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EXPERIMENTAL AND THEORETICAL STUDY OF THE BUCKLING OF NARROW THIN PLATES ON AN ELASTIC FOUNDATION UNDER COMPRESSION

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Abstract: The paper describes the processes of elastic deformation of thin films under mechanical loading. The film is modeled longitudinally by a compressed plate arranged on an elastic foundation. A computer model of the buckling of the narrow thin plate with a delamination portion located on an elastic foundation is constructed. This paper also touches upon the supercritical behavior of the plate–substrate system. The experiments on the axial compression of a metal strip adhered to a rubber plate are performed, and 2 to 7 buckling modes are obtained therein. The critical loads and buckling modes obtained in the numerical calculations are compared with the experimental data. It is shown that there is the possibility of progressive delamination of the metal plate from the foundation if the critical load is exceeded. It is found that the use of the proposed approach, which, in contrast to other approaches, accounts for the elastic deformation of the substrate, causes the dependence between the critical bending stress and the stiffness of the foundation.

Keywords: thin films, delamination, elastic foundation, buckling, nonlinear deformation.

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INTRODUCTION

Thin films are widely used in modern microelectronics when it comes to designing integrated circuits, microsensors, gauges, etc. The use of new chemical and physical methods of depositing thin films allows obtaining heterostructures with unique functional characteristics and also various composite materials with gradient structures and properties. In industry, there is the method of depositing films on a large area, which allows creating functional materials with nonlinear electrophysical, optical, tribological, and other parameters [1].

Thin films can be solid or liquid. The composition, structure, and properties of the thin film and main material from which it formed may vary. Solid thin films include oxide films on metal surfaces and artificial film coatings formed on various materials and used in microelectronics for preventing corrosion, improving the appearance of products, etc. [2].

The manufacture of microelectronic devices is complicated by a problem of stability of thin films on a substrate. Vacuum deposition of metals on the surface is accompanied with deposition of thin films, which, in the process of growth or under thermomechanical effects, are known to have stresses caused by the difference of thermal and mechanical characteristics of the film and the substrate. Under the stresses, the film and the substrate undergo deformations of varying degrees, but, as they are rigidly connected, the equilibrium state can be obtained if the substrate–film system should be capable of bending. The bending of thin films on a substrate is similar to the classical case of the Euler elastic instability of the rod, which is under the action of longitudinal compressive forces [3]. When the critical value of the compressive force is exceeded, the rod buckles and then bends at an

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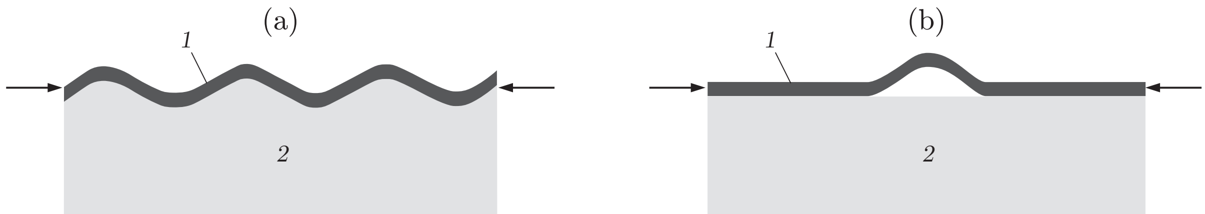

Fig. 1. Deformation of thin films on a substrate under compression: (a) embossing of the film on a nonrigid substrate; (b) delamination of the film on a rigid substrate; (*1*) film; (*2*) substrate.

arbitrarily small perturbation caused by initial imperfections (eccentricity of the applied force, heterogeneity of the material in the cross section, etc.). The bending that minimizes the elastic energy of the rod is dependent on the type of fixation of its ends.

Shugurov and Panin [4] described the elastic deformation of metal films under mechanical loading. It was shown that the longitudinal compression of thin films on a nonrigid substrate causes folds in the films and leads to coherent deformation of the substrate. In the case of a rigid substrate, the presence of compressive stresses leads to elastic bending of the film with local delamination from the substrate. Figure 1 shows the deformation of thin films on a substrate under compression [4].

