ISSN 0021-3640, JETP Letters, 2019, Vol. 110, No. 1, pp. 15–24. © Pleiades Publishing, Inc., 2019.
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OPTICS
AND LASER PHYSICS

Ultrashort Optical Pulses and Their Generation in Resonant Media (Scientific Summary)

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Received May 4, 2019; revised May 18, 2019; accepted May 18, 2019

Our recent studies of methods for obtaining ultrashort light pulses and analysis of the impact of ultrashort optical pulses on classical and quantum micro-objects with the identification of the determining role of the degree of unipolarity of pulses (maximum for strictly unipolar pulses) in the efficiency of direct laser acceleration of charged particles and excitation of atoms and molecules have been reviewed. Special attention has been paid to the coherent mode locking regime in lasers, which is implemented when the laser pulse duration is less than the relaxation time of the working transition in media with resonance amplification and absorption (laser analog of the phenomenon of self-induced transparency). Experimental data on coherent mode locking in a titanium–sapphire laser with a cell with rubidium vapor at self-induced transparency in rubidium have been presented. The interaction of ultrashort pulses in resonant media has also been analyzed.

DOI: 10.1134/S0021364019130071

1. INTRODUCTION

Ultrashort (with a few cycles) attosecond electromagnetic pulses in the vacuum ultraviolet [1–5] and in the optical range [6–8] have already been obtained. Low durations and high peak intensities make such pulses a unique tool to study and control the dynamics of elementary processes in atoms and matter [4–6]. The problem of obtaining extremely strong laser fields, capable of revealing the nonlinearity of even electron-positron vacuum in the future, is also adjacent to this topic [9]. It is important in these matters to clarify the requirements for the characteristics of laser pulses, determining the effectiveness of the impact on various objects.

Typically, ultrashort pulses are obtained by passive mode locking in lasers [1] and attosecond pulses are generated using a set of high harmonics that occur when a high-power femtosecond pulse is applied to a target medium with their subsequent superposition [2]. However, such methods have significant drawbacks. In particular, the method of obtaining attosecond pulses through the generation of high-order harmonics has low energy efficiency and requires large and expensive experimental facilities. This makes it relevant to search for alternative approaches to the generation of ultrashort pulses.

This review mainly presents the results of our recent studies of both the efficiency of the effect of ultrashort pulses on elementary objects and the possibility of generation of ultrashort pulses directly in a compact laser. The widely discussed possibilities of shortening pulses down to subcyclic or (quasi-)unipolar ones consisting (mainly) of a field outburst of the same polarity are also considered.

2. ELECTRIC AREA OF THE PULSE AND ITS IMPACT ON ELEMENTARY OBJECTS

An electromagnetic pulse will be characterized by the degree of its unipolarity:

$$\xi(\mathbf{r}) = \frac{\left|\int_{-\infty}^{+\infty} \mathbf{E}dt\right|}{\int_{-\infty}^{+\infty} |\mathbf{E}|dt}. \quad (1)$$

Here, **r** is the position vector (coordinates of the spatial point), **E** is the electric field strength, and t is the time. The degree of unipolarity can vary from 0 (bipolar pulse) to 1 (unipolar pulse). A pulse with a degree of unipolarity close to 1 will be called quasi-unipolar: $1 - \xi \ll 1$.

The numerator of Eq. (1) is the absolute value of the electric area of the pulse

$$\mathbf{S}_E = \int_{-\infty}^{+\infty} \mathbf{E} dt. \quad (2)$$

Recall that, within the electrodynamics of continuous media, this quantity has the property

$$\mathrm{curl}\mathbf{S}_E = 0, \quad (3)$$

which leads to the conservation rule in problems of one-dimensional propagation (plane waves and field strengths depend only on the longitudinal coordinate Z) [10–15]

$$\frac{d}{dz}\mathbf{S}_E = 0. \quad (4)$$

The mechanical meaning of the electric area is clarified when it is compared with the moment (impulse) of the Lorentz force acting on a nonrelativistic (the velocity much less than the speed of light in vacuum) charge q: $\mathbf{F} = q\mathbf{E}$. Accordingly, the impulse of the force is $\mathbf{N} = \int_{-\infty}^{+\infty} \mathbf{F} dt = q\mathbf{S}_E$. Thus, the electric area is the moment (impulse) of the force acting on a unit charge by the field [16]. This fact emphasizes the importance of the electric area as a criterion for the efficiency of the impact of the pulse on the charged particle, because the moment of the force determines the change in the momentum of the particle in mechanics (Newton's second law).

