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EFFECT OF THE THERMAL RELAXATION AND MAGNETIC FIELD ON GENERALIZED MICROPOLAR THERMOELASTICITY

M. I. A. Othman[a] **and Y. Q. Song**[b]

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Abstract: The model of generalized micropolar magneto-thermoelasticity for a thermally and perfectly conducting half-space is studied. The initial magnetic field is parallel to the boundary of the half-space. The formulation is applied to the generalized thermo-elasticity theories of Lord and Shulman, Green and Lindsay, as well as to the coupled dynamic theory. The normal mode analysis is used to obtain expressions for the temperature increment, the displacement, and the stress components of the model at the interface. By using potential functions, the governing equations are reduced to two fourth-order differential equations. By numerical calculation, the variation of the considered variables is given and illustrated graphically for a magnesium crystal micropolar elastic material. Comparisons are performed with the results predicted by the three theories in the presence of a magnetic field.

Keywords: micropolar electromagneto-thermoelasticity, Lord–Shulman theory, Green–Lindsay theory, normal mode analysis.

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INTRODUCTION

In recent years, micropolar elasticity gained great importance due to the large-scale development and utilization of composite, reinforced, and coarse-grained materials. The general theory of linear micropolar elasticity was given by Eringen [1]. Under this theory, solids can undergo macro-deformations and micro-rotations. The motion in this kind of solids is completely characterized by the displacement vector and the rotation vector, while the motion in the case of the classical elasticity is characterized by the displacement vector only. Micropolar solids can support the coupled stresses in addition to force stresses. Metals, polymers, composites, soils, rocks, and concrete is typical materials of this kind. The micropolar theory was extended to include thermal effects by Tauchert [2] and Nowacki and Olszak [3]. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves, i.e., in the case of elastic vibrations characterized by high frequencies and small wavelengths, the influence of the body microstructure becomes significant. This influence of the microstructure results in the development of new types of waves, not found in the classical theory of elasticity. The difference between the micropolar theory and the classical theory is the introduction of an independent microrotation vector. Moreover, there exists not only the traditional stress tensor, but also the coupled stress tensor in the micropolar theory. Many researches have studied the micropolar theory under the generalized thermoelastic theory [4–6].

[a]Faculty of Science, Department of Mathematics, Zagazig University, P.O. Box 44519, Zagazig, Egypt. [b]MOE Key Laboratory for Strength and Vibration, School of Aerospace, Xi' an Jiaotong University, Xi'an, 710049, People's Republic of China; m_i_a_othman@yahoo.com; yqsong@mail.xjtu.edu.cn. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 57, No. 1, pp. 126–135, January–February, 2016. Original article submitted December 4, 2012; revision submitted June 3, 2013.

Biot [7] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations of the diffusion type for both theories, which predict infinite speeds of propagation for heat waves, are contrary to physical observations. Hetnarski and Ignaczack [8] examined five generalizations to the coupled theory and obtained a number of important analytical results.

Lord and Shulman [9] were the first researchers who introduced the theory of generalized thermoelasticity with one relaxation time by postulating a new law of heat conduction to replace the classical Fourier law. This new law contains the heat flux vector as well as its time derivative. It also contains a new constant that acts as a relaxation time. The heat equation of this theory is of the wave-type, ensuring finite speeds of propagation of heat and elastic waves. The remaining governing equations for this theory, namely, the equation of motion and constitutive relations, are the same as those for the coupled and the uncoupled theories. The second generalization to the coupled theory is known as the generalized theory with two relaxation times [10]. A more explicit version was then introduced by Green and Laws [11], Green and Lindsay [12], and independently by Suhubi [13]. In this theory, the temperature change rates are considered among the constitutive variables. This theory also predicted finite speeds of propagation as in the Lord–Shulman theory. It differs from the latter in that Fourier's law of heat conduction is not violated if the body under consideration has a center of symmetry. Othman [14] established the model of two-dimensional equations of generalized thermoelasticity with one and two relaxation times under the effect of rotation, magnetic field, and dependence of the modulus of elasticity on the reference temperature.

The aim of this work is to study the thermal relaxation and magnetic field effects and to compare the three theories of thermoelasticity interaction in a micropolar medium for small times.

1. FORMULATION OF THE PROBLEM

We consider an isotropic, homogeneous, linear, thermally and perfectly conducting micropolar thermoelastic medium. The whole body is at a constant temperature T_0, and it is acted on throughout by a constant magnetic field $\boldsymbol{H} = (0, H_0, 0)$, which is parallel to the Y axis, and by a thermal field T. The medium is deformed because of the thermal and magnetic fields, which results in the emergence of an induced magnetic field $\boldsymbol{h} = (0, h, 0)$ and an induced electric field $\boldsymbol{E}$. We assume that h, E, T, and H are functions of the x and z coordinates and the time t and are independent of the y coordinate. So the displacement vector has the components $u_x = u(x, z, t)$, $u_y = 0$, and $u_z = w(x, z, t)$, while the micro-rotation vector has the form $\boldsymbol{\omega}_i = (0, \omega_2, 0)$.

