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Josephson Effect in SIFS Tunnel Junctions with Domain Walls in the Weak Link Region[¶]

In memory of V.F. Gantmakher

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We study theoretically the properties of SIFS type Josephson junctions composed of two superconducting (S) electrodes separated by an insulating layer (I) and a ferromagnetic (F) film consisting of periodic magnetic domains structure with antiparallel magnetization directions in neighboring domains. The two-dimensional problem in the weak link area is solved analytically in the framework of the linearized quasiclassical Usadel equations. Based on this solution, the spatial distributions of the critical current density, J_C, in the domains and critical current, I_C, of SIFS structures are calculated as a function of domain wall parameters, as well as the thickness, d_F, and the width, W, of the domains. We demonstrate that $I_C(d_F, W)$ dependencies exhibit damped oscillations with the ratio of the decay length, ξ_1, and oscillation period, ξ_2, being a function of the parameters of the domains, and this ratio may take any value from zero to unity. Thus, we propose a new physical mechanism that may explain the essential difference between ξ_1 and ξ_2 observed experimentally in various types of SFS Josephson junctions.

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It is well known that properties of Josephson structures with ferromagnetic (F) material in a weak link region depends on relation between the complex decay length, ξ ($\xi^{-1} = \xi_1^{-1} + i\xi_2^{-1}$) and geometrical parameters of these junctions [1–3]. If F metal is in the dirty limit and exchange energy, H, sufficiently exceeds the critical temperature of superconducting (S) electrodes, πT_C, then from Usadel equations it follows that $\xi_1 \approx \xi_2$. However, it was demonstrated experimentally [4–12] that there could be a noticeable difference between ξ_1 and ξ_2. Previously the difference has been attributed either to the presence of strong paramagnetic scattering in the F layer [7], or to violation of the dirty limit conditions in ferromagnetic material [12, 13]. However, application of the first of the mechanisms for the experimental data interpretation requires the existence of unreasonably strong paramagnetic scattering in the weak link material [7]. The relation between an electron mean free, ℓ, and ξ_1, ξ_2 in typical experimental situation is also closer to the dirty limit conditions, $\ell \lesssim \xi_1, \xi_2$ rather than to the clean one.

In this article, we prove that the existence of a ferromagnetic domain walls in F layer can also lead to appearance of substantial differences between ξ_1 and ξ_2 even in the absence of strong scattering by paramagnetic impurities, and under the fulfilment of the dirty limit conditions in the F material.

MODEL

Consider multilayered SIFS structure presented in Fig. 1. It consists of superconductor electrode (S), insulator (I), and FS bilayer as an upper electrode. We assume that the F film has a thickness, d_F, and that it subdivides into domain structure with antiparallel direction of magnetization vector in the neighboring domains. The width of the domains is W and they separated by atomically sharp domain walls oriented perpendicular to SF interfaces. Due to periodicity of the structures, we, without any loss of generality, can perform our analysis within its half of the period, which is from $-W/2$ to $W/2$. This element is enlarged in Fig. 1. It consists of two halves of domains and domain wall separating them.

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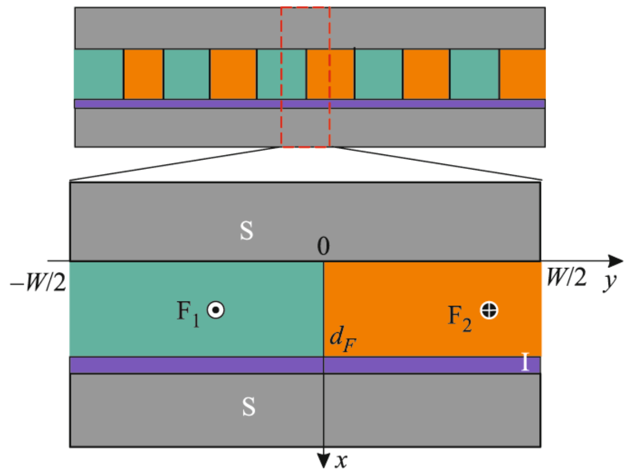

Fig. 1. (Color online) Geometry of the considered SIFS Josephson junction and its enlarged part, which includes two halves of domains and domain wall separating them. The insulating barrier I has a small transparency (shown by a blue line).

