ISSN 0021-8944, Journal of Applied Mechanics and Technical Physics, 2016, Vol. 57, No. 4, pp. 652–665.

GENERALIZED THERMOELASTIC PROBLEM OF AN INFINITE BODY WITH A SPHERICAL CAVITY UNDER DUAL-PHASE-LAGS

R. Karmakar, A. Sur, and M. Kanoria

UDC 539.3

Abstract: The aim of the present contribution is the determination of the thermoelastic temperatures, stress, displacement, and strain in an infinite isotropic elastic body with a spherical cavity in the context of the mechanism of the two-temperature generalized thermoelasticity theory (2TT). The two-temperature Lord–Shulman (2TLS) model and two-temperature dual-phase-lag (2TDP) model of thermoelasticity are combined into a unified formulation with unified parameters. The medium is assumed to be initially quiescent. The basic equations are written in the form of a vector matrix differential equation in the Laplace transform domain, which is then solved by the state-space approach. The expressions for the conductive temperature and elongation are obtained at small times. The numerical inversion of the transformed solutions is carried out by using the Fourier-series expansion technique. A comparative study is performed for the thermoelastic stresses, conductive temperature, thermodynamic temperature, displacement, and elongation computed by using the Lord–Shulman and dual-phase-lag models.

Keywords: two-temperature generalized thermoelasticity, dual-phase-lag model, state-space approach, vector–matrix differential equation.

DOI: 10.1134/S002189441604009X

INTRODUCTION

Gurtin and William [1, 2] suggested that there are no a priori grounds for assuming that the second law of thermodynamics for continuous bodies involves only a single temperature, i.e., it is more logical to assume a second law in which the entropy contribution due to the heat conduction is governed by one temperature and that due to heat supply by another. Chen and Gurtin [3] and Chen et al. [4, 5] formulated a theory of heat conduction in deformable bodies, which depends on two distinct temperatures: conductive temperature and thermodynamic temperature. For time-independent situations, the difference between these two temperatures is proportional to the heat supply; in the absence of any heat supply, the two temperatures are identical [4]. For time-dependent problems, however, and for wave propagation problems in particular, the two temperatures are in general different, regardless of the presence of the heat supply. The key element that sets the two-temperature thermoelasticity (2TT) apart from the classical theory of thermoelasticity (CTE) is the material parameter $a \geq 0$, called the temperature discrepancy [4]. Specifically, if $a = 0$, then $\phi = \theta$, and the field equations of the 2TT reduce to those of the CTE.

The linearized version of the two-temperature theory (2TT) was studied by many authors. Warren and Chen [6] investigated wave propagation in the two-temperature theory of thermoelasticity. Lesan [7] established the uniqueness and reciprocity theorems for the 2TT. Puri and Jordan [8] studied propagation of plane waves under the 2TT. The existence, structural stability, and spatial behaviour of the solution in the 2TT was discussed by

University of Calcutta, 92 A.P.C. Road, 700009 Kolkata, West Bengal, India; karmakar.reshmi21@gmail.com; abhiksur4@gmail.com; k_mri@yahoo.com. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 57, No. 4, pp. 91–106, July–August, 2016. Original article submitted February 3, 2014; revision submitted August 12, 2014.

Quintanilla [9]. It should be pointed out that both the CTE and 2TT suffer from the so-called paradox of heat conduction, i.e., the prediction that a thermal disturbance at some point in a body is felt instantly, but unequally, throughout the body.

During the last five decades, nonclassical thermoelasticity theories involving hyperbolic-type heat transport equations admitting finite speeds for thermal signals have been formulated. According to these theories, heat propagation is to be viewed as a wave phenomenon rather than a diffusion phenomenon. Various approaches were applied to obtain wave-type heat conduction equations by different researchers.

Lord and Shulman [10] formulated the generalized thermoelasticity theory by introducing one relaxation time in Fourier's law of the heat conduction equation and, thus, transforming the heat conduction equation into a hyperbolic type. The uniqueness of the solution for this theory was proved under different conditions in [11–14].

Green and Lindsay [15] introduced one more theory, called the temperature-rate-dependent theory, which involves two relaxation times. In this model, Fourier's law of heat conduction is left unchanged, but the classical energy equation and stress–strain temperature relation are modified.

Later on, Green and Naghdi [16] developed three models for generalized thermoelasticity of a homogeneous isotropic material, which are labelled as models I, II, and III. If the respective theories are linearized, model I reduces to the classical heat conduction theory (based on Fourier's law). The linearized versions of models II and III permit propagation of thermal waves at finite speeds. Model II, in particular, exhibits a feature that is not present in the other established thermoelastic models as it does not sustain dissipation of thermal energy [17]. In this model, the constitutive equations are derived by starting with the reduced energy equation and by including the thermal displacement gradient among other constitutive variables. The third Green–Naghdi model [17] admits the dissipation of energy. Problems concerning with the generalized thermoelasticity proposed by Green and Naghdi [17, 18] were studied by Mallik and Kanoria [19, 20]. The next generalization of the thermoelasticity theory is known as the dual-phase-lag thermoelasticity developed by Tzou [21]. Tzou considered microstructural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase in the lattice temperature is delayed due to phonon–electron interactions at the macroscopic level. Tzou introduced two-phase-lags to both the heat flux vector and the temperature gradient.

According to this model, the classical Fourier's law $\boldsymbol{q} = -k\nabla T$ is replaced by the modified law $\boldsymbol{q}(P, t + \tau_q) = -k\nabla T(P, t + \tau_T)$, where the temperature gradient ∇T at a point P of the material at the time $t + \tau_T$ corresponds to the heat flux vector $\boldsymbol{q}$ at the same point at the time $t + \tau_q$. Here k is the thermal conductivity of the material. The delay time τ_T is interpreted as that caused by the microstructural interactions and is called the phase-lag of the temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of thermal inertia and is called the phase-lag of the heat flux. The case with $\tau_q = \tau_T = 0$ corresponds to the classical Fourier's law. If $\tau_q = \tau$ and $\tau_T = 0$, Tzou refers to the model as a single-phase-lag model. Roychoudhuri [22] studied one-dimensional thermoelastic wave propagation in an elastic half-space in the context of the dual-phase-lag model. Quintanilla [23–25] solved several problems on the basis of the dual-phase-lag model. He also considered the exponential stability and the effect of the delay parameters [26, 27]. Prasad et al. [28] studied propagation of a finite thermal wave in the context of the dual-phase-lag model.

Youssef [29] developed the theory of two-temperature generalized thermoelasticity based on the Lord–Shulman model. Under the two-temperature theory, the dual-phase-lag model is modified as $\boldsymbol{q}(P, t + \tau_q) = -k\nabla\phi(P, t + \tau_T)$, where ϕ is the conductive temperature. Mukhopadhyay and Kumar [30] studied propagation of thermoelastic waves in an infinite medium with a cylindrical cavity. Variational and reciprocal principles were considered by Kumar et al. [31]. The uniqueness and growth of solutions in the two-temperature generalized thermoelastic theories were investigated by Magane and Quintanilla [32]. Banik and Kanoria [33] studied thermoelastic interactions in an infinite body with a spherical cavity under the 2TT. Thermoelastic interactions in an infinite body under the 2TT in the context of the fractional heat equation were analyzed by Sur and Kanoria [34]. Mondal et al. [35] studied dual-phase-lag thermoelastic interaction due to variable thermal conductivity.

