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CONDENSED MATTER

Weak Universality in the Disordered Two-Dimensional Antiferromagnetic Potts Model on a Triangular Lattice

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The critical behavior of the disordered two-dimensional antiferromagnetic Potts model with the number of spin states $q = 3$ on a triangular lattice with disorder in the form of nonmagnetic impurities is studied by the Monte Carlo method. The critical exponents for the susceptibility γ, magnetization β, specific heat α, and correlation radius ν are calculated in the framework of the finite-size scaling theory at spin concentrations $p = 0.90$, 0.80, 0.70, and 0.65. It is found that the critical exponents increase with the degree of disorder, whereas the ratios γ/ν and β/ν do not change, thus holding the scaling equality $\frac{2\beta}{\nu} + \frac{\gamma}{\nu} = d$. Such behavior of the critical exponents is related to the weak universality of the critical behavior characteristic of disordered systems. All results are obtained using independent Monte Carlo algorithms, such as the Metropolis and Wolff algorithms.

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1. INTRODUCTION

Phase transitions and critical phenomena in systems with disorder in the form of quenched nonmagnetic impurities, random bonds, and random fields are addressed in a huge number of papers (see [1–3]). The Harris criterion [4] clarifies the principally important problem concerning a change in the critical behavior because of the introduction of a small number of immobile ("quenched") impurities. According to this criterion, in the case of $d\nu > 2$, where d is the dimensionality of the system and ν is the critical exponent of the correlation radius, impurities do not affect the critical exponents. The Harris criterion is inapplicable to the two-dimensional Ising model, where $d\nu = 2$. The detailed analysis of this case [5] indicates that impurities affect only the behavior of the specific heat, whereas the other thermodynamic and correlation functions retain their critical behavior.

At the same time, there are some indications that impurities produce a quite different effect up to the change in the order of a phase transition in the case of spin systems undergoing a first order phase transition in the absence of impurities [6, 7]. Such a change in the phase transition is indeed observed experimentally in liquid crystals in the presence of aerogel [8]. For low-dimensional systems ($d \leq 2$) described by the Potts model with $q > q_c(d)$ (q_c is the critical number of spin states and d is the dimensionality), as small as possible disorder is sufficient for changing the phase transition from first order to second order [7, 9]. For homogeneous systems with $d \geq 3$, described by the Potts models exhibiting the first order phase transition, the situation can be quite different. In such case, the introduced quenched disorder can give rise to a tricritical point p^* below which a second order phase transition occurs and above it the phase transition is of the first order [10–12].

However, it is still unclear whether the critical exponents of disordered low-dimensional systems which exhibit the first order phase transition in the homogeneous state are universal, i.e., whether they are independent of the concentration of impurities up to the percolation threshold or they vary continuously with the increase in the concentration of impurities. The main task of our work is to solve this problem.

The results of different experimental and theoretical studies in the case of finite concentrations of magnetic impurities at different lattices are not so unambiguous. The results of [13, 14] confirm the independence of the critical exponents for the magnetization and susceptibility in the two-dimensional Ising model of the degree of disorder. At the same time, Kim [15] argues that the critical exponents for the two-dimensional ferromagnetic Potts model with $q = 3$ depend on the concentration of random bonds. Moreover, it was found in [16] that the quenched disorder introduced to the spin system within the two-dimensional Ising model does not affect the universality in the critical behavior of the critical amplitudes up to the percolation threshold.

In contrast to [13–16], we study here the critical behavior of the disordered two-dimensional antiferromagnetic Potts model with $q = 3$ on a triangular lattice with disorder conventionally introduced in the form of quenched nonmagnetic impurities. This model is of interest since the numerous studies for various lattices reveal a nontrivial feature. Namely, the triangular lattice turns out to be the only lattice exhibiting a phase transition at the antiferromagnetic coupling between the nearest neighbors. Moreover, the model under study describes many physical characteristics of multicomponent alloys, adsorbed films, and liquid crystals in a porous aerogel material [17].

2. MODEL AND THE METHOD OF ITS STUDY

The classical Ising model involves N discrete objects referred to as lattice sites; each of them can correspond to one of two states (Fig. 1a). Generalizations of the Ising model to the cases where the number of possible directions of spin is larger than two ($q \geq 2$) are the Potts models. Therefore, while formulating the two-dimensional diluted antiferromagnetic Potts model with the number of spin states $q = 3$, we should have in mind its following specific features.