We will suppose that the condition of dirty limit is fulfilled for all metals and that effective electron-phonon coupling constant is zero in F material. We will assume further that either temperature T is close to the critical temperature of superconducting electrodes T_C or the suppression parameters $\gamma_{BS} = R_{BS}\mathcal{A}_{BN}/\xi_F$ at SF interface is large enough to permit the use of the linearized Usadel equations in F film of the structure. We will characterize the FF interface (domain wall) by the suppression parameter $\gamma = 1$, and the suppression parameter $\gamma_{BF} = R_{BF}\mathcal{A}_{BF}/\rho_F\xi_F$, which can take any value. Here, R_{BS}, R_{BF} and $\mathcal{A}_{BN}$, $\mathcal{A}_{BF}$ are the resistances and areas of the SF and FF interfaces, S, and $\xi_F = (D_F/2\pi T_C)^{1/2}$ are the decay lengths of S, F materials, while ρ_S and ρ_F are their resistivities, D_F is diffusion coefficient in the F metal.

Under the above conditions the proximity problem in the SF part of SIFS junction ($0 \le x \le d_F$) reduces to solution of the set of linearized Usadel equations [1–3, 14]

$$\left\{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right\}F_F-\tilde{\Omega}_+F_F = 0, \quad 0\le y\le \frac{W}{2}, \tag{1}$$

$$\left\{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right\}F_F-\tilde{\Omega}_-F_F = 0, \quad -\frac{W}{2}\le y\le 0, \tag{2}$$

where $\Omega = \omega/\pi T_C$, $\tilde{\Omega}_\pm = |\Omega| \pm ih\mathrm{sgn}(\omega)$, $h = H/\pi T_C$, H is the exchange energy of the ferromagnetic material, and $\omega = \pi T(2n + 1)$ are Matsubara frequencies. The spatial coordinates in (1), (2) are normalized on decay length ξ_F. To write these equations we have chosen the x and y axis in the directions perpendicular and parallel to the SF plane and put the origin in the middle of SF interface to the point, which belongs to the domain wall (see Fig. 1).

Equations (1), (2) must be supplemented by the boundary conditions [15]. They have the form

$$\begin{aligned}\gamma_{BS}\frac{\partial}{\partial x}F_F &= -G_0\frac{\Delta}{\omega}, \quad x = 0, \quad -\frac{W}{2}\le y\le\frac{W}{2},\\ \frac{\partial}{\partial x}F_F &= 0, \quad x = d_F, \quad -\frac{W}{2}\le y\le\frac{W}{2}.\end{aligned} \tag{3}$$

At FF interface ($y = 0$, $0 \le x \le d_F$) and in the middle of the domains ($y = \pm W/2$, $0 \le x \le d_F$) we also have

$$\gamma_{BF}\frac{\partial}{\partial y}F_F(x,+0) = F_F(x,+0)-F_F(x,-0), \tag{4}$$

$$\begin{aligned}\frac{\partial}{\partial y}F_F(x,+0) &= \frac{\partial}{\partial y}F_F(x,-0)\\ \frac{\partial}{\partial y}F_F(x,W/2) &= \frac{\partial}{\partial y}F_F(x,-W/2) = 0.\end{aligned} \tag{5}$$

Here, W is the width of the domains, $G_0 = \omega/\sqrt{\omega^2+\Delta^2}$, Δ is the modulus of the order parameter of superconducting electrodes. The critical current density, J_C, of SIFS Josephson junction is determined by s-wave superconducting correlations at IF interface, which is even function of the Matsubara frequencies

$$\frac{eJ_CR_N}{2\pi T_C} = \frac{T}{WT_C}\sum_{\omega>0}\frac{G_0\Delta}{\omega}\Phi(y), \tag{6}$$

where $\Phi(y) = [F_{F,+\omega}(d_F, y) + F_{F,-\omega}(d_F, y)]/2$, while the full critical current, I_C, is the result of integration of $J_C(y)$ over width of the junction.

$$\frac{eI_CR_N}{2\pi T_C} = \frac{T}{WT_C}\sum_{\omega>0}\frac{G_0\Delta}{\omega}\int_{-W/2}^{W/2}\Phi(y)dy. \tag{7}$$

Here, R_N is the normal junction resistance.