The basic governing equations of a micropolar magneto-thermoelastic medium in the absence of body forces, body coupled forces, and heat sources are written as

$$\rho \ddot{u}_i = \sigma_{ir,r} + F_i, \qquad \varepsilon_{irp}\sigma_{rp} + m_{ri,r} = j\rho\ddot{\omega}_i, \tag{1}$$

where ρ is the density, j is the micro inertia moment, ε_{irp} is an alternative tensor defined as

$$\varepsilon_{irp} = n_i \cdot (n_r \times n_p),$$

and F_i is the Lorentz force given in the form

$$F_i = \mu_0(\boldsymbol{J} \times \boldsymbol{H})_i \tag{2}$$

($\boldsymbol{J}$ is the current density vector).

The displacement relations are presented as

$$e_{ir} = \frac{1}{2}(u_{i,r} + u_{r,i}), \qquad e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}. \tag{3}$$

We begin our consideration with the linearized equations of electromagnetism, valid for slowly moving media

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \dot{\boldsymbol{D}}; \tag{4}$$

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}; \tag{5}$$

$$\boldsymbol{E} = -\dot{\boldsymbol{u}} \times \boldsymbol{B},$$

$$\operatorname{div} \boldsymbol{B} = 0, \quad \operatorname{div} \boldsymbol{D} = 0, \qquad \boldsymbol{B} = \mu_0 \boldsymbol{H}, \quad \boldsymbol{D} = \varepsilon_0 \boldsymbol{E}.$$

The components of the magnetic intensity vector in the medium are

$$H_1 = H_3 = 0, \qquad H_2 = H_0 + h(x, z, t). \tag{6}$$

Taking into account Eqs. (3)–(5), we write Eq. (2) in the form

$$F_1 = \mu_0 H_0^2 \frac{\partial e}{\partial x} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 u}{\partial t^2}, \qquad F_2 = 0,$$

$$F_3 = \mu_0 H_0^2 \frac{\partial e}{\partial z} - \mu_0^2 H_0^2 \varepsilon_0 \frac{\partial^2 w}{\partial t^2}, \tag{7}$$

where μ_0 is the magnetic permeability and ε_0 is the dielectric constant. It follows from Eqs. (6) and (7) that

$$h = -H_0 e. \tag{8}$$

The components of the force stress and coupled stress have the form

$$\sigma_{ir} = \lambda u_{p,p}\delta_{ir} + \mu(u_{i,r} + u_{r,i}) + k(u_{r,i} - \varepsilon_{irp}\omega_p) - \hat{\gamma}\Big(1 + \nu \frac{\partial}{\partial t}\Big) T\delta_{ir},$$

$$m_{ir} = \alpha\omega_{p,p}\delta_{ir} + \beta\omega_{i,r} + \gamma\omega_{r,i}, \qquad i = 1, 2, 3,\ p = 1, 2, 3,\ r = 1, 2, 3, \tag{9}$$

where $\hat{\gamma} = (3\lambda + 2\mu + k)\alpha_T$, λ, μ, k, α, β, and γ are the elastic coefficients, α_T is the coefficient of linear thermal expansion, δ_{ir} is the Kronecker delta function, T_0 is the reference temperature chosen so that $|T - T_0|/T_0 \ll 1$, and T is the absolute temperature.

Substituting Eqs. (7)–(9) into Eqs. (1), we obtain

$$\rho\ddot{u}_i = (\lambda + \mu)u_{r,ri} + (\mu + k)u_{i,rr} - k\varepsilon_{irp}\omega_{p,r} - \hat{\gamma}\Big(1 + \nu\frac{\partial}{\partial t}\Big)T_{,i} + F_i; \tag{10}$$

$$\rho j\ddot{\omega}_i = \gamma\omega_{i,rr} + k\varepsilon_{irp}u_{p,r} - 2k\omega_i. \tag{11}$$

The generalized heat conduction equation is

$$KT_{,ii} = \rho C_E\Big(n_1 + \tau_0\frac{\partial}{\partial t}\Big)\dot{T} + \hat{\gamma}T_0\Big(n_1 + n_0\tau_0\frac{\partial}{\partial t}\Big)\dot{e}, \tag{12}$$

where K is the thermal conductivity, C_E is the specific heat at constant stress, and n_0, n_1, τ_0, and ν are constants. In Eqs. (9)–(11), a comma followed by a suffix denotes a material derivative with respect to the corresponding coordinate; a superposed dot denotes the derivative with respect to time; the suffices i and r take the values of x and z.

Equations (10)–(12), which are the field equations of the generalized linear micropolar magneto-thermoelasticity, yield the coupled theory equations and also the equations of the Lord–Shulman and Green–Lindsay generalizations.