In this work, we investigate the thermoelastic stresses, conductive temperature, and thermodynamic temperature in an infinite isotropic elastic body having a spherical cavity by using the two-temperature generalized thermoelasticity theory in the context of the 2TLS and 2TDP models. The generalized coupled thermoelasticity theories are combined into a unified formulation with unified parameters. The governing equations of the two-temperature generalized thermoelasticity theory are obtained in the Laplace transform domain, which are then solved by using the state-space approach [36]. The inversion of the transform solution is carried out numerically by applying a method based on the Fourier-series expansion technique [37]. A complete and comprehensive analysis of the results is performed for the 2TLS and 2TDP models. These results are also compared with those of the one-temperature Lord–Shulman model [38].

1. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider an isotropic infinite solid having a spherical cavity of radius R. A problem with spherical symmetry is solved in a spherical coordinate system (r, ϑ, φ) with the origin of the center of the cavity. Therefore, all the considered functions depend only on the radial coordinate r and time t. It follows that the displacement vector $\boldsymbol{u}$, thermodynamic temperature θ, and conductive temperature ϕ can be presented as

$$\boldsymbol{u} = (u(r,t), 0, 0), \qquad \theta = \theta(r,t), \qquad \phi = \phi(r,t).$$

The strain tensor components are

$$e_{rr} = \frac{\partial u}{\partial r}, \qquad e_{\vartheta\vartheta} = e_{\varphi\varphi} = \frac{u}{r}.$$

The cubical dilatation is given by

$$e = \frac{\partial u}{\partial r} + 2\,\frac{u}{r} = \frac{1}{r^2}\,\frac{\partial}{\partial r}\,(r^2 u). \tag{1.1}$$

The stress–strain and stress–temperature relations for the present problem are

$$\sigma_{rr} = 2\mu\,\frac{\partial u}{\partial r} + \lambda e - \gamma\theta; \tag{1.2}$$

$$\sigma_{\vartheta\vartheta} = \sigma_{\varphi\varphi} = 2\mu\,\frac{u}{r} + \lambda e - \gamma\theta. \tag{1.3}$$

In the context of the two-temperature generalized thermoelasticity based on the Lord–Shulman and dual-phase-lag theories, the equation of motion in the absence of body forces and the heat conduction equation for a linearly isotropic generalized thermoelastic solid in the absence of heat sources in the body are, respectively,

$$(\lambda + 2\mu)\,\frac{\partial e}{\partial r} - \gamma\,\frac{\partial \theta}{\partial r} = \rho\,\frac{\partial^2 u}{\partial t^2}; \tag{1.4}$$

$$k\Big(t_1 + t_2\,\frac{\partial}{\partial t} + t_3\,\frac{\partial^2}{\partial t^2}\Big)\nabla^2\phi = \Big(1 + t_4\,\frac{\partial}{\partial t} + t_5\,\frac{\partial^2}{\partial t^2}\Big)(\rho c_E\dot{\theta} + \gamma T_0 \dot{e}), \tag{1.5}$$

where ρ is the density, λ and μ are the Lamé parameters, $\gamma = (3\lambda + 2\mu)\alpha_t$ (α_t is the coefficient of linear thermal expansion), T_0 is the reference temperature, c_E is the specific heat at constant strain, ∇^2 is the Laplacian given in our case by

$$\nabla^2 = \frac{1}{r^2}\,\frac{\partial}{\partial r}\Big(r^2\,\frac{\partial}{\partial r}\Big),$$

t_i are unified parameters corresponding to a particular model ($t_1 = 1$, $t_2 = 0$, $t_3 = 0$, $t_4 = \tau_0$, and $t_5 = 0$ for the 2TLS model; $t_1 = 1$, $t_2 = \tau_T$, $t_3 = \tau_T^2/2$, $t_4 = \tau_q$, and $t_5 = \tau_q^2/2$ for the 2TDP model), and τ_0 is the relaxation time.

The relation between the conductive temperature ϕ and the thermodynamic temperature θ is

$$\phi - \theta = a\nabla^2\phi, \tag{1.6}$$

where $a > 0$ is the two-temperature parameter. Introducing the dimensionless variables

$$r' = \frac{r}{\varkappa}\Big(\frac{\lambda + 2\mu}{\rho}\Big)^{1/2}, \quad u' = \frac{u}{\varkappa}\Big(\frac{\lambda + 2\mu}{\rho}\Big)^{1/2}, \quad t' = \frac{t}{\varkappa}\,\frac{\lambda + 2\mu}{\rho},$$

$$\tau_0 = \frac{\tau_0}{\varkappa}\,\frac{\lambda + 2\mu}{\rho}, \quad \theta' = \frac{\theta}{T_0}, \quad \phi' = \frac{\phi}{T_0}, \quad \sigma' = \frac{\sigma}{2\mu + \lambda},$$

we can write Eqs. (1.2)–(1.6) in the form

$$\sigma_{rr} = \beta^2 e + (1-\beta^2)\,\frac{\partial u}{\partial r} - \alpha\theta; \tag{1.7}$$

$$\sigma_{\vartheta\vartheta} = \sigma_{\varphi\varphi} = (1-\beta^2)\,\frac{u}{r} + \beta^2 e - \alpha\theta; \tag{1.8}$$

$$\frac{\partial e}{\partial r} - \alpha\,\frac{\partial \theta}{\partial r} = \frac{\partial^2 u}{\partial t^2}; \tag{1.9}$$

$$\Big(t_1 + t_2 G\,\frac{\partial}{\partial t} + t_3 G^2\,\frac{\partial^2}{\partial t^2}\Big)\nabla^2\phi = \Big(1 + t_4 G\,\frac{\partial}{\partial t} + t_5 G^2\,\frac{\partial^2}{\partial t^2}\Big)(\dot\theta + \varepsilon\dot e); \tag{1.10}$$

$$\phi - \theta = \omega\nabla^2\phi, \tag{1.11}$$

where $t_1 = 1$, $t_2 = 0$, $t_3 = 0$, $t_4 = \tau_0/G$, and $t_5 = 0$ for the 2TLS model, $t_1 = 1$, $t_2 = \tau_T/G$, $t_3 = \tau_T^2/(2G^2)$, $t_4 = \tau_q/G$, and $t_5 = \tau_q^2/(2G^2)$ for the 2TDP model, $G = (\lambda + 2\mu)/(\rho\varkappa)$, $\varepsilon = \gamma\varkappa/k$, $\alpha = \gamma T_0/(\lambda + 2\mu)$, $\omega = a(\lambda + 2\mu)/(\rho\varkappa^2)$, and $\beta^2 = \lambda/(\lambda + 2\mu)$; the primes at the dimensionless variables are omitted.