SOLUTION OF USADEL EQUATIONS IN FS ELECTRODE

Solution of two-dimensional boundary value problem (1)–(5) in the F layer ($0 \le x \le d_F$) is convenient to find in the form of the Fourier series expansion

$$F_F(x,y) = \sum_{n=-\infty}^{\infty}A_n(y)\cos\frac{\pi nx}{d_F}, \quad 0\le y\le\frac{W}{2}, \tag{8}$$

$$F_F = \sum_{n=-\infty}^{\infty}B_n(y)\cos\frac{\pi nx}{d_F}, \quad -\frac{W}{2}\le y\le 0, \tag{9}$$

where

$$A_n(y) = \frac{Z}{q_+^2} + a_n \cosh\left[q_+\left(y - \frac{W}{2}\right)\right], \tag{10}$$

$$B_n(y) = \frac{Z}{q_-^2} + b_n \cosh\left[q_-\left(y + \frac{W}{2}\right)\right], \tag{11}$$

and coefficients a_n and b_n

$$a_n = -\left[\frac{1}{q_+^2} - \frac{1}{q_-^2}\right]\frac{Zq_-S_-}{\delta}, \quad q_\pm = \sqrt{\tilde{\Omega}_\pm + \left(\frac{\pi n}{d_F}\right)^2}, \tag{12}$$

$$b_n = \left[\frac{1}{q_+^2} - \frac{1}{q_-^2}\right]\frac{Zq_+S_+}{\delta}, \quad Z = \frac{\Delta G_0}{\gamma_{BS} d_F \omega} \tag{13}$$

are determined from boundary conditions (4). Here, the coefficients δ, $C_\pm$, and $S_\pm$ are defined by the expressions

$$\delta = q_- q_+ \gamma_{BF} S_+ S_- + q_- C_+ S_- + q_+ S_+ C_-, \tag{14}$$

$$C_\pm = \cosh\left(\frac{q_\pm W}{2}\right), \quad S_\pm = \sinh\left(\frac{q_\pm W}{2}\right). \tag{15}$$

Taking into account the symmetry relation $q_-(-\omega) = q_+(\omega)$ for s-wave superconducting component in the F layer at $x = d_F$ it is easy to get

$$\Phi(y \geq 0) = \frac{Z}{2} \times \sum_{n=-\infty}^{\infty} (-1)^n \left[\frac{1}{q_+^2} + \frac{1}{q_-^2} - \left[\frac{1}{q_+^2} - \frac{1}{q_-^2}\right]\frac{\delta_+}{\delta}\right], \tag{16}$$

$$\Phi(y \leq 0) = \frac{Z}{2} \times \sum_{n=-\infty}^{\infty} (-1)^n \left[\frac{1}{q_+^2} + \frac{1}{q_-^2} - \left[\frac{1}{q_+^2} - \frac{1}{q_-^2}\right]\frac{\delta_-}{\delta}\right], \tag{17}$$

$$\delta_\pm = q_- S_- \cosh\left(q_+ \frac{2y \mp W}{2}\right) - q_+ S_+ \cosh\left(q_- \frac{2y \mp W}{2}\right).$$

Finally for the critical current from (7), (16), and (17) we have

$$\frac{eI_C R_N}{2\pi T_C} = \frac{T}{2WT_C} \sum_{\omega > 0} \frac{ZG_0 \Delta}{\omega} S(\omega), \tag{18}$$

$$S(\omega) = \sum_{n=-\infty}^{\infty} (-1)^n \left[\frac{W}{q_+^2} + \frac{W}{q_-^2} - \frac{2S_- S_+ (q_-^2 - q_+^2)^2}{\delta q_+^3 q_-^3}\right].$$

It is seen that the critical current can be represented as the sum of two terms. The first is the contributions from individual domains separated by fully opaque FF wall

$$\frac{eI_{C1} R_N}{2\pi T_C} = \frac{T}{T_C} \times \sum_{\omega > 0} \frac{G_0^2 \Delta^2}{\gamma_{BS} \omega^2} \mathrm{Re} \frac{1}{\sqrt{\tilde{\Omega}_+} \sinh\left(d_F \sqrt{\tilde{\Omega}_+}\right)}, \tag{19}$$

while the second

$$\frac{eI_{C2} R_N}{2\pi T_C} = \frac{4h^2 T}{W d_F T_C} \sum_{\omega > 0} \frac{G_0^2 \Delta^2}{\gamma_{BS} \omega^2} \times \sum_{n=-\infty}^{\infty} \frac{(-1)^n S_- S_+}{q_+^3 q_-^3 \delta} \tag{20}$$

gives the contribution from the domain wall. Here, $\mathrm{Re}(a)$ denotes the real part of a.