Equations of the coupled dynamic theory are obtained from Eqs. (10) and (12) at $n_1 = 1$ and $\tau_0 = \nu = 0$. In this case, Eqs. (10) and (12) take the form

$$\rho\ddot{u}_i = (\lambda + \mu)u_{r,ri} + (\mu + k)u_{i,rr} - k\varepsilon_{irp}\omega_{p,r} - \hat{\gamma}T_{,i} + \mu_0(\boldsymbol{J} \times \boldsymbol{H})_i; \tag{13}$$

$$KT_{,ii} = \rho C_E\dot{T} + \hat{\gamma}T_0\dot{e}.$$

The equations of the Lord–Shulman theory follow from Eqs. (10) and (12) at

$$n_1 = n_0 = 1, \qquad \nu = 0.$$

In this case, Eq. (10) coincides with Eq. (13), while Eq. (12) takes the form

$$KT_{,ii} = \Big(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\Big)(\rho C_E T + \hat{\gamma} T_0 e).$$

The equations of the Green–Lindsay theory follow from Eqs. (10) and (12) at

$$n_1 = 1, \qquad n_0 = 0, \qquad \nu \geqslant \tau_0 > 0,$$

where ν and τ_0 are two relaxation times. In this case, Eqs. (10) and (12) have the form

$$\rho \ddot{u}_i = (\lambda + \mu) u_{r,ri} + (\mu + k) u_{i,rr} - k\varepsilon_{irp}\omega_{p,r} - \hat{\gamma}\Big(1 + \nu \frac{\partial}{\partial t}\Big)T_{,i} + \mu_0 (\boldsymbol{J} \times \boldsymbol{H})_i,$$

$$KT_{,ii} = \rho C_E \Big(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\Big)T + \hat{\gamma} T_0 \frac{\partial e}{\partial t}.$$

The equations for the generalized thermoelasticity theory result from Eqs. (10) and (12) at $k = \omega_i = H_0 = 0$. In this case, Eqs. (10) and (12) reduce to

$$\rho \ddot{u} = (\lambda + \mu)\frac{\partial e}{\partial x} + (\mu + k)\nabla^2 u - k\frac{\partial \omega_2}{\partial z} - \hat{\gamma}\Big(1 + \nu\frac{\partial}{\partial t}\Big)\frac{\partial T}{\partial x} + \mu_0 H_0^2 \frac{\partial e}{\partial x} - \mu_0^2 H_0^2 \varepsilon_0 \ddot{u},$$

$$\rho \ddot{w} = (\lambda + \mu)\frac{\partial e}{\partial z} + (\mu + k)\nabla^2 w + k\frac{\partial \omega_2}{\partial x} - \hat{\gamma}\Big(1 + \nu\frac{\partial}{\partial t}\Big)\frac{\partial T}{\partial z} + \mu_0 H_0^2 \frac{\partial e}{\partial z} - \mu_0^2 H_0^2 \varepsilon_0 \ddot{w}, \tag{14}$$

$$\rho j \ddot{\omega}_2 = \gamma \nabla^2 \omega_2 - 2k\omega_2 + k\Big(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\Big).$$

The constitutive relations can be written as

$$\sigma_{xx} = (\lambda + 2\mu + k)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \hat{\gamma}\Big(1 + \nu\frac{\partial}{\partial t}\Big)T,$$

$$\sigma_{zz} = (\lambda + 2\mu + k)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \hat{\gamma}\Big(1 + \nu\frac{\partial}{\partial t}\Big)T,$$

$$\sigma_{xz} = \mu\frac{\partial u}{\partial z} + (\mu + k)\frac{\partial w}{\partial x} + k\omega_2, \quad \sigma_{zx} = \mu\frac{\partial w}{\partial x} + (\mu + k)\frac{\partial u}{\partial z} - k\omega_2,$$

$$m_{xy} = \gamma\frac{\partial \omega_2}{\partial x}, \qquad m_{zy} = \gamma\frac{\partial \omega_2}{\partial z}.$$

For convenience, the following dimensionless variables are introduced:

$$x_i' = \frac{\omega^*}{c_0}x_i, \quad u_i' = \frac{\rho c_0 \omega^*}{\hat{\gamma} T_0}u_i, \quad t' = \omega^* t, \quad \tau_0' = \omega^* \tau_0, \quad \nu' = \omega^* \nu, \quad \theta = \frac{T}{T_0},$$

$$\sigma_{ir}' = \frac{\sigma_{ir}}{\hat{\gamma} T_0}, \quad m_{ir}' = \frac{\omega^*}{c_0 \hat{\gamma} T_0}m_{ir}, \quad \omega_2' = \frac{\rho c_0^2}{\hat{\gamma} T_0}\omega_2, \quad \omega^* = \frac{\rho C_E c_0^2}{K}, \quad c_0^2 = \frac{\lambda + 2\mu + k}{\rho}, \quad i = 1, 3.$$