It is necessary to take into account the Lorentz force component depending on the charge velocity **v** in the relativistic equation of motion of the charge [17]

$$\frac{d\mathbf{p}}{dt} = q\mathbf{E} + \frac{q}{c}[\mathbf{v} \times \mathbf{H}]. \quad (5)$$

Here, **H** is the magnetic field strength. In this case, the momentum of the particle is related to its velocity as $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$, where m is the mass of the particle. The electromagnetic pulse is simulated by a linearly polarized plane wave propagating in vacuum along the z axis. Then, the argument of the strengths **E** and **H** in Eq. (5) is a combination $z(t) - ct$, where $z(t)$ is the longitudinal coordinate of the particle.

The result of the action of sufficiently short electromagnetic pulses on the particle should not depend on the specific shape of the pulse. Therefore, the pulse can be considered as rectangular with the electric area coinciding with that of the real pulse. In this case, the energy E_q transferred to the initially stationary particle by the electromagnetic pulse is specified by the expression [18]

$$E_q = mc^2[1 + S_E^2/(2S_0^2)], \quad S_0 = mc/q. \quad (6)$$

The numerical solution of the equation of motion (5) shows that this relation is also valid with a high accuracy for electromagnetic pulses with numerous different time profiles. According to Eq. (6), the direct laser acceleration of charged particles is determined by the electric area of the laser pulse rather than by its energy. Bipolar pulses even with a high energy act on the charge in different directions at stages with different polarities of the field and, therefore, are less effective [18]. We also point out that this consideration excludes the region of the ultrarelativistic acceleration regime, in which quantum electrodynamic effects and radiation inhibition (Lorentz drag forces) become essential [17].

We now discuss the effect of ultrashort electromagnetic pulses on the simplest quantum objects [16, 19–22]. In this case, it is necessary to consider the mean value of the momentum in the state with the wavefunction $\psi(\mathbf{r}, t)$:

$$\langle \mathbf{p} \rangle = \langle \psi | \mathbf{p} | \psi \rangle = -i\hbar \int \psi^* \nabla \psi d\mathbf{r}. \quad (7)$$

The evolution of the wavefunction is described by the nonrelativistic Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = H_0 \psi + V(\mathbf{r}, t)\psi, \quad (8)$$

where H_0 is the Hamiltonian of the system in the absence of radiation and V is the interaction potential of an atom with electromagnetic radiation; $V = q\mathbf{E}\mathbf{r}$ for an atomic object in the electric dipole approximation. Before the interaction with the electromagnetic pulse, the wavefunction is $\psi = \psi_0$ and the average value of the mechanical momentum $\langle \mathbf{p}_0 \rangle = \langle \psi_0 | \mathbf{p} | \psi_0 \rangle = -i\hbar \int \psi_0^* \nabla \psi_0 d\mathbf{r}$. Assuming that the duration of the electromagnetic pulse is much less than the inverse frequency of transitions in the system unperturbed by radiation and its amplitude exceeds the intensity of the atomic field, we obtain the wavefunction after the end of the pulse in the Migdal sudden perturbation approximation [23] in the form

$$\psi = \exp\left(-i\frac{q}{\hbar}\mathbf{S}_E\mathbf{r}\right)\psi_0. \quad (9)$$

According to Eq. (9), this wavefunction is the product of the initial wavefunction ψ_0 and the wavefunction of a plane wave with momentum $-q\mathbf{S}_E$ (the minus sign appears because the electron charge is negative). The change in the electron momentum under the action of the electromagnetic pulse can be found using Eq. (9) [16]:

$$\langle \mathbf{p} \rangle = \langle \mathbf{p}_0 \rangle - q\mathbf{S}_E. \quad (10)$$

This is consistent with the conclusion of classical mechanics about the determining value of the electric area of the electromagnetic pulse in the problem of acceleration of charged particles.

In a similar approximation for the hydrogen atom excited by a short electromagnetic pulse with a large amplitude, it is possible to obtain the probability that the atom initially being in the ground state will remain in it after the end of the pulse [19]:

$$w_0 = \frac{1}{\left(1+\left(\frac{\hbar S_E}{2mq}\right)^2\right)^4}. \quad (11)$$

This probability decreases monotonically with the growth of the electric area. In this case, the efficiency of excitation of atoms by short electromagnetic pulses with a high peak amplitude is determined by the electric area of such pulses.