Expression (19) reproduces the well-known result previously obtained for single-domain SIFS structures [16–18] thereby demonstrating the independence of the critical current on the orientation of the domains magnetization vectors, if they are collinear oriented and the FF interface is fully opaque for electrons.

LIMIT OF LARGE γ_{BF}

For large values of suppression parameter $\gamma_{BF} \gg \max\{1, (Wq_\pm)^{-1}\}$ expression (20) transforms to

$$\frac{eI_{C2} R_N}{2\pi T_C} = \frac{4h^2 T}{W d_F T_C} \sum_{\omega > 0} \frac{G_0^2 \Delta^2}{\gamma_{BF} \gamma_{BS} \omega^2} \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{q_+^4 q_-^4}. \tag{21}$$

The sum over n in Eq. (21) can be calculated analytically using the theory of residues

$$\frac{eI_{C2} R_N}{2\pi T_C} = \frac{4hT}{WT_C} \sum_{\omega > 0} \frac{G_0^2 \Delta^2}{\gamma_{BF} \gamma_{BS} \omega^2} S_1, \tag{22}$$

$$S_1 = \mathrm{Re}\left\{\frac{i}{\tilde{\Omega}_+^{3/2}}\left[\frac{1}{\cosh(d_F \sqrt{\tilde{\Omega}_+})} + \frac{d_F \sqrt{\tilde{\Omega}_+}}{\sinh(d_F \sqrt{\tilde{\Omega}_+})}\right]\right\}.$$

It is seen that I_{C2} is vanished as $(\gamma_{BF} W)^{-1}$ with an increase in $\gamma_{BF} W$ product and scales on the same characteristic lengths ξ_1, ξ_2 as the critical current for single-domain SIFS structures (19).

LIMIT OF SMALL γ_{BF}

In the opposite limit, $\gamma_{BF} \ll \max\{1, (Wq_{\pm})^{-1}\}$, we have

$$\frac{eI_{C2}R_N}{2\pi T_C} = \frac{8h^2T}{Wd_FT_C}\sum_{\omega>0}\frac{G_0^2\Delta^2}{\gamma_{BS}\omega^2}S_2, \tag{23}$$

$$S_2 = \sum_{n=-\infty}^{\infty}\frac{(-1)^n S_-S_+}{q_+^3q_-^3(q_-C_+S_- + q_+S_+C_-)}.$$

It is seen that in full agreement with the result obtained in [19] in the considered limit of large domain width, $W \gg \mathrm{Re}(q_{\pm})$,

$$\frac{eI_{C2}R_N}{2\pi T_C} = \frac{4h^2T}{Wd_FT_C}\sum_{\omega>0}\frac{G_0^2\Delta^2}{\gamma_{BS}\omega^2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{q_+^3q_-^3(q_- + q_+)} \tag{24}$$

contribution to the critical current from domain wall region falls as W^{-1} and decays in the scale of ξ_1.

LIMIT OF SMALL DOMAIN WIDTH

In the opposite case, $W \ll \mathrm{Re}(q_{\pm})$, presentation of the critical current as a sum of I_{C1} and I_{C2} is not physically reasonable and for I_C from (18) we get

$$\frac{eI_CR_N}{2\pi T_C} = \frac{T}{2T_C}\sum_{\omega>0}\frac{G_0^2\Delta^2}{\gamma_{BS}d_F\omega^2}S_3, \tag{25}$$

$$S_3 = \sum_{n=-\infty}^{\infty}(-1)^n\left[\frac{(q_-^2 + q_+^2)\gamma_{BW} + 4}{(q_-^2q_+^2\gamma_{BW} + q_-^2 + q_+^2)}\right],$$

where $\gamma_{BW} = \gamma_{BF}W/2$. It is seen that for $\gamma_{BW} \gg 1$ expression (25) transforms to (19) and $I_C = I_{C1}$, while in the limit $\gamma_{BW} \to 0$ from (25) it follows that the critical current