Equations (12) and (13) take the form

$$\ddot{u} = \frac{\lambda + \mu}{\rho c_0^2}\frac{\partial e}{\partial x} + \frac{\mu + k}{\rho c_0^2}\nabla^2 u - \frac{k}{\rho c_0^2}\frac{\partial \omega_2}{\partial z} - \Big(1 + \nu\frac{\partial}{\partial t}\Big)\frac{\partial \theta}{\partial x} + \frac{\mu_0 H_0^2}{\rho c_0^2}\frac{\partial e}{\partial x} - \frac{\mu_0^2 H_0^2 \varepsilon_0}{\rho}\ddot{u},$$

$$\ddot{w} = \frac{\lambda + \mu}{\rho c_0^2}\frac{\partial e}{\partial z} + \frac{\mu + k}{\rho c_0^2}\nabla^2 w + \frac{k}{\rho c_0^2}\frac{\partial \omega_2}{\partial x} - \Big(1 + \nu\frac{\partial}{\partial t}\Big)\frac{\partial \theta}{\partial z} + \frac{\mu_0 H_0^2}{\rho c_0^2}\frac{\partial e}{\partial z} - \frac{\mu_0^2 H_0^2 \varepsilon_0}{\rho}\ddot{w}, \tag{15}$$

$$\nabla^2 \omega_2 - \frac{\rho j c_0^2}{\gamma}\ddot{\omega}_2 - \frac{2kc_0^2}{\gamma \omega^{*2}}\omega_2 + \frac{kc_0^2}{\gamma \omega^{*2}}\Big(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\Big) = 0,$$

$$\nabla^2\theta = \Big(n_1 + \tau_0\,\frac{\partial}{\partial t}\Big)\dot\theta + \frac{\hat\gamma^2 T_0}{K\rho\omega^*}\,\Big(n_1 + n_0\tau_0\,\frac{\partial}{\partial t}\Big)\dot e$$

(the primes at the dimensionless variables are omitted).

We introduce the displacement potentials φ and ψ by the relations

$$\begin{gathered} u = \varphi_{,x} + \psi_{,z}, \qquad w = \varphi_{,z} - \psi_{,x}, \\ e = \nabla^2\varphi, \qquad \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \nabla^2\psi, \end{gathered} \tag{16}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

Substituting Eqs. (16) into Eqs. (15), we obtain

$$\begin{gathered} \Big(\alpha_0\,\frac{\partial^2}{\partial t^2} - \beta_0^2\nabla^2\Big)\varphi = -\Big(1 + \nu\,\frac{\partial}{\partial t}\Big)\theta, \\ \Big(\alpha_0\,\frac{\partial^2}{\partial t^2} - a_1\nabla^2\Big)\psi = -a_2\omega_2, \\ \Big(\nabla^2 - a_4\,\frac{\partial^2}{\partial t^2} - 2a_3\Big)\omega_2 + a_3\nabla^2\psi = 0, \\ \nabla^2\theta - \Big(n_1 + \tau_0\,\frac{\partial}{\partial t}\Big)\frac{\partial\theta}{\partial t} - \varepsilon\Big(n_1 + n_0\tau_0\,\frac{\partial}{\partial t}\Big)\nabla^2\frac{\partial\varphi}{\partial t} = 0, \end{gathered} \tag{17}$$

where

$$\alpha_0 = 1 + \frac{\mu_0^2 H_0^2\varepsilon_0}{\rho}, \quad \beta_0^2 = 1 + \frac{\mu_0 H_0^2}{\rho c_0^2}, \quad a_1 = \frac{\mu + k}{\rho c_0^2}, \quad a_2 = \frac{k}{\rho c_0^2}, \quad a_3 = \frac{k c_0^2}{\gamma\omega^{*2}},$$

$$a_4 = \frac{\rho j c_0^2}{\gamma}, \qquad \varepsilon = \frac{\hat\gamma^2 T_0}{\rho K\omega^*}.$$

2. NORMAL MODE ANALYSIS

The solution for the considered physical variables can be decomposed in terms of normal modes as

$$[\theta, \omega_2, \varphi, \psi, \sigma_{ij}, m_{ij}](x, z, t) = [\theta^*, \omega_2^*, \varphi^*, \psi^*, \sigma_{ij}^*, m_{ij}^*](z)\exp{(\omega t + iax)}, \tag{18}$$

where ω is the frequency and a is the wave number in the x direction. In view of Eq. (18), Eqs. (17) take the form

$$(D^2 - a^2 - b^2\omega^2)\varphi^* - n_2\theta^* = 0; \tag{19}$$

$$(D^2 - a^2 - a_5\omega^2)\psi^* - a_6\omega_2^* = 0; \tag{20}$$

$$(D^2 - a^2 - 2a_3 - a_4\omega^2)\omega_2^* + a_3(D^2 - a^2)\psi^* = 0; \tag{21}$$