(i) The sites of the triangular lattice should contain either spins S_i, which can correspond to one of $q \geq 2$ states, or nonmagnetic impurities (vacancies, see Fig. 1b). The nonmagnetic impurities are randomly distributed over the lattice sites and are fixed at them (quenched disorder).

(ii) The energy of the pair interaction has one value if the interacting sites correspond to the same states (independent of the specific type of the state) and another value if the sites correspond to different states (again independent of the specific type of the states). The binding energy for two states is zero if one of the interacting sites contains a nonmagnetic impurity.

The microscopic Hamiltonian of such system involving the aforementioned features can be represented in the form [18]

$$H = -\frac{1}{2}J\sum_{i,j}\rho_i\rho_j\cos\theta_{i,j}. \quad (1)$$

Here, J is the parameter characterizing the antiferromagnetic exchange interaction between the nearest-neighbor spins ($J < 0$); $\rho_i = 1$ if the ith site is occupied by a magnetic atom and $\rho_i = 0$ if the ith site contains a nonmagnetic impurity; $\theta_{i,j} = 2\pi n/q$ is the angle between the interacting spins S_i and S_j and can acquire

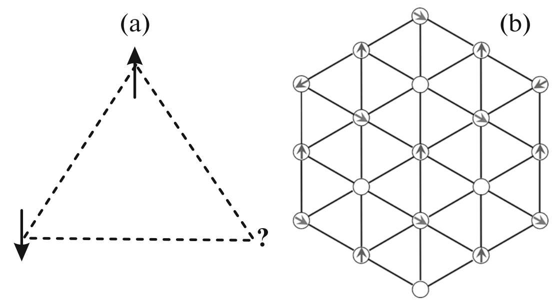

Fig. 1. (a) Frustrations in the Ising model. (b) Antiferromagnetic Potts model containing impurities with the number of spin states $q = 3$ on the triangular lattice.

three values 0°, 120°, and 240° at $n = 0$, 1, and 2, respectively; and the spinS_i can correspond to one of $q = 3$ states $S_i = 1$, 2, and 3.

In the absence of the structural disorder, it is known that such model exhibits a weak first order phase transition [19], as is predicted by the mean-field theory [18]. A weak disorder introduced to this model can induce a second order phase transition, as was rigorously proven in [7]. This result was confirmed by numerous studies [20–23], as well as by the experiments on the superfluid transition of ^{4}He in a porous aerogel material [8, 24]. Moreover, for homogeneous systems with $d \geq 3$ exhibiting the first order phase transition, the introduced disorder gives rise to the tricritical point p^* [10–12, 25, 26].

The analysis of the critical behavior of low-dimensional disordered systems by conventional theoretical and experimental methods is a very complicated task because it is hardly possible to prepare high-quality samples with accurately specified and uniformly distributed concentrations of impurities. Moreover, most of the standard theoretical approaches are inapplicable to disordered systems [1, 2]. Therefore, such low-dimensional systems described by microscopic Hamiltonians can be rigorously and consistently studied by the Monte Carlo (MC) methods. The MC methods provide an opportunity to study the critical parameters of spin systems of any degree of complexity at any controlled values of the concentration of impurities. In this work, we use the cluster Wolff algorithm [27] combined with the classical Metropolis algorithm [28] of the MC method. These algorithms are described in more detail in [29, 30]. The results obtained are tested by these independent algorithms and a good convergence of the algorithms was achieved. The combination of these algorithms leads to an appreciable saving of the computation time.