$$\frac{eI_CR_N}{2\pi T_C} = \frac{T}{T_C}\sum_{\omega>0}\frac{G_0^2\Delta^2}{\gamma_{BS}\omega^2\sqrt{\Omega}\sinh(d_F\sqrt{\Omega})} \tag{26}$$

is independent on exchange energy and falls with increase in d_F in the same scale as it is for SINS devices. Previously it was found that such transformation of decay length takes place in a vicinity of domain wall [20–33]. In particular, it was shown that if a sharp domain wall is parallel [22, 24] or perpendicular to SF interface [33–35] and the thickness of ferromagnetic layers, $d_F \lesssim \xi_F$, then for antiparallel direction of magnetization the exchange field effectively averages out, and the decay length of superconducting correlations becomes close to that of a single nonmagnetic N metal $\xi_F = \sqrt{D_F/2\pi T_C}$. The same effect may also take place in S–FNF–S variable thickness bridges [36, 37].

For arbitrary values of γ_{BW} the sum over n in (25) can also be calculated analytically. The denominator in (25) has the poles at

$$n = \pm i\frac{d_F}{\pi}\sqrt{\Omega + \frac{1 \pm \sqrt{1 - \gamma_{BW}^2h^2}}{\gamma_{BW}}}.$$

Application of the residue theorem to the summation of the series in n in the expression (25) leads to

$$\frac{eI_CR_N}{2\pi T_C} = \frac{T}{2T_C}\sum_{\omega>0}\frac{G_0^2\Delta^2}{\gamma_{BS}\omega^2}\frac{\gamma_{BM}}{\sqrt{1 - \gamma_{BM}^2h^2}}S_4, \tag{27}$$

$$S_4 = \frac{q}{\sqrt{\Omega + p}\sinh(d_F\sqrt{\Omega + p})} - \frac{p}{\sqrt{\Omega + q}\sinh(d_F\sqrt{\Omega + q})},$$

$$p = \frac{1 - \sqrt{1 - \gamma_{BW}^2h^2}}{\gamma_{BW}}, \quad q = \frac{1 + \sqrt{1 - \gamma_{BW}^2h^2}}{\gamma_{BW}}. \tag{28}$$

It is seen that for $\gamma_{BW}h \le 1$ s-wave superconducting correlations decay exponentially into the F metal without any oscillations with two characteristic scales, $\xi_{11} = \xi_F(\Omega + p)^{-1/2}$, and, $\xi_{12} = \xi_F(\Omega + q)^{-1/2}$. If γ_{BW} tends to zero then one of the damping characteristic scale ξ_{11} goes to that $\xi_F\Omega^{-1/2}$ of SINF junctions (see (26)), while the other ξ_{12} goes to zero. With γ_{BW} increase, ξ_{11} reduces, whereas ξ_{12} increases, so that at $\gamma_{BW}h = 1$ they become equal to each other $\xi_{11} = \xi_{12} = \xi_F(\Omega + h)^{-1/2}$. Further increase in $\gamma_{BW}h$ leads to appearance of the damped oscillations in $I_C(d_F)$ dependence with the ratio

$$\frac{\xi_1}{\xi_2} = \frac{\sqrt{\gamma_{BW}^2h^2 - 1}}{\sqrt{(\gamma_{BW}\Omega + 1)^2 + \gamma_{BW}^2h^2 - 1} + \Omega_{\gamma_{BW}} + 1}, \tag{29}$$

which increases monotonically from zero at $\gamma_{BW}h = 1$ up to that of single domain SIFS junctions

$$\frac{\xi_1}{\xi_2} = \frac{h}{\sqrt{\Omega^2 + h^2} + \Omega}, \tag{30}$$

in the limit $\gamma_{BW} \to \infty$.

From (29), (30) we can conclude that the existence of domain structure in the F layer of SIFS devices can significantly modify the relation between ξ_1 and ξ_2 extracted from experimental studies of $I_C(d_F)$ dependence in SIFS tunnel junctions.

This conclusion is valid not only in the limit of small domain width.

