to the Ox axis was $-5°$.

There were five host array models under consideration:

(1) homogeneous medium with the following elastic characteristics: the propagation velocity of longitudinal waves is $V_l = 4270$ m/s, and the propagation velocity of transverse waves is $V_t = 2135$ m/s;

(2) layered medium with horizontal boundaries and the elastic characteristics given in the table (h is the thickness of the layer);

(3) layered medium with an inclined boundary between layers 1 and 2, given by the expression

$$Z = -0.01X - 450;$$

(4) layered medium with the curvilinear boundary between layers 2 and 3, given by the expression

$$Z = -1500 + 100 \sin(\pi(X - 5000)/2);$$

5) layered medium obtained as a result of combining models 3 and 4.

The density of the host array in all models was assumed to be constant and equal to 2500 kg/m^3. Figure 1 shows the diagram of the layered fractured medium cross-sectioned by the plane Oxz.

Characteristics of the elastic layers that the host array consists of

Layer number	h, m	V_l, m/s	V_t, m/s
1	500	3500	1750
2	1000	4500	2250
3	300	5000	2500
4	100	4000	2000
5	300	5500	2750
6	800	5500	2750

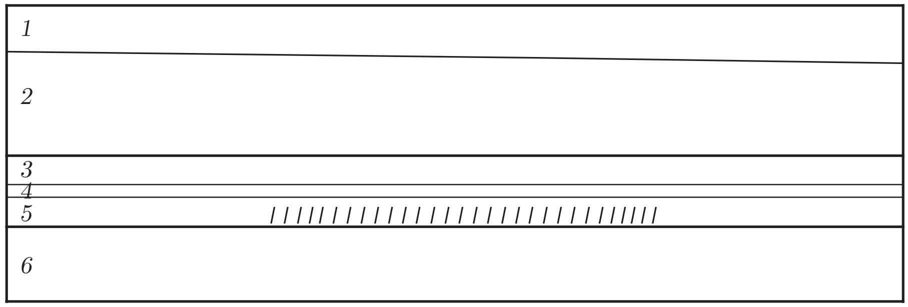

Fig. 1. Diagram of the layered fractured medium cross-sectioned by the plane Oxz: (*1–6*) layer numbers; the hatchet region denotes the cluster of fluid-saturated macrocracks.

MATHEMATICAL MODEL AND NUMERICAL METHOD

The dynamic processes in the medium were described using the mathematical model of the linear-elastic isotropic body with explicitly manifested contact boundaries between layers and crack surfaces. The dynamic behavior of the infinitesimal volume of the medium is described by the elasticity equations [19] of the form

$$\rho \frac{\partial V_i}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j}, \qquad \frac{\partial \sigma_{ij}}{\partial t} = q_{ijkl} \frac{\partial e_{kl}}{\partial t}, \quad q_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}). \tag{1}$$

Here ρ is the density of the medium; V is the velocity of the medium; σ is the strain tensor; x is the coordinate; q is the fourth order tensor that determines the rheology of the medium; e is the strain tensor; δ is the Kronecker symbol; λ and μ are the Lame parameters.

It should be noted that the analytical solution can be obtained only in the simplest cases (a homogeneous medium, a point pressure source, or a plane wave). The defining system is hyperbolic, and its numerical solution is carried out by the grid-characteristic method. The scheme of the fourth-order accuracy in space was used, and it had the following form for a one-dimensional transfer equation:

$$v_m^{n+1} = v_m^n - \sigma(\Delta_1 - \sigma(\Delta_2 - \sigma(\Delta_3 - \sigma\Delta_4))),$$

$$\Delta_1 = (-2v_{m+2}^n + 16v_{m+1}^n - 16v_{m-1}^n + 2v_{m-2}^n)/24,$$

$$\Delta_2 = (-v_{m+2}^n + 16v_{m+1}^n - 30v_m^n + 16v_{m-1}^n - v_{m-2}^n)/24,$$

$$\Delta_3 = (2v_{m+2}^n - 4v_{m+1}^n + 4v_m^n - 2v_{m-2}^n)/24,$$

$$\Delta_4 = (v_{m+2}^n - 4v_{m+1}^n + 6v_m^n - 4v_{m-1}^n + v_{m-2}^n)/24,$$

$$\sigma = c\tau/h.$$

The monotonicity of the scheme was ensured by using the grid-characteristic criterion of monotonicity

$$\min\{v_m^n, v_{m-1}^n\} \leqslant v_m^{n+1} \leqslant \max\{v_m^n, v_{m-1}^n\}.$$