The calculations are performed for the systems with periodic boundary conditions. We study the systems with linear sizes $L \times L = N$, L = 20–120. The initial configurations are specified in such a way that all nearest neighbors of the spin under study are in dif-

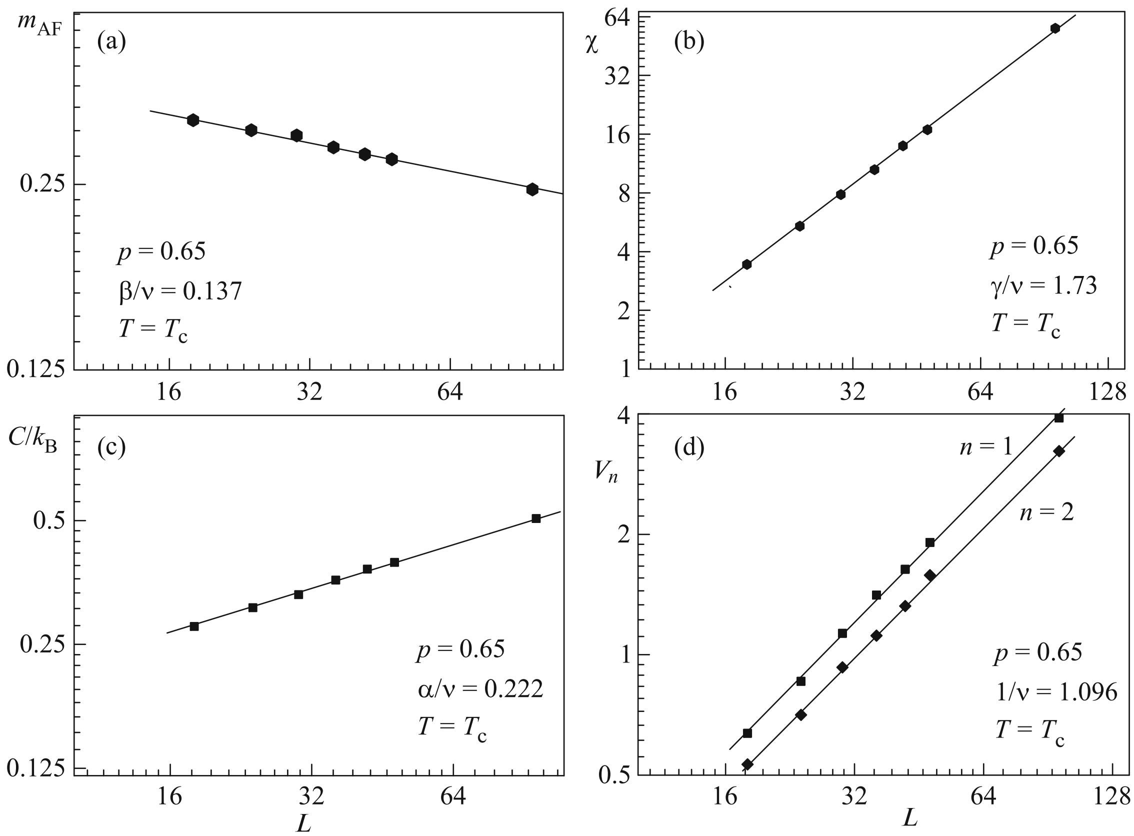

Fig. 2. (a) Order parameter m_{AF}, (b) susceptibility χ, (c) specific heat C, and (d) parameter V_n for the two-dimensional strongly diluted three-vertex antiferromagnetic Potts model versus the linear size L of the system at $p = 0.65$ and $T = T_c$.

ferent states. The frustration observed in the antiferromagnetic Ising model ($q = 2$) (see Fig. 1a) is absent for the Potts model with $q = 3$ (see Fig. 1b). To drive the system to the equilibrium state, we reject the nonequilibrium segment with the length τ_0 from the system with the linear size L. Then, we perform the averaging over the part of a Markovian chain with the length $\tau = 200\tau_0$. For the largest system, $L = 120$ and $\tau_0 = 2 \times 10^3$ MC steps per spin. In addition, we perform averaging over different initial disordered spin configurations. For the spin systems with the concentration of spins $p = 0.90$–0.65, we perform the averaging over 1000–25000 configurations with different realizations of disorder. These data are used to calculate the average values of thermodynamic parameters.

3. RESULTS OF SIMULATIONS

The effect of quenched magnetic disorder on the phase transitions in the two-dimensional antiferromagnetic Potts model on the triangular lattice was studied in [22, 23, 31, 32]. It was demonstrated that the first order phase transition changes to the second order phase transition at the concentrations of impurities $c \geq 0.1$, $c = 1 - p$. Moreover, the method of the fourth order Binder cumulants [33] and the histogram analysis of the data [34] allowed determining the critical temperatures for the weakly diluted regime (at spin concentrations $p = 0.90$ and 0.80) and for the strongly diluted regime (at spin concentrations $p = 0.70$ and 0.65). The critical temperatures $T_c(p)$ of the diluted systems determined by such a method in units of $|J|/k_B$ are $T_c(0.90) = 0.79(1)$, $T_c(0.80) = 0.65(2)$, $T_c(0.70) = 0.42(3)$, and $T_c(0.65) = 0.35(4)$.

