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Inhomogeneous States in a Nonlinear Self-Focusing Medium Generated by a Nonlinear Defect

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A model of a planar defect with nonlinear properties, which separates media with a Kerr-type nonlinearity, has been considered. It has been found that new steady states appear in a medium with self-focusing because of the nonlinearity of the defect, which do not occur in the case of a linear defect. The energies of such states have been obtained in an analytical form. The conditions for existence of such states have been determined depending on the characteristics of the defect and medium.

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The study of nonlinear surface waves propagating along the interfaces between media with different physical characteristics is topical in view of their wide application in optical data storage systems [1–3]. The theoretical description of nonlinear waves in media with defects widely involves the nonlinear Schrödinger equation, which contains a cubic (with respect to the desired field) term for a medium with the Kerr effect [3]. In particular, localized states at the interface between nonlinear and linear media were considered within various models in [4–6] and the effect of the spatial dispersion of the medium on localization near the defect was analyzed in [7].

In this work, I propose a generalization of the model of a thin defect layer with Kerr nonlinearity in its bulk proposed in [8, 9]. The main aim of this work is to determine the energies of steady states appearing in the considered system exclusively because of the nonlinearity of the defect.

Let us consider a simple model of the interface between two crystal media. This interface is assumed to be flat and thin as compared to the localization lengths of interface-induced perturbations of the characteristics of the media. Let the plane of the defect pass through the coordinate origin and lie in the yOz plane perpendicular to the Ox axis.

Let us describe perturbations that are uniformly distributed along the plane of the defect and are inhomogeneous in the normal direction within a one-dimensional model described by the stationary nonlinear Schrödinger equation:

$$E\psi = -\frac{1}{2m}\psi''_{xx} + \Omega(x)\psi + \gamma(x)|\psi|^2\psi + U(x)\psi, \quad (1)$$

where E is the energy of a stationary state; m is the effective mass of an excitation; $\Omega(x) = \Omega_1$ at $x < 0$ and $\Omega(x) = \Omega_2$ at $x > 0$, where $\Omega_{1,2}$ are constants; and the parameter of nonlinearity has the form $\gamma(x) = -\gamma_1$ at $x < 0$ and $\gamma(x) = -\gamma_2$ at $x > 0$, where $\gamma_{1,2}$ are constant. The parameter of nonlinearity $\gamma(x)$ is negative for media with self-focusing (attraction) and is positive for media with defocusing (repulsion). In this work, only media with self-focusing, which corresponds to $\gamma_{1,2} > 0$, are considered.

The nonlinear properties of a planar defect are described by a one-dimensional potential in the form [8, 9]

$$U(x) = \{U_0 + W_0|\psi|^2\}\delta(x), \quad (2)$$

where $\delta(x)$ is the Dirac delta function, U_0 is the linear-approximation coupling constant of the excitation with the defect located at the coordinate origin, and W_0 is the parameter of nonlinearity of the defect, which is positive in the case of defocusing and negative in the case of self-focusing in a thin defect layer.

It is noteworthy that the nonlinear equation with the term given by Eq. (2) was used to formulate a model of an optical system where the periodic modulation of the linear refractive index is combined with a single nonlinear defect [10, 11]. An example is the considered physical model of a nonlinear optical medium with the Kerr effect containing periodically distributed defect layers whose refractive index strongly differs from the refractive index of the optical medium of other layers between them [12]. If the layers are perpendicular to the Oz axis, the electric field vector **E** directed along the Oy axis satisfies Maxwell's equation $n^2(z,\mathbf{E})\mathbf{E}''_{tt} = c^2\Delta\mathbf{E}$, where $n(z,\mathbf{E})$ is the refractive index. The refractive index for a medium exhibiting the Kerr effect can be represented in the form $n(z,\mathbf{E}) = n_0 + n_1 - \sigma\alpha|E|^2$, where n_0 and n_1 are the linear refractive indices in the wide layer and narrow layer of the waveguide, respectively; σ is the parameter that is -1 and $+1$ in the "focusing" medium and "defocusing" medium, respectively; and α is the nonlinearity coefficient of the medium.