$$(D^2 - a^2 - n_3)\theta^* - n_4(D^2 - a^2)\varphi^* = 0, \tag{22}$$

where

$$D = \frac{d}{dz}, \quad n_2 = \frac{1 + \nu\omega}{\beta_0^2}, \quad n_3 = \omega(n_1 + \tau_0\omega), \quad n_4 = \varepsilon\omega(n_1 + n_0\tau_0\omega),$$

$$b^2 = \frac{\alpha_0}{\beta_0^2}, \quad a_5 = \frac{\alpha_0}{a_1}, \quad a_6 = \frac{a_2}{a_1}.$$

Substituting Eq. (22) into Eq. (19) and Eq. (21) into Eq. (20), we obtain

$$(D^4 - AD^2 + B)(\varphi^*, \theta^*) = 0, \qquad (D^4 - CD^2 + E)(\psi^*, \omega_2^*) = 0, \tag{23}$$

where

$$A = 2a^2 + n_3 + b^2\omega^2 + n_2 n_4; \tag{24}$$

$$B = a^4 + (n_3 + b^2\omega^2 + n_2 n_4)a^2 + n_3 b^2\omega^2; \tag{25}$$

$$C = 2a^2 + 2a_3 + a_5\omega^2 + a_4\omega^2 - a_3 a_6, \qquad E = a^4 + (a_5\omega^2 + 2a_3 + a_4\omega^2 - a_6 a_3)a^2 + a_5\omega^2(2a_3 + a_4\omega^2).$$

The solution of Eqs. (23), which is bounded for $z > 0$, is sought in the form

$$\varphi^* = \sum_{n=1}^{2} A_n\, \mathrm{e}^{-k_n z}, \qquad \theta^* = \sum_{n=1}^{2} A_n R_n\, \mathrm{e}^{-k_n z}, \qquad \psi^* = \sum_{n=1}^{2} A_{n+2}\, \mathrm{e}^{-k_{n+2} z}, \qquad \omega_2^* = \sum_{n=1}^{2} A_{n+2} R_{n+2}\, \mathrm{e}^{-k_{n+2} z},$$

where A_n and A_{n+2} $(n = 1, 2)$ are parameters depending on a and ω; the values of R_n are calculated by the formulas

$$R_{1,2} = \frac{1}{n_2}\,(k_{1,2}^2 - a^2 - b^2\omega^2), \qquad R_{3,4} = \frac{1}{a_6}\,(k_{3,4}^2 - a^2 - a_5\omega^2);$$

$k_{1,2}^2$ and $k_{3,4}^2$ are the roots of Eqs. (23):

$$k_{1,2}^2 = \frac{1}{2}\Big(A \pm \sqrt{A^2 - 4B}\,\Big), \qquad k_{3,4}^2 = \frac{1}{2}\Big(C \pm \sqrt{C^2 - 4E}\,\Big).$$

3. SPECIAL CASES OF THE MICROPOLAR MAGNETO-THERMOELASTICITY THEORY

Equations (9)–(12) are the field equations of the micropolar generalized magneto-thermoelastic theory, which has several special cases. The equations of the generalized micropolar magneto-thermoelasticity theory are derived from Eqs. (9)–(12) at

$$n_1 = 1, \quad \tau_0 = \nu = 0, \quad n_2 = \beta_0^{-2}, \quad n_3 = \omega, \quad n_4 = \varepsilon\omega.$$

In this case, Eqs. (24) and (25) take the form

$$A = 2a^2 + \omega + \frac{\alpha_0\omega^2}{\beta_0^2} + \frac{\varepsilon\omega}{\beta_0^2}, \qquad B = a^4 + \Big(\omega + \frac{\alpha_0\omega^2}{\beta_0^2} + \frac{\varepsilon\omega}{\beta_0^2}\Big)a^2 + \frac{\alpha_0\omega^3}{\beta_0^2}.$$

The equations of the Lord–Shulman theory follow from Eqs. (9)–(12) at

$$n_1 = n_0 = 1, \quad \tau_0 > 0, \quad \nu = 0, \quad n_2 = \beta_0^{-2}, \quad n_3 = \omega(1 + \tau_0\omega), \quad n_4 = \varepsilon\omega(1 + \tau_0\omega).$$

In this case, Eqs. (24) and (25) take the form

$$A = 2a^2 + \omega(1 + \tau_0\omega) + \frac{\alpha_0\omega^2}{\beta_0^2} + \frac{\varepsilon\omega(1 + \tau_0\omega)}{\beta_0^2},$$

$$B = a^4 + \Big(\omega(1 + \tau_0\omega) + \frac{\alpha_0\omega^2}{\beta_0^2} + \frac{\varepsilon\omega(1 + \tau_0\omega)}{\beta_0^2}\Big)a^2 + \frac{\alpha_0\omega^3(1 + \tau_0\omega)}{\beta_0^2}.$$

