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FREE VIBRATIONS OF AN ANISOTROPIC CYLINDRICAL FIBERGLASS SHELL REINFORCED BY ANNULAR RIBS AND CONTAINING FLUID FLOW

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Abstract: This paper presents the results of determining the free vibration frequency of a structurally anisotropic, cylindrical fiberglass shell reinforced by annular ribs and containing flowing fluid. Boundary Navier conditions are imposed on the ends of the shell. Natural vibration frequencies are calculated as dependences of the frequency on the fiberglass winding angle and fluid flow velocity for different values of the wave formation parameters and the parameters characterizing the geometric dimensions of the shell.

Keywords: anisotropic ribbed shell, free vibrations, natural vibration frequencies.

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In the design of thin-walled shell structures widely used in the aviation, space engineering, and various industrial areas, an important problem is the dynamic calculation of the stress–strain state of these structures. In studies of shell dynamics, it is necessary to determine the eigenfrequencies and modes of small vibrations, with the lower frequencies of the spectrum being of greatest interest. The thin-walled shell is reinforced by ribs to increase its stiffness, which significantly improves its strength with a slight increase in the mass of the structure even if the ribs have a small height. The results of determining the natural frequencies of axisymmetric vibrations of smooth orthotropic cylindrical shells in an endless elastic medium containing fluid are presented in [1, 2]. The stability and vibrations of isotropic shells under static and dynamic loading are studied in [3]. Natural vibrations of an isotropic cylindrical shell reinforced by longitudinal and cross-sectional systems of ribs and containing flowing fluid in an infinite elastic medium are considered in [4–7].

This paper presents the results of determining the free vibration frequencies of a structurally anisotropic cylindrical fiberglass shell reinforced by annular ribs and containing flowing fluid. It is assumed that the annular ribs are spaced equally apart on the outer surface of the shell.

FORMULATION OF THE PROBLEM

The total energy of elastic deformation of an anisotropic cylindrical shell loaded by external forces can be expressed as

$$J = \frac{1}{2}\,R^2 \int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2} (N_{11}\varepsilon_{11} + N_{22}\varepsilon_{22} + N_{12}\varepsilon_{12} - M_{11}\chi_{11} - M_{22}\chi_{22} - M_{12}\chi_{12})\,dx\,dy$$

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$$+\frac{1}{2}\sum_{j=1}^{k_2}\int\limits_{x_1}^{x_2}\Big[E_jF_j\Big(\frac{\partial u_j}{\partial x}\Big)^2+E_jJ_{yj}\Big(\frac{\partial^2 w_j}{\partial x^2}\Big)^2+E_jJ_{zj}\Big(\frac{\partial^2 v_j}{\partial x^2}\Big)^2+G_jJ_{\mathrm{cr}j}\Big(\frac{\partial\varphi_{\mathrm{cr}j}}{\partial x}\Big)^2\Big]\,dx$$

$$+\rho_0h\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}\Big[\Big(\frac{\partial u}{\partial t}\Big)^2+\Big(\frac{\partial v}{\partial t}\Big)^2+\Big(\frac{\partial w}{\partial t}\Big)^2\Big]\,dx\,dy-\int\limits_{x_1}^{x_2}\int\limits_{y_1}^{y_2}q_zw\,dx\,dy$$

$$+\sum_{j=1}^{k_2}\rho_jF_j\int\limits_{y_1}^{y_2}\Big[\Big(\frac{\partial u_j}{\partial t}\Big)^2+\Big(\frac{\partial v_j}{\partial t}\Big)^2+\Big(\frac{\partial w_j}{\partial t}\Big)^2+\frac{J_{\mathrm{cr}j}}{F_j}\Big(\frac{\partial\varphi_{\mathrm{cr}j}}{\partial t}\Big)^2\Big]\,dy. \qquad (1)$$