Equation (1.9) can be expressed in the form

$$\frac{\partial^2 e}{\partial t^2} = \nabla^2 e - \alpha\nabla^2\theta. \tag{1.12}$$

The initial and regularity conditions for the present problem are

$$t = 0,\ r \geqslant R:\quad u = \theta = \phi = 0, \qquad t = 0:\quad \frac{\partial u}{\partial t} = \frac{\partial \theta}{\partial t} = 0,$$

$$t = 0,\ r \to \infty:\quad u = \theta = \phi = 0.$$

Equations (1.10)–(1.12) are subjected to the thermal boundary condition (the internal surface $r = R$ is subjected to the thermal shock)

$$\phi(R, t) = \phi_R = F(t), \tag{1.13}$$

where

$$F(t) = \begin{cases} \phi_0, & t > 0, \\ 0, & t < 0, \end{cases}$$

and to the mechanical boundary condition (there is no cubical dilatation on the internal surface $r = R$)

$$e(R, t) = e_R = 0. \tag{1.14}$$

2. METHOD OF THE SOLUTION

Applying the Laplace transform

$$\bar f(s) = \int_0^\infty f(t)\,\mathrm{e}^{-st}\,dt, \qquad \mathrm{Re}(s) > 0$$

to Eqs. (1.7)–(1.11), we obtain

$$\bar\sigma_{rr} = \beta^2\bar e + (1-\beta^2)\,\frac{\partial \bar u}{\partial r} - \alpha\bar\theta; \tag{2.1}$$

$$\bar\sigma_{\varphi\varphi} = \bar\sigma_{\psi\psi} = (1-\beta^2)\,\frac{\bar u}{r} + \beta^2\bar e - \alpha\bar\theta;$$

$$\nabla^2\bar e = s^2\bar e + \alpha\nabla^2\bar\theta; \tag{2.2}$$

$$(t_1 + t_2 Gs + t_3 G^2 s^2)\nabla^2 \bar{\phi} = (1 + t_4 Gs + t_5 G^2 s^2) s(\bar{\theta} + \varepsilon \bar{e}); \tag{2.3}$$

$$\bar{\phi} - \bar{\theta} = \omega \nabla^2 \bar{\phi}. \tag{2.4}$$

The boundary conditions (1.13) and (1.14) in the transformed domain take the form

$$\bar{\phi}(R, s) = \bar{F}(s) = \frac{\phi_0}{s}; \tag{2.5}$$

$$\bar{e}(R, s) = \bar{e} = 0. \tag{2.6}$$

It follows from Eqs. (2.3) and (2.4) that

$$\bar{\theta} = \frac{1}{1 + a_3\omega}\, \bar{\phi} - \frac{a_3 \varepsilon \omega}{1 + a_3 \omega}\, \bar{e}, \tag{2.7}$$

where

$$a_3 = \frac{s(1 + t_4 Gs + t_5 G^2 s^2)}{t_1 + t_2 Gs + t_3 G^2 s^2}.$$

Substituting $\bar{\theta}$ from Eq. (2.7) into Eq. (2.3), we obtain

$$\nabla^2 \bar{\phi} = L_1 \bar{\phi} + L_2 \bar{e}. \tag{2.8}$$

Using Eqs. (2.8) and (2.2), we find

$$\nabla^2 \bar{e} = M_1 \bar{\phi} + M_2 \bar{e}, \tag{2.9}$$

where

$$L_1 = \frac{a_3}{1 + a_3\omega}, \qquad L_2 = \frac{a_3 \varepsilon}{1 + a_3 \omega},$$

$$M_1 = \frac{\alpha a_3}{(1 + a_3\omega)[1 + a_3\omega(1 + \alpha\varepsilon)]}, \qquad M_2 = \frac{s^2(1 + a_3\omega)^2 + \alpha a_3 \varepsilon}{(1 + a_3\omega)[1 + a_3\omega(1 + \alpha\varepsilon)]}.$$

The differential equations (2.8) and (2.9) can be written in the vector–matrix form as

$$\nabla^2 \bar{V}(r, s) = A(s) \bar{V}(r, s), \tag{2.10}$$

where

$$\bar{V}(r, s) = \begin{pmatrix} \bar{\phi}(r, s) \\ \bar{e}(r, s) \end{pmatrix}, \qquad A(s) = \begin{pmatrix} L_1 & L_2 \\ M_1 & M_2 \end{pmatrix}.$$

3. STATE-SPACE APPROACH

The formal solution of Eq. (2.10) can be written in the form

$$\bar{V}(r, s) = \frac{R}{r}\, \mathrm{e}^{-\sqrt{A(s)}(r-R)}\, \bar{V}(R, s), \tag{3.1}$$

where

$$\bar{V}(R, s) = \begin{pmatrix} \bar{\phi}(R, s) \\ \bar{e}(R, s) \end{pmatrix}.$$

The characteristic equation of the matrix $A(s)$ takes the form

$$k^2 - k(L_1 + M_2) + (L_1 M_2 - L_2 M_1) = 0. \tag{3.2}$$

The roots of Eq. (3.2) satisfy the following relations:

$$k_1 + k_2 = L_1 + M_2, \qquad k_1 k_2 = L_1 M_2 - L_2 M_1.$$

The spectral decomposition of the matrix $A(s)$ is

$$A(s) = k_1 E_1 + k_2 E_2,$$

where E_1 and E_2 are called the projectors of $A(s)$ and satisfy the conditions

$$E_1 + E_2 = I, \qquad E_1 E_2 = Z, \qquad E_i^2 = E_i, \quad i = 1, 2$$

(I is the identity matrix and Z is the zero matrix). Therefore, we obtain

$$\sqrt{A(s)} = \sqrt{k_1}\, E_1 + \sqrt{k_2}\, E_2,$$

where

$$E_1 = \frac{1}{k_1 - k_2} \begin{pmatrix} L_1 - k_2 & L_2 \\ M_1 & M_2 - k_2 \end{pmatrix}, \qquad E_2 = \frac{1}{k_1 - k_2} \begin{pmatrix} k_1 - L_1 & -L_2 \\ -M_1 & k_1 - M_2 \end{pmatrix}.$$

Thus, we have

$$B(s) = \sqrt{A(s)} = \frac{1}{\sqrt{k_1} + \sqrt{k_2}} \begin{pmatrix} L_1 + \sqrt{k_1 k_2} & L_2 \\ M_1 & M_2 + \sqrt{k_1 k_2} \end{pmatrix}.$$

Now solution (3.1) can be written as

$$\bar{V}(r, s) = \frac{R}{r}\, \mathrm{e}^{-B(s)(r-R)}\, \bar{V}(R, s). \tag{3.3}$$

To find the form of the matrix $\exp\,[-B(s)(r-R)]$, we now apply the Cayley–Hamilton theorem. The characteristic equation of the matrix $B(s)$

$$m^2 - m(\sqrt{k_1} + \sqrt{k_2}\,) + \sqrt{k_1}\sqrt{k_2} = 0$$

has the roots

$$m_1 = \sqrt{k_1}\,, \qquad m_2 = \sqrt{k_2}.$$

The Taylor series expansion for the matrix exponential in Eq. (3.3) has the following form [39]:

$$\mathrm{e}^{-B(s)(r-R)} = \sum_{n=0}^{\infty} \frac{(-B(s)(r-R))^n}{n}. \tag{3.4}$$

We can express the matrix B^2 and the higher orders of the matrix B in terms of the matrix B and the second-order unit matrix I [39]. Then the infinite series in Eq. (3.4) can be presented as

$$\mathrm{e}^{-B(s)(r-R)} = b_0(r, s) I + b_1(r, s) B(s),$$

where b_0 and b_1 are coefficients depending on r and s.

