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Raman Scattering by Plasma Oscillations in Quantum Rings

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Resonant Raman scattering in quantum rings with a sufficiently large number of conduction electrons has been studied. The cross section for Raman scattering accompanied by the excitation of a one-dimensional plasmon in a ring has been determined in the self-consistent field approximation.

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Inelastic light scattering from low-dimensional systems is widely used to study collective electronic excitations. In [1–3], this method was used to study dispersion laws of two-dimensional plasmons in quantum wells and multilayer superlattices. Dispersion laws for an ensemble of quantum wires were studied in [4–7]. A plasmon in the mentioned systems is characterized by two-dimensional or one-dimensional momentum and has a continuous spectrum. A specific situation arises for quantum rings, where the orbital quantum number $l = \pm 1, \pm 2...$ serves as the longitudinal momentum in a wire, the plasmon spectrum becomes discrete, and the application of a magnetic field results in Aharonov–Bohm-type effects.

This work is devoted to the theoretical study of Raman scattering by magnetoplasmons in quantum rings. We analyze resonant inelastic light scattering in a quantum ring with numerous conduction electrons.

The differential scattering cross section (per quantum ring) can be represented in the form [8–10]

$$\frac{d^2\sigma}{d\omega d\Omega} = -\frac{\omega_2}{\omega_1}\frac{n_\omega + 1}{\pi}\mathrm{Im}(G), \qquad (1)$$

where the function G is given by the expression

$$G = \int_0^\infty dt e^{i\omega t}(-i\langle[\mathcal{N}^+(t), \mathcal{N}(0)]\rangle). \qquad (2)$$

Here, $n_\omega = (\exp(\omega/T) - 1)^{-1}$ is the Bose function; ω_1 and ω_2 are the frequencies of incident and scattered light, respectively; $\omega = \omega_1 - \omega_2$; $\mathcal{N} = \sum_{\beta'\mu';\beta\mu}\Gamma_{\beta'\mu';\beta\mu}c^+_{\beta'\mu'}c_{\beta\mu}$ is the effective "density" operator; $c^+_{\beta\mu}$ and $c_{\beta\mu}$ are the creation and annihilation operators of an electron in the single-particle state $(\beta\mu)$ in the conduction band (β is the set of orbital quantum numbers and $\mu = \pm 1$ is the spin index), respectively; $\Gamma_{\beta'\mu';\beta\mu}$ is the scattering matrix element; and $\hbar = 1$ is taken hereinafter. It is assumed that the total Hamiltonian of the system includes the Coulomb interaction between electrons in the conduction band. Below, we consider the case of resonant scattering involving the spin-split branch of the valence band. In this case, $\Gamma_{\beta'\mu';\beta\mu}$ has the form [11]

$$\Gamma_{\beta'\mu';\beta\mu} = \frac{e^2P^2}{3c^2}\sum_\gamma \frac{C^*_{\beta',\gamma}(\mathbf{q}_1)C_{\beta,\gamma}(\mathbf{q}_2)}{\varepsilon^{\mathrm{v}}_\gamma - \varepsilon^{\mathrm{c}}_{\beta'} + \omega_1 - E_{\mathrm{g}}}\Big((\mathbf{e}_1\mathbf{e}^*_2)\delta_{\mu'\mu} + i(\mathbf{a}\sigma)_{\mu'\mu}\Big), \qquad (3)$$

where E_{g} is the effective width of the band gap, $\mathbf{q}_1$ ($\mathbf{q}_2$) is the projection of the wave vector of incident (scattered) light on the plane of the quantum ring, $\mathbf{e}_1$ ($\mathbf{e}_2$) is the polarization vector of incident (scattered) light, $\mathbf{a} = [\mathbf{e}_1, \mathbf{e}^*_2]$, σ is the vector of the Pauli matrices, P is the Kane parameter, and $\varepsilon^{\mathrm{c}}_\beta$ and $\varepsilon^{\mathrm{v}}_\gamma$ are the single-particle energies of the electron in the conduction and valence bands, respectively, independent of the spins (spin–orbit interaction in the conduction band and Zeeman splitting are neglected). The function $C_{\beta,\gamma}(\mathbf{q})$ is given by the formula

$$C_{\beta,\gamma}(\mathbf{q}) = \int \psi^{\mathrm{v}*}_\gamma(\mathbf{r})\psi^{\mathrm{c}}_\beta(\mathbf{r})e^{-i\mathbf{qr}}d\mathbf{r}. \qquad (4)$$