The equations of the Green–Lindsay theory follow from Eqs. (9)–(12) at

$$n_1 = 1, \quad n_0 = 0, \quad \nu \geqslant \tau_0 > 0, \quad n_2 = (1 + \nu\omega)/\beta_0^2, \quad n_3 = \omega(1 + \tau_0\omega), \quad n_4 = \varepsilon\omega.$$

In this case, Eqs. (24) and (25) take the form

$$A = 2a^2 + \omega(1 + \tau_0\omega) + \frac{\alpha_0\omega^2}{\beta_0^2} + \frac{\varepsilon\omega(1 + \nu\omega)}{\beta_0^2},$$

$$B = a^4 + \Big(\omega(1 + \tau_0\omega) + \frac{\alpha_0\omega^2}{\beta_0^2} + \frac{\varepsilon\omega(1 + \nu\omega)}{\beta_0^2}\Big)a^2 + \frac{\alpha_0\omega^3(1 + \tau_0\omega)}{\beta_0^2}.$$

Special cases of Eqs. (9)–(12) can be also obtained by setting $k = \alpha = \beta = \gamma = 0$ or $H_0 = 0$.

In order to determine the parameters A_n and A_{n+2} $(n = 1, 2)$, we need to consider the boundary conditions:

$$\sigma_{zz} = \sigma_{zx} = m_{zy} = 0, \qquad \theta = f(x, t), \qquad z = 0.$$

With the help of these boundary conditions, the amplitudes of the expressions for the displacement components, force stresses, coupled stress, and temperature field are written as

$$u^* = \frac{1}{\Delta}\Big(ia\Delta_1\,\mathrm{e}^{-k_1 z} + ia\Delta_2\,\mathrm{e}^{-k_2 z} - k_3\Delta_3\,\mathrm{e}^{-k_3 z} - k_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$w^* = -\frac{1}{\Delta}\Big(k_1\Delta_1\,\mathrm{e}^{-k_1 z} + k_2\Delta_2\,\mathrm{e}^{-k_2 z} + ia\Delta_3\,\mathrm{e}^{-k_3 z} + ia\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$\theta^* = \frac{1}{\Delta}\Big(R_1\Delta_1\,\mathrm{e}^{-k_1 z} + R_2\Delta_2\,\mathrm{e}^{-k_2 z}\Big), \qquad \omega_2^* = \frac{1}{\Delta}\Big(R_3\Delta_3\,\mathrm{e}^{-k_3 z} + R_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$\sigma_{zz}^* = \frac{1}{\Delta}\Big(f_1\Delta_1\,\mathrm{e}^{-k_1 z} + f_2\Delta_2\,\mathrm{e}^{-k_2 z} + iab_5k_3\Delta_3\,\mathrm{e}^{-k_3 z} + iab_5k_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$\sigma_{zx}^* = \frac{1}{\Delta}\Big(-iab_6k_1\Delta_1\,\mathrm{e}^{-k_1 z} - iab_6k_2\Delta_2\,\mathrm{e}^{-k_2 z} + g_3\Delta_3\,\mathrm{e}^{-k_3 z} + g_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$\sigma_{xz}^* = \frac{1}{\Delta}\Big(-iab_6k_1\Delta_1\,\mathrm{e}^{-k_1 z} - iab_6k_2\Delta_2\,\mathrm{e}^{-k_2 z} + g_1\Delta_3\,\mathrm{e}^{-k_3 z} + g_2\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$\sigma_{xx}^* = \frac{1}{\Delta}\Big(f_3\Delta_1\,\mathrm{e}^{-k_1 z} + f_4\Delta_2\,\mathrm{e}^{-k_2 z} - iab_5k_3\Delta_3\,\mathrm{e}^{-k_3 z} - iab_5k_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

$$m_{xy}^* = -\frac{1}{\Delta}\Big(b_4k_3R_3\Delta_3\,\mathrm{e}^{-k_3 z} + b_4k_4R_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big), \quad m_{zy}^* = \frac{ia}{\Delta}\Big(b_4R_3\Delta_3\,\mathrm{e}^{-k_3 z} + b_4R_4\Delta_4\,\mathrm{e}^{-k_4 z}\Big),$$

where

$$\Delta = a^2b_5^2k_3k_4(k_2R_1 - k_1R_2)(R_3 - R_4) - (f_2R_1 - f_1R_2)(g_4k_3R_3 - g_3k_4R_4),$$

$$\Delta_1 = \theta_0 b_4[f_2(g_3k_4R_4 - g_4k_3R_3) - a^2b_5b_6k_2k_3k_4(R_4 - R_3)],$$

$$\Delta_2 = -\theta_0 b_4[f_1(g_3k_4R_4 - g_4k_3R_3) - a^2b_5b_6k_1k_3k_4(R_4 - R_3)],$$

$$\Delta_3 = ia\theta_0b_4b_6k_4R_4(f_2k_1 - f_1k_2), \quad \Delta_4 = ia\theta_0b_4b_6k_3R_3(f_1k_2 - f_2k_1),$$