The characteristic roots m_1 and m_2 of the matrix B must satisfy Eq. (3.4). Therefore, we have

$$\mathrm{e}^{-P_1(r-R)} = b_0 + b_1 P_1; \tag{3.5}$$

$$\mathrm{e}^{-P_2(r-R)} = b_0 + b_1 P_2. \tag{3.6}$$

Solving Eqs. (3.5) and (3.6), we find the coefficients b_0 and b_1:

$$b_0 = \frac{P_1\, \mathrm{e}^{-P_2(r-R)} - P_2\, \mathrm{e}^{-P_1(r-R)}}{P_1 - P_2}, \qquad b_1 = \frac{\mathrm{e}^{-P_1(r-R)} - \mathrm{e}^{-P_2(r-R)}}{P_1 - P_2}.$$

Hence, Eq. (3.4) can be written as

$$\mathrm{e}^{-B(s)(r-R)} = H(r, s) = [h_{ij}(r, s)], \qquad i, j = 1, 2, \tag{3.7}$$

where

$$h_{11} = \frac{(P_1^2 - l)\, \mathrm{e}^{-P_2(r-R)} - (P_2^2 - l)\, \mathrm{e}^{-P_1(r-R)}}{P_1^2 - P_2^2}, \qquad h_{12} = \frac{\varepsilon l\, \mathrm{e}^{-P_1(r-R)} - \mathrm{e}^{-P_2(r-R)}}{P_1^2 - P_2^2},$$

$$h_{21}=\frac{M_2\,e^{-P_1(r-R)}-e^{-P_2(r-R)}}{P_1^2-P_2^2},\qquad h_{22}=\frac{(P_1^2-M_1)\,e^{-P_2(r-R)}-(P_2^2-M_1)\,e^{-P_1(r-R)}}{P_1^2-P_2^2}.$$

Hence, from Eqs. (3.3) and (3.7), we obtain

$$\bar{V}(r,s)=\frac{R}{r}\,[h_{ij}(r,s)]\bar{V}(R,s). \tag{3.8}$$

Applying the Laplace transform to Eqs. (1.3) and (1.4), we obtain

$$\bar{\phi}(R,s)=\bar{F}(s)=\frac{\phi_0}{s}; \tag{3.9}$$

$$\bar{e}(R,s)=\bar{e}=0. \tag{3.10}$$

Hence, using the boundary conditions (2.5) and (2.6), we find the solutions for $\bar{\phi}$ and $\bar{e}$ from Eq. (3.8):

$$\bar{\phi}=\frac{R\bar{F}(s)}{r(P_1^2-P_2^2)}\,[(P_1^2-L_1)\,e^{-P_2(r-R)}-(P_2^2-L_1)\,e^{-P_1(r-R)}]; \tag{3.11}$$

$$\bar{e}=\frac{R\bar{F}(s)M_1}{r(P_1^2-P_2^2)}\,[e^{-P_1(r-R)}-e^{-P_2(r-R)}]. \tag{3.12}$$

In view of Eqs. (3.11) and (3.12), Eq. (2.8) yields

$$\bar{\theta}=\frac{R\bar{F}(s)}{r(1+a_3\omega)(P_1^2-P_2^2)}\,[(P_1^2-L_1)\,e^{-P_2(r-R)}-(P_2^2-L_1)\,e^{-P_1(r-R)}]. \tag{3.13}$$

Applying the Laplace transform to Eq. (1.1), using Eq. (3.10), and then integrating, we find the radial displacement

$$\bar{u}=\frac{R\bar{F}(s)M_1}{r^2(P_1^2-P_2^2)}\Big(\frac{P_2r+1}{P_2^2}\,e^{-P_2(r-R)}-\frac{P_1r+1}{P_1^2}\,e^{-P_1(r-R)}\Big). \tag{3.14}$$

The stress component σ_{rr} in the Laplace transform domain is obtained from Eqs. (2.1) and (3.12)–(3.14):

$$\begin{aligned}\bar{\sigma}_{rr}=\frac{R\bar{F}(s)}{r^3P_1^2P_2^2(P_1^2-P_2^2)}\Big[&P_2^2\,e^{-P_1(r-R)}\Big(\beta^2M_1r^2P_1^2+2(1-\beta^2)M_1(P_1r+1)+M_1r^2(1-\beta^2)+\frac{\alpha(P_2^2-L_1)}{1+a_3\omega}\Big)\\&-P_1^2\,e^{-P_2(r-R)}\Big(\beta^2M_1r^2P_2^2+2(1-\beta^2)M_1(P_2r+1)+M_1r^2(1-\beta^2)+\frac{\alpha(P_1^2-L_1)}{1+a_3\omega}\Big)\Big].\end{aligned} \tag{3.15}$$

Equations (3.11)–(3.15) complete our solution in the Laplace transform domain.

4. DERIVATION AND ANALYSIS OF SMALL TIME SOLUTIONS

Under dual-phase-lag heat conduction, Eqs. (2.2)–(2.4) take the form

$$N\nabla^2\bar{\phi}=M\bar{\theta}+\varepsilon M\bar{e},\qquad \bar{\phi}-\bar{\theta}=\omega\nabla^2\phi,\qquad \nabla^2\bar{e}=s^2\bar{e}+\alpha\nabla^2\bar{\theta},$$

where $M=s(1+\tau_qs+\tau_q^2s^2/2)$ and $N=1+\tau_Ts$.

This leads to the following equation for $\bar{\phi}$ and $\bar{e}$:

$$[(N+M\omega+\alpha\varepsilon M\omega)\nabla^4-(s^2N+M+\alpha\varepsilon M+s^2M\omega)\nabla^2+s^2M](\bar{\phi},\bar{e})=0.$$

The biquadratic equation

$$(N+M\omega+\alpha\varepsilon M\omega)P^4-(s^2N+M+\alpha\varepsilon M+s^2M\omega)P^2+s^2M=0$$

has the roots

$$\begin{aligned}P_{1,2}^2=\frac{1}{2(N+M\omega+\alpha\varepsilon M\omega)}\Big(&s^2(N+M\omega)+M(1+\alpha\varepsilon)\\&\pm\sqrt{[s^2(N+M\omega)+M(1+\alpha\varepsilon)]^2-4Ms^2(N+M\omega+\alpha\varepsilon M\omega)}\Big).\end{aligned} \tag{4.1}$$

Applying the inverse Laplace transform to Eqs. (3.11)–(3.15), we find ϕ, e, θ, u, and σ_{rr}.

The roots $P_{1,2}$ depend on the Laplace transform parameter s; for this reason, it is difficult to determine the inverse transform. As the second sound effects (propagation of thermal waves with a finite speed) are short-lived, it is sufficient to derive and analyze the solutions for small times t. This is done by taking the Laplace parameter s to be large.

Thus, we obtain the following results for large values of s:

$$P_1 \simeq \frac{s}{v_1} + \frac{1}{2}\frac{\lambda_2}{\lambda_1}\frac{1}{s} + \frac{1}{2v_1}\Big(\frac{\lambda_3}{\lambda_1} - \frac{1}{4}\frac{\lambda_2^2}{\lambda_1^2}\Big)\frac{1}{s}$$

if the plus sign is taken in Eq. (4.1) and

$$P_2 \simeq \frac{s}{v_2} + \frac{1}{2}\frac{\mu_2}{\mu_1}\frac{1}{s} + \frac{1}{2v_2}\Big(\frac{\mu_3}{\mu_1} - \frac{1}{4}\frac{\mu_2^2}{\mu_1^2}\Big)\frac{1}{s}$$

if the minus sign is taken in Eq. (4.1). Here we have

$$\lambda_1 = 2A, \quad \lambda_2 = B + \frac{B}{A} - \frac{4A}{\tau_T}, \quad \lambda_3 = C + \frac{C}{A} - \frac{4A[\tau_T + \omega(1+\alpha\varepsilon)]}{\tau_q^2\omega(1+\alpha\varepsilon)} + \frac{8A}{\tau_q^2},$$

$$\mu_1 = \tau_q, \qquad \mu_2 = -2, \quad \mu_3 = \Big(C - \frac{C}{A} - AF\Big)\tau_q - \frac{2}{\tau_q\omega}\,\frac{\tau_T + \omega(1+\alpha\varepsilon)}{1+\alpha\varepsilon},$$

$$v_1 = \Big(\frac{\omega(1+\alpha\varepsilon)\tau_q^2}{\lambda_1}\Big)^{1/2}, \quad v_2 = \Big(\frac{2\omega(1+\alpha\varepsilon)\tau_q^2}{\lambda_1}\Big)^{1/2},$$

$$A = \frac{\omega\tau_q^2}{2}, \quad B = \omega\tau_q, \quad C = \omega + \frac{1}{2}\tau_q^2(1+\alpha\varepsilon), \quad F = -\omega\tau_q^4(1+\alpha\varepsilon).$$