Here, $\psi^{\mathrm{c}}(\mathbf{r})$ and $\psi^{\mathrm{v}}(\mathbf{r})$ are the single-particle envelops of the electron wavefunctions in the conduction and valence bands, respectively. Below, we consider the case of polarization light scattering in the $\mathbf{e}_1 \parallel \mathbf{e}_2$ geometry. In this case, $\Gamma_{\beta'\mu';\beta\mu} = \tilde{\Gamma}_{\beta'\beta}\delta_{\mu'\mu}$.

To take into account the Coulomb interaction in the self-consistent field approximation following [8], we represent the function G given by Eq. (2) in the form

$$G = 2\sum_{\beta'\beta} g_{\beta'\beta}\tilde{\Gamma}_{\beta'\beta}, \tag{5}$$

where $g_{\beta'\beta}$ satisfies the equation

$$g_{\beta'\beta} - 2\Pi_{\beta\beta'}\sum_{\alpha'\alpha} V_{\beta'\beta;\alpha\alpha'} g_{\alpha'\alpha} = \Gamma_{\beta'\beta}\Pi_{\beta\beta'}. \tag{6}$$

Here,

$$\Pi_{\beta\beta'} = \frac{f(\varepsilon^{c}_{\beta}) - f(\varepsilon^{c}_{\beta'})}{\varepsilon^{c}_{\beta} - \varepsilon^{c}_{\beta'} + \omega + i\delta} (\delta = +0), \tag{7}$$

where $f(\varepsilon) = (\exp((\varepsilon - \zeta)/T) + 1)^{-1}$ is the Fermi distribution function, ζ is the chemical potential, and T is the temperature, and

$$V_{\beta'\beta;\alpha\alpha'} = \int d\mathbf{r} d\mathbf{r}' \frac{e^2}{\kappa|\mathbf{r} - \mathbf{r}'|} \psi^{c*}_{\beta'}(\mathbf{r})\psi^{c}_{\beta}(\mathbf{r})\psi^{c*}_{\alpha}(\mathbf{r}')\psi^{c}_{\alpha'}(\mathbf{r}') \tag{8}$$

is the matrix element of the Coulomb potential, where κ is the background dielectric constant.

The single-particle wavefunction and energy of the electron in the one-dimensional ring in the transverse magnetic field have the form ($\mathbf{r} = (\rho, \varphi)$ are the polar coordinates in the ring plane)

$$\psi^{c}_{m} = u_{c}(\rho)\frac{e^{im\varphi}}{\sqrt{2\pi}}; \quad \varepsilon^{c}_{m}(\Phi) = B_e(m + \Phi)^2. \tag{9}$$

Here, $u_c(\rho)$ is the size-quantized function limiting the radial motion of the electron $(u_c(\rho))^2 = \delta(\rho - R)$, R is the ring radius), $B_e = 1/(2m_eR^2)$, m_e is the effective mass of the electron, Φ is the magnetic flux in units of the flux quantum $\Phi_0 = 2\pi c/e$, and $m = 0, \pm1, \pm2...$ is the angular momentum of the electron. The single-particle wavefunction and energy of the electron in the valence band have the form

$$\psi^{v}_{n} = u_{v}(\rho)\frac{e^{in\varphi}}{\sqrt{2\pi}}; \quad \varepsilon^{v}_{n}(\Phi) = -B_h(n + \Phi)^2. \tag{10}$$

Here, $B_h = 1/(2m_hR^2)$, where m_h is the effective mass of the hole.

The substitution of Eq. (4) into Eq. (3) gives

$$\tilde{\Gamma}_{m'm} = \frac{e^2P^2}{3c^2}\sum_{n} J_{m'-n}(q_2R)J_{m-n}(q_1R) \times \frac{i^{m-m'}e^{im\theta_{\mathbf{q}_2} - im'\theta_{\mathbf{q}_1}}e^{in\theta}}{\varepsilon^{v}_{n}(\Phi) - \varepsilon^{c}_{m'}(\Phi) + \omega_1 - E_g}, \tag{11}$$

where $J_k(x)$ is the Bessel function, $\theta_{\mathbf{q}_{1,2}}$ are the polar angles of the vectors $\mathbf{q}_{1,2}$, and θ is the angle between $\mathbf{q}_1$ and $\mathbf{q}_2$.