θ_0 is the amplitude of the function $f(x,t) = \theta_0\exp(\omega t + iax)$, and

$$f_n = \begin{cases} k_n^2 - a^2b_1 - (1+\nu\omega)R_n, & n = 1,2, \\ b_1k_{n-2}^2 - a^2 - (1+\nu\omega)R_{n-2}, & n = 3,4, \end{cases}$$

$$g_n = \begin{cases} b_2k_{n+2}^2 + a^2b_3 + (b_3 - b_2)R_{n+2}, & n = 1,2, \\ b_3k_n^2 + a^2b_2 - (b_3 - b_2)R_n, & n = 3,4, \end{cases}$$

$$b_1 = \frac{\lambda}{\rho c_0^2}, \quad b_2 = \frac{\mu}{\rho c_0^2}, \quad b_3 = \frac{\mu + k}{\rho c_0^2}, \quad b_4 = \frac{\gamma\omega^{*2}}{\rho c_0^4}, \quad b_5 = 1 - b_1, \quad b_6 = b_2 + b_3.$$

4. NUMERICAL RESULTS

For numerical evaluations, the values of the relevant parameters for the magnesium crystal are taken at $T_0 = 297$ [15]: $\lambda = 9.4\cdot 10^{10}$ N/m^2, $\mu = 4\cdot 10^{10}$ N/m^2, $k = 10^{10}$ N/m^2, $\rho = 1.74\cdot 10^3$ kg/m^3, $j = 0.2\cdot 10^{-15}$ m^2, $C_E = 1.04\cdot 10^3$ J$\cdot$kg/K, $\hat{\gamma} = 6.493\cdot 10^{-8}$ K^{-1}, $\gamma = 0.779\cdot 10^{-9}$ N, $K = 1.7\cdot 10^2$ J/(m$\cdot$s$\cdot$K), $\omega = \omega_0 + i\xi$, $\omega_0 = 2$, and $\xi = 1$. The other constants of the problem are taken as $L = 4$, $\theta_0 = 1$, $b = 1$, and $a = 2$; the dimensionless relaxation times are taken as $\nu = 0.03$ and $\tau_0 = 0.02$. The real parts of the displacement vector components $u(x,z,t)$ and $w(x,z,t)$, temperature $\theta(x,z,t)$, micro-rotation vector components $\omega_2(x,z,t)$, force stress tensor components $\sigma_{xx}(x,z,t)$, $\sigma_{xz}(x,z,t)$, and $\sigma_{zz}(x,z,t)$, and coupled stress tensor components $m_{xy}(x,z,t)$ and $m_{zy}(x,z,t)$ are calculated under different conditions.

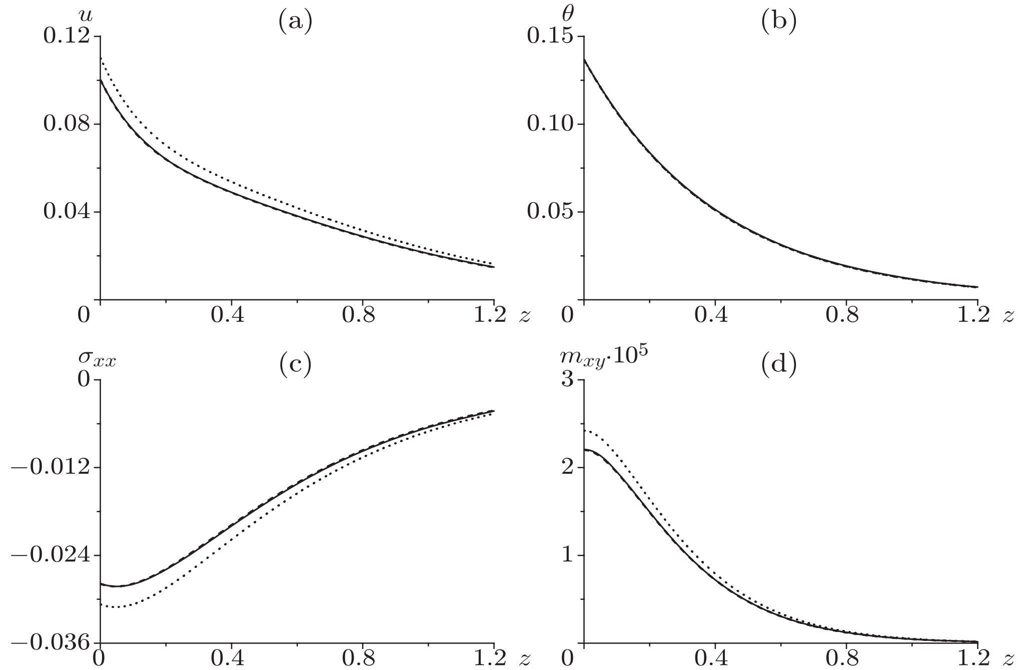

Fig. 1. Distributions of the displacement u (a), temperature θ (b), force stress σ_{xx} (c), and coupled stress m_{xy} (d) at $x = 0$ and $\alpha_0 = 1.1$, which were obtained by using different theories: coupled dynamic theory (solid curves), Lord–Shulman theory (dashed curves), and Green–Lindsay theory (dotted curves).