If the film connection to the substrate is not strong enough or weakened by some defects at the interface, the local delamination of the film from the substrate is possible (see Fig. 1b) due to the occurrence of tensile stresses in the bending regions due to buckling of the thin film during its compression by longitudinal forces. Brittle fracture under compression was described in many papers, but some features of the fracture mechanism remain unexplored [5]. Below is a model of quasibrittle delamination of a narrow thin plate from an elastic substrate under axial compression, which takes into account the possibility of progressive delamination of the plate from the foundation if the critical load is exceeded. This work ignores the transverse wave formation occurring in the longitudinal compression of thin wide plates whose size is of the same order. The computer modeling data and the results of field experiments are compared.

1. MODELING OF THE BUCKLING OF THE LONGITUDINALLY COMPRESSED PLATE ARRANGED ON THE ELASTIC FOUNDATION

In the solution of the nonlinear problems of deformation of thin-walled structures (beams, plates, and shells), it is very important to select the constitutive equation of elasticity with small deformation of the body. The bending of the plate is usually accompanied by the condition of smallness of deformations, despite the fact that displacements and rotations can be sufficiently great. Therefore, the equations for description of the bending of thin-walled structures with elastic deformation are reasonable to be formulated with the use of constitutive equations of hyperelastic material as only this material is guaranteed to preserve the potential energy of internal forces on closed strain paths in the space of the strain tensor components [6].

The constitutive equations for the hyperelastic material in the Kirchhoff–Saint-Venant model have the form

$$S = \frac{\partial W(E)}{\partial E}, \qquad W \equiv \frac{1}{2}\, E : C^E : E.$$

Here $W(E)$ is the specific strain energy, S is the second Piola–Kirchhoff stress tensor, E is the Green–Lagrange strain tensor, $C^E = \lambda C_{\rm I} + \mu(C_{\rm II} + C_{\rm III})$ is the fourth-order elasticity tensor connecting variation rates of the tensors E and S by the relation $\dot S = C^E : \dot E$ ($C_{\rm I}$, $C_{\rm II}$, and $C_{\rm III}$ are the basic isomers of the fourth-order tensor [6]), λ and μ are the Lamé constants expressed through Young's modulus E and Poisson's ratio ν as follows:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{E}{2(1+\nu)}.$$

Figure 1b shows the deformation of the thin film on the substrate, in the case where delamination already occurred at the interface portion. We study the stability of such a film on the basis of the model of the longitudinally compressed plate partially resting on the elastic foundation (Fig. 2). Due to the symmetry of the problem, we

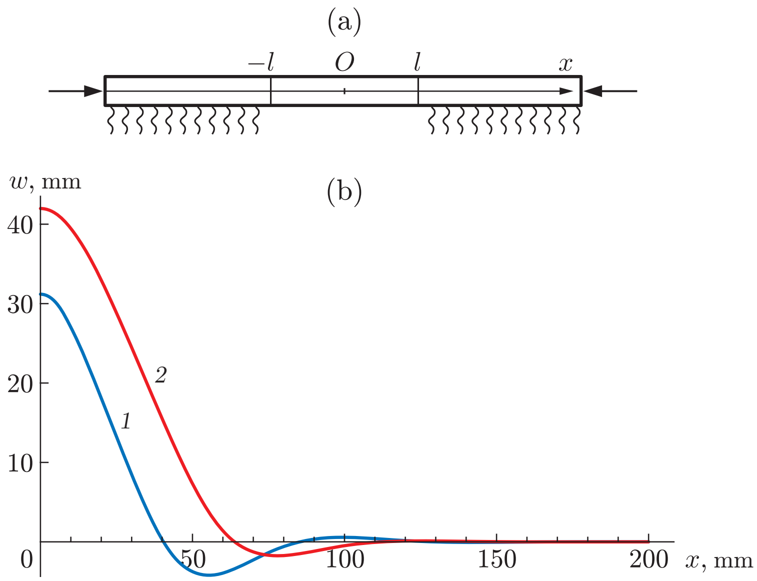

Fig. 2. Loading (a) and buckling mode (b) of the plate with a delamination portion, located on an elastic foundation: $l = 25$ (*1*) and 50 mm (*2*).