Equation (6) acquires the form

$$g_{m+l,m} - 2\Pi_{m+l,m}V^{(l)}\sum_{k} g_{k+l,k} = \tilde{\Gamma}_{m+l,m}\Pi_{m,m+l}, \tag{12}$$

where the Fourier transform $V^{(l)}$ of the Coulomb potential (taking into account the periodicity of the potential in the angular variable) is given by the expression

$$V^{(l)} = \frac{e^2}{\kappa}\int_0^{2\pi} \frac{d\varphi e^{-il\varphi}}{2\pi\sqrt{4R^2\sin(\varphi/2)^2 + d^2}}. \tag{13}$$

Here, d is the cutoff parameter of about the width or thickness of the ring.

The solution of Eq. (12) for $g_{m+l,m}$ has the form

$$g_{m+l,m} = \Pi_{m,m+l}V^{(l)}\frac{2\sum_{k}\tilde{\Gamma}_{k+l,k}\Pi_{k,k+l}}{1 - 2V^{(l)}\sum_{k}\Pi_{k,k+l}} + \tilde{\Gamma}_{m+l,m}\Pi_{m,m+l}. \tag{14}$$

According to Eqs. (1), (5), and (14), the scattering cross section is given by the expression

$$\frac{d^2\sigma}{d\omega d\Omega} = \frac{\omega_2}{\omega_1}\frac{n_\omega + 1}{\pi}\sum_{l} S_l,$$

where

$$S_l = -\mathrm{Im}\left\{\frac{V^{(l)}\bar{K}_l(\omega)K_l(\omega)}{1 - V^{(l)}\Pi_l(\omega)} + 2\sum_{m}|\tilde{\Gamma}_{m+l,m}|^2\Pi_{m,m+l}(\omega)\right\}. \tag{15}$$

Here,

$$\Pi_l(\omega) = 2\sum_{m}\Pi_{m,m+l}(\omega),$$
$$K_l(\omega) = 2\sum_{m}\tilde{\Gamma}_{m+l,m}\Pi_{m,m+l}(\omega), \tag{16}$$
$$\bar{K}_l(\omega) = 2\sum_{m}\tilde{\Gamma}^{*}_{m+l,m}\Pi_{m,m+l}(\omega).$$

The denominator in the first term of Eq. (15) is the effective longitudinal permittivity $\varepsilon_l(\omega)$. The zeros of this function of ω, i.e., the roots of the equation $1 - V^{(l)}\Pi_l(\omega) = 0$, constitute the spectrum of the plasmons in the ring $\omega^{(l)}_{p}$. In contrast to the ordinary one-dimensional system, the plasmon in the one-dimensional ring is characterized by the angular momentum $l = \pm1, \pm2...$ rather than by the wave vector. The quantity S_l is the contribution to the cross section from scattering involving a plasmon with the angular momentum l.

Below, we consider only the backscattering geometry. In this case, $\mathbf{q}_2 \simeq -\mathbf{q}_1$, $\theta = \pi$. The further calculations involve the following explicit expression for K_l:

$$K_l(\omega, \Phi; \Delta) = \frac{e^2 P^2}{3c^2}$$
$$\times \sum_n \sum_m \frac{(-1)^n i^l e^{-il\theta_{q_1}} J_n(q_1 R) J_{n-l}(q_1 R)}{\Delta - \Delta_{n,m}(\Phi) + i\lambda} \tag{17}$$
$$\times \frac{f(\varepsilon_m^c(\Phi)) - f(\varepsilon_{m+l}^c(\Phi))}{\varepsilon_m^c(\Phi) - \varepsilon_{m+l}^c(\Phi) + \omega + i\delta};$$

$$\Delta_{n,m}(\Phi) = \varepsilon_{m+l}^c(\Phi) - \varepsilon_{n+m}^v(\Phi) \equiv B_e((m + l + \Phi)^2 + s(n + m + \Phi)^2). \tag{18}$$

Here, λ and δ are the phenomenological damping parameters, $s = m_e/m_h$, $\Delta = \omega_1 - E_g$ is the resonance detuning, and ω is the Raman frequency shift. The expression for $\bar{K}_l$ is obtained from Eq. (17) by changing $\lambda \to -\lambda$.