If monotonicity criterion was not satisfied, we used a limiter that reduced the order of the decision to the third in the case of large gradients:

$$\hat{v}_m^{n+1} = \begin{cases} \max\{v_m^n, v_{m-1}^n\}, & v_m^{n+1} > \max\{v_m^n, v_{m-1}^n\}, \\ \min\{v_m^n, v_{m-1}^n\}, & v_m^{n+1} < \min\{v_m^n, v_{m-1}^n\}, \\ v_m^{n+1}, & \min\{v_m^n, v_{m-1}^n\} \leqslant v_m^{n+1} \leqslant \max\{v_m^n, v_{m-1}^n\}. \end{cases}$$

In the case of an inclined (curved) boundary and the presence of cracks in the layers, each layer was described by a structural parallelepipedal or hexahedral grid. All layers, except for layer 5, correspond to the homogeneous isotropic medium, so system (1) is solved for them. Layer 5 contains cracks described by a fixed set of nodes located on their edges. During the calculation, the stress tensor and velocity vector values in the nodes are additionally corrected in order to ensure free sliding along the plane of the crack and the unimpeded passage of the longitudinal waves incident along the normal to the plane of the crack. Between the boundary nodes of the grids belonging to different layers, there is a condition of complete adherence that ensures the continuity of the velocity vector and the fulfillment of Newton's third law in the nodes. A more detailed description of the grid-characteristic numerical method on structural computational grids that contain fluid-saturated cracks is given in [18].

The above-described mathematical model and numerical method are implemented in the software package developed at the Moscow Institute of Physics and Technology. As the computational complexity of the solved problem is great, the acceptable calculation time can be ensured by parallelization of the computational algorithm on the basis of the MPI technology. The effectiveness of the parallelization amounted to about 80 %. The calculations were carried out on a supercomputer containing 30 processor cores and having a RAM of 128 GB. The duration of one calculation was approximately 10 h.

RESULTS OF NUMERICAL SIMULATION

The disturbance source during simulation was a longitudinal wave with a length of 150 m (rectangular pulse) set at a depth of 200 m and propagating vertically downward along the Oz axis. On the upper boundary of the rectangular parallelepiped, there is a set of three-component gauges located at an interval of 40 m. The calculated grid contained 75 000 000 nodes.

Figure 2 shows the wave pattern (velocity modulus distribution in the volume of the medium at a fixed time) for model 3.

Lately, seismic prospecting has often been carried out using three-component seismometers [20]. Figure 3 presents the surface wave fields constructed according to the vertical component of the displacement velocity vector (models 1 and 2). In the case of a homogeneous medium, there is a clearly visible signal coming from the fractured structure, while, in the case of a layered medium, it is completely covered by multiply reflected waves.

In the case of a plane longitudinal wave incident along the normal to the interface of the elastic media, the wave reflection coefficient P-S is zero, which means that there is no exchange wave with the horizontal displacement component. Thus, on the surface wave field constructed according to the horizontal component of the displacement velocity vector, there should be no multiply reflected waves that impede the interpretation of the wave pattern. Figure 4 presents the surface wave fields constructed according to the horizontal component of the displacement velocity vector V_x. The main difference of model 2 from model 1 is in the presence of an additional plane signal at the times $t = 1.90$; 2.25 s because of which the wave that passed through the fractured cluster has the horizontal