consider a 200-mm long, 10-mm wide, and 0.1-mm thick half of the plate with a delamination portion of a length $l = 25$ mm. The plate is partially resting on an elastic foundation of a length of 175 mm. The symmetry condition $u = 0$ is on its left side (u is the horizontal displacement), and the condition of hinged support is on its right side. The plate is subjected to the longitudinal compression in the mode of rigid loading, i.e., the horizontal displacement $u = -20$ mm is set on the right side. The law of load rise in time is considered to be linear, and time in quasi-static problems is understood as some monotonically increasing loading parameter.

The plate is split into 50 000 20-node hexagonal elements of size of $0.2 \times 0.2 \times 0.1$ mm with quadratic approximation of displacements. We choose the following mechanical characteristics of the material: $E = 2{\cdot}10^5$ MPa, $\nu = 0.35$, and the stiffness coefficient of the elastic foundation is $c = 0.0016$ N/mm^3. The computer modeling of the plate deformation is carried out with the use of the MSC.Marc 2012 package [7].

Figure 2 shows the deflection diagrams w of the plate with a delamination section of a length $l = 25$ and 50 mm. One can observe the buckling of the plate and the formation of three half-waves. The buckling modes in Fig. 2 are presented in the reference configuration, so the coordinate of the right end of the plate is equal to $x = 200$, despite the fact that it shifted by 20 mm. Note that the mathematical model yields only the trivial solution where $w = 0$ (the plate is compressed without buckling, i.e., it remains flat). In order to obtain the bending modes illustrated in Fig. 2, it is necessary to introduce initial perturbations in the form of small lateral forces of the order of 10^{-7} N applied to the front surface of the plate.

If the longitudinal stresses σ are greater than the critical stresses σ^*, then the plate buckles, which is accompanied by the formation of three half-waves (Fig. 2). The calculation results for the delamination section of a length $l = 25$, 38, and 50 mm show that the critical stresses decrease as l increases. This allows concluding that exceeding the critical load may cause progressive delamination. Moreover, the proposed model accounts for the effect of elastic deformation of the substrate; the critical stress is $\sigma^* = 5.169$ MPa for $c = 0.0016$ N/mm^3 and $\sigma^* = 1.837$ MPa for $c = 0.0004$ N/mm^3.

2. EXPERIMENTAL STUDIES

The experimental study of the buckling modes of flexible plates on the elastic foundation was carried out with the use of an experimental device (Fig. 3) created specifically for this research. The plates were polished steel strips of different length ($L = 80$–140 mm), thickness ($h = 0.2$–0.6 mm), and width ($a = 8$–10 mm). The elastic foundation

(a) (b)