The internal forces and moments can be represented as

$$N_{ij}=\int\limits_{-h/2}^{h/2}(\sigma_{ij}+zw_{ij})\,dz,\qquad M_{ij}=-\int\limits_{-h/2}^{h/2}(\sigma_{ij}+zw_{ij})z\,dz, \qquad (2)$$

where

$$w_{11}=B_{11}\chi_{11}+B_{12}\chi_{22}+B_{16}\chi_{12},\qquad w_{22}=B_{12}\chi_{11}+B_{22}\chi_{22}+B_{26}\chi_{12},$$

$$w_{21}=w_{12}=B_{16}\chi_{11}+B_{22}\chi_{22}+B_{66}\chi_{12}.$$

In view of (2), the stresses σ_{ij} and strains ε_{ij} in the middle surface are defined as follows:

$$\sigma_{11}=B_{11}\varepsilon_{11}+B_{12}\varepsilon_{22}+B_{16}\varepsilon_{12},\qquad \sigma_{22}=B_{12}\varepsilon_{11}+B_{22}\varepsilon_{22}+B_{26}\varepsilon_{12},$$

$$\sigma_{12}=B_{16}\varepsilon_{11}+B_{26}\varepsilon_{22}+B_{66}\varepsilon_{12},$$

$$\varepsilon_{11}=\frac{\partial u}{\partial x},\qquad \varepsilon_{22}=\frac{\partial v}{\partial y}+w,\qquad \varepsilon_{12}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},$$

$$\chi_{11}=\frac{\partial^2 w}{\partial x^2},\qquad \chi_{22}=\frac{\partial^2 w}{\partial y^2},\qquad \chi_{12}=-2\,\frac{\partial^2 w}{\partial x\,\partial y}.$$

The elastic constants depend on the fiberglass winding angle φ and are given by the formulas

$$\begin{aligned}
B_{11}&=b_{11}\cos^4\varphi+b_{22}\sin^4\varphi+(b_{66}+0.5b_{12})\sin^2 2\varphi,\\
B_{22}&=b_{11}\sin^4\varphi+b_{22}\cos^4\varphi+(b_{66}+0.5b_{12})\sin^2 2\varphi,\\
B_{12}&=(b_{11}+b_{22}-4b_{66})\sin^2\varphi\cos^2\varphi+b_{12}(\sin^4\varphi+\cos^4\varphi),\\
B_{66}&=-(b_{11}+b_{22}-2b_{12})\sin^2\varphi\cos^2\varphi+b_{66}\cos^2 2\varphi,\\
B_{26}&=(1/2)(b_{22}\cos^2\varphi-b_{11}\sin^2\varphi)\sin 2\varphi-(1/6)(b_{12}+2b_{66})\sin 4\varphi,\\
B_{16}&=(1/2)(b_{22}\sin^2\varphi-b_{11}\cos^2\varphi)\sin 2\varphi-(1/6)(b_{12}+2b_{66})\sin 4\varphi.
\end{aligned} \qquad (3)$$

In (1)–(3), b_{11}, b_{22}, b_{12}, and b_{66} are the basic elastic moduli of the orthotropic material, φ is the angle between the fiberglass winding direction and the circumferential direction, R is the radius of the middle surface of the shell, h is the thickness of the shell, u, v, and w are the displacement components of points of the middle surface of the shell, x_1 and x_2 are the coordinates of the curvilinear edges of the shell, q_z is the fluid pressure on the shell, F_j, J_{zj} and J_{yj}, and $J_{\mathrm{cr}j}$ are the area and moments of inertia of the cross section of the jth cross bar relative to the axis Oz and the axis parallel to the axis Oy and passing through the center of gravity of the section and its moment of inertia in torsion, respectively, E_j and G_j are the elastic and shear moduli of the material of the jth cross bar, respectively, t is time, $t_1=\omega_0t$, $\omega_0=\sqrt{b_{11}/[(1-\nu^2)\rho_0R^2]}$, and ρ_0 and ρ_i are the densities of the materials of the shell and the jth cross bar, respectively.

The equations of motion for an anisotropic shell reinforced by annular ribs and loaded by radial forces and containing flowing fluid were obtained based on the Ostrogradskii–Hamilton principle of stationary action

$$\delta W = 0, \tag{4}$$

where $W = \int\limits_{t'}^{t''} L\,dt$ is the action after Hamilton, L is the Lagrangian, and t' and t'' are the specified arbitrary times.