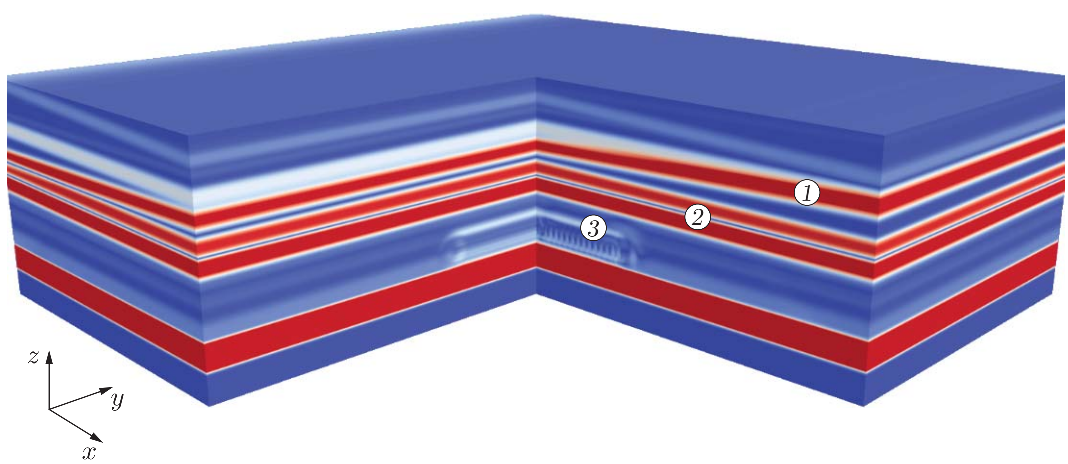

Fig. 2. Wave pattern for model 3: the dark regions are the regions in which the velocity modulus is zero, the light regions are the regions in which the velocity modulus is maximal; (*1*) the wave formed as a result of reflection from the inclined boundary and daylight surface; (*2*) the pair of waves formed as a result of reflection from the upper and lower boundaries of layer 4; (*3*) the scattered waves reflected from the fractured cluster.

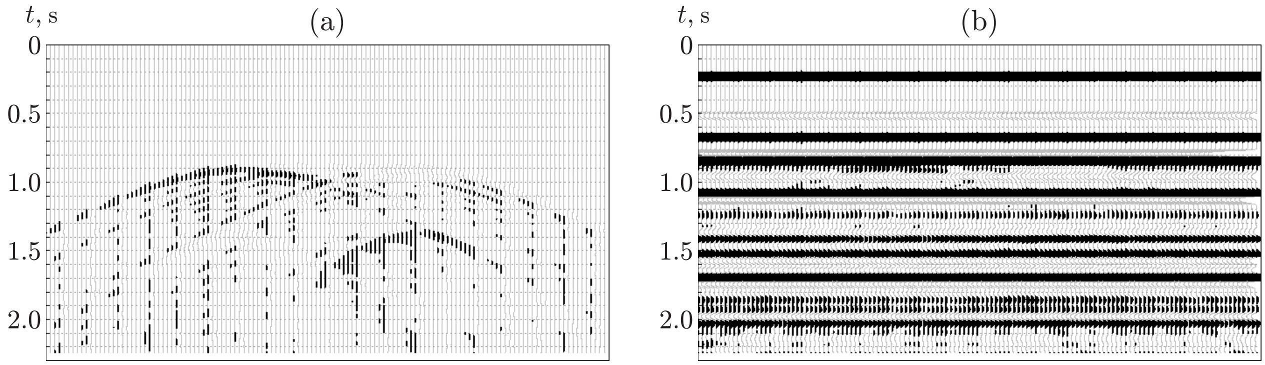

Fig. 3. Seismograms constructed according to the vertical component of the displacement velocity vector V_z for model 1 (a) and model 2 (b).