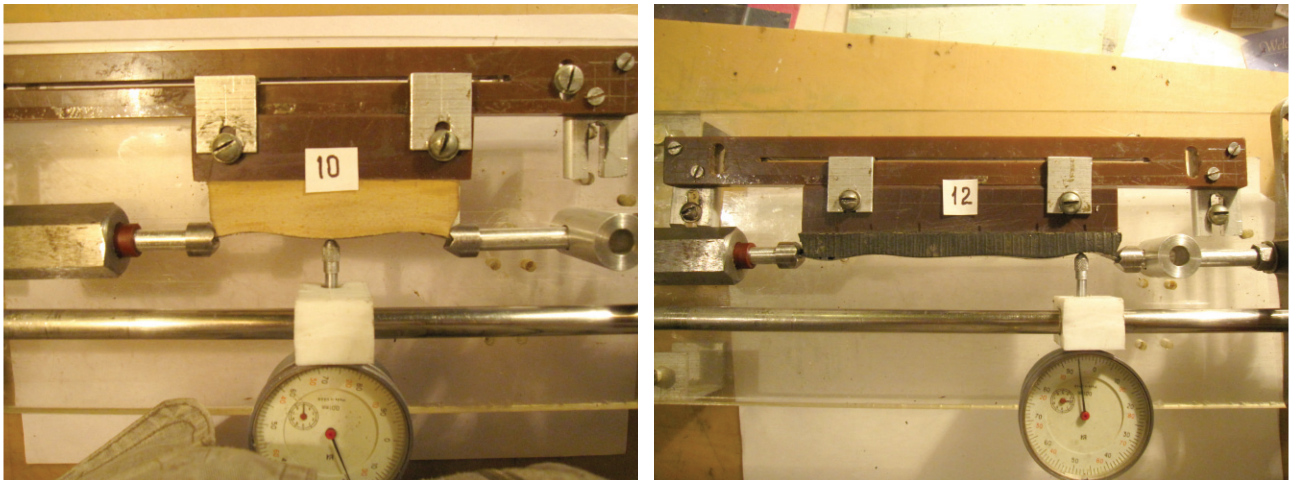

Fig. 3. Compression of the plates arranged on an elastic foundation with the formation of three half-waves (a) and six half-waves (b): (a) elastic foundation is made of vacuum rubber ($L = 80$ mm, $h = 0.25$ mm, $a = 8$ mm, and $b = 18.5$ mm); (b) elastic foundation is made of cellular rubber ($L = 135$ mm, $h = 0.2$ mm, $a = 10$ mm, and $b = 10$ mm).

glued to the strips was a cellular or vacuum rubber with Young's moduli $E = 0.6$ and 2.3 MPa and Poisson's ratios $\nu = 0.39$ and 0.47, respectively; accordingly, the rubber thickness $b = 10$–24 mm. To ensure the stability of the deformation process in the middle plane, a rigid plate made of micarta was glued to the rubber. The longitudinal load on the steel strip was applied symmetrically relative to its ends and measured with a dynamometer with a scale division of 0.025 N. As the longitudinal load increased, we observed from 2 to 7 buckling modes, depending on the strip length. With each fixed load, we measured the transverse displacement of the steel plate at the points corresponding the tops of the half-waves with an accuracy of up to 0.01 mm.

3. COMPUTER SIMULATION

We consider the problem of deformation of the steel plate glued to the elastic foundation under the effect of longitudinal compressive load in a two-dimensional formulation under the plane strain condition. The plate is simulated by a layer of 8-node rectangular elements with quadratic displacement approximation, and the foundation is split into 4-node rectangular elements with linear displacement approximation. The effect of the micarta plate on the rubber is taken into account assuming that the plate is fixed-ended. The horizontal displacements $u = 1$ mm are applied to the end surfaces of the plate, and the hinge support conditions are set in the middle nodes. The gluing conditions are set at the plate–foundation interface, i.e., we consider the rigid contact of two deformed bodies. As shown by the preliminary numerical experiments, the use of standard finite elements in the solution of the problem of the contact of deformed bodies with significantly different characteristics leads to undesirable calculation effects, such as the penetration of elements through a fixed boundary. On the contact surface of two adjacent elements, the equilibrium condition is satisfied only for the element as a whole. Therefore, there is a need to use the Herrmann elements, which combine the approximation of nodal displacements with the additional average stress function.

Figure 4 shows the calculated buckling modes with the formation of four and seven half-waves. As in Section 1, we obtain a nontrivial solution by introducing initial perturbations into the problem in the form of small transverse forces of the order of 10^{-7} N applied to the face of the plate. The comparison of the experimental data and computer simulation results shows that they agree qualitatively. Depending on the number of half-waves, the error in determining the plate deflections at the convexity and concavity points of the half-waves is 10–30% for the cellular rubber and 15–50% for the vacuum rubber. It should be noted that the experimental data and calculation for the vacuum rubber differ more significantly than for the cellular rubber. Apparently, this is due to the use of the inadequate material model.