In this work, we calculate the static critical exponents of the order parameter β, susceptibility γ, specific heat α, and correlation radius ν in the framework of the finite-size scaling (FSS) theory [35], using the critical temperatures obtained in [22, 23, 31, 32] within a fairly broad range of the studied concentration of impurities, $0.1 \leq c \leq 0.35$. The relations following from this theory suggest that, for a rather large system with the periodic boundary conditions at $T = T_c$, the order parameter m_{AF}, susceptibility χ, and parameter V_n determining the critical exponent ν behave as [36]

$$m_{AF} \sim L^{-\beta/\nu}, \tag{2}$$

$$\chi \sim L^{\gamma/\nu}, \tag{3}$$

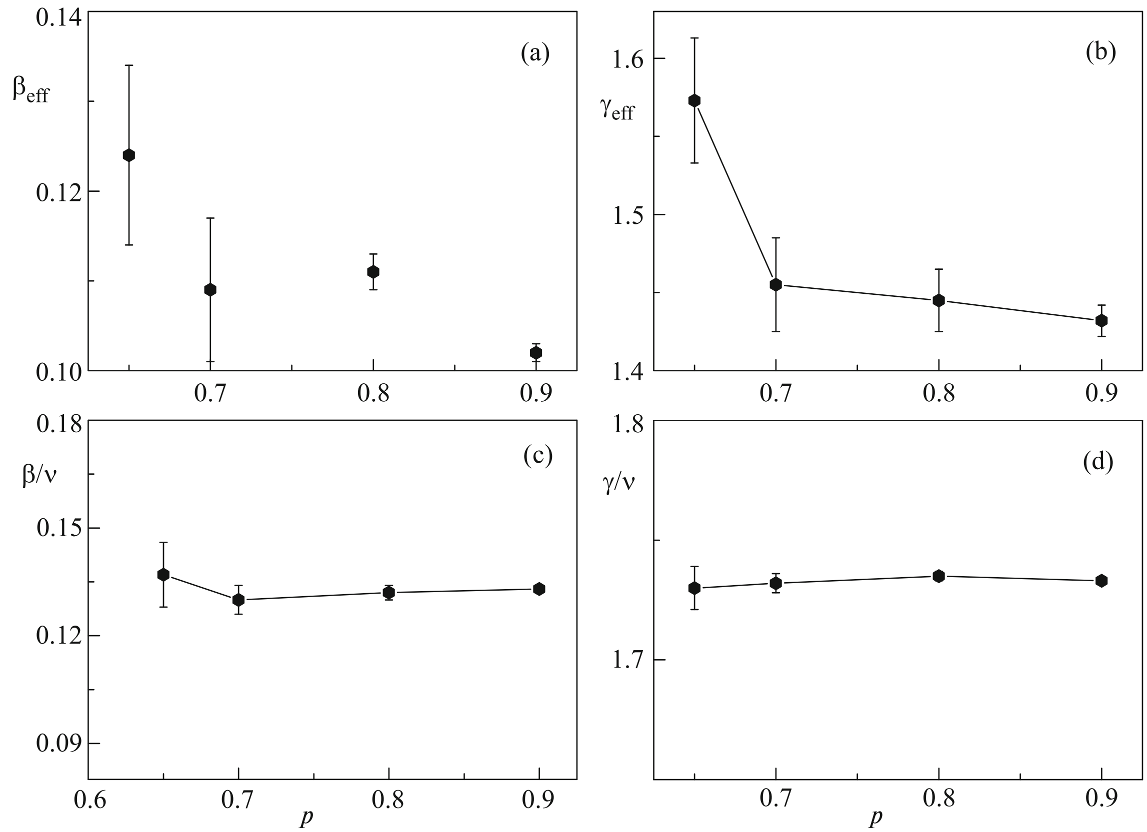

Fig. 3. Effective critical exponents of the (a) order parameter β_{eff} and (b) susceptibility γ_{eff}, as well as (c) β/ν and (d) γ/ν ratios versus the concentration of spins p.