Assuming that the flow velocity is U and the perturbations of this velocity are small, we use the wave equation for the perturbed velocity potential $\tilde{\varphi}$ [8]

$$\Delta\tilde{\varphi} - \frac{1}{a_0^2}\Big(\frac{\partial^2\tilde{\varphi}}{\partial t^2} + 2U\,\frac{\partial^2\tilde{\varphi}}{R\,\partial\xi\,\partial t} + U^2\,\frac{\partial^2\tilde{\varphi}}{R^2\,\partial\xi^2}\Big) = 0. \tag{5}$$

On the shell–fluid interface, the continuity condition for the radial velocities and pressures is satisfied. The impermeability condition of the shell wall has the following form [8, 9]:

$$v_r\Big|_{r=R} = \frac{\partial\tilde{\varphi}}{\partial r}\Big|_{r=R} = -\Big(\omega_0\,\frac{\partial w}{\partial t_1} + U\,\frac{\partial w}{R\,\partial\xi}\Big). \tag{6}$$

The radial fluid pressure on the shell is expressed as

$$q_z = -p\Big|_{r=R}. \tag{7}$$

Completing the expression for the total energy of the shell (1) and the equation of fluid motion (5) with contact conditions (6), (7), we obtain the problem of natural vibrations of an anisotropic cylindrical shell reinforced by cross ribs and containing flowing fluid.

SOLUTION OF THE PROBLEM OF NATURAL VIBRATIONS OF THE SHELL

It is assumed that the ends of the shell are simply supported and can be moved in the longitudinal direction, and their displacements in the circumferential direction are zero. These boundary conditions are called the Navier conditions [10]. Thus, for $x = 0$ and $x = L_1$, we have $v = 0$, $w = 0$, $M_x = 0$, and $N_x = 0$. The displacement vector components of the middle surface of the shell will be sought in the form

$$\begin{gathered} u = u_0\cos\chi\xi\cos n\theta\sin\omega_1 t_1, \qquad v = v_0\sin\chi\xi\sin n\theta\sin\omega_1 t_1, \\ w = w_0\sin\chi\xi\cos n\theta\sin\omega_1 t_1. \end{gathered} \tag{8}$$

Here u_0, v_0, and w_0 are unknown constants, L_1 is the length of the shell, M_x is the bending moment, N_x is the longitudinal strain, and χ and n are the wavenumbers of the longitudinal and circumferential directions, respectively.

The perturbed velocity potential $\tilde{\varphi}$ is given by the formula

$$\tilde{\varphi}(\xi, r, \theta, t_1) = f(r)\cos n\theta\cos n\theta\cos\omega_1 t_1. \tag{9}$$

Using (9), from (6) and (5) we obtain

$$\begin{gathered} \tilde{\varphi} = -\Phi_{\alpha n}\Big(\omega_0\,\frac{\partial w}{\partial t_1} + U\,\frac{\partial w}{R\partial\xi}\Big), \\ p = \Phi_{\alpha n}\rho_m\Big(\omega_0^2\,\frac{\partial^2 w}{\partial t_1^2} + 2U\omega_0\,\frac{\partial^2 w}{R\,\partial\xi\,\partial t_1} + U^2\,\frac{\partial^2 w}{R^2\partial\,\xi^2}\Big), \end{gathered} \tag{10}$$

where

$$\Phi_{\alpha n} = \begin{cases} I_n(\beta r)/I'_n(\beta r), & M_1 < 1, \\ J_n(\beta_1 r)/J'_n(\beta_1 r), & M_1 > 1, \\ R^n/(nR^{n-1}), & M_1 = 1, \end{cases}$$

$M_1 = (U + \omega_0 R\omega_1/\alpha)/a_0$, $\beta^2 = R^{-2}(1 - M_1^2)\chi^2$, $\beta_1^2 = R^{-2}(M_1^2 - 1)\chi^2$, I_n are modified Bessel functions of the first kind of order n, and J_n are Bessel functions of order n.