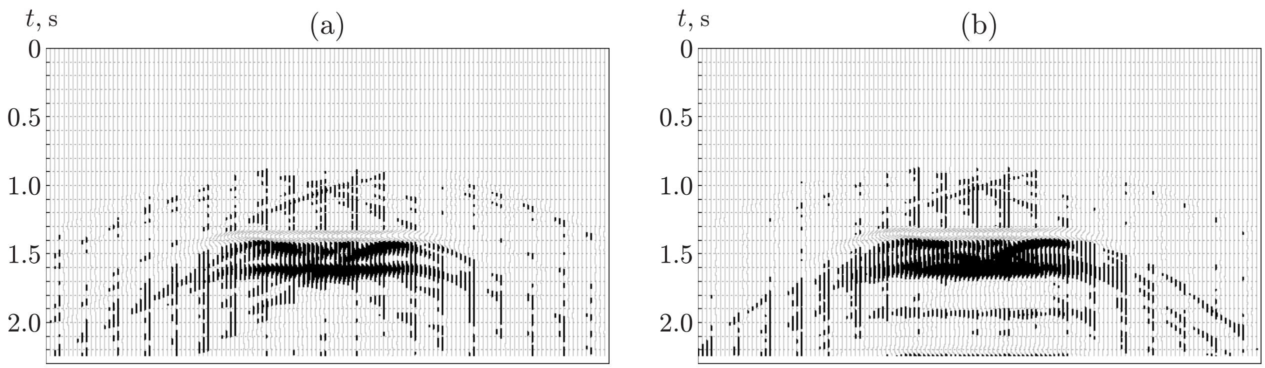

Fig. 4. Seismograms constructed according to the horizontal component of the displacement velocity vector V_x for model 1 (a) and model 2 (b).

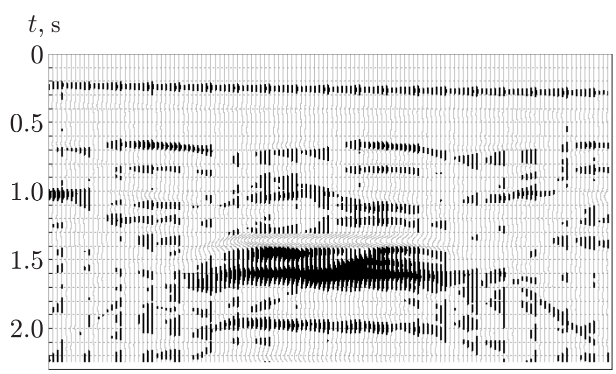

Fig. 5. Seismogram constructed according to the horizontal component of the displacement velocity vector V_x for model 5.

component of the displacement velocity vector. It follows from the results of the numerical calculation that, when constructing a seismogram, it is reasonable to detect the horizontal component of the displacement velocity vector for plane-parallel fractured media.

We carried out the numerical simulation of the wave field for all five host array models. The most complex signal was observed for model 5 due to the presence of non-horizontal boundaries with the largest angle of slope. It is of interest to isolate the scattered component (response of the fractured geological object) of the elastic field under these conditions. Figure 5 shows the surface wave field constructed according to the horizontal component of the displacement velocity vector for model 5. The wave reflected from the inclined boundary is seen at $t \approx 0.25$ s, the wave reflected from the curved boundary is seen at $t \approx 0.6$ s, and the scattered component clearly detected in the case of one-dimensional array is slightly seen in the interval $t \approx 0.6$–1.3 s. However, at $t \approx 1.5$–1.6 s, the main signal from the cluster of cracks easily observed on the seismogram arrives at the gauges. In the case of curved boundaries, the multicomponent detection also helps finding the waves reflected from fractured inclusions, which can correspond to the regions of accumulation of oil and gas at deposits or local fracture regions in composite materials.

CONCLUSION

This paper describes the simulation of the propagation of elastic waves in a multilayer elastic fractured medium. This was carried out with the use of numerical grid-characteristic method on hexahedral computational grids, which made it possible to calculate the theory of elasticity in the three-dimensional problem during a given time. The results obtained by the detected three-component gauges located on the surface were used to construct surface wave fields. It is shown on the basis of the analysis of these fields that, when constructing seismograms for the media consisting of plane parallel layers, it is reasonable to use the horizontal component of the displacement velocity vector, in which case it is possible to detect the scattered component of the signal that arises in the reflection from the cluster of cracks not only in the case of homogeneous or plane-parallel host array, but also in the presence of curvilinear contact boundaries between the layers.

The results of computer simulation can be used, for example, when choosing an effective placement system for sources-receivers at the stage of planning of field research or in testing the techniques of inverse problems of seismic exploration (migration and inversion). The described method can be also used to determine defects in composite materials and layered structures.

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