Thus, for large values of s, we have

$$P_1 \simeq \frac{s}{v_1} + \frac{1}{2}\frac{\lambda_2}{\lambda_1}\frac{1}{v_1}, \qquad P_2 \simeq \frac{s}{v_2} + \frac{1}{2}\frac{\mu_2}{\mu_1}\frac{1}{v_2}.$$

After simplifications, we obtain the following relations for large values of s:

$$\frac{M_1}{P_2^2 - P_1^2} \simeq \frac{1}{s^5}\frac{\delta_1}{L_0} + \frac{1}{s^6}\frac{\delta_2 + \delta_1 M_0}{L_0},$$

$$\frac{P_1^2 - L_1}{s(P_2^2 - P_1^2)} \simeq \frac{1}{L_0}\Big(\omega + \frac{1}{v_1^2}\Big)\frac{1}{s} + \frac{1}{s^2}\Big[-\frac{1}{L_0} - \frac{M_0}{L_0}\Big(\omega + \frac{1}{v_1^2}\Big)\Big],$$

$$\frac{P_2^2 - L_1}{s(P_2^2 - P_1^2)} \simeq \frac{1}{L_0}\Big(\omega + \frac{1}{v_2^2}\Big)\frac{1}{s} + \frac{1}{s^2}\Big[-\frac{1}{L_0} - \frac{M_0}{L_0}\Big(\omega + \frac{1}{v_2^2}\Big)\Big].$$

In these relations, we have

$$L_0 = \frac{1}{v_2^2} - \frac{1}{v_1^2}, \qquad M_0 = \frac{\mu_2}{\mu_1 v_2^2} - \frac{\lambda_2}{\lambda_1 v_1^2},$$

$$\delta_1 = \frac{2\alpha}{\omega^2\tau_q^2(1+\alpha\varepsilon)}, \qquad \delta_2 = \delta_1\Big(\tau_T + \frac{2}{\tau_q} - \frac{2\tau_q}{\omega} - \frac{2\tau_q}{\omega(1+\alpha\varepsilon)}\Big).$$

Finally, we obtain the following solutions for the conductive temperature ϕ and strain e fields in the Laplace transform domain for large values of s:

$$\bar\phi(\xi,s) \simeq R\phi_0\Bigg(\Big\{\frac{1}{L_0}\Big(\omega + \frac{1}{v_2^2}\Big)\frac{1}{s} - \Big[\frac{1}{L_0} + \frac{M_0}{L_0}\Big(\omega + \frac{1}{v_2^2}\Big)\Big]\frac{1}{s^2}\Big\}\exp\Big(-\Big(\frac{s}{v_1} + \frac{\lambda_2}{2\lambda_1 v_1}\Big)\xi\Big)$$

$$- \Big\{\frac{1}{L_0}\Big(\omega + \frac{1}{v_1^2}\Big)\frac{1}{s} - \Big[\frac{1}{L_0} + \frac{M_0}{L_0}\Big(\omega + \frac{1}{v_1^2}\Big)\Big]\frac{1}{s^2}\Big\}\exp\Big(-\Big(\frac{s}{v_2} + \frac{\mu_2}{2\mu_1 v_2}\Big)\xi\Big)\Bigg),$$

$$\bar{e}(\xi,s)\simeq R\phi_0\Big[\Big(\frac{\delta_1}{L_0}\,\frac{1}{s^5}+\frac{1}{s^6}\,\frac{\delta_2+\delta_1M_0}{L_0}\Big)\exp\Big(-\Big(\frac{s}{v_2}+\frac{\mu_2}{2\mu_1v_2}\Big)\xi\Big)$$

$$-\Big(\frac{\delta_1}{L_0}\,\frac{1}{s^5}+\frac{1}{s^6}\,\frac{\delta_2+\delta_1M_0}{L_0}\Big)\exp\Big(-\Big(\frac{s}{v_1}+\frac{\lambda_2}{2\lambda_1v_1}\Big)\xi\Big)\Big]$$

($\xi = r - R$). Finally, applying the inverse Laplace transforms, we obtain

$$\phi(\xi,t)\simeq R\phi_0\Bigg(\exp\Big(-\frac{\lambda_2}{2\lambda_1v_1}\,\xi\Big)\Big\{\frac{1}{L_0}\Big(\omega+\frac{1}{v_2^2}\Big)-\Big(t-\frac{\xi}{v_1}\Big)\Big[\frac{1}{L_0}+\frac{M_0}{L_0}\Big(\omega+\frac{1}{v_2^2}\Big)\Big]\Big\}H\Big(t-\frac{\xi}{v_1}\Big)$$

$$-\exp\Big(-\frac{\mu_2}{2\mu_1v_2}\,\xi\Big)\Big\{\frac{1}{L_0}\Big(\omega+\frac{1}{v_1^2}\Big)-\Big(t-\frac{\xi}{v_2}\Big)\Big[\frac{1}{L_0}+\frac{M_0}{L_0}\Big(\omega+\frac{1}{v_1^2}\Big)\Big]\Big\}H\Big(t-\frac{\xi}{v_2}\Big)\Bigg); \tag{4.2}$$

$$e(\xi,t)=\frac{R}{r}\,\phi_0\Big\{\exp\Big(-\frac{\mu_2}{2\mu_1v_2}\,\xi\Big)\Big[\frac{\delta_1}{L_0}\,\frac{1}{24}\Big(t-\frac{\xi}{v_2}\Big)^4+\frac{1}{120}\Big(t-\frac{\xi}{v_2}\Big)^5\,\frac{\delta_2+\delta_1M_0}{L_0}\Big]H\Big(t-\frac{\xi}{v_2}\Big)$$

$$-\exp\Big(-\frac{\lambda_2}{2\lambda_1v_1}\,\xi\Big)\Big[\frac{\delta_1}{L_0}\,\frac{1}{24}\Big(t-\frac{\xi}{v_1}\Big)^4+\frac{1}{120}\Big(t-\frac{\xi}{v_1}\Big)^5\,\frac{\delta_2+\delta_1M_0}{L_0}\Big]H\Big(t-\frac{\xi}{v_1}\Big)\Big\}. \tag{4.3}$$

The small time solutions (4.2) and(4.3) for the conductive temperature and elongation reveal the existence of two waves. The expressions for ϕ and e are composed of two parts, and each part corresponds to a wave propagating with a finite speed. The speed of the wave corresponding to the first part is v_1 and that corresponding to the second part is v_2.

By means of direct inspection of solutions (4.2) and (4.3), we find that the elongation e is continuous, whereas the temperature ϕ is discontinuous at the wave fronts. The finite jumps experienced by the temperature at the wave fronts are found as

$$[\phi]\Big|_{\xi=tv_1}=R\phi_0\exp\Big(-\frac{\lambda_2}{2\lambda_1v_1}\,\xi\Big)\,\frac{1}{L_0}\Big(\omega+\frac{1}{v_2^2}\Big),\qquad [\phi]\Big|_{\xi=tv_2}=-R\phi_0\exp\Big(-\frac{\mu_2}{2\mu_1v_2}\,\xi\Big)\frac{1}{L_0}\Big(\omega+\frac{1}{v_1^2}\Big).$$

The finite jumps are not constant, but they decay exponentially with distance from the cavity boundary. The velocity v_2 corresponds to the modified speed of thermal signals, and v_1 corresponds to the modified elastic dilatational wave speed. As $v_1 < v_2$, the faster wave is a predominantly modified Tzou wave (T-wave) and the slower one is a predominantly modified elastic wave (E-wave).