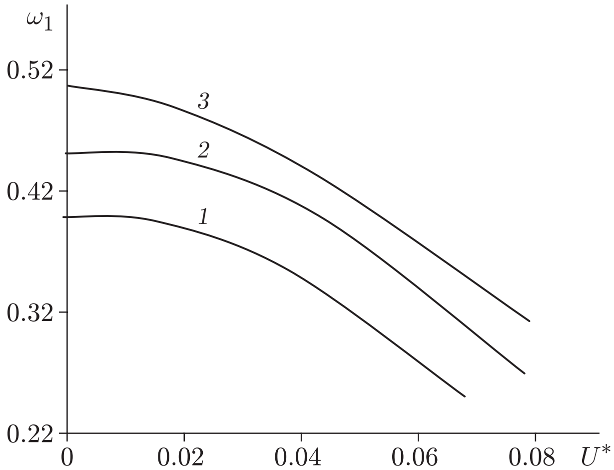

Fig. 1.

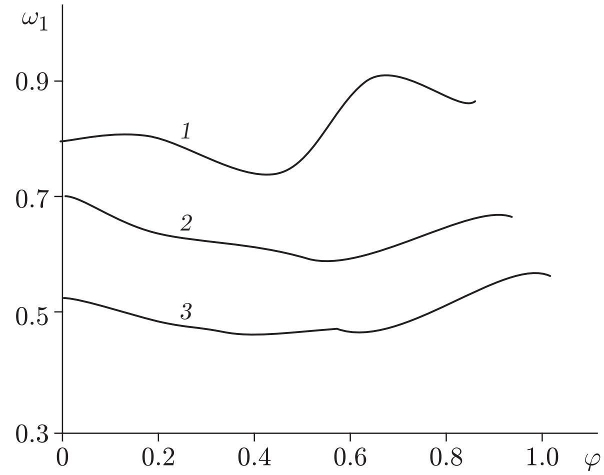

Fig. 2.

Fig. 1. Vibration frequency versus flow velocity in a cylindrical shell reinforced by a system of cross ribs for $\chi = 1$, $n = 4$, $\varphi = 0.55$, and $b_{11}/b_{22} = 0.75$ (*1*), 1.00 (*2*), and 1.25 (*3*).

Fig. 2. Free vibration frequency versus fiberglass winding angle φ for $\chi = 1$, $n = 4$, and $h/R = 0.001$ (*1*), 0.002 (*2*), and 0.004 (*3*).

Next, condition (7) is replaced by the condition $q_z = -p$ [p is the pressure defined by formula (10)]. In view of (8), the pressure p can be expressed as

$$p = \frac{\rho_m \Phi_{\alpha n}}{\rho_0 \omega_0^2 h} (\omega_0^2 \omega_1^2 + 2\omega_0 \omega_1 \chi U + \chi^2 U^2) w. \qquad (11)$$

Substitution of (11) and (8) into (4) reduces the problem to a homogeneous system of linear algebraic equations of the third order

$$a_{i1} u_0 + a_{i2} v_0 + a_{i3} w_0 = 0 \qquad (i = 1, 2, 3). \qquad (12)$$

The expressions for the coefficients a_{i1}, a_{i2}, and a_{i3} ($i = 1, 2, 3$) are cumbersome and are therefore not given in this paper. A nontrivial solution of the system of third-order linear algebraic equations (12) exists only if its main determinant is zero. As a result, determining the frequency ω_1 is reduces to the solution of the transcendental equation.

Presented below are the results of calculations of the eigenfrequency of ω_1.

The geometric and physical parameters of the reinforced shell of woven fiberglass [10] and the fluid are assumed to have the following values: $\rho_0/\rho_m = 0.105$, $\rho_0 = \rho_j = 1850$ kg/m^3, $L_1 = 10^4$ mm, $b_{11} = 18.3$ GPa, $b_{12} = 2.77$ GPa, $b_{22} = 25.2$ GPa, $b_{66} = 3.5$ GPa, $\xi_1 = 1$, $h_j = 1.39$ mm, $R = 160$ mm, $h = 0.45$ mm, $F_j = 5.75$ mm^2, $I_{xj} = 19.9$ mm^4, $I_{\mathrm{cr}j} = 0.48$ mm^4, $k_2 = 20$, and $E_j = b_{11}$.

Figure 1 shows the frequency ω_1 as a function of the relative velocity of the flow $U^* = U/c$ ($c = \omega_0 R$). It can be seen that increasing the velocity leads to a reduction in the frequency of vibrations of the system.

Figure 2 shows the free vibration frequency as a function of the winding angle for various ratios h/R. It can be seen that the ratio h/R has a significant effect on the dependence $\omega_1(\varphi)$.

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