$$V_n \sim L^{1/\nu}. \tag{4}$$

The parameter V_n can have the form

$$V_i = \frac{\langle m_{AF}^i E\rangle}{\langle m_{AF}^i\rangle} - \langle E\rangle \quad (i = 1, 2), \tag{5}$$

$$V_3 = \frac{dU_L}{d\beta} = \frac{1}{3\langle m_{AF}^2\rangle^2} \times\left[\langle m_{AF}^4\rangle\langle E\rangle - 2\frac{\langle m_{AF}^4\rangle\langle m_{AF}^4 E\rangle}{\langle m_{AF}^2\rangle} + \langle m_{AF}^4 E\rangle\right], \tag{6}$$

where $\beta = 1/T$ and T is the temperature.

As a rule, to fit the temperature dependence of the specific heat as a function of L, other expressions are used, e.g., [37]

$$C_{max}(L) = C_{max}(L = \infty) - AL^{\alpha/\nu}, \tag{7}$$

where A is some factor.

To calculate the critical exponents β, γ, α, and ν, we plot m_{AF}, χ, C, and V_n versus L. In Figs. 2a–2d, we show log–log plots of the order parameter m_{AF}, susceptibility χ, specific heat C, and parameter V_n determining the critical exponent of the correlation radius versus the linear lattice size L for the two-dimensional antiferromagnetic strongly diluted Potts model on the triangular lattice at $T = T_c$ and $p = 0.65$. The calculation error is indicated in Fig. 3. Note that the data obtained for all studied thermodynamic parameters deviate only slightly from a straight line at small values of L. It is evident that the number of initial configurations used and the sizes $L \geq 20$ of the system under study allow us to achieve the asymptotic critical regime. It is very important that the exponent ν is calculated directly from the results of our numerical simulation, whereas in many other works, it was usually determined from different scaling relations. The ratios of critical exponents α/ν, β/ν, γ/ν, and $1/\nu$ and these exponents themselves for different p values obtained at the corresponding $\nu(p)$ value are listed in Table 1. Figure 3 shows the corresponding plots of effective critical exponents (a) β_{eff} and (b) γ_{eff}, as well as the ratios of critical exponents (c) β/ν and (d) γ/ν, versus the spin concentration p. This figure demonstrates that the effective critical exponents (a) β_{eff} and (b) γ_{eff} appreciably vary when the concentration of magnetic sites changes from $p = 0.90$ to 0.65, whereas the ratios of critical exponents β/ν and γ/ν remain constant within the statistical error. The main result of this study is the conclusion that the ratios of critical exponents β/ν

Table 1. Critical exponents for the two-dimensional disordered three-vertex antiferromagnetic Potts model on the triangular lattice determined using the theory of finite-size scaling

p	β/ν	γ/ν	α/ν	$1/\nu$	ν	β	γ	α	$2\beta/\nu + \gamma/\nu = 2$
0.90	0.133	1.733	0.351	1.136	0.880	0.117	1.430	0.310	1.999
0.80	0.132	1.735	0.402	1.203	0.831	0.111	1.451	0.333	1.999
0.70	0.130	1.732	0.381	1.189	0.841	0.109	1.462	0.319	1.992
0.65	0.137	1.730	0.222	1.096	0.912	0.124	1.571	0.202	2.004

and γ/ν are independent of the degree of dilution, thus keeping the scaling equality $\frac{2\beta}{\nu} + \frac{\gamma}{\nu} = d$. As is well known from the current literature, this property of the behavior of critical exponents is due to the "weak universality" characteristic of diluted systems [38].

4. CONCLUSIONS

To summarize, the critical behavior of the two-dimensional disordered antiferromagnetic Potts model with the number of spin states $q = 3$ on the triangular lattice with disorder in the form of quenched nonmagnetic impurities has been studied using a single method. The numerical simulations have indicated that the ratios β/ν and γ/ν of the critical exponents are independent of the concentration of nonmagnetic impurities c within the range from 0.10 to 0.35, which is due to weak universality in the disordered model under study.

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