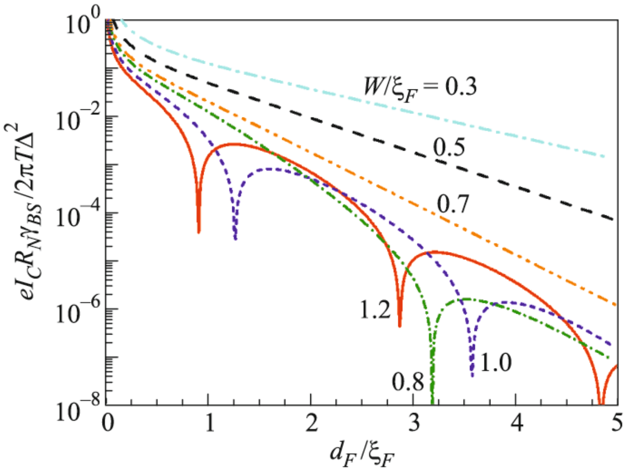

Fig. 2. (Color online) Critical current of the SIFS Josephson junction versus the thickness of the F layer d_F calculated numerically from (18) for $T = 0.5T_C$, $H = 10\pi T_C$, $\gamma_{BF} = 0$ and for a set of widths $W/\xi_F = 0.3, 0.5, 0.7, 0.8, 1, 1.2$.

ARBITRARY VALUES OF THE DOMAIN WIDTH

For arbitrary values of the width of the magnetic domains to calculate, the dependence of $I_C(d_F)$ is necessary to use the general expression (18). Figure 2 gives the $I_C(d_F)$ curves calculated for $H = 10\pi T_C$, $\gamma_{BF} = 0$ and for a set of widths W/ξ_F. It is seen that in full accordance with the analytical analysis given above for W smaller than $0.78\xi_F$, I_C falls monotonically with W increase. At $W \gtrsim 0.78$ there is a transformation from a monotonic dependence of $I_C(d_F)$ to a damped oscillatory one. It is interesting to note that the critical current in the vicinity of the transition decays even faster than for large W. To illustrate this result, we make a fit of the calculated curves by the simple expression

$$I_C(d_F) = A\exp(-d_F/\xi_1)\cos(d_F/\xi_2 + \varphi),$$

which is ordinary used for estimation of the decay lengths ξ_1 and ξ_2 from an experimental data [38, 39]. At the first step, we define ξ_2

$$\xi_2 = (d_{F2} - d_{F1})/\pi$$

from the positions of the first, d_{F1}, and the second, d_{F2}, 0–π transitions in I_C (d_F) dependence and put

$$\varphi = \pi/2 - d_{F1}/\xi_2$$

in order to get $I_C(d_{F1}) = 0$. The decay length ξ_1 is determined from the ratio of magnitudes of critical current taken in two points having equal phase of oscillation:

$$\xi_1 = \pi\xi_2 \ln\left[\frac{I_C(d_{F1} + \xi_2\pi/2)}{I_C(d_{F2} + \xi_2\pi/2)}\right]$$

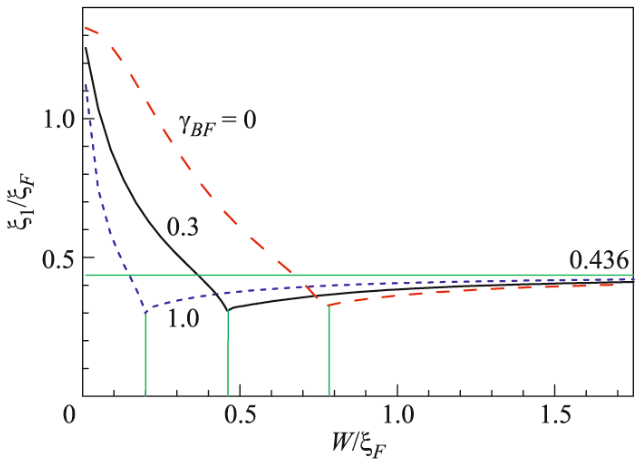

Fig. 3. (Color online) Decay length ξ_1 versus the domain width W calculated at $T = 0.5T_C$, $H = 10\pi T_C$, and $\gamma_{BF} = 0$, 0.3, 1.