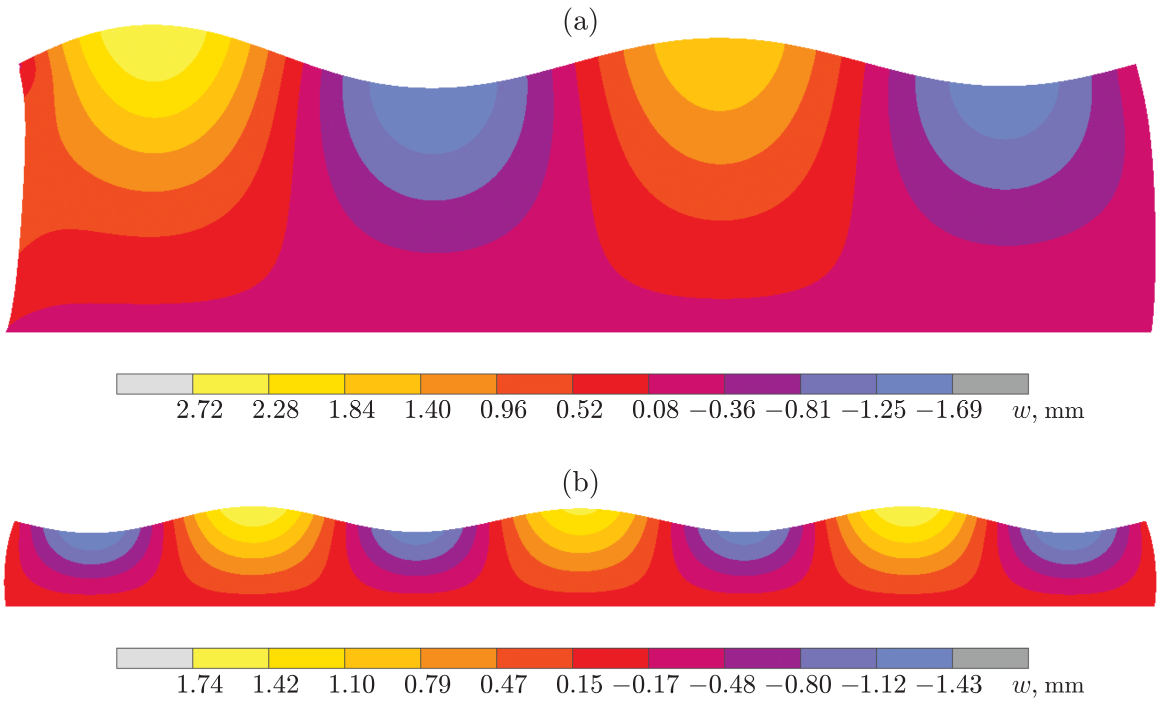

Fig. 4. Calculated buckling modes of the plates arranged on an elastic foundation with the formation of four half-waves (a) and seven half-waves (b): (a) elastic foundation is made of vacuum rubber ($L = 80$ mm, $h = 0.25$ mm, $a = 8$ mm, and $b = 18.5$ mm); (b) elastic foundation is made of cellular rubber ($L = 138$ mm, $h = 0.2$ mm, $a = 10$ mm, and $b = 10$ mm).