Another example is the excitation of molecular vibrations by short electromagnetic pulses. The exact solution of the Schrödinger equation is known for the model of a quantum harmonic oscillator with a frequency ω_0 swung by a force with an arbitrary time dependence [24]. The following simple expression for the probability of excitation of the nth oscillator level by a pulse can be obtained for the case of electromagnetic pulses short compared to the period of free oscillations $2\pi/\omega_0$ of the oscillator [20]:

$$w_n = \frac{1}{n!}\tilde{S}_E^{2n}\exp(-\tilde{S}_E^2), \quad (12)$$

where

$$\tilde{S}_E = \frac{qS_E}{\sqrt{2m\hbar\omega_0}}.$$

Expression (12), as well as Eq. (11), indicates a monotonic increase in excitation with an increase in the electric area of the pulse at $n = 0$. However, this increase is exponential rather than power-law because of the different nature of the spectrum of energy levels. The population of the nth excited level ($n \geq 1$) depends now nonmonotonically on the electric area with the maximum at $\tilde{S}_E^2 = n$. A determining role of the electric area of the pulse in the effectiveness of the impact is again a common feature. Excitation is nonresonant because the pulse duration is small.

It should also be noted that the exact solution of the Schrödinger equation for a many-electron atom interacting with a short electromagnetic pulse is obtained in the electric dipole and sudden perturbation approximations in [25, 26]. In addition, we mention work [27], where the features of the excitation of multilevel quantum systems by an electromagnetic pulse whose duration is less than the inverse frequency of transitions between the levels were considered taking into account the members of higher orders of H_0 in Eq. (8). As a result, it was shown that second order effects allow the effective nonresonant excitation of transitions even in the case of zero electric area of the pulse. In particular, it was demonstrated that this mechanism can be used to create population inversion in multilevel quantum systems and to excite and dissociate molecules under the action of femtosecond pulses.

3. ON THE POSSIBILITY OF GENERATION OF UNIPOLAR ELECTROMAGNETIC PULSES

d'Alembert's equation allows pulses with an arbitrary intensity profile moving at the speed of light c for the one-dimensional propagation of radiation (plane waves), which accurately describes, for example, radiation in single-mode fibers. Thus, Maxwell's equations are consistent with unipolar electromagnetic pulses, in which the main polarization component of the electric field does not change the sign. This solves the problem that the constant component corresponding to the unipolar pulse has a frequency $\omega = 0$ and therefore cannot spread. In fact, the phase velocity v of plane monochromatic waves in the radiation spectrum is equal to the ratio of the frequency and wavenumber k; in vacuum, $v = \omega/k = c$.

It is still unclear how a unipolar or quasi-unipolar pulse can be created. One of the most important mechanisms here is the generation of radiation by accelerated moving charges. Apparently, the possibility of generation of three-dimensional unipolar electromagnetic pulses through this mechanism was shown for the first time theoretically in [28, 29] and experimentally in [30]. To date, quasi-unipolar half-cycle pulses have been demonstrated experimentally in the terahertz frequency range [31–34]. Quasi-unipolar half-cycle (200 as in duration) pulses with a high peak amplitude (above 10^{12} V/m) can be obtained in the visible and ultraviolet spectral ranges in a gas, where high-power femtosecond laser pulses form a thin layer of relativistic electrons, which then passes through an inclined target [8]. A number of other approaches were discussed in detail in [35–40] and in references therein. Before listing some of them, we note that the rule of conservation of the electric area given by Eq. (4) seemingly prohibits the conversion of standard bipolar pulses into unipolar ones at one-dimensional radiation propagation. However, this rule concerns the total area of all pulses that are present in the problem. Therefore, it is possible to divide the initial standard bipolar pulse with zero electric area into subpulses with nonzero areas of opposite signs that propagate in different directions or at different velocities and are thus easily separated from each other. In particular, calculations in [41, 42] showed that a unipolar half-cycle pulse is reflected from a thin metal or dielectric film when a bipolar single-cycle pulse is incident on it. It was also shown in [43] that a quasi-unipolar pulse can be reflected when a bipolar pulse is

incident on a layer of a medium with quadratic nonlinearity.