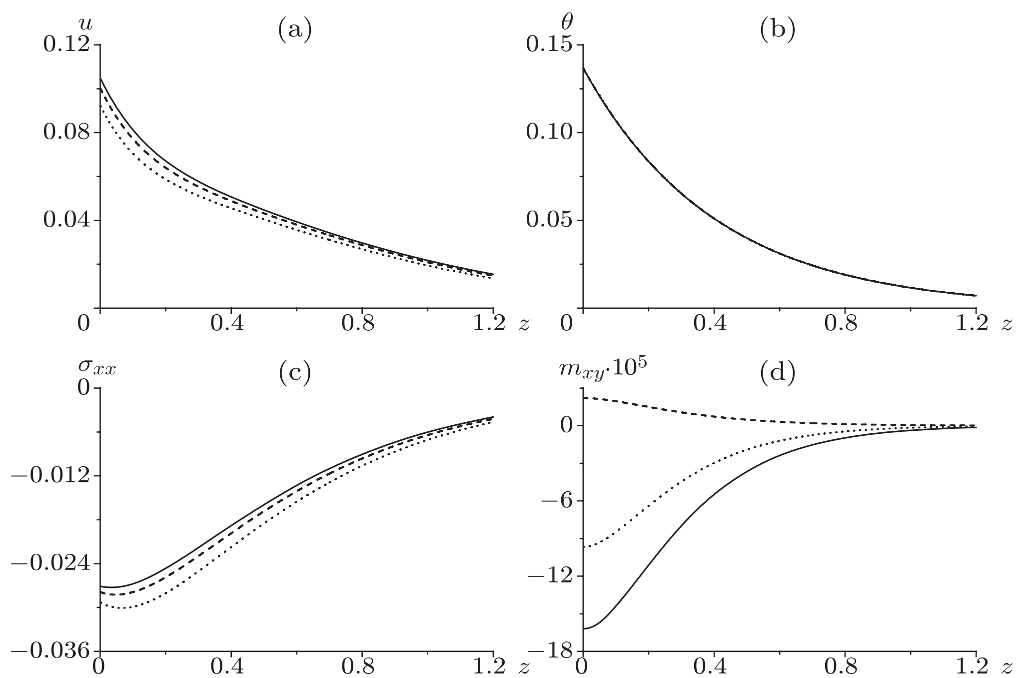

Fig. 2. Distributions of the displacement u (a), temperature θ (b), force stress σ_{xx} (c), and coupled stress m_{xy} (d) at $x = 0$ and different values of α_0: $\alpha_0 = 1.0$ (solid curves), 1.1 (dashed curves), and 1.3 (dotted curves).

Figure 1 shows the real parts of the field variables u, θ, σ_{xx}, and m_{xy} along the z axis for $x = 0$, $t = 0.02$, and $\alpha_0 = 1.1$ obtained by using different theories. It is seen that the temperature distributions under the three theories have almost the same behavior. The difference in the values of the displacement, stress, and couple stress predicted by the dynamic couples theory and the Lord–Shulman theory is very small. However, the difference in the distributions of the field variables predicted by the coupled dynamic theory and the Green–Lindsay theory is remarkable. For the field variable σ_{xx}, the curves corresponding to the Green–Lindsay theory are lower than those predicted by the coupled dynamic and Lord–Shulman theories. The curves for the field variables u and m_{xy} corresponding to the Green–Lindsay theory are higher than those predicted by the coupled dynamic and Lord–Shulman theories.

Figure 2 depicts the effect of the magnetic field on the distributions of the field variables u, θ, σ_{xx}, and m_{xy} predicted by the Lord–Shulman theory at $\alpha_0 = 1.0$ (no allowance for the magnetic field), $\alpha_0 = 1.1$, and $\alpha_0 = 1.3$. The magnetic field has a strong effect on all field variables (except for temperature). The values of the field variables u and σ_{xx} decrease with an increase in the magnetic field.

CONCLUSIONS

A fundamental solution of the system of equations for a generalized micropolar magneto-thermoelastic medium is obtained, and the effect of the thermal relaxation and magnetic field on the displacement, temperature, force stress, and coupled stress is studied.

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