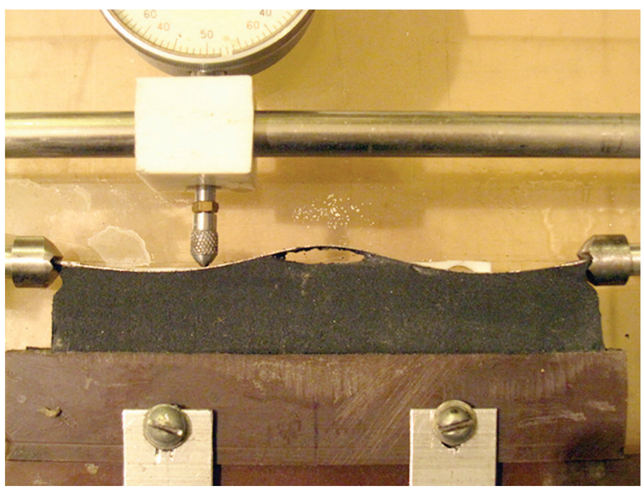

Fig. 5. Delamination of the steel plate glued to the cellular rubber, with the formation of the three half-waves ($l = 10$ mm, $L = 120$ mm, $h = 0.25$ mm, $a = 8$ mm, and $b = 20$ mm).

4. MODELING OF THE DELAMINATION OF THE PLATE FROM THE ELASTIC SUBSTRATE

We consider the possibility of progressive delamination of the longitudinally compressed plate from the foundation in excess of the critical load. Figure 5 shows a steel plate with a delamination section, arranged on the foundation made of cellular rubber. It follows from the results of the theoretical analysis of the problem of stability of the longitudinally compressed plate and from the experimental data that the deflection amplitude decreases with