5. NUMERICAL INVERSION OF THE LAPLACE TRANSFORM

Let us find the inverse Laplace transform numerically. Let $\bar{f}(r,s)$ be the Laplace transform of a function $f(r,t)$. Then, the inversion formula for the Laplace transform can be written as

$$f(r,t)=\frac{1}{2\pi i}\int\limits_{d-i\infty}^{d+i\infty}\mathrm{e}^{st}\,\bar{f}(r,s)\,ds, \tag{5.1}$$

where d is an arbitrary real number greater than the real part of all the singularities of the function $\bar{f}(r,s)$.

Assuming that $s = d + iw$ in Eq. (5.1), we obtain

$$f(r,t)=\frac{\mathrm{e}^{dt}}{2\pi}\int\limits_{-\infty}^{\infty}\mathrm{e}^{itw}\,f(r,d+iw)\,dw.$$

Expanding the function $h(r,t) = \mathrm{e}^{-dt}\,f(r,t)$ into the Fourier series in the interval $[0, 2T]$, we obtain the approximate formula [37]

$$f(r,t)=f_\infty(r,t)+E_D,$$

where

$$f_\infty(r,t) = \frac{1}{2}\,c_0 + \sum_{k=1}^{\infty} c_k, \qquad 0 \leqslant t \leqslant 2T; \tag{5.2}$$

$$c_k = \frac{e^{dt}}{T}\, e^{ik\pi t/T}\, \bar f\Big(r, d + \frac{ik\pi t}{T}\Big).$$

The discretization error E_D can be made arbitrary small by choosing d large enough [37]. Leaving a finite number of terms in series (5.2), we obtain the approximate value for the function $f(r,t)$:

$$f_N(r,t) = \frac{1}{2}\,c_0 + \sum_{k=1}^{N} c_k, \qquad 0 \leqslant t \leqslant 2T. \tag{5.3}$$

Using Eq. (5.3), we have to take into account a truncation error, which must be added to the discretization error to produce the total approximation error.

Two methods are used to reduce the total error: the Korrecktur method to reduce the discretization error and the ε-algorithm to accelerate the convergence [37].

The function $f(r,t)$ is evaluated in the Korrecktur method by the formula

$$f(r,t) = f_\infty(r,t) - e^{-2dT}\, f_\infty(r, 2T+t) + E'_D,$$

where the discretization error is $|E'_D| \ll |E_D|$. Thus, the approximate value of the function $f(r,t)$ is

$$f_{NK}(r,t) = f_N(r,t) - e^{-2dT}\, f_{N'}(r, 2T+t), \tag{5.4}$$

where $N' < N$ is an integer number.

We now describe the ε-algorithm that is used to accelerate the convergence of the series in Eq. (5.3). Let $N = 2q+1$ (q is a natural number) and let $s_m = \sum_{k=1}^{m} c_k$ be a sequence of the partial sums of series in (5.3). We define the ε-sequence by

$$\varepsilon_{0,m} = 0, \quad \varepsilon_{1,m} = s_m, \quad \varepsilon_{p+1,m} = \varepsilon_{p-1,m+1} + \frac{1}{\varepsilon_{p,m+1} - \varepsilon_{p,m}}, \quad p = 1,2,3,\ldots\ .$$

The sequence $\varepsilon_{1,1}, \varepsilon_{3,1}, \varepsilon_{5,1}, \ldots, \varepsilon_{N,1}$ [37] converges to $f(r,t) + E_D - c_0/2$ faster than the sequence of the partial sums s_m $(m = 1,2,3,\ldots)$.

The actual procedure used to invert the Laplace transform consists of using Eq. (5.4) together with the ε-algorithm. The values of the constants d and T are chosen according to the criterion outlined in [37].

6. NUMERICAL RESULTS AND DISCUSSION

To obtain the solution for the thermal stresses, conductive temperature, and thermodynamic temperature in the space-time domain, we have to apply the Laplace inversion to Eqs. (3.9)–(3.15). This has been done numerically by using the above-described procedure [37]. The numerical code is written in the Fortran-77 programming language.

For the purpose of illustration, we consider a copper-like material with the material constants $k = 386$ N/(K·s), $\alpha_T = 1.78\cdot10^{-5}$ K^{-1}, $c_E = 383.1$ m^2/K, $\eta = 8886.73$ m/s^2, $\mu = 3.86\cdot10^{10}$ N/m^2, $\lambda = 7.76\cdot10^{10}$ N/m^2, $\rho = 8954$ kg/m^3, $\tau_T = 0.015$ s, $\tau_0 = \tau_q = 0.02$ s, $T_0 = 293$ K, $\varepsilon = 0.0168$, $\alpha = 0.0104$, and $F_0 = 1$, which satisfies the stability condition of the dual-phase-lag model $\tau_T/\tau_q > 1/2$.

Figure 1 shows the effect of the radial coordinate on the conductive temperature ϕ, thermodynamic temperature θ, radial stress σ_{rr}, displacement u, and strain e for $t = 0.2$ for the Lord–Shulman and dual-phase-lag models, where $\omega = 0$ and 0.1 correspond to the one-temperature and two-temperature theories, respectively. It is seen that the conductive temperature ϕ reaches the maximum value at the internal surface $r = R$, which satisfies our theoretical boundary condition, and then it gradually decays and asymptotically tends to zero far from the center of the spherical cavity. The magnitude of the conductive temperature is larger for the Lord–Shulman model than for the dual-phase-lag model. The rate of decay of the conductive temperature is faster for the one-temperature theory than that for the two-temperature theory.