Another method for obtaining quasi-unipolar pulses of a controlled shape is based on a coherent control of low-frequency oscillations in a nonlinear Raman-active medium [44–51]. At the same time, the medium is excited by a pair of optical femtosecond pulses that follow with an interval equal to half the period of low-frequency oscillations in the medium. In this case, the first pulse excites low-frequency oscillations of the medium and the second one stops them. As a result, a unipolar polarization pulse in the form of a half-cycle of oscillations occurs in the medium. The radiation generated by it will contain a half-wave and a long front of the opposite polarity. The contribution of this front can be suppressed by a spectral filter. Pulses of various shapes (from rectangular to triangular) can be obtained in the calculations depending on the geometry of the experiment when a time delay in the arrival of half-waves from different parts of the medium to the observation point is introduced into the system. Such a delay can be created, for example, in the case of superluminal excitation of a nonlinear medium by a light spot arising at the inclined incidence of a plane wave front on the line along which the atoms of the medium are located [44, 45]. An alternative variant of superluminal excitation can be realized by rotating a light spot from a source in a circle at a constant speed [46]. The possibility of obtaining a circular medium at the excitation by a superluminal spot created by an ultrafast laser radiation deflector was studied in [50]. Various superluminal sources of electromagnetic radiation were also considered in [52–55]. Recently, the possibility of obtaining a similar (rectangular or triangular) shape in the terahertz range through the coherent control of low-frequency oscillations in a nonlinear medium at the introduction of a delay into the system using a diffraction element was shown in [51].

4. ULTRASHORT PULSES BASED ON THE PHENOMENON OF SELF-INDUCED TRANSPARENCY

Next, we will focus on the most promising, in our opinion, method of generating ultrashort pulses, based on the resonant optical nonlinearity of the medium and the phenomenon of self-induced transparency [56]. This is one of the first studied phenomena of nonlinear optics: pulses of a special shape, whose leading edge excites the atoms of the medium from the ground state to the excited one and the trailing edge returns them to the ground state (the so-called 2π pulses or solitons of self-induced transparency), can pass almost without loss through a medium with a pronounced linear absorption resonance. Although relatively long electromagnetic pulses containing a large number of field oscillations were considered in early works, including the pioneering work [56], the possibility of propagation of ultrashort unipolar solitons of self-induced transparency in a two-level medium disregarding relaxation processes was indicated as early as 1971 [57]. Later, similar unipolar video solitons for ultrashort pulses were also predicted in multilevel quantum media, including nonequilibrium media with dissipation [58] (see also subsequent works [59–67]).

Work [57] again does not answer the question of how to obtain ultrashort and unipolar pulses on the basis of standard relatively long bipolar pulses. The possibility of the formation of ultrashort pulses, including quasi-unipolar ones, in a medium with both resonance absorption and resonance amplification was proposed in [68] and studied in more detail in [12, 69, 70]. At the same time, a standard bipolar femtosecond pulse incident on the medium and propagating in it can be transformed into a quasi-unipolar dissipative soliton of self-induced transparency. The parameters of the medium are chosen such that the pulse is close to the 2π pulse for absorption, which minimizes its loss, and approximately the π pulse that transfers atoms from the upper to the lower operating level, which provides an effective removal of the medium energy, for amplification. In this case, the removal of the inversion in the active medium is accompanied by the shortening of the pulse duration and increase in its amplitude.

The above-considered scheme is cavity-free (single-pass), whereas laser (cavity) schemes are more convenient and are considered below.

5. SELF-INDUCED TRANSPARENCY IN LASERS

The radiation dynamics in a sufficiently long laser (the cavity travel time for light exceeds the relaxation times of the medium) is practically reduced to that for the cavity-free scheme considered above. The discrete frequency spectrum of longitudinal modes, which is inherent in the laser, is insignificant in the case of a long cavity, where this spectrum is almost continuous. Pulses with a steady shape can approximately be considered as dissipative solitons of self-induced transparency if their length is much less than the length of the cavity. However, in the terminology used further, we will consider this situation as a coherent mode locking regime, which was previously predicted for relatively long electromagnetic pulses [71] (see also [72, 73]).

The regime of coherent interaction of radiation with the medium occurs for high-amplitude short electromagnetic pulses. Namely, the electromagnetic pulse duration should be shorter than the relaxation time of the medium polarization, and the Rabi frequency $\Omega_R = dE_0/\hbar$, where d is the dipole transition moment and E_0 is the amplitude of the electric field, should significantly exceed the inverse relaxation time: $\Omega_R \gg T_2^{-1}$ [56, 74, 75]. The use of the mode

locking coherent mechanism allows one to remove the restrictions on the duration of laser pulses imposed by time T_2 and generate short pulses with a duration up to one oscillation cycle directly in the laser owing to the phenomenon of self-induced transparency. This mode was obtained in calculations for ultrashort pulses in a laser with a long cavity [76]. Mode locking in a laser with a coherent absorber due to the formation of 0π pulses in the absorber was experimentally observed in [77, 78]. The phenomenon of self-induced transparency was experimentally demonstrated in a titanium–sapphire laser with a rubidium vapor cell in its cavity [79]. However, self-induced transparency was not the cause of mode locking in this case. It was due to the SESAM saturable absorber. The coherent mode locking regime was experimentally observed for the first time only recently [80].