and normalization constant A

$$A = \frac{I_C d_F}{\exp(-d_F/\xi_1)\cos(d_F/\xi_2 + \varphi)}$$

has been determined by direct calculation of magnitude in the certain point between 0–π transitions. If the position of the second 0–π transition exceeds $10\xi_F$, we suppose that ξ_2 is infinite and $I_C(d_{F1})$ dependence can be fitted by function

$$I_C(d_F) = A\exp(-d_F/\xi_1).$$

The results of the fitting procedure are presented in Figs. 3–5, which give the decay lengths ξ_1 and ξ_2 as well as their ratio ξ_1/ξ_2 calculated at $T = 0.5T_C$, $H = 10\pi T_C$ for a set of suppression parameter $\gamma_{BF} = 0$, 0.3, 1. Thin vertical lines in Figs. 3 and 4 give values on the x-axis, at which there is a transition from a monoto-

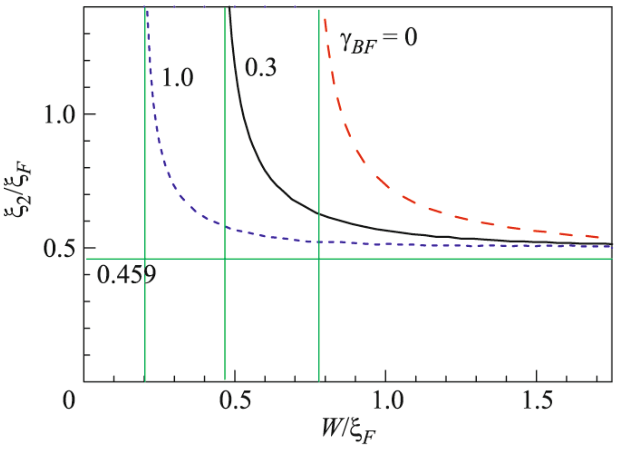

Fig. 4. (Color online) Decay length ξ_2 versus the domain width W calculated at $T = 0.5T_C$, $H = 10\pi T_C$, and $\gamma_{BF} = 0$, 0.3, 1.

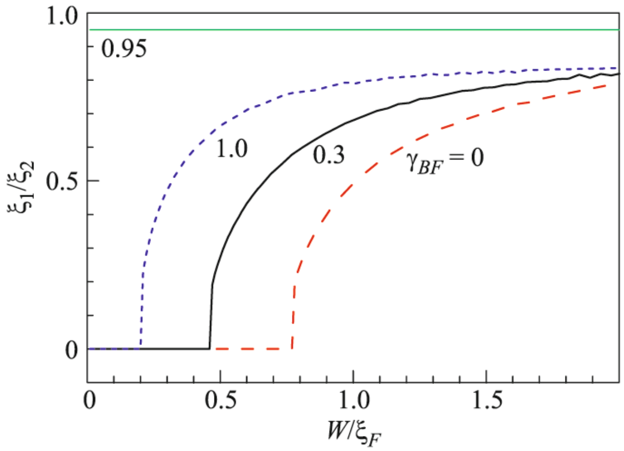

Fig. 5. (Color online) Ratio of decay lengths ξ_1 and ξ_2 versus the domain width W calculated at $T = 0.5T_C$, $H = 10\pi T_C$, and $\gamma_{BF} = 0$, 0.3, 1.

nous exponential decay of $I_C(d_F)$ to the damped oscillation lows. Thin horizontal lines in Figs. 3–5 provide the asymptotic values of ξ_1, ξ_2, and ξ_1/ξ_2 in the limit $W \gg \xi_F$, which are coincide with the magnitudes calculated for single domain SIFS junction for given temperature $T = 0.5\pi T_C$ and exchange energy $H = 10\pi T_C$.

It is seen that the transition point at which monotonic decay of $I_C(d_F)$ dependence transforms to a damped oscillation behavior the smaller the larger is suppression parameter γ_{BF}. Interestingly, decay length ξ_1 near this transition is even smaller compare to its magnitude in the limit of large W.

It is also necessary to note that despite of the fact that the transition takes place at $W < \xi_F$, the difference between ξ_1 and ξ_2, as it follows from Fig. 5, exists even for large domain width: the ratio ξ_1/ξ_2 is only around 0.8 at $W = 4\xi_F$ and very slowly tends to the following from (30) the single domain value 0.95 with W increase. This fact permits us to conclude that the difference between ξ_1 and ξ_2 experimentally observed in SFS Josephson structures based on dilute magnetic alloys can be also the consequence of existence of magnetic domains in the F layer.

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