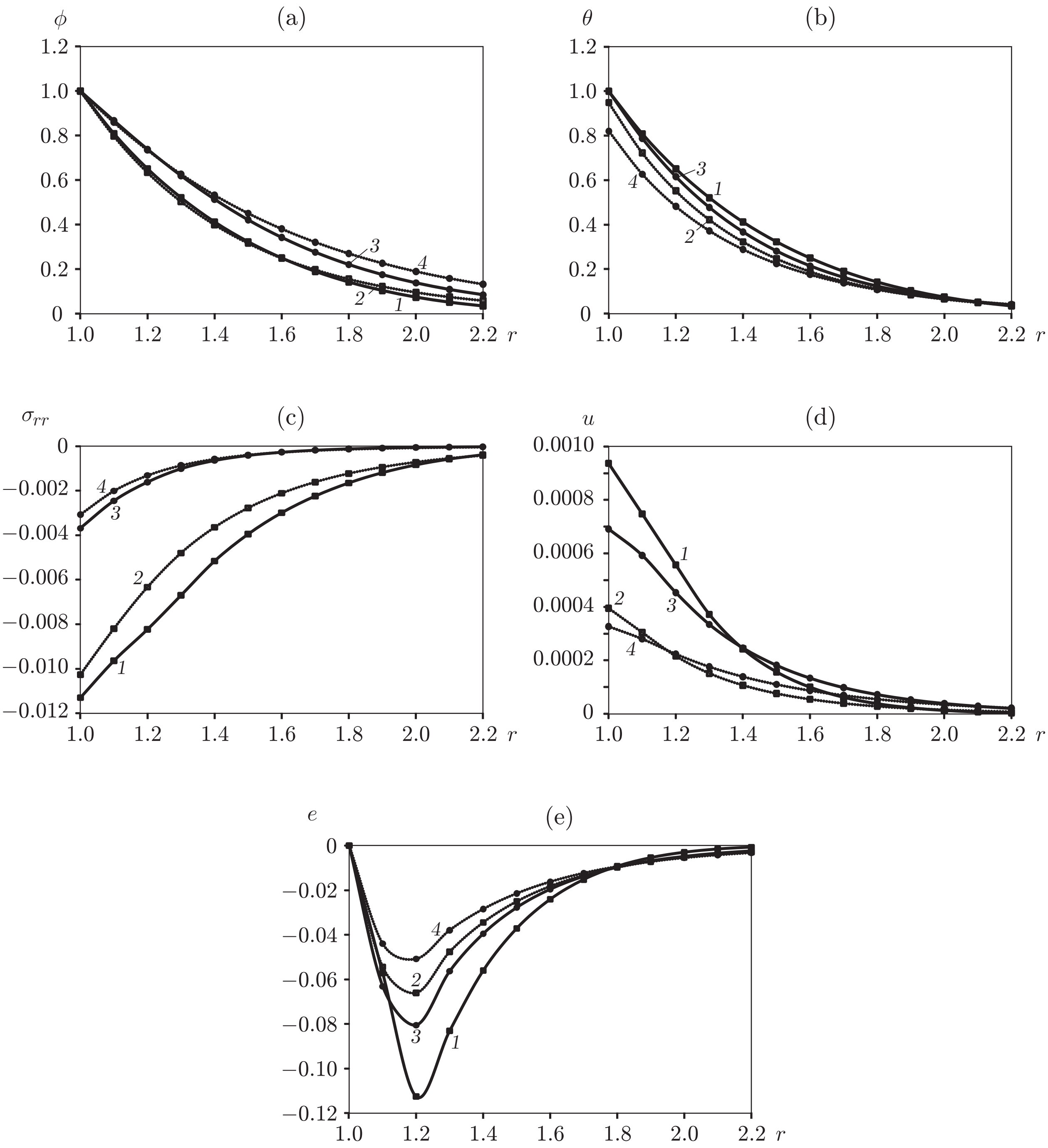

Fig. 1. Effect of the radial coordinate on the conductive temperature (a), thermodynamic temperature (b), radial stress (c), displacement (d), and strain (e) for $t = 0.2$ and $\omega = 0$ and 0.1: one-temperature Lord–Shulman model of thermoelasticity (*1*); two-temperature Lord–Shulman model of thermoelasticity (*2*); one-temperature dual-phase-lag model of thermoelasticity (*3*); two-temperature dual-phase-lag model of thermoelasticity (*4*).

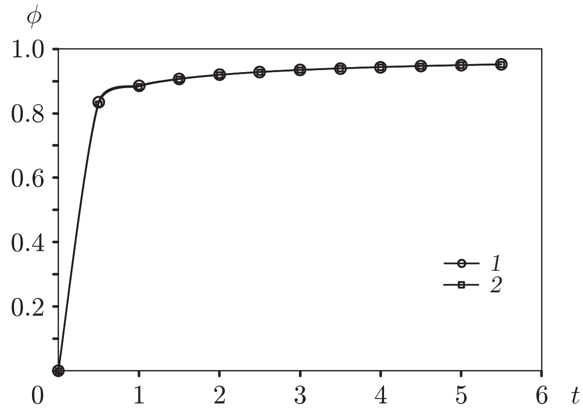

Fig. 2. Conductive temperature versus time predicted by the two-temperature Lord–Shulman thermoelasticity model (*1*) and two-temperature dual-phase-lag model (*2*) for $r = 1.2$ and $\omega = 0.1$.

The temperature θ attains its maximum value at the internal surface $r = 1$ for both one-temperature and two-temperature models, then it asymptotically tends to zero as r increases. The magnitude for the Lord–Shulman model is larger than that for the dual-phase-lag model.

The radial stress σ_{rr} predicted by both models is compressive near the internal surface $r = 1$, and the maximum magnitudes are also attained there. Also, the magnitude of the thermal stress decays gradually as r increases and finally tends to zero far from the cavity. Also, the magnitude of the thermal stress is larger for the Lord–Shulman model than that for the dual-phase-lag model.

The displacement u predicted by the Lord–Shulman model is also larger than that for the dual-phase-lag model. As r increases, the magnitude of u decays and asymptotically tends to zero far from the cavity.

The elongation e is equal to zero at the internal surface $r = 1$ (see Fig. 1e), which agrees with our mechanical boundary condition. The peak value of the elongation is reached at $r \simeq 1.2$. The peak value predicted by the Lord–Shulman model is greater than that of the dual-phase-lag model for both $\omega = 0$ and $\omega = 0.1$.

Figure 2 shows the conductive temperature ϕ versus the time t for $r = 1.2$ and $\omega = 0.1$. It is observed that the conductive temperature increases at first with increasing t and then reaches a steady state.

CONCLUSIONS

The problem of investigating the conductive temperature, thermodynamic temperature, thermal stress, displacement, and elongation in an infinite homogenous isotropic elastic spherical shell is considered in the light of two-temperature generalized thermoelasticity in the context of the dual-phase-lag heat conduction equation. The Laplace transform is used to write the basic equations in the form of a vector–matrix differential equation, which is then solved by the state-space approach. The numerical inversion of the Laplace transform is performed by using the Fourier's series expansion technique [37]. The analysis of the results permits some concluding remarks.

In some practical relevant problems, particularly heat transfer problems involving very small time intervals and very high heat fluxes, the hyperbolic equations give significantly different results than the parabolic equations. According to this phenomenon, the lagging behavior in heat conduction in solids should not be ignored, particularly when the elapsed times during the transient process are very small (about 10^{-7} s) or the heat flux is very high. Whenever such a physical situation arises, it is convenient to use the dual-phase lag model of generalized thermoelasticity.

When very high heat fluxes arise in the body for a very small time interval (about 10^{-7} s), then the heat conduction in the deformable solid depends on two distinct temperatures: conductive temperature ϕ and thermodynamic temperature θ. As the thermodynamic temperature θ is a function of the conductive temperature and its first two spatial gradients, i.e., $\theta = \hat{\theta}(\phi, \nabla\phi, a^2\phi)$ [3], it is convenient to choose the two-temperature theory rather than the one-temperature theory. For time-independent situations, the difference between these two temperatures is proportional to the heat supply; in the absence of any heat supply, the two temperatures are identical. In this case, the solutions predicted by the two-temperature generalized thermoelasticity theory are continuous, which seems to be physically plausible. Thus, the two-temperature thermoelasticity model is more realistic than the one-temperature model.

The small time solutions for the conductive temperature ϕ and strain e reveal the existence of two waves propagating with a finite speed.

The expressions for ϕ and e are composed of two parts, each corresponding to a wave propagating with a finite speed. For the conductive temperature, the speed of the wave corresponding to the first part is v_1 and that corresponding to the second part is v_2. For the strain e, the speed of the wave corresponding to the first part is v_2 and that corresponding to the second part is v_1. Both waves decay exponentially with distance. The temperature ϕ and strain e are identically equal to zero for $x > v_2 t$. This implies that, at a given instant of time t^e, the points of the solid beyond the faster wave front ($x = v_2 t^e$) do not experience any disturbances. This observation confirms that the two-temperature dual-phase-lag theory is also a wave thermoelasticity theory, like other generalized thermoelasticity theories. The domain $0 < x < v_2 t^e$ is the domain of influence of the disturbance at a given time, contrary to the result that this domain extends and the effects occur instantaneously everywhere in the solid, as predicted by the classical thermoelasticity theory. The results obtained in this work agree with the results of [38].