Further in this section, special attention is paid to obtaining short pulses in lasers owing to the phenomenon of self-induced transparency (coherent mode locking) in compact short lasers and to the latest experimental observations of such a regime.

Coherent mode locking in two-section lasers. It was assumed in the first theoretical works [71–73, 76] on coherent mode locking in lasers with a ring cavity that the mixture of amplifying and absorbing centers is uniformly distributed in the optical cavity and an external seed pulse is necessary to initiate the generation (rigid excitation of the generation, which is necessary for the stability of dissipative solitons in the cavity-free scheme). The self-start of generation is impossible in such a system. The existence of the coherent mode locking regime, when the absorbing and amplifying media are separated in space, which corresponds to the situation realized in practice, was shown in subsequent works [81–83]. The self-start of generation is possible in such a system. In addition, a laser with a linear cavity in which the oncoming waves propagate poses the question about the effect of population gratings on the generated pulses.

It is necessary to use the complete system of the Maxwell–Bloch equations to simulate the generation in a laser with a two-level medium and a linear cavity. When generating a single transverse laser mode, this system has the form

$$\frac{\partial \rho_{12}(z,t)}{\partial t} = -\frac{\rho_{12}(z,t)}{T_2} + i\omega_0\rho_{12}(z,t) - \frac{i}{\hbar}d_{12}E(z,t)n(z,t), \quad (13)$$

$$\frac{\partial n(z,t)}{\partial t} = -\frac{n(z,t)-n_0(z)}{T_1} + \frac{4}{\hbar}dE(z,t)\mathrm{Im}\rho_{12}(z,t), \quad (14)$$

$$P(z,t) = 2N\mathrm{Re}\rho_{12}(z,t), \quad (15)$$

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E(z,t)}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P(z,t)}{\partial t^2}. \quad (16)$$

Bloch equations (13) and (14), which can be applied to both the amplifying and absorbing media with the corresponding replacement of parameters, describe the evolution of the off-diagonal element of the density matrix ρ_{12} and population difference n between the ground and excited states of the two-level system. The propagation of radiation along the z axis is described by wave equation (16). It contains the polarization of the medium related to the off-diagonal element of the density matrix ρ_{12} according to Eq. (15). The other parameters of the model are the frequency ω_0 of the resonance transition of a two-level medium, the population difference n_0 in the absence of an electric field, the concentration of particles N, and the relaxation time T_1 of the population difference.

An extremely short linear cavity with metal mirrors was considered. These mirrors are simulated by the Drude scheme [84]:

$$\ddot{r} + \gamma\dot{r} = \frac{e}{m}E, \quad (17)$$

$$P = N_{\mathrm{el}}er, \quad (18)$$

where γ is the friction (drag) coefficient of electrons in the metal, e is the elementary charge, and N_{el} is the concentration of free electrons in the metal. The displacement of electrons r in the metal in the Drude model under the action of an electric field E is described by Eq. (17). The polarization that is caused by electron displacements is described by Eq. (18).

The system of equations (13)–(18) was solved numerically for different relaxation times of the medium, cavity lengths, and concentrations of absorbing and amplifying particles. The results of the calculations showed the possibility of generating ultrashort pulses with a duration down to one oscillation cycle. The calculations illustrated in Fig. 1 show that the introduction of the absorber leads to the generation of single-cycle pulses.

We experimentally studied the coherent mode locking regime in two-section dye [77, 78] and titanium–sapphire [80] lasers with cells filled with coherent absorbers of molecular iodine and rubidium vapors, respectively, inside the cavities. The mode locking regime in the dye laser was due to the coherent interaction of laser radiation with transitions in iodine vapor filling the cell. In this case, 0π pulses (with zero envelope area [74, 85]) were generated. In our case, they usually had two intensity peaks during the double bypass of the cavity (see Fig. 2). Numerical simulation showed a satisfactory qualitative coincidence of the pulse shape with that observed in the experiment [78].