In the situation with a fixed number of electrons, the chemical potential ζ is a function of the magnetic flux Φ, which is different for different numbers of electrons in the ring. There are the four specific cases where the number of electrons is $N = 4j, 4j + 1, 4j + 2, 4j + 3$ (j is a nonnegative integer). The relation of the number of electrons to the chemical potential has the form

$$N = 2\sum_m f(\varepsilon_m^c(\Phi)). \tag{19}$$

According to Eq. (19), $\zeta(\Phi)$ is a periodic (with a period of 1) even function of Φ. The solution of Eq. (19) in the limit of low temperature in the Φ interval of (–1/2, 1/2) gives

$$\zeta(\Phi) = \begin{cases} (N/4)^2 + \Phi^2 & \text{for } N = 4j, \\ ([N/4] + |\Phi|)^2 & \text{for } N = 4j + 1, \\ (N/4)^2 + (|\Phi| - 1/2)^2 & \text{for } N = 4j + 2, \\ ([N/4] + 1 - |\Phi|)^2 & \text{for } N = 4j + 3, \end{cases} \tag{20}$$

where $[x]$ is the integer part of the number x.

The indicated four N values are specific because the states of the electron in the ring at $\Phi = 0$ and in the absence of the Zeeman contribution to the energy are doubly degenerate (in spin) at $m = 0$ and are quadruply degenerate (in spin and sign of m) at $m \neq 0$. At $N = 4j + 2$, the situation of the "completely filled shell" occurs, so that the Fermi level at $T = 0$, $\Phi \ll 1$ lies in the gap between the $m = j$ and $m = -(j + 1)$ levels (for positive Φ). Such a situation corresponds to a dielectric spectrum. The same situation appears at $N = 4j$, but the Fermi level in this case lies between the $m = j$ and $m = -j$ levels in the gap narrow at a low flux. A "metallic" spectrum arises at odd N values, when the Fermi level at $T = 0$ coincides with a certain degenerate but incompletely filled level. These dependences on the number of electrons in the system are manifested in the magnetic flux dependence of the Raman scattering cross section. We numerically calculated the cross section for N = 58, 59, 60, and 61. It can be shown that the quantities $\omega_p^{(l)}$ are also periodic functions of Φ with a period of 1 and satisfy the relation

$$\omega_p^{(-l)}(\Phi) = \omega_p^{(l)}(-\Phi). \tag{21}$$

The quantities S_l have the same properties. It is noteworthy that the sum of partial cross sections $\sum_l S_l$ is obviously an even function of Φ.

The inelastic scattering cross section was calculated as a function of Φ in the interval of (–1/2, 1/2). We considered the contribution to the cross section from the excitation of the plasmon with the angular momentum $l = 1$. At typical values of the frequency of the incident light (for GaAs $\omega_1 \simeq 1.86$ eV) and ring radius ($R \sim 10^{-6}$–10^{-5} cm), the argument of the Bessel functions in Eq. (17) is small ($q_1 R \ll 1$). These functions decrease rapidly with an increase in n and $n - 1$. For this reason, only terms with $n = 0$ and 1 in the sum over n are taken into account when calculating the cross section. Furthermore, Eq. (17) has a resonance denominator. The resonance situation arises when $\Delta = \Delta_{n,m}(\Phi)$, which occurs at certain n, m, and Φ values. The main contribution to the sum over m in Eq. (17) comes from m values for which the difference $f(\varepsilon_m^c(\Phi)) - f(\varepsilon_{m+l}^c(\Phi))$ is noticeably nonzero. For the chosen N values, $m = 14$, ± 15, and -16 contribute to the cross section.

We calculated the magnetic flux dependence of the quantity $S_1(\omega, \Phi; \Delta)$ at the frequency shift corresponding to the plasmon peak; i.e., ω was changed to $\omega_p^{(1)}$. The calculations were performed with a particular detuning $\Delta = 292.5B_e$. This Δ value is resonant at $n = 1$, $m = 14$ and $n = 1$, $m = -16$ for $\Phi = 0$; at $n = 0$, $m = 14$ for $\Phi = 0.225$; and at $n = 0$, $m = -16$ for $\Phi = 0.237$. These terms are leading in sums in Eq. (17) and it is sufficient to retain only these terms to calculate the cross section.