Let us introduce the complex function $\psi = E_1 + iE_2$, where E_1 and E_2 are slowly varying functions of z and t related to the electric field strength by the expression $\mathbf{E} = \mathbf{e}_y\{E_1(z,t)\cos(kx - \omega t) + E_2(z,t)\sin(kx - \omega t)\}$, which describes a monochromatic wave with the wave vector $\mathbf{k} = \mathbf{e}_x k$ and frequency $\omega = ck/n_0$. In terms of the dimensionless time (t in units of $2n_0/\alpha\omega$) and coordinate ($x' = zk/(n_0/\alpha)^{1/2}$; below, $x' = x$), the function ψ at $n_1 \ll n_0$ and $\alpha|\psi|^2 \ll n_0$ satisfies the standard nonlinear Schrödinger equation $i\psi'_t = -\psi''_{xx} + 2\sigma|\psi|^2\psi + F(x)$, where $F(x) = \Sigma\{U_0 + W_0|\psi|^2\}\delta(x - 2an)\psi$, $U_0 = -2hn_1/n_0$, $W_0 = -a\beta/n_0$, h is the width of the waveguides, $2a$ is the distance between them, and β is the nonlinearity coefficient inside the waveguides. The condition that the width of waveguides is much smaller than the distance between them allows considering a point interaction described by a Dirac delta function. In the case of weak coupling between plane-parallel waveguides, the amplitude of the field in them is much larger than the average field amplitude in the entire crystal. For this reason, it was proposed to take into account nonlinear terms inside the waveguides [11].

A new physical model leading in the limiting case to the nonlinear Schrödinger equation (ultraquantum limit described by the Gross–Pitaevskii equation) with both a finite-width potential well and a short-range delta-function potential was recently proposed in [13]. The potential well specified by Eq. (2) makes it possible to take into account the properties of a trap for small-amplitude exciton vibrational states that are due to the nonlinear exciton–exciton interaction, to obtain field distributions described by solutions by the nonlinear Schrödinger equation in terms of elementary functions, and to analyze the conditions of their existence and localization.

The solution of the nonlinear Schrödinger equation (1) with the potential (2) is equivalent to the solution of the contact boundary value problem for the nonlinear Schrödinger equation with zero potential:

$$\psi''_{xx} + 2m(E - \Omega(x) + \gamma(x)\psi^2)\psi = 0, \quad (3)$$

with two matching boundary conditions at the point $x = 0$:

$$\psi(-0) = \psi(+0) = \psi(0), \quad (4)$$

$$\psi'(+0) - \psi'(-0) = 2m\psi(0)\{U_0 + W_0|\psi(0)|^2\}. \quad (5)$$

The nonlinear boundary condition (5) is obtained by integrating both sides of Eq. (1) with the potential (2) with respect to x over the small interval $[-\varepsilon;\varepsilon]$ and tending of ε to zero [8]. In [9], the existence of localized states in nonlinear media with focusing and defocusing with a nonlinear defect was demonstrated and their stability was analyzed. This work is devoted to more general states described by periodic solutions of the nonlinear Schrödinger equation.

For the self-focusing medium at $E < \min\{\Omega_1, \Omega_2\}$, the nonlinear Schrödinger equation (3) has the spatially periodic solution

$$\psi_j(x) = kq_{cj}(m\gamma_j)^{-1/2}\mathrm{cn}(q_{cj}(x - x_{cj}), k), \quad (6)$$

where $q_{cj}^2 = 2m(\Omega_j - E)/(2k^2 - 1)$ and k is the modulus of the elliptic function cn ($1 > k^2 > 1/2$). Here and below, $j = 1$ and 2 specify the characteristics of the crystal to the left ($x < 0$) and right ($x > 0$) of the defect plane, respectively.

According to the boundary conditions (4) and (5),

$$\eta q_{c1}\mathrm{cn}(q_{c1}x_{c1}, k) = q_{c2}\mathrm{cn}(q_{c2}x_{c2}, k), \quad (7)$$

$$D_{c1} - D_{c2} = mU_0 + W_0k^2q_{c1}^2\mathrm{cn}^2(q_{c1}x_{c1}, k)/\gamma_1, \quad (8)$$

where $\eta = (\gamma_2/\gamma_1)^{1/2}$, $D_{cj} = q_{cj}\mathrm{sn}(q_{cj}x_{cj}, k)/\mathrm{sn}(q_{cj}x_{cj} + K(k), k)/2$, and $K(k)$ is the complete elliptic integral of the first kind.