Further experiments were carried out at the installation with a titanium–sapphire laser containing an absorbing cell with rubidium vapor. Details of the installation are described in [80]. We give a brief scheme and the main results of the experiments. The spectrally selective elements (Lyot filter and Fabry–

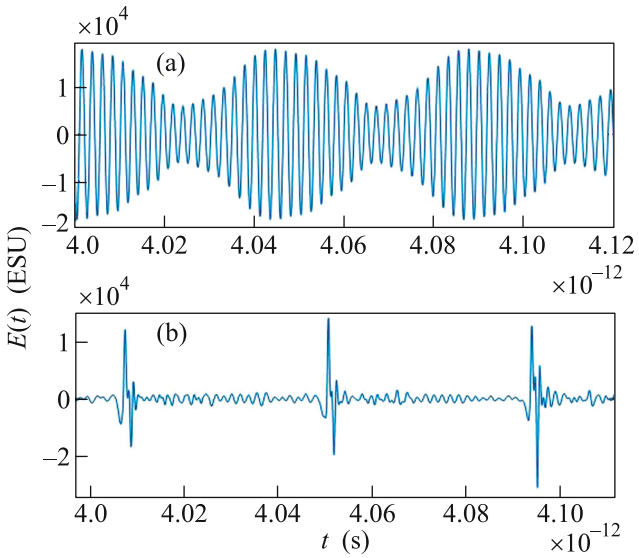

Fig. 1. (Color online) Calculated time dependence of the electric field strength at the output of the laser. (a) Laser without absorber in the cavity. (b) Ultrashort pulses in the coherent mode locking regime when the absorber is placed in the cavity. The cavity length is 6.3 μm, the amplifier length is 2.45 μm, the absorber length is 2.1 μm, and the transition wavelength in the amplifying and absorbing media is 700 nm. The parameters of the amplifying medium (subscript g) and absorbing medium (subscript a) are $N_g = 2.5\times10^{20}$ cm^{-3}, $d_g = 5$ D, $T_{g1} = 10^{-13}$ s, $T_{g2} = 10^{-14}$ s, $N_a = 2\times10^{20}$ cm^{-3}, $d_a = 10$ D, $T_{a1} = 8\times10^{-14}$ s, and $T_{a2} = 8\times10^{-14}$ s.

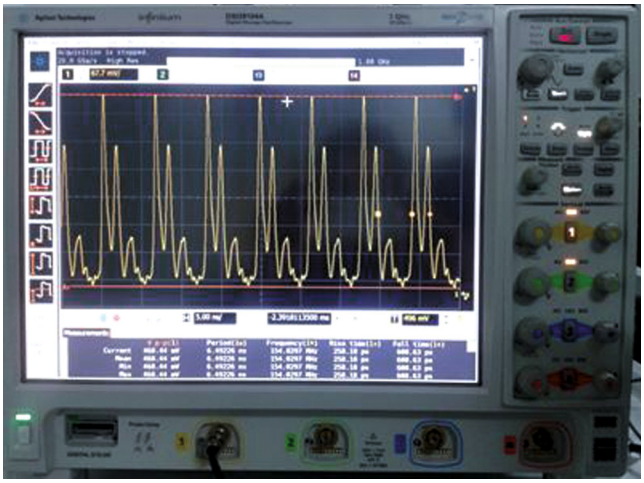

Fig. 2. (Color online) Time dependence of the intensity of the dye laser generation in the mode locking regime at a wavelength of 585.181 nm. The horizontal axis scale is 5 ns/division.

Perot etalon) were located in the cavity of the laser to adjust the generation wavelength. Mode locking occurred when the generation wavelength was tuned to the 794 nm DI and 780 nm DII lines of rubidium. The mode locking regime was always accompanied by intense luminescence of rubidium vapor in the cell and this was the rubidium vapor that caused its occurrence, because when the cell was cooled by liquid nitrogen and the concentration of absorbing atoms became extremely low, locking disappeared. The mode locking regime was not observed if the generation wavelength did not coincide with the DI and DII resonance transitions in rubidium. Note that the mode locking regime was observed in the linear and circular configurations of the cavity. The peculiarity of the circular cavity was that both unidirectional generation and generation of two oncoming waves with switching between them were possible.