Figure 1 shows the dependences of the plasmon frequency with $l = 1$ on the magnetic flux. Plasmons with $l \neq 0$ are intersubband and correspond to single-particle virtual $m \to m + l$ transitions of conduction electrons. The plasmon with $l = 0$ does not exist in the ring, which means that the frequency of the one-dimensional plasmon in a quantum wire vanishes at zero momentum. The $N = 58$ case is a separate case of

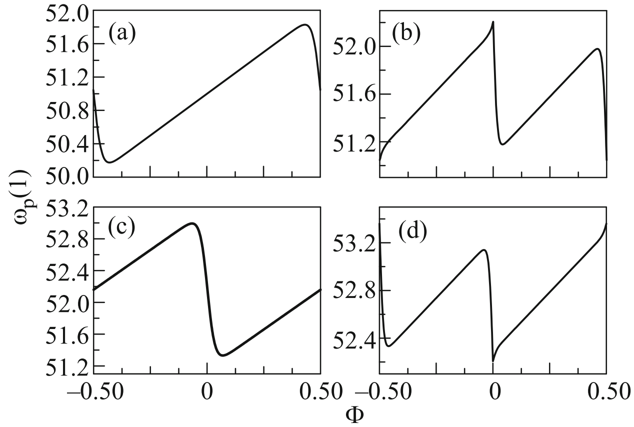

Fig. 1. Magnetic-flux dependence of the plasmon frequency with $l = 1$ (in units of B_e) at the parameters $R = 10a_e$; $T = 0.5B_e$; $d = a_e$, where a_e is the effective Bohr radius of the electron; $\delta = B_e$; and $N =$ (a) 58, (b) 59, (c) 60, and (d) 61.

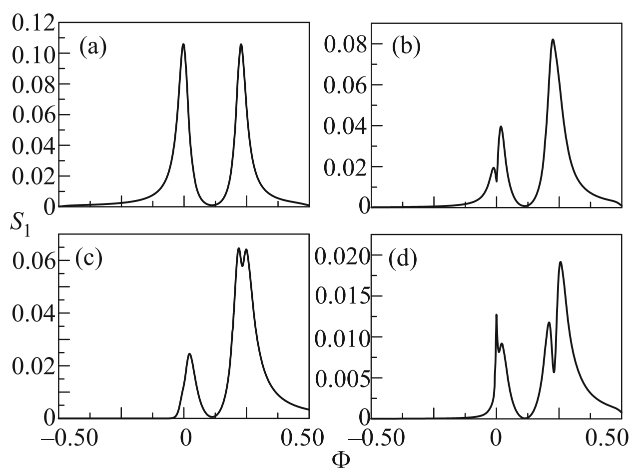

Fig. 2. Magnetic-flux dependence of the plasmon peak amplitude in the scattering cross section (in units of $e^4P^4/(9c^4B_e^3)$) at the parameters $q_1R = 0.47$, $\lambda = B_e$; the other parameters are the same as in Fig. 1; $N =$ (a) 58, (b) 59, (c) 60, and (d) 61.

the “filled shell,” when the Fermi level at $\Phi = 0$ lies in the gap with a width of $(2|m_0| + 1)B_e$, where m_0 is the number of the last filled level; $|m_0| = 14$ at $N = 58$. The dependences $\omega_p(\Phi)$ in the remaining three cases (two odd N values and one of the $4j$ type) are similar to each other because the “dielectric” gap for $N = 60$ also vanishes at $\Phi \to 0$; i.e., the situation becomes “metallic.” The same specificity of the $N = 58$ case, as well as similarity (except for fine details) of the remaining three N values, is also seen in Fig. 2, where the dependences of the amplitudes of the plasmon peak in the scattering cross section on the magnetic flux are shown.

To summarize, it has been shown that collective oscillations of electrons in a quantum ring are manifested in the spectra of resonant inelastic light scattering. Raman frequency shifts correspond to intersubband plasmons, which are characterized by the discrete quantum number $l = \pm 1, \pm 2, \ldots$. The scattering cross section in a magnetic field demonstrates an Aharonov–Bohm-type effect, being a periodic function of the magnetic flux through the ring with the period Φ_0. This cross section as a function of Φ changes significantly at a change in the number of electrons in the ring by unity (even at $N \gg 1$) because of the degeneracy multiplicity of the single-electron spectrum at $\Phi = 0$ (2 for $m = 0$ and 4 for $m \neq 0$).

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