The energy of the state for which $x_{c2} = x_{c1} = 0$ can be determined in an explicit form. Then, the relation $q_{c2} = \eta q_{c1}$ follows from Eq. (7), and Eq. (8) yields

$$q_{c1}^2 = -\gamma_1 mU_0/W_0k^2. \quad (9)$$

Therefore, such a state is possible only at opposite signs of the parameters of the defect. The energy is obtained from Eq. (9) in the form

$$E = \Omega_1 + \gamma_1 mU_0(2k^2 - 1)/2mW_0k^2. \quad (10)$$

The modulus of the elliptic function is expressed in terms of the parameters of the crystal and defect as

$$k^2 = \frac{U_0(\gamma_2 - \gamma_1)}{2\{U_0(\gamma_2 - \gamma_1) + W_0(\Omega_2 - \Omega_1)\}}. \quad (11)$$

Expression (11) at $k \to 1$ gives the following condition of localization of the state: $U_0/W_0 = (\Omega_1 - \Omega_2)/(\gamma_2 - \gamma_1)$. In this case, Eq. (6) gives the localized state described by the function $\psi(x) = \psi_0/\cosh\{(m\gamma_1)^{1/2}(1 + \theta(x)\eta)\psi_0 x\}$ vanishing at infinity, where $\theta(x)$ is the Heaviside step function and $\psi_0 = (-U_0/W_0)^{1/2}$.

States of such a form with the energy (10) are possible only when the planar defect separates crystals with the nonlinearity characteristics different in magnitude (but not in sign, $\gamma_1 \neq \gamma_2$). Furthermore, the existence of these states is exclusively due to the nonlinear properties of the defect because they do not appear at $W_0 = 0$.

The nonlinear Schrödinger equation (3) has another spatially periodic solution

$$\psi_j(x) = q_{dj}(m\gamma_j)^{-1/2}\mathrm{dn}(q_{dj}(x - x_{dj}), k), \qquad (12)$$

where dn is the elliptic function and $q_{dj}^2 = 2m(\Omega_j - E)/(2 - k^2)$.

The substitution of Eq. (12) into the boundary conditions (4) and (5) yields the expressions

$$\eta q_{d1}\mathrm{dn}(q_{d1}x_{d1}, k) = q_{d2}\mathrm{dn}(q_{d2}x_{d2}, k), \qquad (13)$$

$$D_{d1} - D_{d2} = mU_0 + W_0 q_{d1}^2 \mathrm{dn}^2(q_{d1}x_{d1}, k)/\gamma_1, \qquad (14)$$

where $D_{dj} = k^2 q_{dj}\mathrm{sn}(q_{dj}x_{dj}, k)\mathrm{sn}(q_{dj}x_{dj} + K(k), k)/2$.

The energy of the state for which $x_{d2} = x_{d1} = 0$ can be determined in an explicit form. Then, the relation $q_{d2} = \eta q_{d1}$ follows from Eq. (13), and Eq. (14) yields the relation $q_{d1}^2 = -\gamma_1 mU_0/W_0$. Consequently, similar to the states of the first type, such a state is possible only at the opposite signs of the parameters of the defect. From these expressions, the energy is obtained in the form

$$E = \Omega_1 + \gamma_1 mU_0(2 - k^2)/2mW_0, \qquad (15)$$

and the modulus of the elliptic function is given by the expression

$$k^2 = 2\frac{U_0(\gamma_1 - \gamma_2) + W_0(\Omega_1 - \Omega_2)}{U_0(\gamma_1 - \gamma_2)}. \qquad (16)$$

The product of elliptic moduli specified by Eqs. (11) and (16) is equal to unity. The same condition of localization of the state follows from Eq. (16). Similar to the states of the first type, such states are possible only when the planar defect separates crystals with different nonlinearity characteristics and has nonlinear properties ($W_0 \neq 0$).

To summarize, it has been found that the interface with nonlinear properties between nonlinear self-focusing crystals can generate two types of spatially nonuniform periodic steady states describing excitations of media, which exist exclusively because of the nonlinear properties of the defect. These new states appear in the case of the defocusing nonlinearity of the defect ($W_0 > 0$) and the attractive defect ($U_0 < 0$) or in the case of the self-focusing nonlinearity of the defect ($W_0 < 0$) and the repulsive defect ($U_0 > 0$). Moreover, such states can exist only when the defect separates self-focusing media with parameters of nonlinearity differing in magnitude ($\gamma_1 \neq \gamma_2$).

It should be emphasized that the model proposed in this work is a generalization of the model considered in [7, 8], within which new types of steady states, which cannot exist in a nonlinear medium with a "linear" defect, have been obtained.

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