Oscillograms of the intensity of radiation emitted from the laser in the mode locking regime are given in Fig. 3. An analysis of the experimental data showed that the mode locking regime was caused precisely by the phenomenon of self-induced transparency in rubidium vapor rather than by the phenomenon of a Kerr lens and saturation of absorption in rubidium vapor. This is evidenced by the fact that the mode locking regime was observed under the conditions of low power generation, when the effect of the Kerr lens could not occur. In addition, the duration of the lasing pulses in our experiments was several nanoseconds and hundreds of picoseconds, which is shorter than the relaxation time $T_2 = 2T_1$, where $T_1 = 27$ ns for rubidium. Therefore, the interaction of pulses with rubidium vapor was coherent. Finally, numerical estimates showed that the measured pulse durations corresponded to 2π pulses of self-induced transparency [80]. As far as we know, this is the first experimental demonstration of the coherent mode locking regime due to the phenomenon of self-induced transparency in the absorber; this regime has been discussed only theoretically [71–73, 76, 81–83, 86, 87]. Although the experimentally obtained pulses are not yet ultrashort, it is important that the confirmation of the feasibility of this approach opens the way for the advancement of extreme laser physics to this area.

6. PROPAGATION AND INTERACTION OF ULTRASHORT PULSES IN RESONANT MEDIA

Studies of the propagation of short pulses are reviewed in [35–37, 88, 89]. However, they usually consider the situation where the central frequency of pulses is far from the frequency of the resonance transition in the medium, whereas the problems of the resonant, coherent interaction of light with the medium are not discussed. This section presents recent results on the coherent propagation of extremely short pulses in a resonant medium.

The coherent interaction of two short pulses with resonant media (pulse duration is shorter than the

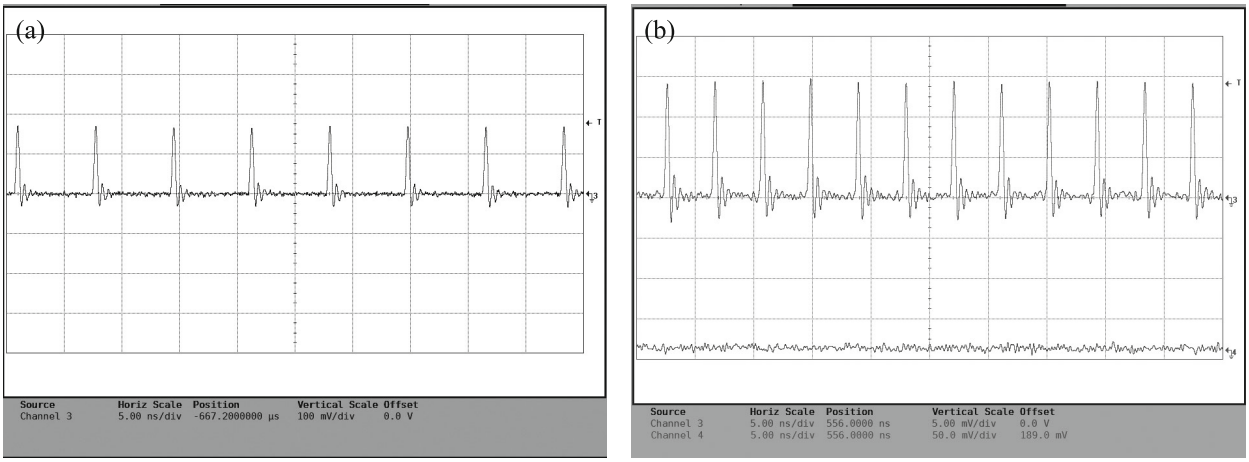

Fig. 3. Oscillograms of titanium–sapphire laser generation pulses with a cell containing rubidium vapor in the mode locking regime on the rubidium DI line in the case of the (a) linear cavity and (b) ring cavity in the unidirectional generation mode. The horizontal axis scale is 5 ns/division.

relaxation times T_1 and T_2 in the resonant media) allows the induction of the population difference gratings when the pulses do not overlap in the medium (see [90–92], as well as [93] and references cited therein). Previously, this possibility was studied experimentally and theoretically for a sequence of long pulses when the concepts of a slowly changing field envelope are applicable [90–93]. The creation of gratings is caused by the interaction of incident pulses with the polarization waves of the medium induced by the previous pulse that passed the medium [93]. When a short pulse (with the duration shorter than T_2) passes through the medium, a wave of macroscopic polarization of the medium occurs. These waves exist for the time T_2. If the second pulse is introduced in a time interval shorter than T_2 into the medium in the direction opposite to the first pulse leaving the medium, then a harmonic population difference grating appears in the medium as a result of the interaction of the second pulse with the polarization wave. In the traditional approach, such gratings are created experimentally at the interference of long quasi-monochromatic beams intersecting in the medium [94]. The diffraction of light on such gratings finds numerous applications in optics and spectroscopy [94]. In particular, the relaxation time T_2 of the polarization of the medium was measured in the experiments [90–93] on the creation of such gratings at coherent interaction of pulses not intersecting in the medium.