We are grateful to Prof. S. C. Bose of the Department of Applied Mathematics, University of Calcutta for his valuable suggestions and guidance in preparation of the paper.

REFERENCES

1. M. E. Gurtin and W. O. Williams, "On the Clausius–Duhem Inequality," Z. Angew. Math. Phys. **7**, 626–633 (1966).
2. M. E. Gurtin and W. O. Williams, "An Axiomatic Foundation for Continuum Thermodynamics," Arch. Rational Mech. Anal. **26**, 83–117 (1967).
3. P. J. Chen and M. E. Gurtin, "On a Theory of Heat Conduction Involving Two Temperatures," Z. Angew. Math. Phys. **19**, 614–627 (1968).
4. P. J. Chen, M. E. Gurtin, and W. O. Williams, "A Note on Non Simple Heat Conduction," Z. Angew. Math. Phys. **19**, 969–970 (1968).
5. P. J. Chen, M. E. Gurtin, and W. O. Williams, "On the Thermodynamics of Non-Simple Elastic Materials with Two Temperatures," Z. Angew. Math. Phys. **20**, 107–112 (1969).
6. W. E. Warren and P. J. Chen, "Wave Propagation in Two Temperatures Theory of Thermoelasticity," Acta Mech. **16**, 83–117 (1973).
7. D. Lesan, "On the Linear Coupled Thermoelasticity with Two Temperatures," Z. Angew. Math. Phys. **21**, 583–591 (1970).
8. P. Puri and P. M. Jordan, "On the Propagation of Harmonic Plane Waves under the Two-Temperature Theory," Int. J. Eng. Sci. **44**, 1113–1126 (2006).
9. R. Quintanilla, "On Existence, Structural Stability, Convergence and Spatial Behavior in Thermoelasticity with Two Temperatures," Acta Mech. **168**, 61–73 (2004).
10. H. Lord and Y. Shulman, "A Generalized Dynamical Theory of Thermoelasticity," J. Mech. Phys. Solids **15**, 299 (1967).
11. J. Ignaczak, "Uniqueness in Generalized Thermoelasticity," J. Thermal Stresses **2**, 171–175 (1979).
12. J. Ignaczak, "A Note on Uniqueness in Thermoelasticity with One Relaxation Time," J. Thermal Stresses **5**, 257–263 (1982).
13. R. S. Dhaliwal and H. Sherief, "Generalized Thermoelasticity for Anisotropic Media," Quart. Appl. Math. **33**, 1–8 (1980).
14. H. H. Sherief, "On Uniqueness and Stability in Generalized Thermoelasticity," Quart. Appl. Math. **45**, 773–778 (1987).
15. A. E. Green and K. A. Lindsay, "Thermoelasticity," J. Elasticity **2**, 1–7 (1972).
16. A. E. Green and P. M. Naghdi, "A Re-Examination of the Basic Results of Thermomechanics," Proc. Roy. Soc. London, Ser. A **432**, 171–194 (1992).

17. A. E. Green and P. M. Naghdi, “Thermoelasticity without Energy Dissipation,” J. Elasticity **31**, 189–208 (1993).
18. A. E. Green and P. M. Naghdi, “On Undamped Heat Waves in an Elastic Solid,” J. Thermal Stresses **15**, 252–264 (1992).
19. S. H. Mallik and M. Kanoria, “Two Dimensional Problem in Generalized Thermoelasticity for a Rotating Orthotropic Infinite Medium with Heat Sources,” Indian J. Math. **49** (1), 47–70 (2007).
20. S. H. Mallik and M. Kanoria, “A Unified Generalized Thermoelastic Formulation: Application to an Penny Shaped Crack Analysis,” J. Thermal Stresses **32**, 945–965 (2009).
21. D. Y. Tzou, “A Unified Approach for Heat Conduction from Macro to Micro-Scales,” Trans. ASME, J. Heat Transfer **117**, 8–16 (1995).
22. S. K. Roychoudhury, “One-Dimensional Thermoelastic Waves in Elastic Half-Space with Dual-Phase-Lag Effect,” J. Mech. Mater. Struct. **2**, 489–502 (2007).
23. R. Quintanilla and C. O. Horgan, “Spatial Behaviour of Solutions of the Dual-Phase-Lag Heat Equation,” Math. Methods Appl. Sci. **28**, 43–57 (2005).
24. R. Quintanilla and R. Racke, “A Note on Stability in Dual-Phase-Lag Heat Conduction,” Int. J. Heat Mass Transfer **49**, 1209–1213 (2006).
25. R. Quintanilla, “A Well-Posed Problem for the Three-Dual-Phase-Lag Heat Conduction,” J. Thermal Stresses **32**, 1270–1278 (2009).
26. R. Quintanilla, “Exponential Stability in the Dual-Phase-Lag Heat Conduction Theory,” J. Non-Equib. Thermodyn. **27**, 217–227 (2002).
27. R. Quintanilla, “A Condition on the Delay Parameters in the One-Dimensional Dual-Phase-Lag Thermoelastic Theory,” J. Thermal Stresses **26**, 713–721 (2003).
28. R. Prasad, R. Kumar, and S. Mukhopadhyay, “Propagation of Harmonic Plane Waves under Thermoelasticity with Dual-Phase-Lags,” Int. J. Eng. Sci. **48**, 2028–2043 (2010).
29. H. M. Youssef, “Theory of Two-Temperature Generalized Thermoelasticity,” IMA J. Appl. Math. **71**, 1–8 (2006).
30. S. Mukhopadhyay and R. Kumar, “Thermoelastic Interactions on Two Temperature Generalized Thermoelasticity in an Infinite Medium with a Cylindrical Cavity,” J. Thermal Stresses **32**, 341–360 (2009).
31. R. Kumar, R. Prasad, and S. Mukhopadhyay, “Variational and Reciprocal Principles in Two-Temperature Generalized Thermoelasticity,” J. Thermal Stresses **33**, 161–171 (2010).
32. A. Magane and R. Quintanilla, “Uniqueness and Growth of Solutions in Two-Temperature Generalized Thermoelastic Theories,” J. Math. Mech. Solids **14**, 622–634 (2009).
33. S. Banik and M. Kanoria, “Two Temperature Generalized Thermoelastic Interactions in an Infinite Body with a Spherical Cavity,” Int. J. Thermophys. **32**, 1247–1270 (2011).
34. A. Sur and M. Kanoria, “Fractional Order Two-Temperature Thermoelasticity with Finite Wave Speed,” Acta Mech. **223**, (12), 2685–2701 (2012).
35. S. Mondal, S. H. Mallik, and M. Kanoria, “Fractional Order Two-Temperature Dual-Phase-Lag Thermoelasticity with Variable Thermal Conductivity,” Int. Scholarly Res. Notices **2014**, 646049 (2014).
36. L. Y. Bahar and R. B. Hetnarski, “State Space Approach to Thermoelasticity,” in *Proc. of the 6th Canad. Congress of Applied Mechanics, Vancouver (Canada), May 30 to June 3, 1977* (Univ. of British Columbia, 1977), pp. 17–18.
37. G. Honig and U. Hirdes, “A Method for the Numerical Inversion of Laplace Transform,” J. Comput. Appl. Math. **10**, 113–132 (1984).
38. S. Banik and M. Kanoria, “Two-Temperature Generalized Thermoelastic Interactions in an Infinite Body with a Spherical Cavity,” Int. J. Thermophys. **32** (6), 1247–1270 (2011); DOI: 10.1007/s10765-011-1002-2.
39. L. Y. Bahar and R. B. Hetnarski, “State Space Approach to Thermoelasticity,” J. Thermal Stresses **1** (1), 135–145 (1978).