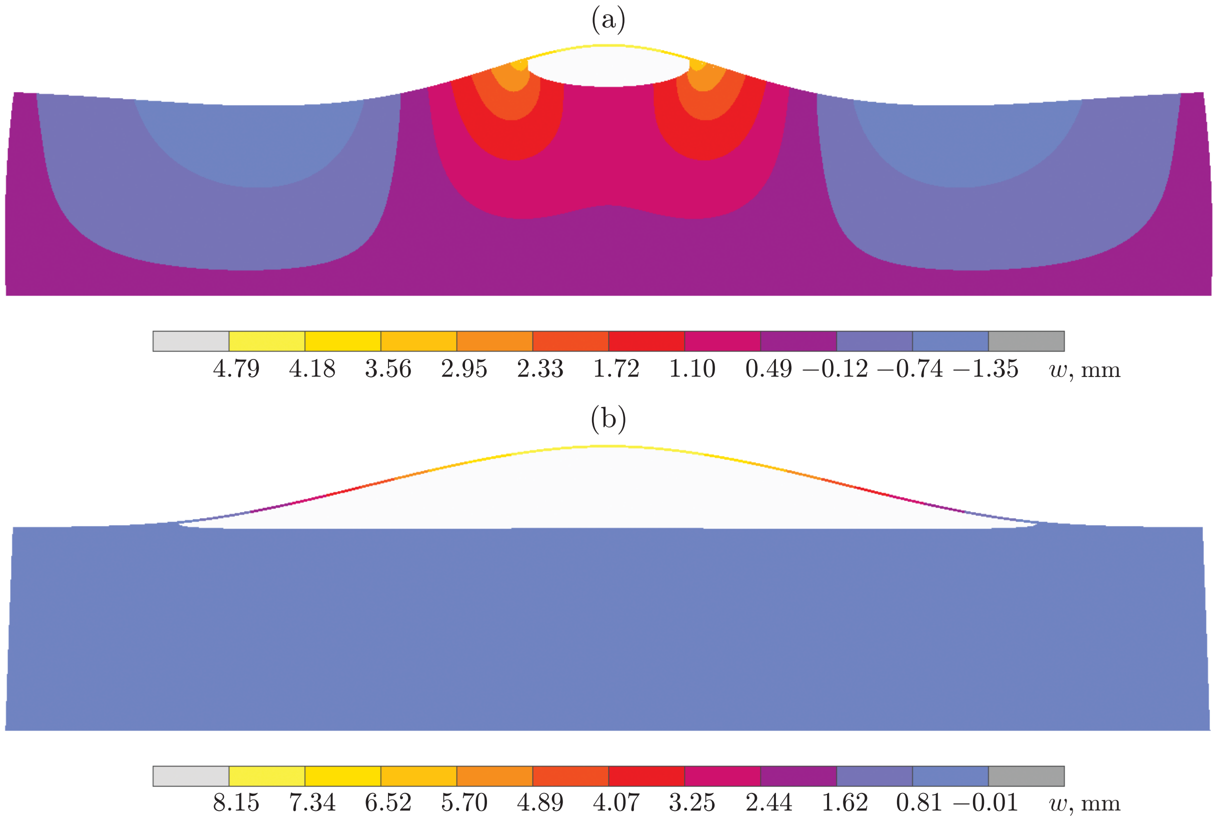

Fig. 6. Modeling of the delamination of the plate from the substrate at $t = 0.7659$ (a) and 0.7666 (b).

an increasing number of half-waves, so the substrate delaminates at the time of the occurrence of the first buckling modes (no more than 2 or 3 half-waves). The delamination is also affected by the plate length, the mechanical properties of the substrate, and the strength properties of the glue joint. In the experiments, the metal plate was glued to the rubber by a 3-component adhesive glue based on a solvent-free epoxy resin. Such glue consisted of a binder (ED-20 epoxy resin), plasticizer (dibutyl phthalate), and hardener (polyethylene polyamine). Depending on the concentration of components, the strength properties of the glue can be varied. The plasticizer is especially important when gluing metal and rubber for flexibility of the joint under large deformations. In the experiments described in Section 2, the adhesive strength was sufficient to prevent delamination of the rubber in the formation of bending shapes in the longitudinally compressed plate. In the experiment, whose results are shown in Fig. 5, the adhesive strength of the glue was comparable with the strength properties of the cellular rubber, which led to the formation of alternating separation sections in the glue joint and rubber. A delamination section was formed under a load at the time corresponding to Fig. 5. The slightest increase in the longitudinal force cause this portion to expand quickly, almost reaching both ends of the plate.

The modeling of the progressive delamination in the model described in Section 3 was carried out with account for the possibility of fracture of the material along the boundary of the contact between the plate and the foundation upon reaching the critical values of the normal stresses at this boundary. Figure 6 presents the isolines of the vertical displacements of the delaminated plate at times $t = 0.7659$ and 0.7666. The horizontal displacements were applied to the end surfaces of the plate, which equaled to $u = 1$ mm at $t = 1$, i.e., the ends of the plate approached each other at $t = 0.7659$ by a distance equal to 0.7659 mm. The adhesive strength limit was determined in the experiment with uniaxial tension of the glued prismatic samples. Despite the fact that the times $t = 0.7659$ and $t = 0.7666$ are very close, the calculation between these moments includes 50 steps. This leads to substantial differences in the deformed configurations of the plate and foundation. The comparison of the experimental data and numerical suggests that the proposed model adequately describes the progressive delamination of the plate from the substrate.

CONCLUSIONS

The model of quasibrittle delamination of a thin film from an elastic substrate under axial compression is built. The film is modeled by a narrow thin plate arranged on an elastic foundation with a small portion of its delamination from the substrate. The problem of buckling of the thin plate on the elastic foundation in a geometrically nonlinear formulation is solved numerically, and the supercritical behavior of the plate–substrate system is studied. There are large stress gradients at the delamination portion $x = l$ in the bending region of the plate, and they cause further delamination of the plate and according changes in the bending shape. Unlike the known techniques [8, 10], the proposed approach accounts for the elastic deformation of the substrate, i.e., the dependence of the critical bending stresses on the rigidity of the foundation.

The buckling modes of narrow steel plates arranged on the elastic foundation (cellular or vacuum rubber) are experimentally studied. As the longitudinal load increased, from 2 to 7 buckling modes are observed, depending on the strip length. The occurrence of the separation portion, which expanded rapidly even with a slight increase in the longitudinal force, almost reaching both ends of the plate, is experimentally observed. The modeling results show that, the excess of the critical load causes the thin plate to delaminate from the foundation almost instantaneously. The comparison of the experimental data and the results of numerical calculations suggest that the proposed model adequately describes the progressive delamination of the plate from the substrate.

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