The possibility of creating light-induced gratings using extremely short pulses has been studied theoretically quite recently. The possibility of induction, erasure, and ultrafast control of the population difference gratings in a resonant medium by means of a sequence of bipolar attosecond optical pulses that do not overlap in the medium was shown in [95–97]. An example of the dynamics of such gratings is shown in Fig. 4.

The possibility of induction, erasure, and ultrafast control of polarization and population difference gratings by a sequence of subcycle unipolar attosecond pulses was theoretically studied in [98, 99]. In this case, the pulses also do not overlap in the medium.

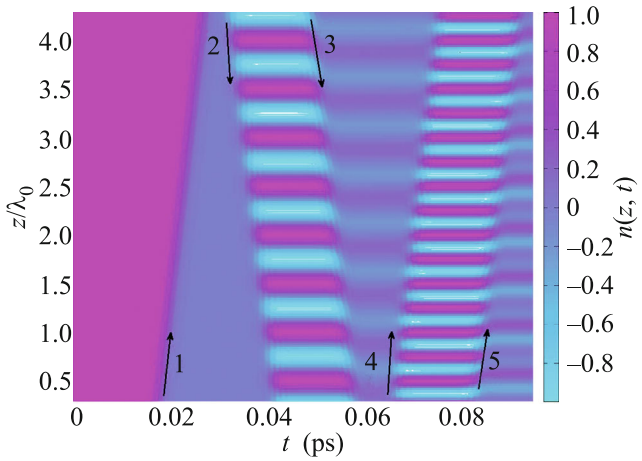

Fig. 4. (Color online) Evolution of light-induced population difference gratings induced by a sequence of attosecond pulses in a two-level resonant medium with a length of $\sim 5\lambda_0$ and the parameters $\lambda_0 = 700$ nm, $d = 5$ D, $N = 10^{17}$ cm^{-3}, $T_1 = 1$ ns, and $T_2 = 5$ ps. The medium was excited by a sequence of bipolar Gaussian pulses $E(t) = E_0 e^{\frac{-t^2}{\tau^2}} \sin \omega_0 t$, where the central frequency ω_0 corresponds to the resonance frequency of the medium, the pulse duration is $\tau = 2.1$ fs, and the amplitude of the field $E_0 = 90000$ ESU. Numbers and arrows indicate the number of the pulse and the direction of its propagation in the medium, respectively. The possibility of the induction, erasing, and multiplication of the spatial frequency of the gratings is seen.

The theoretical analysis was based on the analytical and numerical solution of the complete system of Maxwell–Bloch equations (13)–(16) (without the slowly varying envelope approximation).

As seen in Fig. 4, the state of the resonant medium can be rapidly changed at times about the oscillation period of the light wave. At the same time, it is possible not only to induce but also to erase and multiply the spatial frequency of gratings using a sequence of attosecond pulses. The possibility of creating such gratings opens new potentiality in ultrafast optics involving attosecond pulses. It should also be noted that the capability of induction of such gratings using a sequence of extremely short terahertz pulses, when the resonance frequency of the medium is in the terahertz frequency range, was studied in [100].

The induction of light-induced structures of population difference under the conditions when attosecond pulses overlap in the medium center was studied in [101, 102]. In particular, it was shown in [102] that the medium can be in states with different level populations on different sides of the pulse overlap region. For example, macroscopic polarization waves of the medium are formed with different spatial frequencies and propagate in opposite directions [102].

In conclusion, we note that the McCall–Hahn area theorem is violated for subcycle pulses [103]. However, there are a number of general properties inherent in both ultrashort and longer electromagnetic pulses. For example, the possibility of splitting a subcycle unipolar pulse with a large electric area into a pair of semicircular pulses at coherent propagation in a resonantly absorbing medium, which is similar to the splitting of longer 4π pulses, was theoretically shown in [104]. At the same time, the presented results indicate the need for further refinement of a number of key approaches to the description of generation and propagation of ultrashort optical pulses, which can open new prospects in the development of this direction, physically interesting and potentially important for applications.

This work was supported by the Russian Foundation for Basic Research (project nos. 16-02-00762 and 19-02-00312). Theoretical study of the generation of few-cycle pulses using coherent mode locking in lasers and experimental studies of the coherent mode locking regime in the titanium–sapphire laser were supported by the Russian Science Foundation (project no. 19-72-00012). The experiments were carried out using the equipment of the Scientific Park Optical and Laser Methods of Matter Research, St. Petersburg State University.

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Translated by N. Petrov